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The Third St.Petersburg Conference in Spectral Theory

1 – 6 July 2010

Dedicated to the memory of M. Sh. Birman (1928–2009)

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Program

Abstracts

St.Petersburg, 2011

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Alexandre Fedotov, Nikolai Filonov, Alexander Pushnitski

Organizing committee:

Alexandre Fedotov, Nikolai Filonov, Alexander Pushnitski, Tatayna Vinogradova, Nadia Zalesskaya

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The International Conference in Spectral Theory is supported by RFBR and Chebyshev laboratory, Russia, and Nordforsk Network, Norway.

The conference website: http://www.pdmi.ras.ru/EIMI/2011/ST

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Scientific programme

FRIDAY 1 July:

9:30–10:00: REGISTRATION

10:00–11:00: Gerd Grubb (Copenhagen University)
Spectral estimates of resolvent differences

COFFEE BREAK

11:30–12:30: Leonid Friedlander (University of Arizona)
Determinants of elliptic operators

LUNCH

14:30–15:30: Alexei Alexandrov (Steklov Institute, St.Petersburg)
Estimates of operator moduli of continuity

15:35–16:35: Dmitri Yafaev (University of Rennes 1)
Multichannel scattering theory for Hankel operators with discontinuous symbols

COFFEE BREAK

17:00–18:00: Alexander Sobolev (University College London)
A family of anisotropic integral operators and behaviour of its maximal eigenvalue

SATURDAY 2 July:

10:00–11:00: Vladimir Marchenko (Inst. for Low Temperature Physics, Kharkov)
Inverse problem of the theory of small oscillations

COFFEE BREAK

11:30–12:30: Igor Rodnianski (Princeton)
On singularity formation in geometric evolution equations

LUNCH

14:30–15:30: Mihalis Dafermos (Cambridge)
TBA

15:35–16:35: Alexander Komech (Vienna University and IITP, Moscow)
On global attractors of nonlinear hyperbolic PDEs

COFFEE BREAK

17:00–18:00: Elena Kopylova (IITP, Moscow)
On asymptotic stability of kinks for relativistic Ginzburg-Landau equation

SUNDAY 3 July:

FREE DAY

MONDAY 4 July:

10:00–11:00: Leonid Pastur (Inst. for Low Temperature Physics, Kharkov)
Orthogonal polynomials, quasiperiodic Jacobi matrices, and random matrices

COFFEE BREAK

11:30–12:30: Yoram Last (Hebrew University of Jerusalem)
On level spacings for Jacobi operators

LUNCH

14:30–15:30: Evgeni Korotyaev (St.Petersburg State University)
Löwner equation and integrable systems

COFFEE BREAK

16:00–16:20: Nikolai Veniaminov (University of Paris Nord)
Thermodynamic limit for the system of interacting quantum particles in random media

16:25–16:45: Albrecht Seelmann (Johannes Gutenberg-Universität Mainz)
A new estimate in the subspace perturbation problem

16:50–17:20: Ondrej Turek (Kochi University of Technology, Japan)
Quantum graph vertices with equal transmission probabilities

18:00: BOAT TRIP

TUESDAY 5 July:

10:00–11:00: Gian Michele Graf (ETH Zurich)
Adiabatic evolution and dephasing

COFFEE BREAK

11:30–12:30: Ira Herbst (University of Virginia)
Smoothness and analyticity in nonrelativistic QED

LUNCH

14:30–15:30: Akira Iwatsuka (Kyoto Institute of Technology)
Schrödinger operators with singular magnetic fields supported by a circle in $\mathbb{R}^3$

COFFEE BREAK

16:00–16:20: Giuseppe De Nittis (University Paris-13)
Peierls substitution, Hofstadter model, disorder

16:25–16:45: Vladimir Pchelin (St.Petersburg University)
On eigenvalues distribution of a one-dimensional random Schrödinger operator

16:50–17:20: Vsevolod Chernyshev (BMSTU, MSU
Gaussian packets on networks: the case of linearly dependent lengths

18:00 (TBC): CONFERENCE DINNER

WEDNESDAY 6 July:

10:00–11:00: Thomas Hoffmann-Ostenhoff (University of Vienna)
Spectral minimal partitions

COFFEE BREAK

11:30–12:30: Boris Vainberg (University of North Carolina, Charlotte)
On the negative spectrum of low dimensional Schrödinger operators

LUNCH

14:30–15:30: Maxim Skriganov (Steklov Institute, St.Petersburg)
Harmonic analysis of uniformly distributed point sets

15:35–16:35: Michael Belishev (Steklov Institute, St.Petersburg)
s-points in three-dimensional acoustical scattering

Abstracts

Estimates of operator moduli of continuity

Alexei Aleksandrov

St.Petersburg Department of Steklov Institute

Joint work with V. Peller

Let f be a uniformly continuous function on $\mathbb{R}$. Denote by ω_f the modulus of continuity of f. We define the operator modulus of continuity as follows

$$\Omega_f(\delta) \stackrel{\text{def}}{=} \sup\{\|f(A) - f(B)\| : A, B \text{ are self-adjoint}, \quad \|A - B\| \le \delta\}.$$

It is well known that a Lipschitz function does not have to be operator Lipschitz. In other words, the condition $\omega_f(\delta) = O(\delta)$ does not imply that $\Omega_f(\delta) = O(\delta)$ in general.

The following result was obtained in [1].

Theorem. $\Omega_f(\delta) \le \text{const}\ \delta \int_\delta^\infty \frac{\omega_f(t)}{t^2}\, dt$.

In particular, if f is a bounded Lipschitz function, then

$$\Omega_f(\delta) = O(\delta|\log\delta|) \quad \text{for small} \quad \delta > 0.$$

We are going to improve these estimates for some classes functions. For example, if f is a bounded piecewise linear function, then

$$\Omega_f(\delta) = O(\delta \log|\log\delta|) \quad \text{for small} \quad \delta > 0. \tag{1}$$

Moreover, we obtain lower estimates for Ω_f. In particular, we prove that estimate (1) cannot be improved for nonconstant functions f. In other words, for every nonconstant piecewise linear function f there exist $\delta_f, c_f > 0$ such that

$$\Omega_f(\delta) \ge c_f\, \delta \log|\log\delta| \quad \text{for all} \quad \delta \in (0, \delta_f).$$

Finally, we construct a C^∞ function f on $\mathbb{R}$ such that $|f| \le 1$, $|f'| \le 1$, and

$$\Omega_f(\delta) \ge \text{const}\, \delta\sqrt{|\log\delta|} \quad \text{for small} \quad \delta > 0.$$

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s-points in three-dimensional acoustical scattering

Mikhail Belishev and Alexei Vakulenko

St.Petersburg Department of Steklov Institute

The notion of s-points was introduced by the authors in [1] in connection with the control problem for the dynamical system governed by the three-dimensional acoustical equation $u_{tt} - \Delta u + qu = 0$ with a real potential $q \in C_0^\infty(\mathbb{R}^3)$ and controlled by incoming spherical waves. In the generic case, this system is controllable in the relevant sense, whereas $a \in \mathbb{R}^3$ is called an *s-point* (we write $a \in \Upsilon_q$) if the system with the shifted potential $q_a = q(\,\cdot - a)$ *is not controllable*. Such a lack of controllability is related to the subtle physical effect: in the system with the potential q_a, there exist the finite energy waves vanishing in the past and future cones simultaneously. The subject of this paper is the set Υ_q: we reveal its relation to the factorization of the S-matrix, connections with the discrete spectrum of the Schrödinger operator $-\Delta + q$, and the jet degeneration of the polynomially growing solutions to the equation $(-\Delta + q)p = 0$.

References

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Gaussian packets on networks: the case of linearly dependent lengths

Vsevolod Chernyshev

BMSTU, Moscow State University

The talk is devoted to the dynamics and statistics of gaussian packets on a metric graph (see papers [1], [2] and references therein) and their connection with the number of integral points in extending simplices. We consider linearly dependent as well as linearly independent lengths of the edges.

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Peierls substitution, Hofstadter model, disorder

Giuseppe De Nittis

Université de Paris 13

The study of the (integer) *Quantum Hall Effect* (QHE) requires a careful analysis of the spectral properties of the $2D$, single-electron Hamiltonian

$$H_{\Gamma,B} := (-\,\mathrm{i}\,\partial_x - B\ y)^2 + (-\,\mathrm{i}\,\partial_y + B\ x)^2 + V_\Gamma(x,y) \tag{2}$$

where $H_B := H_{\Gamma,B} - V_\Gamma$ is the usual *Landau Hamiltonian* (in symmetric gauge) with *magnetic field* B and V_Γ is a $\Gamma \equiv \mathbb{Z}^2$ *periodic potential* which models the electronic interaction with a crystalline structure. Under usual conditions (e.g., $V_\Gamma \in L^2_{\mathrm{loc}}(\mathbb{R}^2)$) the Hamiltonian (2) is self-adjoint on a suitable subdomain of $L^2(\mathbb{R}^2)$.

A direct analysis of the fine spectral properties of (2) is extremely difficult and one needs resorting to simpler effective models hoping to capture (some of) the main physical features in suitable physical regimes. A very interesting regime (easily accessible to experiments) concerns the limit of a weak magnetic field $B \ll 1$. The common lore, which dates back at the pionering works of R. Peierls, P. G. Harper and D. Hofstadter, says that the "local description" of the spectrum of (2) is "well approximated" by the spectrum of the *Hofstadter* (effective) *model*

$$\left(H^{(B)}_{\mathrm{Hof}}\xi\right)_{n,m} := e^m_B\ \xi_{n+1,m} + \overline{e^m_B}\ \xi_{n-1,m} + \overline{e^n_B}\ \xi_{n,m+1} + e^n_B\ \xi_{n,m-1} \tag{3}$$

with $\{\xi_{n,m}\} \in \ell^2(\mathbb{Z}^2)$ and $e^m_B := \mathrm{e}^{\mathrm{i}\,2\pi m B}$.

The above discussion leaves open some natural questions:

Q.1) In what mathematical sense are $\mathrm{Spec}\big(H_{\Gamma,B}\big)$ and $\mathrm{Spec}\big(H^{(B)}_{\mathrm{Hof}}\big)$ "locally equivalent"?

Q.2) What is the relation between the dynamics induced by $H^{(B)}_{\mathrm{Hof}}$ on $\ell^2(\mathbb{Z}^2)$ and the "true" dynamics induced by $H_{\Gamma,B}$ on $L^2(\mathbb{R}^2)$?

A third question concerns the rôle of the disorder for a full explanation of the QHE. Indeed, it is well known that the introduction of a *random potential* V_ω (e.g., an *Anderson potential*) in (2), leading to

$$H_{\Gamma,B,\omega} := H_{\Gamma,B} + V_\omega, \tag{4}$$

is essential in order to explain the emergence of the quantum Hall plateaus.

Q.3) Is it possible to derive in the case V_ω a "simplified" (i.e., effective) model for $H_{\Gamma,B,\omega}$ which encodes the (main) spectral and dynamical properties of the full model?

In order to fully answer questions Q.1) and Q.2) one needs to prove rigorously the so-called *Peierls substitution* which, in particular, implies the derivation of the effective model (3) from the full model (2). This is an old problem which dates back to the works of J. Bellissard [1] and B. Helffer and J. Sjöstrand [5]. However, these works provide only a partial answer to Q.1) (*local isospectrality*) and no answer for Q.2). A fully satisfactory answer has been given only recently by the author and M. Lein in [4]. In this paper a strong version of the Peierls substitution is derived by means of a joint application of the *Space-adiabatic perturbation theory* (SAPT) developed by G. Panati, H. Spohn and S. Teufel [6] and the *magnetic Weyl quantization* developed by M. Măntoiu and R. Purice [3]. The main result derived in [4] can be stated as follows:

Theorem 1. *Assume the existence of a* $S \subset \mathrm{Spec}\big(H_{\Gamma,B=0}\big)$ *separated from the rest of the spectrum* $\mathrm{Spec}\big(H_{\Gamma,B=0}\big) \setminus S$ *by gaps*[1]*. Then:*

(i) Associated to S *there exists an an orthogonal projection* Π_B *in* $L^2(\mathbb{R}^2)$ *such that for any* $N \in \mathbb{N}$

$$\big\|[H_{\Gamma,B};\Pi_B]\big\| \leqslant C_N\ B^N \qquad \text{if} \qquad B \to 0 \tag{5}$$

where $C_N > 0$ *are suitable constants. The space* $\mathrm{Ran}\ \Pi_B \subset L^2(\mathbb{R}^2)$ *is called* almost-invariant *subspace.*

(ii) There exists a reference *Hilbert space* $\mathcal{H}_{\mathrm{r}}$ *(B-independent), an* effective *(bounded)* operator H_B^{eff} *on* $\mathcal{H}_{\mathrm{r}}$ *and a unitary operator* $U_B : \mathrm{Ran}\ \Pi_B \to \mathcal{H}_{\mathrm{r}}$ *such that for any* $N \in \mathbb{N}$

$$\big\|\big(\mathrm{e}^{\,\mathrm{i}\,tH_{\Gamma,B}} - U_B^{-1}\ \mathrm{e}^{\,\mathrm{i}\,tH_B^{\mathrm{eff}}}\ U_B\big)\Pi_B\big\| \leqslant C'_N\ B^N\ |t| \qquad \text{if} \qquad B \to 0. \tag{6}$$

(iii) If S *corresponds to a single Bloch energy band* E_* *of the periodic operator* $H_{\Gamma,B=0}$*, then* $\mathcal{H}_{\mathrm{r}} \equiv \ell^2(\mathbb{Z}^2)$*. Moreover if the dispersion law for* E_* *can be approximated as* $E_*(k_1,k_2) = 2\cos(k_1) + 2\cos(k_2) + Bf(k_1,k_2)$*, with* k_1 *and* k_2 *the Bloch momenta, then*

$$H_B^{\mathrm{eff}} = H_{\mathrm{Hof}}^{(B)} + \mathcal{O}(B) \qquad \text{if} \qquad B \to 0. \tag{7}$$

Thus, from Theorem 1 one deduces the following answers for Q.1) and Q.2): $\Pi_B H_{\Gamma,B} \Pi_B$ and $H_{\mathrm{Hof}}^{(B)}$ are unitarily equivalent up to an error which goes to zero in the limit $B \to 0$ (*asymptotic unitary equivalence*); the dynamics generated by $H_{\mathrm{Hof}}^{(B)}$ approximates the dynamics generated by $\Pi_B H_{\Gamma,B} \Pi_B$ up to a small error over any macroscopic time-scale $t \in [0,T]$.

As a first attempt to answer question Q.3) one could try to combine SAPT-techniques with the randomness induced by V_ω. However, one of the main ingredients of SAPT is the separation in fast and slow degrees of freedom induced by the periodic structure of $H_{\Gamma,B=0}$. This separation (mathematically highlighted by means of a Bloch-Floquet transform) identifies the fast part of the dynamics with the dynamics inside the fundamental cell of the lattice. The slow part is related to the motion at the boundary of adjacent cells and is controlled by the slow variation of the Bloch momenta induced by the weak, but non-zero, magnetic field $B \ll 1$. In order to include V_ω in this schema, one needs to assume that the randomness produces a perturbation in the periodic structure which lives on a scale larger that the typical lenght of the crystal and which becomes larger and larger when $B \to 0$. In other words SAPT-tecniques are compatible only with B-dependent random potentials of type

$$V_{\omega,B}(x,y) := w_\omega\big(B^{-1}x, B^{-1}x\big) \tag{8}$$

with w_ω suitable random variables. If order to overcome the quite unphysical restriction (8) one has to replace the usual Bloch-Floquet transform with some non-commutative extension.

[1] This assumption can be relaxed by introducing the notion of *adiabatically decoupled* energy subspace, cf. [6] or [4]

An hint in this direction is provided by the Bellissard's idea of replacing the Bloch-Floquet decomposition with the non-commutative notion of crossed product C^*-algebra [2].

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Determinants of Elliptic Operators

Leonid Friedlander

University of Arizona

I will give a historical overview of how the notion of the determinant was being developed, starting from Leibniz. Certain problems arise when one goes from matrices to operators acting in infinite-dimensional spaces. I will discuss how to use different regularization techniques to deal with these problems. In particular, I will discuss determinants of elliptic differential operators, and anomalies that are associated with regularization procedures.

Adiabatic evolution and dephasing

Gian Michele Graf

ETH Zurich

Joint work with Y. Avron, M. Fraas, P. Grech and O. Kenneth

Lindbladians are generators of the effective dynamics of open quantum systems. We focus on dephasing Lindbladians. Like Hamiltonians of isolated quantum mechanical system, but in contrast to generic Lindbladians, they exhibit several stationary states. We will provide examples.

The adiabatic evolution of an isolated quantum mechanical system exhibits no irreversible transitions if its Hamiltonian undergoes a slow, temporary time-dependence. By contrast, if a dephasing Lindbladian undergoes such a change, transitions are typically irreversible. I'll present a formulation of the adiabatic theorem which accounts for the different kinds of transition, though treating Hamiltonian and Lindbladian dynamics on equal footing.

We'll then present an application to transport and linear response theory. In the context of dephasing Lindbladians, the coefficients of dissipative conductance are determined by the Fubini-Study metric, while their non-dissipative counterparts are determined by the adiabatic curvature. If the metric and the (symplectic) curvature form are compatible, in the sense of defining a Khler structure, then the non-dissipative resistance coefficients are immune to dephasing. We give some examples of compatible systems.

Spectral estimates of resolvent differences

Gerd Grubb

Copenhagen University

M. Sh. Birman showed in 1962 that the difference between the resolvents of two selfadjoint realizations of a second-order elliptic operator, on a domain in $\mathbb{R}^n$ with compact boundary, is in the weak Schatten class $\mathfrak{S}_p$ for $p = (n-1)/2$, i.e., it is compact with eigenvalues μ_j that are $O(j^{-2/(n-1)})$. Later works have obtained limit formulas (Weyl-type spectral asymptotic estimates) and have extended the scope to higher-order problems with various boundary conditions. We give an account of the development, including some recent progress: A Weyl-type formula for *mixed problems* for the Laplacian, that measures the area of the part of the boundary where the Neumann condition is imposed. Important tools are here: Krein resolvent formulas and nonstandard pseudodifferential operators.

Smoothness and analyticity of perturbation expansions in QED

I. Herbst

University of Virginia

Joint work with David Hasler

Consider the operators

$$H^{(1)} = \sum_{n=1}^{N} (p_j + gA_\Lambda(\beta x_j))^2 + V(x_1, \cdots, x_N) + H_f \tag{9}$$

$$H^{(2)} = \sum_{n=1}^{N} (p_j^2 + 2gp_j \cdot A_\Lambda(\beta x_j)) + V(x_1, \cdots, x_N) + H_f \tag{10}$$

describing an atom interacting with the quantized electromagnetic field on the Hilbert space $\mathcal{H} = L_a^2(R^{3N}; \mathcal{F})$, the space of N spinless antisymmetric electrons and an arbitrary number of photons. We show under the main assumptions that V is symmetric and rotation invariant, and that if

$$\sum_{n=1}^{N} p_j^2 + V(x_1, \cdots, x_N)$$

has a unique ground state, then for small $g, H^{(1)}$ and $H^{(2)}$ have unique ground states, which have convergent expansions of the form

$$\psi = \sum_{n=0}^{\infty} g^n \psi_\beta$$

where the coefficients are C^∞ in β. Similarly for the ground state energy.

Spectral Minimal Partitions

Thomas Hoffmann-Ostenhof

University of Vienna

Joint work with Bernard Helffer and Susanna Terracini;
some important numerical work was contributed by Virginie Bonnaillie Noël

Consider a domain $\Omega \subset \mathbb{R}^2$ and a partition of Ω into k pairwise disjoint open sets, $D_1, D_2, \ldots, D_k$. Associate to each of the D_i the corresponding lowest Dirichlet eigenvalue $\lambda_1(D_i) = \inf_{u \in C_0^\infty(D_i)} \|\nabla u\|^2 / \|u\|^2$. Take the infimum of $\max_{i \le k} \lambda_1(D_i)$ over all k-partitions and call it $\mathfrak{L}_k(\Omega)$. A partition for which the infimum is a minimum is called a spectral minimal partition $\mathcal{P}_k(\Omega)$.

In this talk we will review some properties of the $\mathfrak{L}_k(\Omega)$ and their associated $\mathcal{P}_k(\Omega)$. In particular, relations of the $\mathfrak{L}_k(\Omega)$ with the spectrum of the Drichlet Laplacian for Ω and the relation between the $\mathcal{P}_k(\Omega)$ with the eigenfunctions and their nodal domains are discussed. Finally, we present a new approach characterizing those $\mathfrak{L}_k(\Omega)$ and $\mathcal{P}_k(\Omega)$ with the help of Aharonov Bohm problems.

Schrödinger operators with singular magnetic fields supported by a circle in $\mathbf{R}^3$

Akira Iwatsuka[1] and Takuya Mine[2]

[1] Kyoto Institute of Technology
[2] Setsunan University

We consider the Schrödinger operator H_ϵ in $\mathbf{R}^3$ with a magnetic field supported in a torus of thickness $\epsilon > 0$, which is generated by a toroidal solenoid. We prove that H_ϵ converges to an operator H_0 in the norm resolvent sense as $\epsilon \to 0$, if we choose the gauge of the vector potential appropriately. The limit operator H_0 is the Schrödinger operator with a singular magnetic field supported on a circle.

A result of this type is obtained in the two-dimensional case by Tamura [4]. Tamura's result is concerned with the Aharonov-Bohm effect (see [1]). The Aharonov-Bohm effect had caused a lot of controversies among physicists until in 1986 a decisive experiment using a toroidal magnetic field enclosed by a superconductive material was carried out by Tonomura et al. ([5], see also [3]).

We have shown in our previous work [2] as an attempt to give a model for Tonomura's experiment that, if we choose the magnetic field and the vector potential appropriately, then H_ϵ converges in the norm resolvent sense to some operator H_0, which is the Schrödinger operator with a singular magnetic field supported on a circle.

We like to present some improvements of this result: First we show the choice of the three dimensional magnetic fields B_ϵ is arbitrary in the sense that, if B_ϵ are of the form given in the condition (A2) below, the two dimensional magnetic field b is arbitrary as far as it satisfies the assumption (A1) below. Second we show "the norm resolvent convergence" of the Schrödinger operators defined on the exterior region $\mathbf{R}^3 \setminus \mathcal{T}_\epsilon$ with Dirichlet boundary conditions as ϵ tends to zero, where $\mathcal{T}_\epsilon$ is the torus of thickness ϵ around a fixed circle and where we should interpret the meaning of the norm resolvent convergence appropriately since the operators considered are defined on different domains.

We like to give some more detailed description about our main theorem. We consider magnetic Schrödinger operators on $\mathbf{R}^3$

$$H_\epsilon = (D - A_\epsilon)^2 = \sum_{j=1}^{3} (D_j - A_{\epsilon,j})^2,$$

where $0 \le \epsilon < \epsilon_0$ (ϵ_0 is some positive constant), $D_j = \frac{1}{i}\partial_j$, $\partial_j = \frac{\partial}{\partial x_j}$, $D = {}^t(D_1, D_2, D_3)$ and $A_\epsilon = {}^t(A_{\epsilon,1}, A_{\epsilon,2}, A_{\epsilon,3})$. The magnetic field $B = {}^t(B_1, B_2, B_3)$ corresponding to a vector potential

$A = {}^t(A_1, A_2, A_3)$ is given by $B = \nabla \times A = {}^t\Big(\partial_2 A_3 - \partial_3 A_2, \partial_3 A_1 - \partial_1 A_3, \partial_1 A_2 - \partial_2 A_1\Big)$. We denote

$$B_\epsilon = \nabla \times A_\epsilon. \tag{11}$$

We shall define our magnetic fields as follows. Let $a > \epsilon_0$ be a constant. We introduce a local coordinate

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} (a + -\tau\cos\phi)\cos\theta \\ (a + -\tau\cos\phi)\sin\theta \\ \tau\sin\phi \end{pmatrix},$$

where $0 \le \tau < a$, $\phi \in \mathbf{R}/2\pi\mathbf{Z}$, $\theta \in \mathbf{R}/2\pi\mathbf{Z}$. We denote the torus $\{\tau < \epsilon\} = \mathcal{T}_\epsilon$ for $0 < \epsilon \le a$, the circle $\{\tau = 0\} = \mathcal{C}$, and the unit vector fields around the x_3 axis $e_\theta = {}^t(-\sin\theta, \cos\theta, 0)$. We need the following conditions

(A1) Let $E := \{\, (x_1,\, x_2) \in \mathbf{R}^2 \,|\, x_1^2 + -x_2^2 < 1 \,\}$ and $b \in C_0^\infty(E; \mathbf{R})$ satisfying

$$\int_E b(x_1, x_2)dx_1dx_2 = 2\pi\alpha, \quad \alpha \in \mathbf{R} \setminus \mathbf{Z}.$$

(A2) For $0 < \epsilon < \epsilon_0$, we assume $B_\epsilon \in C^\infty(\mathbf{R}^3; \mathbf{R})^3$, $\operatorname{supp} B_\epsilon$ is contained in the open torus $\mathcal{T}_\epsilon$ and in this torus

$$B_\epsilon = -\frac{1}{\epsilon^2} b\left(\frac{\tau\cos\phi}{\epsilon}, \frac{\tau\sin\phi}{\epsilon}\right) e_\theta.$$

(A3) For $\epsilon = 0$, we assume $B_\epsilon \in \mathcal{D}'(\mathbf{R}^3; \mathbf{R})^3$ (the vector-valued distributions on $\mathbf{R}^3$) and

$$B_0 = -2\pi\alpha\delta_{\mathcal{C}} e_\theta,$$

where $\delta_{\mathcal{C}}$ is the delta measure on the circle $\mathcal{C}$.

(A4) For $0 < \epsilon < \epsilon_0$, $A_\epsilon \in C_0^\infty(\mathbf{R}^3; \mathbf{R})^3$. For $\epsilon = 0$, $A_0 \in C^\infty(\mathbf{R}^3 \setminus \mathcal{C}; \mathbf{R})^3 \cap L^1(\mathbf{R}^3; \mathbf{R})^3$, and the support of A_0 is compact in $\mathbf{R}^3$. A_ϵ satisfies (11) for $0 \le \epsilon < \epsilon_0$.

Our main result is that, if $\{B_\epsilon\}_{0 \le \epsilon < \epsilon_0}$ are given by (A1)–(A3), then there exist vector potentials $\{A_\epsilon\}_{0 \le \epsilon < \epsilon_0}$ satisfying (A4) such that H_ϵ converges to H_0 in the norm resolvent sense, as ϵ tends to 0. We note that the magnetic potential A_0 does not belong to $L^2(\mathbf{R}^3)$ near the circle $\mathcal{C}$ so that we should define the selfadjoint operator H_0 imposing the Dirichlet conditions on $\mathcal{C}$.

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On Global Attractors of Nonlinear Hyperbolic PDEs

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We consider Klein-Gordon and Dirac equations coupled to U(1)-invariant nonlinear oscillators. The solitary waves of the coupled nonlinear system form two-dimensional submanifold in the Hilbert phase space of finite energy solutions. Our main results read as follows:

Theorem *Let all the oscillators be strictly nonlinear. Then any finite energy solution converges, in the long time limit, to the solitary manifold in the local energy seminorms.*

The investigation is inspired by Bohr's postulates on transitions to quantum stationary states. The results are obtained for:

- 1D KGE coupled to one oscillator [1,2,3], and to finite number of oscillators [4];
- nD KGE and Dirac eqns coupled to one oscillator via mean field interaction [5,6].

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On asymptotic stability of kinks for relativistic Ginzburg-Landau equation

Elena Kopylova

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We consider nonlinear relativistic wave equation in one space dimention

$$\ddot{\psi}(x,t) = \psi''(x,t) + F(\psi(x,t)), \quad x \in \mathbb{R}, \quad F(\psi) = -U'(\psi) \quad (1)$$

where $U(\psi)$ is a Ginzburg-Landau potential: $U(\psi) \sim (\psi^2 - 1)^2/4$.

There exist a "kink" - nonconstant finite energy solution $s(x) \sim \tanh x/\sqrt{2}$ of stationary equation. Then the moving kinks or solitary waves

$$s_{q,v}(t) = s(x - vt - q), \quad q, v \in \mathbb{R}, \quad |v| < 1, \quad \gamma = 1/\sqrt{1 - v^2}$$

are the solutions to equation (1). Our main results are the following asymptotics

$$(\psi(x,t), \dot{\psi}(x,t)) \sim (s_{q_\pm,v_\pm}(x - v_\pm t - q_\pm), \dot{s}_{q_\pm,v_\pm}(x - v_\pm t - q_\pm)) + W_0(t)\Phi_\pm, \quad t \to \pm\infty$$

for solutions to (1) with initial states close to the kink. Here $W_0(t)$ is the dynamical group of the free Klein-Gordon equation, $\Phi_\pm$ are the corresponding asymptotic states, and the remainder converges to zero $\sim t^{-1/2}$ in the "global energy norm" of the Sobolev space $H^1(\mathbb{R}) \oplus L^2(\mathbb{R})$. Crucial role in the proof play our recent results on weighted energy decay for the Klein-Gordon equations.

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Lowner equation and integrable systems

Evgeny Korotyaev

St.Petersburg State University

We study the relationship between the action-angle variables for the KDV (and defocussing NLS) equation on the circle and the Marchenko-Ostrovski parameters. In our analysis a new approach is demonstrated. The main tool is the Löwner type equation for the quasimomentum.

On level spacings for Jacobi operators

Yoram Last

Hebrew University of Jerusalem

The talk will discuss some recent results concerning the spacings of eigenvalues for restrictions of self-adjoint Jacobi operators to large "boxes" and their connections with other spectral and dynamical properties of these operators. A central focus will be the phenomenon of "clock behavior" often associated with absolutely continuous spectrum.

Inverse Problem of the Theory of Small Oscillations

Vladimir Marchenko

Institute for Low Temperature Physics, Kharkov, Ukraine

The small oscillations of a finite system A of material points (particles) interacting between themselves and with an external field are considered. It is assumed that only the oscillations of a part $A_0 \subset A$ are known. Necessary and sufficient conditions for the restoring of the oscillations of the whole system are given. A method (an algorithm) for the restoring is also presented.

Orthogonal Polynomials, Quasiperiodic Jacobi Matrices and Random Matrices

Leonid Pastur

Institute for Low Temperature Physics, Kharkov, Ukraine

We discuss recent results on asymptotics of orthogonal polynomials stressing their spectral aspects and similarity in two cases considered. They concern the polynomials orthonormal on a finite union of disjoint intervals with respect to the Szegö weight and polynomials orthonormal on the whole axis with respect to varying weights and having the same union of intervals as the set of oscillations of asymptotics. In both cases we construct double infinite Jacobi matrices with generically quasiperiodic coefficients and show that each of them is an isospectral deformation of another. Related results on asymptotic eigenvalue distribution of a class of random matrices of large size are also briefly discussed.

On eigenvalues distribution of a one-dimensional random Schrödinger operator

Vladimir Pchelin

The talk is devoted to a study of the counting function of a one-dimensional random Schrödinger operator on a finite length interval with the Dirichlet boundary conditions. The model is the continuous Anderson model, the single site potential is the Dirac mass, the coupling constants are assumed to be iid random variables. We also assume that each of them has a continuous density v, and that $v(0) \neq 0$. It's proven that, under some additional (not too restrictve) conditions, the distribution of the centered and properly normalized counting function converges weakly to the normal distribution as the length of the interval where one considers the Schrödinger operator tends to infinity.

On singularity formation in geometric evolution equations

Igor Rodnianski

Princeton

The talk will review recent results on singularity formation for the Wave and Schrodinger Map problems. In both cases we uncover a large class of solutions, arising from smooth data, which break down in finite time via concentration of a bubble containing a harmonic map.

A new estimate in the subspace perturbation problem

Albrecht Seelmann

J.Gutenberg-Universität Mainz

This talk is based on a joint work with K. A. Makarov

We study the problem of variation of spectral subspaces for bounded linear self-adjoint operators. We obtain a new estimate on the norm of the difference of two spectral projections associated with isolated parts of the spectrum of the perturbed and unperturbed operators.

This improves a result obtained earlier by Kostrykin, Makarov and Motovilov in [Trans. Amer. Math. Soc. (2007)].

Harmonic analysis of uniformly distributed point sets

Maxim Skriganov

Steklov Institute, St.Petersburg

We will discuss applications of harmonic analysis to the theory of uniformly distributed point sets in the unit cube. Such an approach turns out to be highly efficient and allows to look at the discrepancy theory from new standpoints. As an illustration we will give a new proof of the classical result of Chen on the existence of point sets with the minimum order of the L_q-discrepancies. The original proof of this theorem was relaying on rather elaborate combinatorial arguments. We will show that the results of such type are intimately related with lacunarity and statistical independence of certain functional series. Particularly, Chen's theorem turns out to be a corollary of Khinchin's inequality for the Rademacher functions.

Below, we list references that can be useful in the context of this talk.

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A family of anisotropic integral operators and behaviour of its maximal eigenvalue

Boris Mityagin[1] and Alexander Sobolev[2]

[1] Ohio State University
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We study the family of compact integral operators $\mathbf{K}_\beta$ in $L^2(\mathbb{R})$ with the kernel

$$K_\beta(x,y) = \frac{1}{\pi}\frac{1}{1+(x-y)^2+\beta^2\Theta(x,y)},$$

depending on the parameter $\beta > 0$, where $\Theta(x,y)$ is a symmetric non-negative homogeneous function of degree $\gamma \geq 1$. The main result is the following asymptotic formula for the maximal eigenvalue M_β of $\mathbf{K}_\beta$:

$$M_\beta = 1 - \lambda_1\beta^{\frac{2}{\gamma+1}} + o(\beta^{\frac{2}{\gamma+1}}), \beta \to 0,$$

where λ_1 is the lowest eigenvalue of the operator $\mathbf{A} = |d/dx| + \Theta(x,x)/2$. A central role in the proof is played by the fact that $\mathbf{K}_\beta, \beta > 0$, is positivity improving. The case $\Theta(x,y) = (x^2+y^2)^2$ has been studied earlier in the literature as a simplified model of high-temperature superconductivity.

Quantum graph vertices with equal transmission probabilities

Ondřej Turek

Kochi University of Technology, Japan

joint work with Taksu Cheon

Scattering properties of a vertex coupling in quantum graphs is determined by its scattering matrix which generally depends on energy. There exists, however, a family of couplings for which the scattering matrix is energy-independent, namely *scale-invariant vertex couplings*. In our talk, we examine scale-invariant couplings in a vertex of general degree $n > 2$ with the following additional property:

- reflection probability at the i-th edge ($i \in \{1, \ldots, n\}$) is independent of i,
- transmission probability from the i-th to the j-th edge ($i, j \in \{1, \ldots, n\}$, $i \neq j$) is independent of i, j.

We discuss necessary and sufficient conditions of existence of vertex couplings with that property. In particular we show that the maximal value of the ratio (reflection probability)/(transmission probability) equals $(1-2/n)^2$. We characterize all the vertex couplings for which this extremal value is attained; they form an $(n-1)$-parameter family that includes the free coupling as its main representative element.

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On the negative spectrum of low dimensional Schrödinger operators

Stanislav Molchanov and Boris Vainberg

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We will discuss estimates for the number of negative eigenvalues for Schrödinger type operators on Riemannian manifolds, graphs and quantum graphs with the spectral dimension $d \leq 2$. The main example: the Schrödinger operator in R^2 with a potential decaying at infinity. The classical Cwikel-Lieb-Rosenblum (CLR) upper estimates require the corresponding Markov process to be transient, and therefore the dimension to be greater than two. We suggest a simple approach allowing to obtain CLR estimates in low dimensions by transforming the underlying recurrent process into a transient one using partial annihilation. As a result, the estimates for the number of negative eigenvalues are not translation invariant and contain Bargmann type terms. We will also discuss the estimates from below and Lieb-Thirring estimates.

Thermodynamic Limit for the System of Interacting Quantum Particles in Random Media

Nikolaj Veniaminov

Université de Paris 13

Motivation. Random Schrdinger operators have been intensively studied during the last fifty years and a great amount of mathematical knowledge has been accumulated. The vast majority of research has been done for one-particle approximation and only a few works treat the case of a finite number of particles. However, in a reasonable physical setting there is plenty of *interacting* particles (say, electrons in a metal or semiconductor) in a macroscopic sample of material. These particles may be subject to some *quantum statistics* (Pauli exclusion principle for electrons, for example). Moreover, the larger the sample, the greater the number of particles.

In the present talk we are going to address the setting where the number of interacting particles grows (approximately linearly) with the volume of a box to which they are restricted.

Multiparticle operator. Consider one particle random Schrdinger operator

$$H_\omega = -\Delta + V_\omega$$

on $\mathfrak{H} = L^2(\mathbb{R}^d)$ or $l^2(\mathbb{Z}^d)$ for either continuous or discrete case respectively. Here Δ is the free Laplace operator and V_ω is the random potential. For simplicity, one may think of the Anderson model:

$$V_\omega = \sum_{\gamma \in \mathbb{Z}^d} \omega_\gamma u(\cdot - \gamma),$$

where $\{\omega_\gamma\}_{\gamma \in \mathbb{Z}^d}$ is a collection of independent identically distributed random variables with bounded compactly supported density and u is a "nice" compactly supported function (in the discrete setting it can be replaced just by the projector on the origin in canonical basis: $u = (\cdot, \delta_0)\delta_0$).

We remark, that the results that follow are still valid for any $\mathbb{Z}^d$-ergodic family of random operators H_ω and both for discrete and continuous settings. For the purpose of notational simplicity we will treat only the continuous case here.

The restriction of H_ω to a domain $\Lambda \subset \mathbb{R}^d$ with some fixed boundary conditions will be denoted by $H_\omega(\Lambda)$.

For n particles the corresponding Hilbert space (which we will universally denote by $\mathfrak{H}^n$) is given by

- $\bigotimes^n \mathfrak{H} = L^2(\mathbb{R}^{nd})$ for Maxwell-Boltzmann particles (classical statistics),
- $\mathrm{Sym}^n \mathfrak{H} = L^2_+(\mathbb{R}^{nd})$, the subspace of permutation-symmetric functions, for bosons (Bose-Einstein statistics),
- $\bigwedge^n \mathfrak{H} = L^2_-(\mathbb{R}^{nd})$, the subspace of permutation-antisymmetric functions, for fermions (Fermi-Dirac statistics).

The n particles Hamiltonian is constructed as follows:

$$H_\omega(n) = H^0_\omega(n) + W_n,$$

where $H^0_\omega(n)$ is the lift-up of H_ω to $\mathfrak{H}^n$:

$$H^0_\omega(n) = \sum_{i=1}^{n} \underbrace{\mathbf{1}_{\mathfrak{H}} \otimes \ldots \otimes \mathbf{1}_{\mathfrak{H}}}_{i-1 \text{ times}} \otimes H_\omega \otimes \underbrace{\mathbf{1}_{\mathfrak{H}} \otimes \ldots \otimes \mathbf{1}_{\mathfrak{H}}}_{n-i \text{ times}},$$

and W_n is the interaction potential. We assume in the current talk that the interactions are pairwise and translation invariant:

$$W_n(x^1, \ldots, x^n) = \sum_{i<j} U(x^i - x^j).$$

Analogous to the one particle case, we denote by $H_\omega(\Lambda, n)$ the restriction of $H_\omega(n)$ to a n-dimensional domain Λ (alternatively, this restriction could be defined directly through the lift of the restricted operator $H_\omega(\Lambda)$). The operator $H_\omega(\Lambda, n)$ describes n quantum particles (with prescribed statistics) enclosed in the domain Λ that interact via pair potential U and are subject to random background potential V_ω.

Thermodynamic limit. It is physically reasonable that a larger domain Λ implies greater number of particles n inside. Moreover, these two quantities should be related by a certain positive constant $\rho > 0$, the density of particles, such that in the infinite volume limit the ratio $n/|\Lambda|$ tends to ρ.

Once more for the sake of simplicity, let us restrict our attention to rectangular domains $\Lambda = \prod_{j=1}^{d}[a_j, b_j]$. In this case one says that a sequence of domains Λ tends to infinity *in the sense of Fisher* ($\Lambda \to \infty$) if all the sides of the rectangular domain go to infinity with comparable speed:

$$\min_j |b_j - a_j| \to +\infty \quad \text{and} \quad \frac{\min_j |b_j - a_j|}{\max_j |b_j - a_j|} > C > 0.$$

Our aim in a very wide sense is the study of the thermodynamic limit for $H_\omega(\Lambda, n)$, i.e. the limit

$$\Lambda \to \infty, \ \frac{n}{|\Lambda|} \to \rho$$

for strictly positive particle densities $\rho > 0$. More precisely, in the current work the results on the existence of the thermodynamic limit for the basic functionals of $H_\omega(\Lambda, n)$ will be discussed.

A characteristic result. We present now the simplest existence theorem. Let H_ω be defined as above and fix Dirichlet boundary conditions:

$$H_\omega(\Lambda, n) = H_\omega^D(\Lambda, n).$$

Under rather mild assumptions on the random potential V_ω and pair interaction potential U, the operator $H_\omega(\Lambda, n)$ is self-adjoint and has a discrete spectrum bounded from below. We denote by $E_\omega(\Lambda, n)$ its ground state energy.

Theorem 2. *Let the following conditions hold true:*

1. *positive background potential:* $V_\omega \geq 0$*,*
2. *interactions are repulsive:* $U \geq 0$*,*
3. *interactions are tempered:* $|U(x)| \leq A|x|^{-\lambda}$ *for some* $\lambda > d$ *and all* $|x| \geq R_0$*.*

Then the ground state energy per particle admits thermodynamic limit:

$$\frac{E_\omega(\Lambda, n)}{n} \to \varepsilon(\rho) \qquad \text{as } \Lambda \to \infty, \ \frac{n}{|\Lambda|} \to \rho.$$

The convergence is in L^2 *with respect to random variable* ω *and the limiting energy density function* $\varepsilon(\rho)$ *is deterministic (does not depend on* ω*). It is also a nondecreasing function of* ρ *and a convex function of* ρ^{-1}*.*

Remark 1. *The limiting function* ε *depends indeed on the quantum statistics (which we omitted in notations) but the existence is a universal fact.*

Multichannel scattering theory for Hankel operators with discontinuous symbols

Dmitri Yafaev

University of Rennes-1

This talk is based on two joint papers with A. Pushnitski

We develop the scattering theory for a pair of self-adjoint operators $A_0 = A_1 \oplus \cdots \oplus A_N$ and $A = A_1 + \cdots + A_N$ under the assumption that all pair products $A_j A_k$ where $j \neq k$ satisfy conditions of "smooth" type. We show that the absolutely continuous parts of the operators A_0 and A are unitarily equivalent. This yields a smooth version of Ismagilov's theorem known earlier in the trace class framework. Our approach relies on a consideration of resolvent equations which are algebraically similar to Faddeev's equations for the three particle quantum system.

As an application of these general results, we develop the spectral and scattering theory for self-adjoint Hankel operators with piecewise continuous symbols. It turns out that every jump of the continuity of the symbol gives rise to a branch of the absolutely continuous spectrum of a Hankel operator. We construct wave operators relating simple "model" (that is, explicitly diagonalizable) operators for each jump and the given Hankel operator. We show that the set of all these wave operators is asymptotically complete. We also prove that under general assumptions the singular continuous spectrum of a Hankel operator is empty and that its eigenvalues may accumulate only to "thresholds" in the absolutely continuous spectrum.