Russian Academy of Sciences
St.Petersburg Department of Steklov Mathematical Institute
The Euler International Mathematical Institute

St.Petersburg State University
Chebyshev Laboratory

Third Northern Triangular Seminar
St.Petersburg, 11-13 April, 2011

Programme and Abstracts

(sponsor pictures)

Welcome to St.Petersburg!

We are pleased to welcome you to the

Third Northern Triangular Seminar

in St.Petersburg from 11th to 13th April, 2011. The seminar is organized under the auspices of the **Euler International Mathematical Institute** at St.Petersburg Department of Steklov Mathematical Institute (Russian Academy of Sciences) and **Chebyshev Research Laboratory** at St.Petersburg State University. It was sponsored by **Russian Foundation for Basic Research**, grant 11-01-06804.

We are very grateful for the generous support provided by these institutions.

Triangular Seminars represent a forum to young researchers (Ph.D. students and fresh PhD) from Russia, Finland, and Sweden and other countries working in Probability Theory and Mathematical Statistics. The seminar programme includes two mini-courses in the field and a number of participants' talks. The major goal of the seminar is to establish personal contacts between new generations of mathematicians in Northern Europe.

The seminars are co-organized by probabilists from Helsinki, Stockholm, and St.Petersburg (whereas the name "Triangular"). The first seminar of this series was successfully held in Helsinki (March 2009), the second one in Stockholm (March 2010).

We hope you will enjoy this meeting and your time in St.Petersburg!

Organizing Committee

`http://www.pdmi.ras.ru/EIMI/NTS/index.html`

Seminar Programme

Monday, April 11

09.00–09.15. Registration.
09.15–10.00. **Mini-Course 1: D. Chelkak. Schramm–Löwner evolution. I**
10.00–10.15. *Coffee.*
10.15–11.00. **Mini-Course 1: D. Chelkak. Schramm–Löwner evolution. II**.
11.00–11.10. *Break.*
11.10–11.40. Alexeev N. Gaussian random matrices and genus expansion.
11.40–12.10. Nyquist P. On Sanov's theorem for importance sampling.
12.10–14.10. *Lunch.*
14.10–14.40. Butkovsky O. Coupling method for estimating mixing coefficients of Markov processes.
14.40–15.10. Gudmundsson Th. Markov chain Monte Carlo for computing probabilities of rare events in a heavy-tailed random walk.
15.10–15.40. Poster Session:

Aksenova K. On stochastic models of teletraffic with heavy-tailed distributions.
Evsikov I. Numerical methods for BSDEs and nonlinear PDEs.
Romadanova M. American option pricing in stochastic volatility models.
Zhechev V. Functional limit theorem for canonical U-processes of dependent observations.

15.40–16.10. Skopenkov M. Discrete complex analysis on a convex quadrangulation of the plane.
16.10–16.40. Perkkiö A.P. Stochastic programs without duality gaps.
16.40–17.00. *Coffee.*
17.00–17.30. Reshetov S. Estimation of the pseudo-periodic function observed in the stationary noise.
17.30–18.00. Tyurin I. Some new advances in estimating the rate of convergence in CLT.

Tuesday, April 12

09.15–10.00 **Mini-Course 1: D. Chelkak. Schramm–Löwner evolution. III**.
10.00–10.15 *Coffee.*
10.15–10.45. Yakubovich Yu. On the variance of sample size.
10.45–11.15. Korchevsky V. On the strong law of large numbers for sequences of dependent random variables.
11.15–11.30. *Break.*
11.30–12.00. Aro H. Pricing and hedging of mortality-linked cash flows.
12.00–12.30. Tikanmäki H. How to hedge Asian options in fractional Black–Scholes model.
12.30–14.30. *Lunch.*
14.30–15.15. **Mini-Course 2: A. Zaitsev. Strong approximation for independent random vectors. I.**
15.15–15.30. *Break.*
15.30–16.15. **Mini-Course 2: A. Zaitsev. Strong approximation for independent random vectors. II.**
16.15–16.30. *Coffee.*
16.30–17.00. Plaksina N. Equilibrium prices for providers in queueing system.
17.00–17.30. Goricheva R. Regenerative approach for retrial queueing system.
17.30–18.00. Lukashenko O. Gaussian queues in communication networks.
19.00– ... *Conference Dinner.*

Wednesday, April 13

09.15–09.45. Källberg D. Statistical inference for entropy-type density functionals.
09.45–10.15. Abramowicz K. Stratified sampling Monte Carlo quadrature for continuous random functions.
10.15–10.30. *Coffee.*
10.30–11.30. Shmileva E. Extrapolation of alpha-stable random fields.
10.30–11.30. Ivashko A. Optimal double-stopping problem on trajectories.
11.30–11.45. *Break.*
11.45–12.30. **Mini-Course 2: A. Zaitsev. Strong approximation for independent random vectors. III.**

Konrad Abramowicz
Umeå University, Sweden

Stratified sampling Monte Carlo quadrature for continuous random functions

In this talk, we consider the problem of numerical approximation of integrals of continuous random functions over a unit hypercube. We use the stratified Monte Carlo quadrature and measure the approximation performance by the mean square error. The quadrature is defined by N stratified randomly chosen observations with the strata generated by a rectangular grid (or design). We study the class of locally stationary random fields whose local behavior is like a fractional Brownian field in the mean square sense and evaluate the asymptotic approximation accuracy for a sequence of designs for large N. For the Hölder class of random functions we provide an upper bound for the approximation error. Additionally, for a certain class of isotropic random functions with an isolated singularity at the origin, we construct a sequence of designs eliminating the effect of the singularity point.

Ksenia Aksenova
St.Petersburg State University, Russia

On stochastic models of teletraffic with heavy-tailed distributions

Recently, a lot of attention has been paid to mathematical modelling of computer systems based on high-speed connections, such as Internet. Following the approach suggested by I. Kaj and M. Taqqu [1], we consider a stochastic model of teletraffic based on Poisson random measure. We study the behavior of the total load on the system. We show that under appropriate assumptions the finite-dimensional distributions for the scaled workload process converge to those of an α-stable Lévy process with $\alpha \in (0, 1)$.

References

[1] Kaj I., Taqqu M. Convergence to fractional Brownian Motion and to the Telecom process: the integral representation approach. – In: *In and out of Equilibrium. II.*, ser.: Progress in Probability, **60** (2008), 383–427.

Nikita Alexeev
St.Petersburg State University, Russia

Genus expansion for some ensembles of random matrices

We consider a random square $N \times N$ Hermitian Gaussian matrix H with complex entries. It turns out that the k^{th} moment of eigenvalue distribution of the matrix H, which equals to $\frac{1}{N}\mathbb{E}\ \mathrm{Tr}\, H^{2k}$, has a nice topological combinatorial interpretation

$$\mathbb{E}\ \mathrm{Tr}\, H^{2k} = N^{k+1} \sum_{g=0}^{[k/2]} T(k,g) \frac{1}{N^{2g}},$$

where $T(k,g)$ is the number of ways to glue pairwise all the edges of a $2k$-gon so as to produce a surface of a given genus g (see Haagerup and Thorbjornsen). For example, there is only one way to obtain a torus by gluing opposite sides of a square, and so $T(2,1) = 1$. We discuss this theorem and consider some generalizations.

Helena Aro
Aalto University, Finland

Pricing and hedging of mortality-linked cash flows

The uncertain future development of mortality and financial markets affects the operation of every life insurer. In particular, the joint distribution of mortality and investment returns is crucial in pricing and hedging of mortality linked bonds and other life insurance products. This paper proposes simple stochastic models that are well suited for numerical analysis of life insurance products that cannot be hedged with traditional techniques of mathematical finance. The models are calibrated with an extensive data set covering six countries and 56 years. Statistical analysis supports the known dependence of old-age mortality on GDP which, in turn, is connected to many sectors of financial markets. Our models provide a simple quantitative formulation of such connections. Particular attention is paid to the long-term development of mortality rates, which is an important issue in life insurance markets.

Oleg Butkovsky
Moscow State University, Russia

Coupling method for estimating mixing coefficients of Markov processes

The coupling method is a powerful tool for proving ergodicity results. It dates back to Doeblin and was later developed by Vaserstein, Pitman, Griffeath, Goldstein and many others (see [1]).

Let $(X_n^1)_{n\in\mathbb{Z}_+}$ and $(X_n^2)_{n\in\mathbb{Z}_+}$ be homogeneous Markov processes with the same transition functions. A coupling is a bivariate processes $\widetilde{X}_n = \left(\widetilde{X}_n^1, \widetilde{X}_n^2\right)$, such that $\widetilde{X}_n^1 \overset{d}{=} X_n^1$, $\widetilde{X}_n^2 \overset{d}{=} X_n^2$ for all $n \in \mathbb{Z}_+$ and $X_n^1 = X_n^2$ for all $n > n_0(\omega)$. In this paper we modify Vaserstein coupling construction [2] to find a sufficient condition which guarantees that Markov process has exponentially decreasing β-mixing coefficients. We also provide an explicit estimate of decay rate.

Suppose that processes X_n^1 and X_n^2 have transition probability density $p(u,v)$ which is assumed to be continuous with respect to the first argument. Let us define

$$q(u,v) := \int_{-\infty}^{+\infty} p(u,t) \wedge p(v,t)\, dt.$$

We introduce a new Markov process $\eta_n = (\eta_n^1, \eta_n^2)$ with transition probability density $\varphi(x,y) := \varphi_1(x,y_1)\varphi_2(x,y_2)$, where $x = (x^1, x^2)$, $y = (y^1, y^2)$ and

$$\varphi_1(x,u) := (1 - q(x^1,x^2))^{-1} \left(p(x^1,u) - p(x^1,u) \wedge p(x^2,u)\right),$$
$$\varphi_2(x,u) := (1 - q(x^1,x^2))^{-1} \left(p(x^2,u) - p(x^1,u) \wedge p(x^2,u)\right).$$

We also set $\eta_0 = X_0$.

Consider operator $A : C_b \to C_b$

$$Af(x) := (1 - q(x))\mathbb{E}_x f(\eta_1),$$

and denote its spectral radius by r.

Theorem 1. *It is possible to construct a coupling* $\left(\widetilde{X}_n^1, \widetilde{X}_n^2\right)$ *such that*

$$\mathbb{P}(\widetilde{X}_n^1 \neq \widetilde{X}_n^2) \leq \mathbb{E} \prod_{i=0}^{n-1} (1 - q(\eta_i^1, \eta_i^2)).$$

Moreover, if $r \neq 1$ then for any $\varepsilon > 0$ and sufficiently large $n > N(\varepsilon)$

$$\|X_n^1 - X_n^2\| \leq 2e^{-n(|\ln r| - \varepsilon)},$$

where $\|X - Y\|$ is a total variation distance between random variables X and Y.

Let us denote by $\beta(n)$ β-mixing coefficients of the process X_n^1.

Theorem 2. *Assume that the process X_n^1 has a stationary distribution. If $r \neq 1$ then for any $\varepsilon > 0$*

$$\beta(n) \leq Ce^{-n(|\ln r| - \varepsilon)},$$

for some $C > 0$ and sufficiently large $n > N(\varepsilon)$.

References

[1] H. Thorisson, Coupling, Stationarity, and Regeneration. Probability and its Applications, Springer–Verlag, New York (2000)

[2] L.N. Vaserstein. Markov processes on countable product spaces describing large systems of automata. Problemy Peredaci Informacii 3, 64-72 (1969)

Igor Evsikov

St.Petersburg State University for Architecture and Civil Engineering, Russia

Numerical methods for BSDEs and nonlinear PDEs

By a numerical method based on BSDEs we construct a solution of a nonlinear PDE. Let W_t be a Wiener process defined on a probability space $(\Omega, \mathcal{F}, P)$, let $\mathcal{F}_t$ be its natural filtration, Consider a FBSDE

$$X_t = x + \int_0^t b(s, X_s, Y_s)ds + \int_0^t \sigma(s, X_s, Y_s)dW_s \tag{1}$$

$$Y_t = g(X_T) + \int\limits_t^T f(s, X_s, Y_s, Z_s)ds - \int\limits_t^T Z_s dW_s. \tag{2}$$

and assume that the conditions from [1] are satisfied and there exist $\mathcal{F}_t$–adapted processes (Y_t, Z_t) satisfying (1), (2) such that

$$E[\sup_{0\leqslant t\leqslant T} \mid Y_t|^2 + \int\limits_0^T \mid Z_s|^2\, ds] < \infty.$$

Note that if $u(t,x)$ is a classical solution to

$$\begin{cases} u_t + \dfrac{1}{2}\text{trace}(\sigma\sigma^*(t,x,u)u_{xx}) + u_x b(t,x,u) + f(t,x,u,u_x\sigma(t,x,u)) = 0, \\ u(T,x) = g(x). \end{cases} \tag{3}$$

then $Y_t = u(t, X_t), \quad Z_t = v(t, X_t) \triangleq u_x(t, X_t)\sigma(t, X_t, u(t, X_t))$, solve (2). At the other hand one can use (2) to construct less regular solution to (3).

We construct a numerical solution of (1) – (3).

A natural time discretization [2] yields

$$\begin{cases} X_{i+1}^n \triangleq X_i^n + b(t_i, X_i^n, Y_i^n)h + \sigma(t_i, X_i^n, Y_i^n)\Delta W_{i+1}, \\ Z_i^n \triangleq \dfrac{1}{h} E_{t_i}\{Y_{i+1}^n \Delta W_{i+1}\}, \\ Y_i^n \triangleq E_{t_i}\{Y_{i+1}^n + f(t_i, X_i^n, Y_{i+1}^n, \hat{Z}_i^n)h\}. \end{cases} \tag{4}$$

Here $X_0^n \triangleq x$, $Y_n^n \triangleq g(X_n^n)$, $h \triangleq \frac{T}{n}$ and $t_i \triangleq ih, i = 0, 1, ..., n$, and $\Delta W_{i+1} \triangleq W_{t_{i+1}} - W_{t_i}$. E_t denotes the conditional expectation $E\{\cdot|\mathcal{F}_t\}$. Next we set $W_{t_i}^n := \sqrt{h}\sum_{j=1}^i \varepsilon_j^n$, where $\{\varepsilon_j^n\}_{j=1}^n$ are $\{1, -1\}$–valued i.i.d. with $P\{\varepsilon_j^n = 1\} = P\{\varepsilon_j^n = -1\} = 0.5$.

We consider a simple case $dX_t = dW_t, \quad f(t,x,y,z) = -yz, \quad g(x) = \cos(x)$. Then BSDE has the form

$$dY_t = (Y_t \cdot Z_t)dt + Z_t dW_t, \quad Y_T = \cos(W_T),$$

and $y(t) = U(t,x)$ solves the Cauchy problem for Burgers equation

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} = \frac{1}{2}\frac{\partial^2 U}{\partial x^2}, \quad U(T,x) = \cos(x).$$

We compare numerical results obtained within the above numerical scheme (4), with numerical values of the known explicit solution

$$U(\tau, x) = \frac{\int\limits_{-\infty}^{\infty} \frac{(x-\theta)}{\tau} \exp(-G(\tau, x, \theta)) d\theta}{\int\limits_{-\infty}^{\infty} \exp(-G(\tau, x, \theta)) d\theta},$$

where

$$G(\tau, x, \theta) = \frac{(x-\theta)^2}{2\tau} + \int\limits_0^\theta \varphi(y) dy, \quad \varphi(y) = U(T, y), \quad \tau = T - t.$$

Explicit solution gives us $Y_0 = U(0,0) = 0.5564803023$, and (4) with different values of n gives

n	100	500	2000	4000
Y_0^n	0.55588	0.55636	0.55645	0.55647

References

[1] Peng, S. and Xu, M. Numerical algorithms for 1-d backward stochastic differential equations: Convergence and simulations. http://arxiv.org/abs/math/0611864

[2] Bender, C. and Zhang, J. (2008). Time discretization and Markovian iteration for coupled FBSDEs. *Ann. Appl. Probab.* **18** 143–144.

Ruslana Goricheva

KRC, Petrozavodsk, Russia

Regenerative approach for retrial queuing system

We present a new retrial system which can be used to describe, for instance, behavior of short TCP transfers or ALOHA type multiple access protocol. For a single-server system with no buffer and Poisson input with rate λ (a $M/G/1/1$-type retrial system denoted as Σ) we estimate blocking probability $\mathbb{P}_{orb}$ both in the stability region and in the instability region. Arrivals who find the server busy join the infinite capacity orbit, and then return to the system after exponentially distributed retrial time with rate μ_0. Thus, the total input stream to the server consists of two (generally, dependent) streams: a Poisson $\lambda-$input of primary customers and the input of retrial customers with a rate $\tilde{\mu}_0 \leq \mu_0$ ($\tilde{\mu}_0 = \mu_0$ when the orbit is not empty, and $\tilde{\mu}_0 = 0$, otherwise).

Consider an auxiliary single-server loss (with no buffer) system $\hat{\Sigma}$ as follows with the same Poisson λ-input, the same service times an independent Poisson input with rate μ_0 (μ_0-customers). Arriving customer who find the server busy *leave the system forever and do not affect the future state.* Since $\tilde{\mu}_0 \leq \mu_0$, the server in system Σ is less loaded than the server in $\hat{\Sigma}$. We expect that the probabilities of the server's states (busy or empty) in Σ approach the ones in $\hat{\Sigma}$ when orbit size in Σ increases.

The stationary loss probability $\mathbb{P}_{loss}$ in $\hat{\Sigma}$ always exists and it has been proved that the following condition

$$(\lambda + \mu_0)\mathbb{P}_{loss} < \mu_0, \tag{5}$$

is sufficient (and often necessary) for stability of the orbit in system Σ.

System $\hat{\Sigma}$ regenerates at the instants when the λ-customers find the server empty (we use at these instants the memoryless property of the input of μ_0-customers). Note that this procedure can be extended to a general renewal λ-input.

Under condition (1) the process denoted number of rejected customers in Σ has regenerative increments and we can apply regenerative method to estimate blocking probability $\mathbb{P}_{orb}$. Otherwise we use another type of regenerations (*quasi-regenerations*). As quasi-regenerations of original (unstable) system we can take the instants when arriving primary customer meets an empty server (while the orbit size *may be arbitrary*).

The further purpose is to check by regenerative (or quasi-regenerative) simulation stability/instability of the original system, in particular, comparing estimator $\hat{\mathbb{P}}_{orb}$ with explicit formula for $\mathbb{P}_{loss}$.

Simulation of M/M/1/1 and M/Pareto/1/1 retrial systems shows a remarkable consistence with the known analytical results for the Erlang type retrial systems and shows that the sufficient stability condition is also necessary.

Thorbjorn Gudmundsson
KTH, Stockholm, Sweden

Markov chain Monte Carlo for computing probabilities of rare events in a heavy-tailed random walk.

An algorithm based on Markov chain Monte Carlo (MCMC) is proposed to compute probabilities of rare events. The algorithm is based on sampling from the conditional distribution given the rare event using MCMC and extracting the normalizing constant. The algorithm is proved to be efficient in the class of random walks with subexponentially distributed steps.

Anna Ivashko
KRC, Petrozavodsk, Russia

Optimal double-stopping problem on trajectories

The following optimal stopping problem on trajectories is considered. There is an urn containing m minus balls and p plus balls. We draw from the urn one at a time randomly without replacement. Define $Z_0 = 0$, $Z_n = \sum\limits_{k=1}^{n} X_k$, $1 \le n \le m+p$, where X_k is the value of the ball chosen at the k-th draw. The value -1 is attached to minus ball and value $+1$ to plus ball. The problem of this paper is to stop with maximum probability at the beginning on the minimum of the trajectory formed by $\{Z_n\}_{n=0}^{m+p}$ and then on the maximum. We derive the optimal stopping rule in this problem.

We consider following optimal double-stopping problem on trajectories. Suppose that we have an urn containing m minus balls and p plus balls. We draw from the urn sequentially one at a time without replacement. The value -1 is attached to minus ball and value $+1$ to plus ball. Determine sequence $Z_0 = 0$, $Z_n = \sum\limits_{k=1}^{n} X_k$, $1 \le n \le m+p$, where X_k is the value of the ball chosen at the k-th draw. Each time a ball is drawn, we observe the value of the ball and decide either to stop or continue drawing. The problem of this paper is to stop with maximum probability at the beginning on the minimum of the trajectory formed by $\{Z_n\}_{n=0}^{m+p}$ and then on the maximum. We find the optimal stopping rule in this problem, i.e. we seek the optimal stopping times $(\sigma*, \tau*)$ such that

$$P\{Z_{\sigma*} = \min_{0\le n\le m+p} Z_n,\ Z_{\tau*} = \max_{\sigma*\le l\le m+p} Z_l\}$$
$$= \sup_{(\sigma,\tau)\in C} P\{Z_\sigma = \min_{0\le n\le m+p} Z_n,\ Z_\tau = \max_{\sigma\le l\le m+p} Z_l\},$$

where C is the class of all double-stopping times.

The work is supported by Russian Foundation for Basic Research, project 10-01-00089-a and by the Division of Mathematical Sciences (the program "Mathematical and algorithmic problems of new information systems").

References

1. Shepp L.A. (1969). Explicit solutions to some problems of optimal stopping. *Annals of Mathematical Statistics*, **40**, 993–1010.

2. Tamaki M. (2001). Optimal stopping on trajectories and the ballot problem, *Journal of Applied Probability*, **38**, 2001, 946–959.

3. Mazalov V.V., Tamaki M. (2007). Duration problem on trajectories, *Stochastics: An International Journal of Probability and Stochastic Processes*, **79(3–4)**, 211–218.

David Källberg
Umeå University, Sweden

Statistical inference for entropy-type density functionals

Entropy and its various generalizations are widely used in mathematical statistics, physics, information theory, and signal processing for characterizing the amount of information in probability distributions. In addition, related density functionals are applied to measure distance between distributions, for example, Bregman distance for image matching in image analysis problems. We consider estimators of some entropy-type (integral) functionals for discrete and continuous distributions based on the number of epsilon-close random vectors in the corresponding i.i.d. samples. We show some asymptotic properties of these estimators (e.g., consistency and asymptotic normality). These proposed estimators can be used in various problems in computer science and mathematical statistics (e.g., approximate matching for random databases; record linkage; image matching; distribution identification problems).

Valery Korchevsky
St. Petersburg State University, Russia

On the strong law of large numbers for sequences of dependent random variables

We present new sufficient conditions for the applicability of the strong law of large numbers to sequences of random variables without the independence conditions. No certain type of dependence is assumed. Only conditions related to moments of random variables and their sums are used. These results are generalizations of some results of N. Etemadi proved under more restrictive conditions.

Oleg Lukashenko
KRC, Petrozavodsk, Russia

Gaussian queues in communication networks

Nowadays Gaussian processes are well-recognized models to describe the traffic dynamics of a wide class of modern telecommunication networks. We discuss the application of the simulation to estimate the loss or overflow probability in a queueing system with a finite or infinite buffer, which is fed by a Gaussian input. In particular, we are interested in systems with small or moderate size buffers, because the infinite buffer approximation is not a realistic assumption in real networks. In this framework, the loss rate prediction may represent a useful tool to define new call admission control algorithms in Quality of Service (QoS) supporting network architectures and for QoS routing protocols.

Pierre Nyquist
KTH, Stockholm, Sweden

On Sanov's theorem for importance sampling

A large deviations principle for the weighted empirical measures arising in Monte Carlo simulation with importance sampling is studied. The result is applied to theoretically quantify the performance of simulation algorithms based on importance sampling.

Ari-Pekka Perkkiö
Aalto University, Finland

Stochastic programs without duality gaps

We study dynamic stochastic optimization problems parameterized by a random variable. Such problems arise in many applications in operations research and mathematical finance. We give sufficient conditions for the existence of solutions and the absence of a duality gap. Our proof uses extended dynamic programming equations, whose validity is established under new relaxed conditions that generalize certain no-arbitrage conditions from mathematical finance.

This is a joint work with Professor Teemu Pennanen.

Nina Plaksina
Petrozavodsk State University, Russia

Equilibrium in prices for providers in queueing system

The work is devoted to investigation of a queueing system model. There are two subsystems M/M/1 in the model and two different services (products) are proposed. These services are complementary and the customer has no benefit from obtaining just one of them. We find equilibrium prices and provider profits depending on service time. A simulation model realizing a competition between providers is developed.

Sergey Reshetov
St.Petersburg State University of Information Technologies, Mechanics and Optics, Russia

Estimation of the pseudo-periodic function observed in the stationary noise

Suppose we observe a random process $Y(t)$ on the large interval $[-T, T]$:

$$Y(t) = s(t) + X(t), \tag{1}$$

where $s \in \mathcal{L}_*(\Lambda)$ is an unknown function we wish to estimate, $X(t)$ is a generalized Gaussian stationary process with zero mean and the spectral density f.
Let $\mathcal{L}_*(\Lambda) = \mathcal{L}_*(\Lambda, C, \beta)$ be the compact subset of the Banach space $\mathcal{L}(\Lambda)$ of the pseudo-periodic functions with the spectral set Λ:

$$\mathcal{L}_*(\Lambda) = \left\{ s(t) = \sum_{u \in \Lambda} a_u \, e^{iut} : \sum_{u \in \Lambda} |a_u|^2 (|u| + 1)^{2\beta} \leq C, \, \beta > 1/2 \right\}.$$

Λ is supposed to be a countable set such that $\tau = \inf\limits_{u \neq v;\, u,v \in \Lambda} |u - v| > 0$.
The spectral density f is unknown, but we assume that $f \in A_2(\mathcal{M})$ and $f \in B_\gamma$, where $A_2(\mathcal{M})$ consists of the nonnegative functions $g = g(t)$ for which

$$\sup_I \frac{1}{|I|} \int_I g(t) dt \frac{1}{|I|} \int_I \frac{1}{g(t)} \, dt \leq \mathcal{M}, \tag{2}$$

with the supremum over all intervals I and $B_\gamma = B_\gamma\left(c_1,\, c_2,\, T_0,\, \beta,\, \Lambda\right),\ \gamma > -1$ consists of the nonnegative functions $g = g(t)$ for which

$$0 < c_2 \le M^{2\beta} \sum_{u\in\Lambda,\, |u|\le M} \int_{|u-t|\le\frac{1}{T}} g(t)\,dt\ \le c_1 < \infty \tag{3}$$

for any $T > T_0,\ M = T^{\frac{1}{1+\gamma+2\beta}}$.

We define the minimax risk in this problem as

$$\mathscr{R}^*\left(\mathcal{L}_*\left(\Lambda\right),\, T\right) = \inf_{\widetilde{s}} \sup_{s\in\mathcal{L}_*(\Lambda)} \mathbf{E}\left\|s-\widetilde{s}\right\|^2_{\mathcal{L}},$$

where the infimum is taken over all estimators $\widetilde{s}$.

Theorem. *Suppose we are given (1) and the spectral density f satisfies (2) and (3). Then there exist such numbers $C_1 = C_1\left(c_1,\, T_0,\, \gamma,\, \beta,\, \tau,\, \mathcal{M},\, C\right) < \infty$ and $C_2 = C_2\left(c_1,\, c_2,\, T_0,\, \gamma,\, \beta,\, \tau,\, \mathcal{M},\, C\right) > 0$ that for any $T > T_0$ we obtain*

$$C_1\, T^{-\frac{2\beta}{2\beta+\gamma+1}} > \mathscr{R}^*\left(\mathcal{L}_*\left(\Lambda\right),\ ,T\right) > C_2\, T^{-\frac{2\beta}{2\beta+\gamma+1}}.$$

References.

1. N. N. Centsov, *Estimation of an unknown density of the distribution on the observations* - Doklady Academii Nauk SSSR **147**, (1962), p. 45–48(in Russian).

2. M. S. Pinsker, *The optimal filtration of the square-integrable signals on the Gaussian noise.* - Problemy peredachy informatsii, **16(2)**, (1980), p. 52–68(in Russian).

3. I. A. Ibragimov, R. Z. Has'minskii, *Nonparametric estimation of the value of a linear functional in Gaussian white noise.* - Theory Probab. Appl. **29**, (1984), p. 1-32.

4. D. L. Donoho, R. C. Liu, B. MacGibbon, *Minimax Risk Over Hyperrectangles, and Implications.* - The Annals of Statistics, Vol.**18**, No. 3, (1990), p. 1416–1437.

Maria Romadanova
St.Petersburg State University for Architecture and Civil Engineering, Russia

American option pricing in stochastic volatility models

We are interested in pricing of American options in various financial market models.

We consider the Heston model when the stock dynamics is given by a system of stochastic equations

$$dS(\vartheta) = S(\vartheta)[rd\vartheta + \sqrt{v(\vartheta)}dw_1(\vartheta)], \quad S(t) = s > 0, \quad 0 \le t \le \vartheta \le T, \quad (1)$$

$$dv(\vartheta) = \kappa_1(\theta_1 - v(\vartheta))d\vartheta + \sigma_1\sqrt{v(\vartheta)}dw_2(\vartheta), \quad v(t) = v > 0. \quad (2)$$

Here $w_k(\vartheta) \in R^1$ are $\mathcal{F}_t$-measurable Wiener processes, $k = 1, 2$, defined on a probability space $(\Omega, \mathcal{F}, P)$ with a filtration $\mathcal{F}_t$, $E[dw_1(\vartheta)dw_2(\vartheta)] = \rho d\vartheta$, while r, κ_1, θ_1 σ_1 and correlation coefficient ρ are given constants, $|\rho| < 1$. For simplicity we assume that P is a martingale measure.

The price $F(t, s, v)$ of an American option with a contract function Φ is given by

$$F(t, s, v) = \sup_{\tau \in \mathcal{T}_{[t,T]}} E_{t,s,v}\{e^{-r(\tau - t)}\Phi(S(\tau))\}, \quad (3)$$

where $\mathcal{T}_{[t,T]}$ is a set of stopping time $\tau \in [t, T]$ with respect to $\mathcal{F}_t$. In particular, a contract function for a put option has the form $\Phi(s) = [K - s]_+ = \max(K - s, 0)$, constant $K > 0$.

An alternative definition

$$F(t, s, v) = \sup_{S^*(\tau,v), \tau \in [t,T]} E_{t,s,v}\{e^{-r(\tau_{S^*} - t)}[K - S(\tau_{S^*})]_+\}, \quad (4)$$

where τ_{S^*} is the first exit time when the process S hits the optimal execution boundary $S^*(\vartheta, v)$, $\vartheta \in [t, T]$ allows to include explicitly an unknown function $S^*(\vartheta, v)$ to be defined in a process of solution. This explains the possibility to describe $F(t, s, v)$ for a put option as a solution of a free boundary value problem for a parabolic equation

$$F = K - s, \quad \frac{\partial F}{\partial t} + \mathcal{L}F - rF < 0, \quad 0 \le s \le S^*(t, v), \quad (5)$$

$$F > K - s, \quad \frac{\partial F}{\partial t} + \mathcal{L}F - rF = 0, \quad S^*(t, v) < s < \infty, \quad (6)$$

where $\mathcal{L}$ is a linear operator acting as follows

$$\mathcal{L}F = rs\frac{\partial F}{\partial s} + \kappa_1(\theta_1 - v)\frac{\partial F}{\partial v} + \frac{1}{2}s^2 v\frac{\partial^2 F}{\partial s^2} + \frac{1}{2}\sigma_1^2 v\frac{\partial^2 F}{\partial v^2} + \rho s\sigma_1 v\frac{\partial^2 F}{\partial s\partial v}. \quad (7)$$

Boundary conditions at $s = S^*(t, v)$ are stated as a continuity condition for $F(t, s, v)$ and its derivative in s and

$$F(t, S^*(t, v), v) = [K - S^*(t, v)]_+, \quad \frac{\partial F}{\partial s}(t, S^*(t, v), v) = -1. \quad (8)$$

A substitution

$$s = Ke^{x+\alpha y}, \quad Kf(t, x, y) = F(t, s, v), \quad K\varphi(x) = \Phi(s), \quad (9)$$

leading to dimensionless variables allows to reduce the problem (5)-(6) to a problem with constant (in x) coefficients and eliminate mixed derivatives.

We start with consideration of a perpetual American put option. In this case the function $f(t, x, y) \equiv f(x, y)$ does not depend on t and satisfies the boundary problem

$$(\mathcal{L}_1 - r)f(x, y) = 0, \quad x > h(y), \quad f(x, y) = (1 - e^{x+\alpha y})_+, \quad x \le h(y), \quad (10)$$

$$f(h, y) = 1 - e^{h+\alpha y}, \quad \frac{\partial}{\partial x}f(x, y)|_{x=h} = -e^{h+\alpha y}, \quad (11)$$

where operator $\mathcal{L}_1$ has the form

$$\mathcal{L}_1 f = a(y)\frac{\partial f}{\partial x} + \frac{y}{2}\frac{\partial^2 f}{\partial x^2} + \kappa(\theta - y)\frac{\partial f}{\partial y} + \frac{\sigma^2 y}{2}\frac{\partial^2 f}{\partial y^2}. \quad (12)$$

After discretization in y-space we reduce equation (10) to a free boundary value problem for a system of parabolic equations

$$\frac{1}{2}y_j\frac{\partial^2}{\partial x^2}f_j(x) + a(y_j)\frac{\partial}{\partial x}f_j(x) - q_j(x)f_j(x) + \sum_{k\neq j}\lambda_{jk}f_k(x) = 0, \quad x > h_j, \quad (13)$$

$$f_j(x) = (1 - e^{x+\alpha y_j}), \quad x \le h_j. \quad (14)$$

To obtain numerical results for the price of the option we apply a method based on the Wiener-Hopf factorization developed in [1]-[2].

Then we consider American put option with finite maturity

$$\partial_t f(t,x,y) + (\mathcal{L}_1 - r)f(t,x,y) = 0, \quad x > h(t,y), \tag{15}$$

$$f(t,x,y) > 1 - e^{x+\alpha y}, \quad x > h(t,y), \tag{16}$$

$$\partial_t f(t,x,y) + \mathcal{L}_1 f(t,x,y) - rf(t,x,y) \leq 0, \quad x \leq h(t,y), \tag{17}$$

$$f(t,x,y) = (1 - e^{x+\alpha y})_+, \quad x \leq h(t,y), \tag{18}$$

$$f(t,h,y) = 1 - e^{h+\alpha y}, \quad \frac{\partial}{\partial x} f(t,x,y)|_{x=h(t,y)} = -e^{h+\alpha y}, \tag{19}$$

where operator $\mathcal{L}_1$ has the form (12). Given a maturity date T we divide the period $[0,T]$ into N subperiods $0 \leq t_0 < t_1 < \cdots < t_n < \cdots < t_N = T$ and apply the Carr randomization technique. We use the time derivative $\partial_t f$ approximation by $\frac{f(t_{n+1},x,y)-f(t_n,x,y)}{\Delta_t}$ and apply the same discretization in y-space as in the case of a perpetual American option.

As a result the discretized form of (15)-(18) has the form

$$\begin{aligned} & \frac{f_j^{n+1}(x) - f_j^n(x)}{\Delta_t} + \frac{1}{2} y_j \frac{\partial^2}{\partial x^2} f_j^n(x) + a(y_j) \frac{\partial}{\partial x} f_j^n(x) - q_j(x) f_j^n(x) \\ + & \sum_{k \neq j} \lambda_{jk} f_k^n(x) = 0, \quad x > h_j^n, \end{aligned} \tag{20}$$

$$f_j^n(x) = (1 - e^{x+\alpha y_j}), \quad x \leq h_j^n, n < N, \quad f_j^N(x) = (1 - e^{x+\alpha y_j})_+, \quad n = N \tag{21}$$

and next we proceed as in the perpetual case.

We obtain numerical results and compare them with the correspondent results in the Black-Scholes model and the Merton model where jumps of a stock price are allowed.

References

[1] Boyarchenko S., Levendorskii S., Irreversible Decisions under Uncertainty. Studies in Economic Theory, v.27 (2007).

[2] Boyarchenko S. I., Levendorskii S. Z. American Options in the Heston Model With Stochastic Interest Rate. (2008). http://ssrn.com/abstract=1031282.

Elena Shmileva
Chebyshev Laboratory, St.Petersburg, Russia

Extrapolation of alpha-stable random fields

We are going to discuss several extrapolation methods for alpha-stable random fields. We will remind shortly the kriging extrapolation method that is well-known in geostatistics and works well until distributions have finite second moment. Then we will propose several extrapolation methods that give estimates for random fields with stable marginal distributions different from gaussian. For these methods we use a specific dependence measure for alpha-stable random variables instead of the covariance.

Mikhail Skopenkov
IITP RAS, Moscow, Russia, & KAUST, Thuwal, Saudi Arabia

Discrete complex analysis on a convex quadrangulation of the plane

Complex analysis on an arbitrary graph lying in the complex plane and having convex quadrilateral faces is developed. A function on the vertices is called discrete holomorphic, if for each face the difference quotients along the two diagonals are equal. This generalizes the definition by R. Isaacs, R. Duffin, and C. Mercat.

We prove that the Dirichlet boundary value problem for the real part of a discrete holomorphic function has a unique solution. In the case when each face has orthogonal diagonals we prove that this solution converges to a harmonic function in the scaling limit (under certain regularity assumptions). This solves a problem of S. Smirnov [4,Question 1].

This was proved earlier by R. Courant–K. Friedrichs–H. Lewy for square lattices [2] and by D. Chelkak–S. Smirnov [1] for rhombic lattices. The result provides a new approximation algorithm for numerical solution of the Dirichlet problem and also some probabilistic corollaries. The proof is based on energy estimates inspired by alternating-current networks theory [3]. Several open problems are stated.

References

[1] D. Chelkak and S. Smirnov, Discrete complex analysis on isoradial graphs, preprint (2008), http://arxiv.org/abs/0810.2188v1.

[2] R. Courant, K. Friedrichs, H. Lewy, Über die partiellen Differenzengleichungen der mathematischen Physik, Math. Ann. 100 (1928), 32–74. Russian transl.: Russ. Math. Surveys 8 (1941), 125–160.

[3] M. Prasolov and M. Skopenkov, Tilings by rectangles and alternating current, J. Combin. Theory A 118:3 (2011), 920–937, http://arxiv.org/abs/1002.1356.

[4] S. Smirnov, Discrete Complex Analysis and Probability, Proc. Intern. Cong. Math., Hyderabad, India, 2010.

Heikki Tikanmäki
Aalto University, Finland

How to hedge Asian options in fractional Black–Scholes model

We prove change of variable formulas [Itô formulas] for both arithmetic and geometric averages of geometric fractional Brownian motion. They are valid for all convex functions, not only for smooth ones. Moreover, they can be used for obtaining hedges (but not prices) for Asian options in fractional Black–Scholes model. We get explicit hedges in some cases where explicit hedges are not known even in the ordinary Black–Scholes model.

References

[1] Heikki Tikanmaki. How to hedge Asian options in fractional Black–Scholes model. Preprint (2011). http://arxiv.org/abs/1102.5182.

Ilya Tyurin
Moscow State University

Some new advances in estimating the rate of convergence in CLT

We improve our method developed in [1]-[3]. The convergence rate in CLT is investigated in terms of a wide class of probability metrics including ideal metrics introduced by V.M. Zolotarev. Namely, optimal estimates of the proximity between a probability distribution and its zero bias transformation are derived. An improvement of the zero bias coupling technique allows to establish optimal rates of convergence in CLT for sums of independent random variables with finite moments of the order $2+\delta$. Moreover, the classical Berry-Esseen theorem is improved as well as its analogue for the case of non-identically distributed summands.

The research is partially supported by RFBR grant 10-01-00397a.

References

[1] I.S. Tyurin (2009), On the accuracy of the Gaussian approximation, Doklady Mathematics, vol. 80, no. 3, 840-842.

[2] I.S. Tyurin (2010), On the convergence rate in Lyapunov's theorem, Teor. Veroyatnost. i Primenen., vol. 55, no. 2, 250-270.

[3] I.S. Tyurin (2010), Some new results concerning the rate of convergence in Lyapunov's theorem, 10th Vilnius Conference on Probab. Theory, Abstracts, p. 279.

Yuri Yakubovich
St.Petersburg State University
& St.Petersburg University of Film and Television, Russia

On the variance of sample size

Let K_n be the number of distinct values occurring among n random values sampled independently from a discrete distribution. We investigate the possible behavior of its variance as the number of sampled values n grows to infinity. In particular we show that if $\operatorname{Var} K_n$ remains bounded and tends to a finite limit, this limit is necessarily a natural number.

This talk is based on the joint work with L. Bogachev and A. Gnedin.

Vasiliy Zhechev
Novosibirsk State University, Russia

Functional limit theorem for canonical U-processes of dependent observations

The work is devoted to proving the functional limit theorem for a sequence of normalized U-statistics (so-called U-process) of an arbitrary order with canonical (degenerate) kernels, defined on the samples from a sequence of stationary connected observations satisfying φ-mixing. The corresponding limit distribution is described as an infinite polynomial form of an infinite-dimensional centered Gaussian process with known covariance.
