Y-junction connecting Luttinger liquids

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Euler Symposium, 13 Jul 2011

Outline

- impurity in the Luttinger liquid: old results
- current algebra and summation of RG B-function
- Y-junction, symmetric setup
- RG flows and fixed points, also with magn.flux
- fermionic approach vs. Abelian bosonization

Impurity in the Luttinger liquid

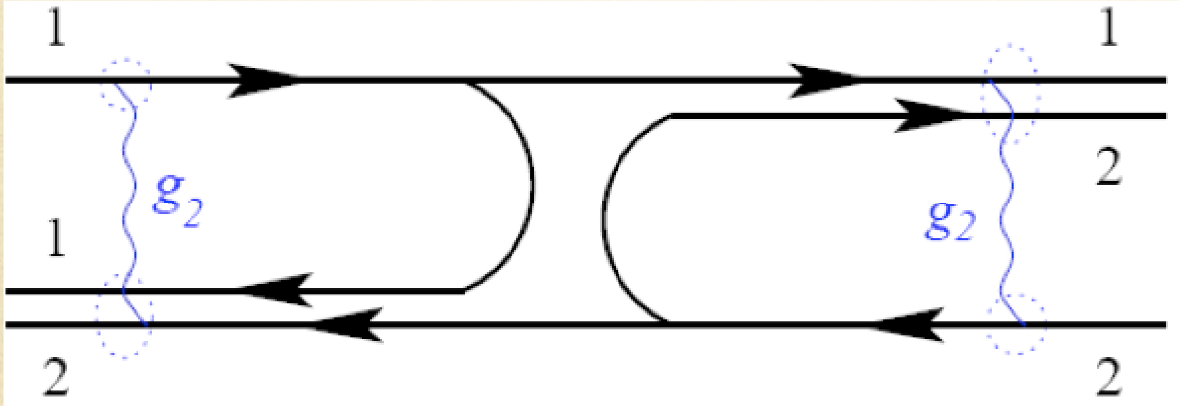
- Kane, Fisher, 1992
- Furusaki, Nagaosa, 1993
- Fendley, Ludwig, Saleur, 1995
- Lukyanov, Werner, 2007
- Yue, Matveev, Glazman, 1995
- Aristov, Wölfle, 2008

Bosonization

Fermions

Scattering states and current algebra

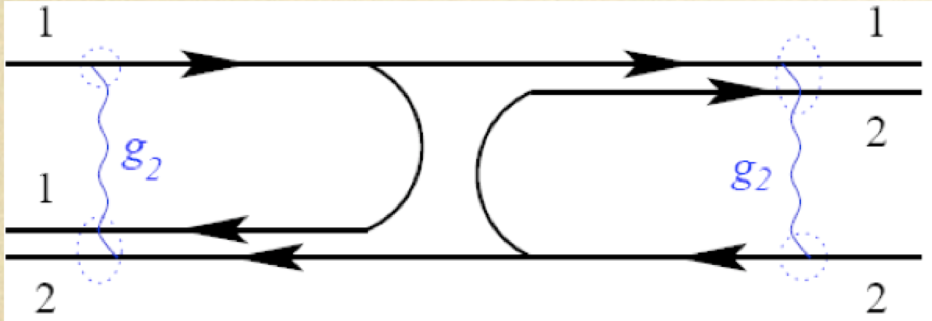
$$\begin{aligned}\psi_k^\dagger(x) &= (e^{ikx}+r_k e^{-ikx})c_{1k}^\dagger + \tilde{t}_k e^{-ikx} c_{2k}^\dagger, \quad x<0 \\ &= t_k e^{ikx} c_{1k}^\dagger + (\tilde{r}_k e^{ikx} + e^{-ikx})c_{2k}^\dagger, \quad x>0\end{aligned}$$



$$\begin{aligned}\rho_{iR}(-x) &= \psi_1^\dagger(-x)\psi_1(-x) = J_0(-x)+J_3(-x) \\ \rho_{oR}(x) &= (t\psi_1^\dagger(x)+\tilde{r}\psi_2^\dagger(-x))(t^*\psi_1(x)+\tilde{r}^*\psi_2(-x)) \\ &= (S.\widehat{J}(x).S^\dagger)_{11} = J_0(x)+\widetilde{J}_3(x)\end{aligned}$$

$$\widehat{J}(x) = \begin{pmatrix}\psi_1^\dagger(x)\psi_1(x) & \psi_1^\dagger(x)\psi_2(-x) \\ \psi_2^\dagger(-x)\psi_1(x) & \psi_2^\dagger(-x)\psi_2(-x)\end{pmatrix} = \begin{pmatrix}J_0+J_3 & J_1-iJ_2 \\ J_1+iJ_2 & J_0-J_3\end{pmatrix}$$

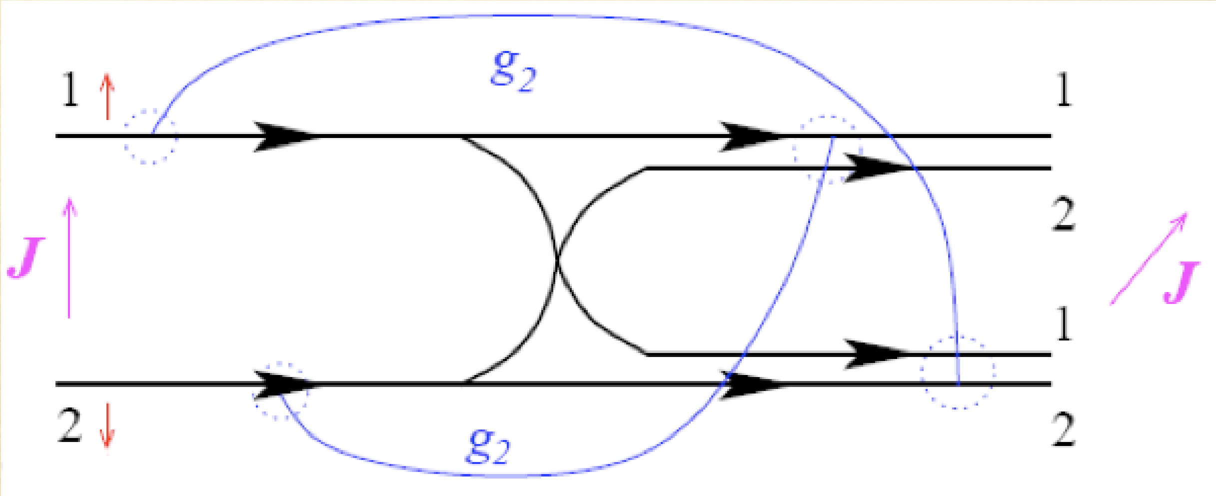
Scattering states and current algebra



$$
\begin{aligned}
H &= 2\pi v_F \int_{-\infty}^{0} dx\,(J_0^2(x) + J_3^2(x)) \\
&\quad + 2\pi v_F \int_0^{\infty} dx\,(J_0^2(x) + \widetilde{J}_3^2(x)) \\
&\quad + 2g_2 \int_a^L dx\,(J_0(-x)J_0(x) - J_3(-x)\widetilde{J}_3(x))
\end{aligned}
$$

$$\widetilde{J}_3 = (R\vec{J})_3, \quad R_{\mu\nu} = \frac{1}{2}Tr[\sigma_\mu.S.\sigma_\nu.S^\dagger]$$

potential barrier is viewed as a local magnetic field rotating the isospin vector of a wave packet, when it passes through the field.



"Nonlocal" interaction: (cf. M.Fabrizio, A.Gogolin, 1995)

Current algebra and renormalization

$$[J_0(x), J_0(y)] = \frac{i}{4\pi}\partial_x \delta(x-y)$$

$$[J_j(x), J_k(y)] = \frac{i}{4\pi}\delta_{jk}\partial_x \delta(x-y) + i\varepsilon_{jkl}J_l(y)\delta(x-y)$$

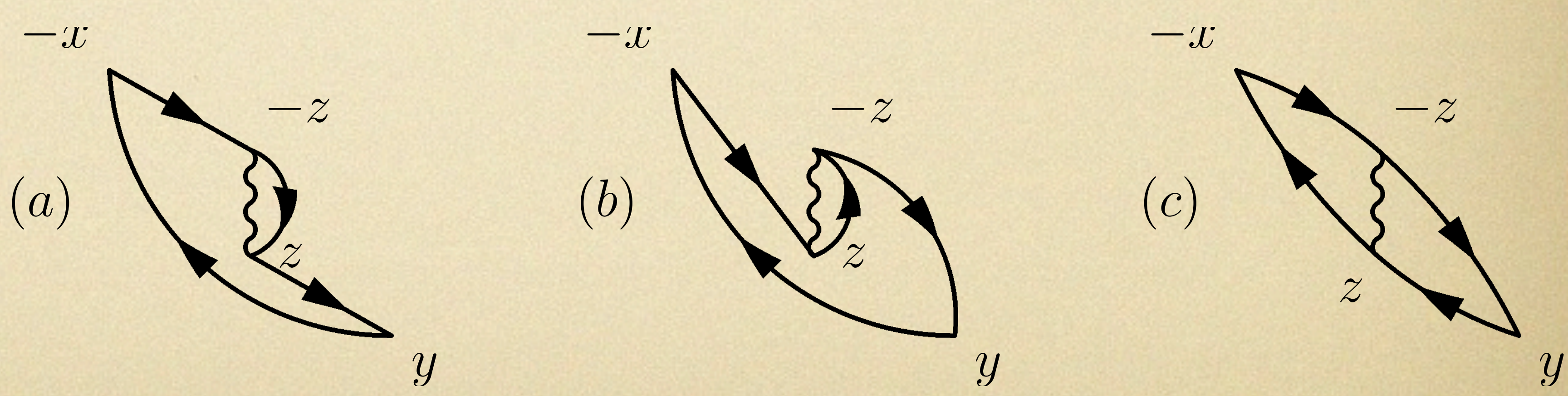
Kac-Moody relations

action:

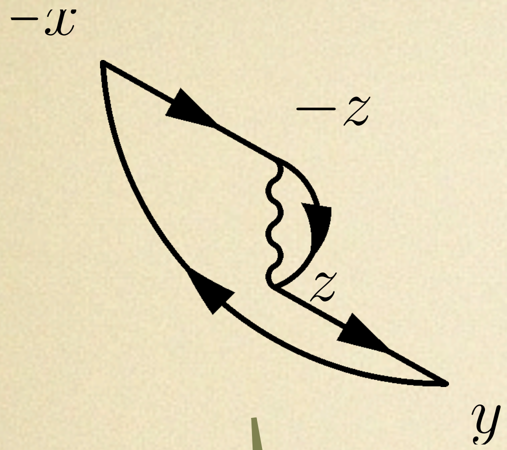
$$-2g_2 \int_a^L dx J_3(-x) R_{3j} J_j(x) \sim \delta B_k J_k(0)$$

$$\delta B_k = \varepsilon_{3jk} R_{3j} \frac{g_2}{\pi v_F} \int_a^L \frac{dx}{2x} = \varepsilon_{3jk} R_{3j} \frac{g_2}{2\pi v_F}\Lambda$$

conductance (transparency)



non-perturbative one-loop RG

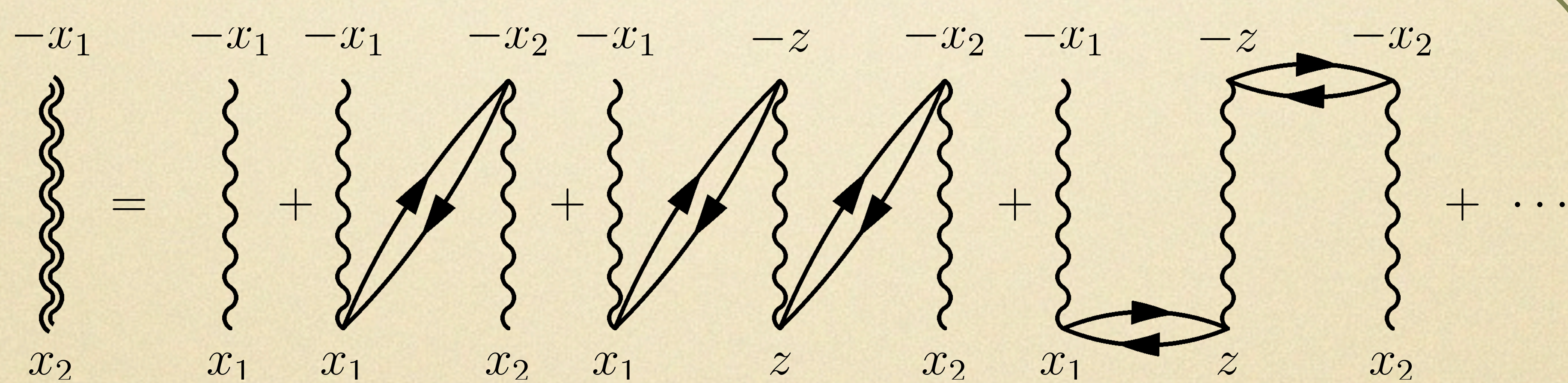


$$\frac{dG}{d\ln(E_F/T)} = \beta_L(G) = -2\,\bar{g}\,G(1-G)$$

$$g \rightarrow \bar{g} = \frac{1-K}{K+(1-K)G}$$

$$K = \sqrt{(1-g)/(1+g)}$$

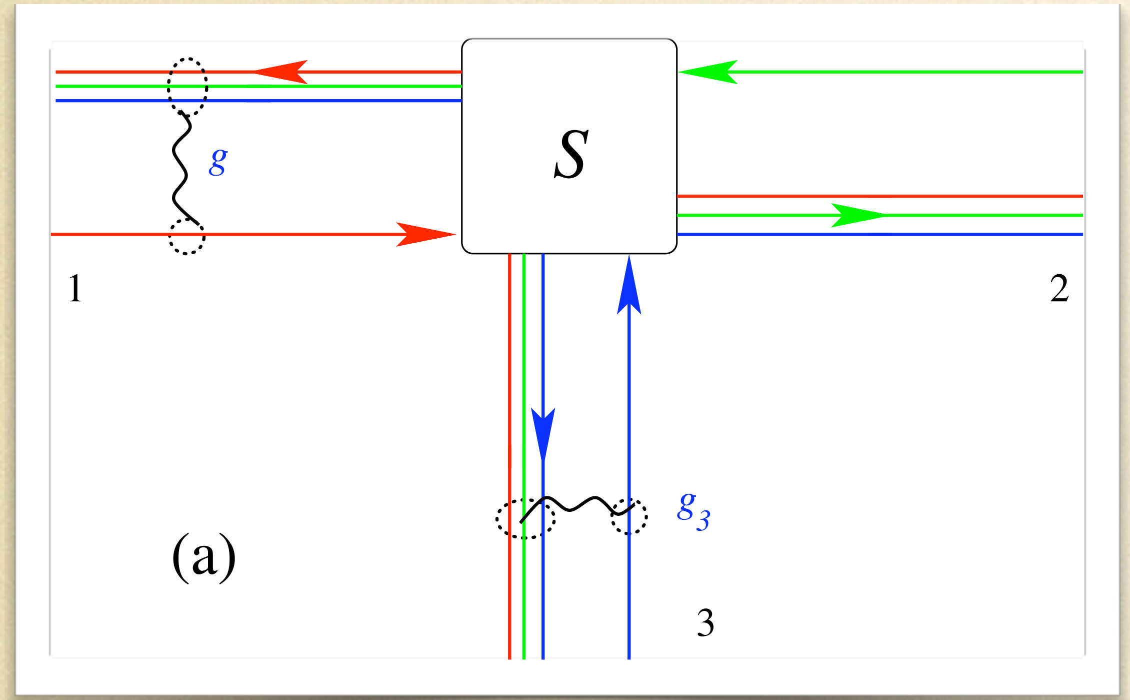
Kane-Fisher, Yue-Matveev-Glazman, Lukyanov-Werner

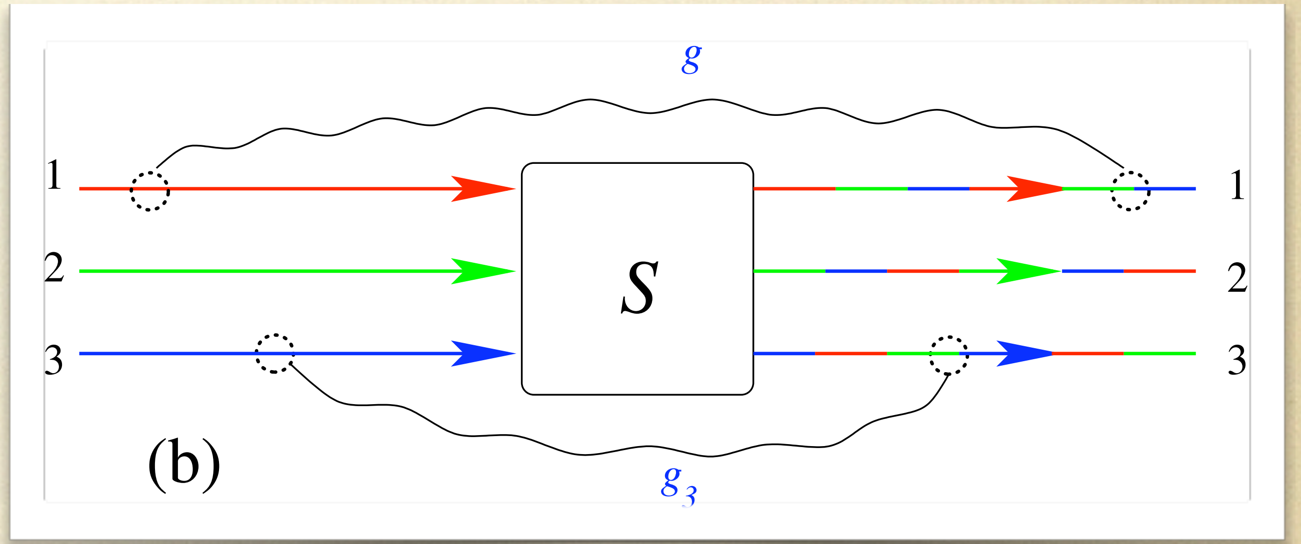


ladder summation, Wiener-Hopf equation

D.A., Wölfle, (PRB 80, 045109 (2009))

Y-junction, symmetric setup





symmetry between 1st and 2nd wire

3rd wire = tunneling tip

$$\mathcal{H} = \int_{-\infty}^{0} dx \sum_{j=1}^{3} \left(v_F \psi^\dagger_{j,in} i \nabla \psi_{j,in} - v_F \psi^\dagger_{j,out} i \nabla \psi_{j,out} \right.$$
$$\left. + 2\pi v_F g_j \psi^\dagger_{j,in} \psi_{j,in} \psi^\dagger_{j,out} \psi_{j,out} \right)$$

$$\psi_{j,in}(x) = \psi_j(x), \quad \psi_{j,out}(x) = S_{jk} \psi_k(-x)$$

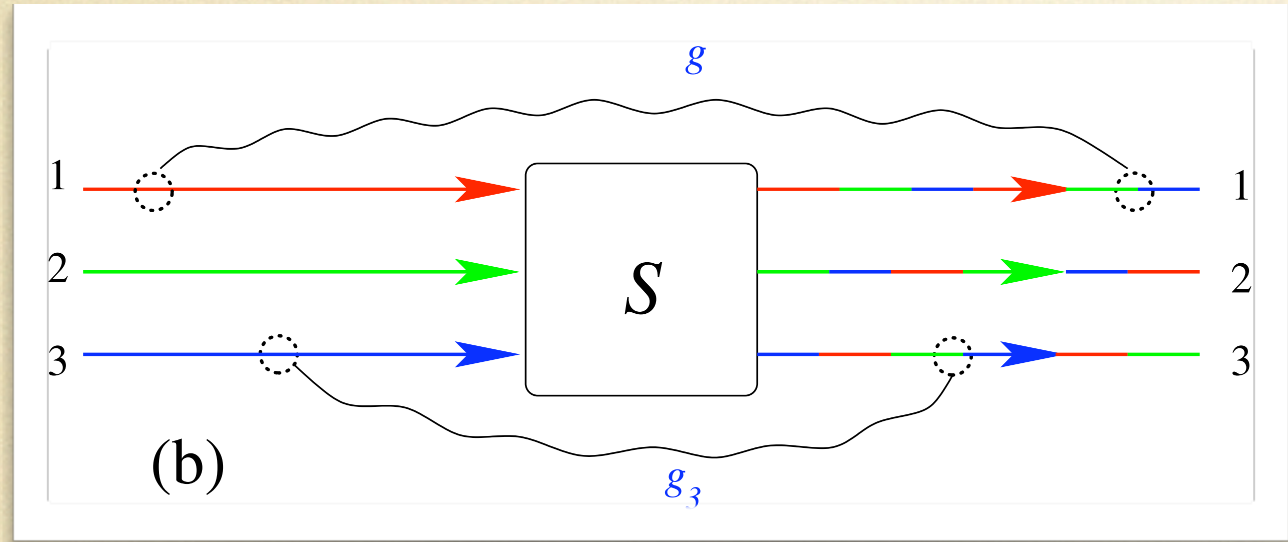
Y-junction, previous studies

- Nayak, Fischer, Ludwig, Lin, *PRB* (1999)
- Oshikawa, Chamon, Affleck, *JStatPhys* (2006)
- Agarwal, Das, Rao, Sen, *PRL* (2009)
- Lal, Rao, Sen, *PRB* (2002)
- Das, Rao, Sen, *PRB* (2004)
- Barnabé-Thériault, Sedeki, Meden, Schönhammer, *PRL, PRB* (2005)

Bosonization

Fermions

Y-junction, symmetric setup



$$\psi_{j,in}(x) = \psi_j(x)$$
$$\psi_{j,out}(x) = S_{jk}\psi_k(-x)$$

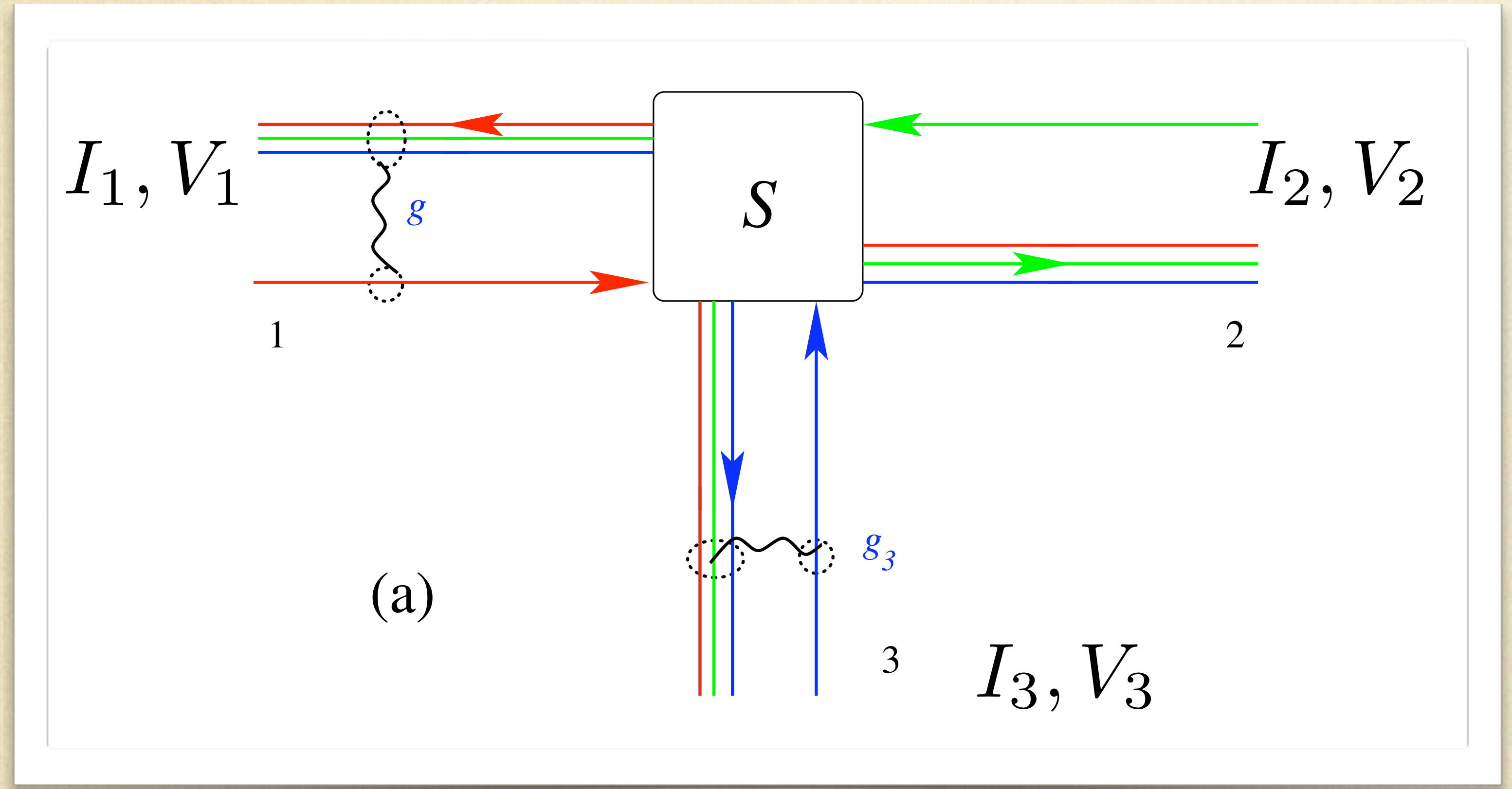
$$S = \begin{pmatrix} r_1 & t_1 & t_2 \\ t_1 & r_1 & t_2 \\ t_2 & t_2 & r_2 \end{pmatrix}$$

impurity: SU(2), Pauli matrices
Y-junction: SU(3) group,
Gell-Mann matrices

$$\begin{aligned} r_1 &= \tfrac{1}{2}(e^{-i\psi} + \cos\theta)e^{i\gamma} \\ t_1 &= \tfrac{1}{2}(-e^{-i\psi} + \cos\theta)e^{i\gamma} \\ t_2 &= \tfrac{i}{\sqrt{2}}\sin\theta \\ r_2 &= \cos\theta e^{-i\gamma} \end{aligned}$$

γ drops out of RG eqs., conductance

S-matrix and conductances



$$I_j = \sum_k G_{jk} V_k$$

$$G_{jk} = \delta_{jk} - |S_{jk}|^2$$

Impurity: $G_{jk} = \begin{pmatrix} 1-|r|^2, & -|t|^2 \\ -|t|^2, & 1-|r|^2 \end{pmatrix} = |t|^2 \begin{pmatrix} 1, & -1 \\ -1, & 1 \end{pmatrix}$

$$\tfrac{1}{2}(I_1 - I_2) = G(V_1 - V_2)$$

Y-junction:

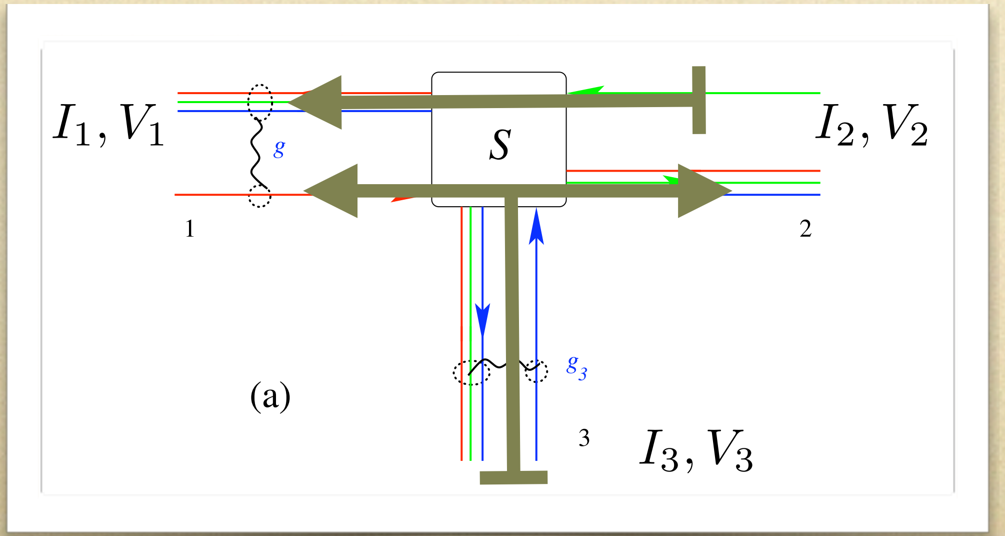
$$\begin{pmatrix} (I_1 - I_2)/2 \\ (I_1 + I_2 - 2I_3)/3 \end{pmatrix} = \begin{pmatrix} G_a, & G_{ab} \\ G_{ba}, & G_b \end{pmatrix} \cdot \begin{pmatrix} V_1 - V_2 \\ (V_1 + V_2 - 2V_3)/2 \end{pmatrix}$$

impurity vs. Y-junction

impurity=2 wires	Y-junction = 3 wires
$(2\text{-}1)^2 = 1$ conductance	$(3\text{-}1)^2 = 4$ conductances
1 RG equation	in general: 4 coupled RG equations

Symmetric setup w/o flux : Gab = Gba =0

$$\begin{aligned}(I_1 - I_2)/2 &= G_a(V_1 - V_2)\\ (I_1 + I_2 - 2I_3)/3 &= G_b(V_1 + V_2 - 2V_3)/2\end{aligned}$$

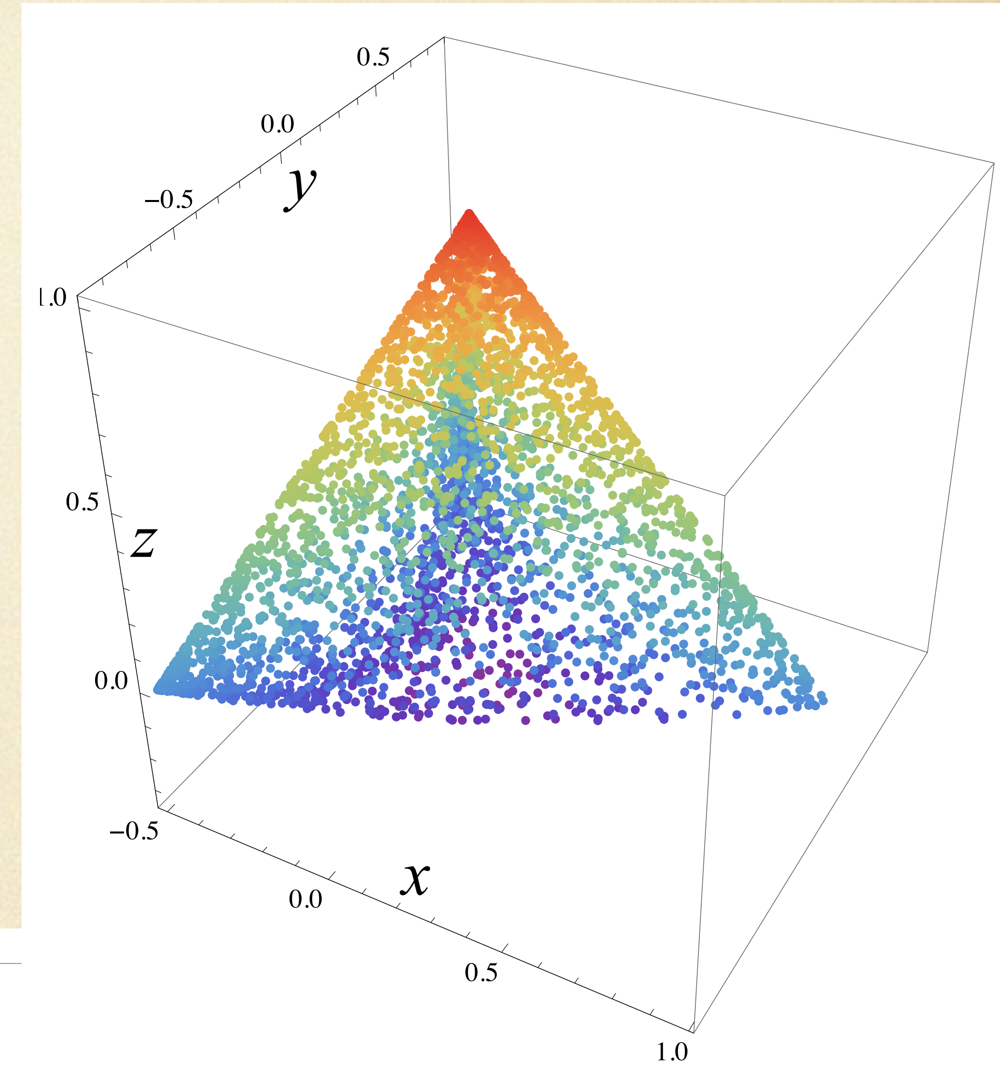


SU(3)- allowed values of conductances

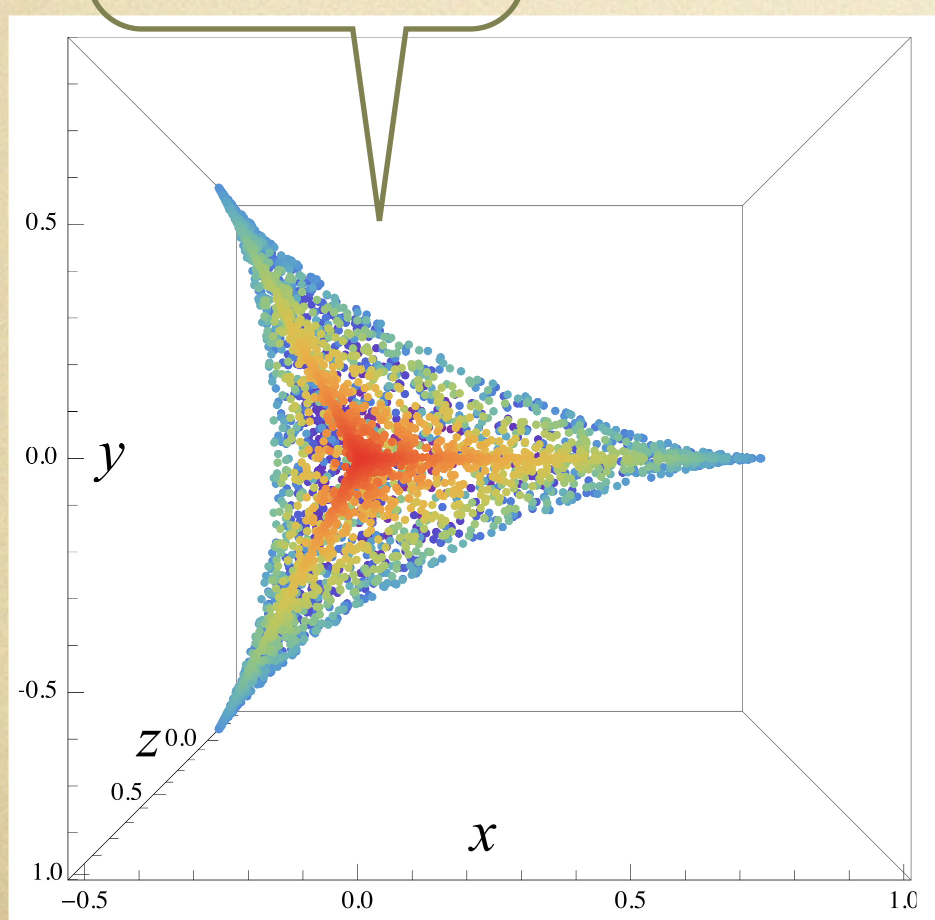
$$\begin{pmatrix} G_{aa} & G_{ab} \\ G_{ba} & G_{bb} \end{pmatrix} \leftrightarrow \begin{pmatrix} z-x, & y+\eta \\ y-\eta, & z+x \end{pmatrix}$$

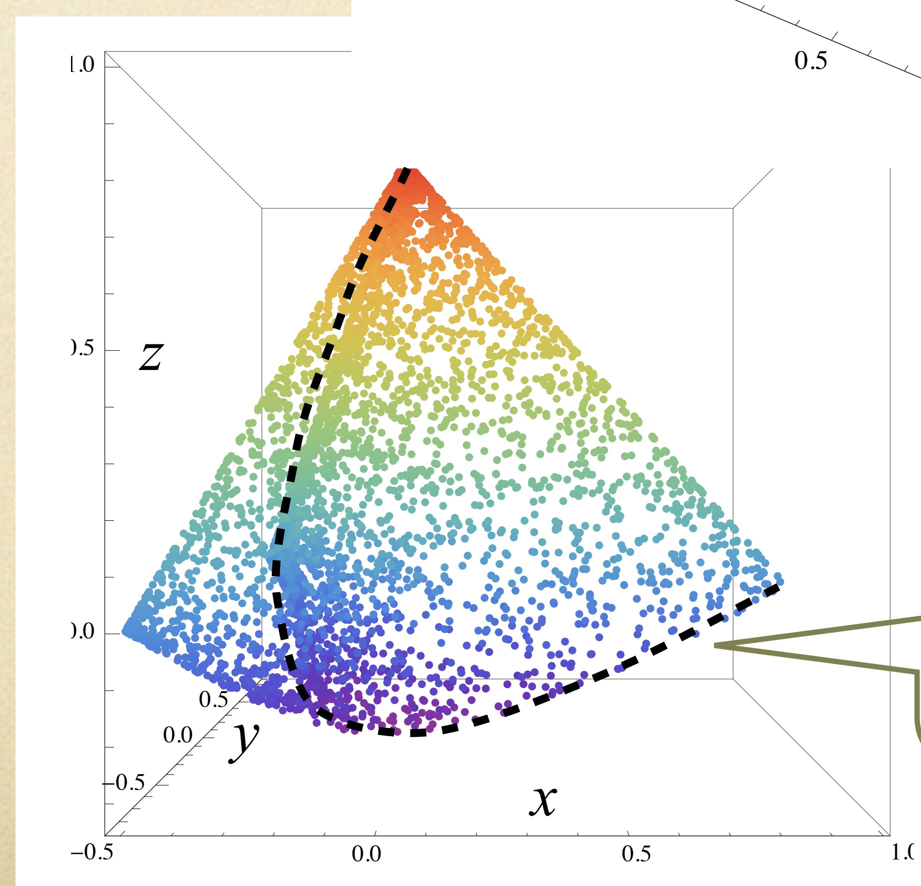
η=0 no magnetic flux

y=0 symmetry 1↔2



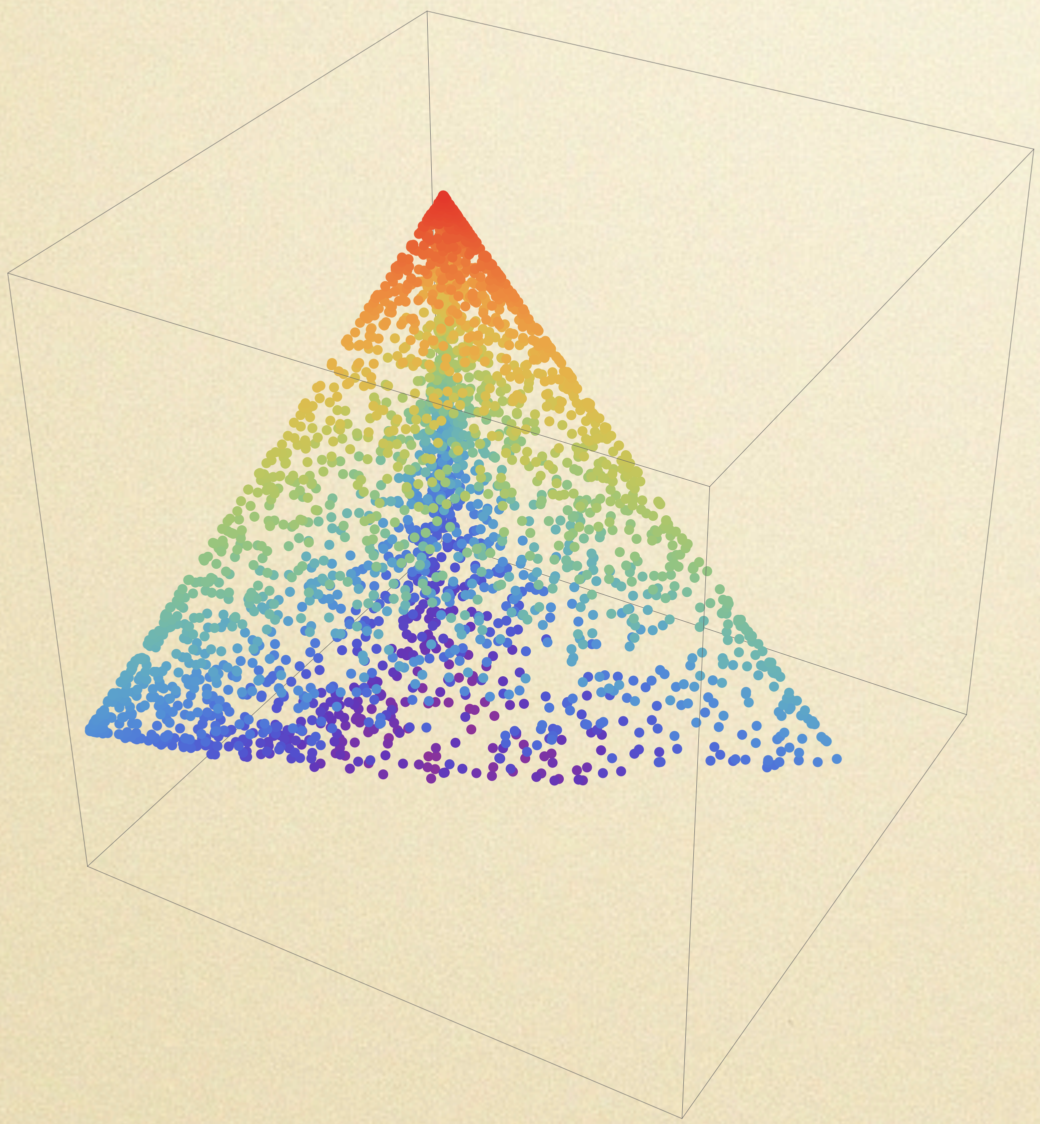
deltoid





parabola

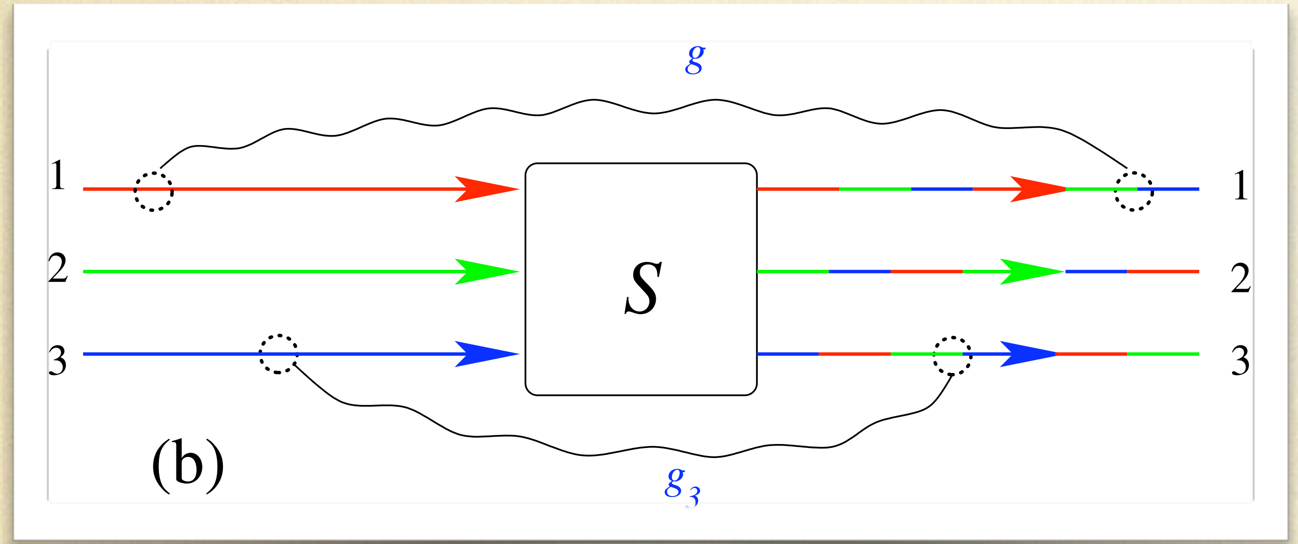
conductances, overall 3D view



3 wires detached
one wire detached
perfect transmission

D.A., PRB 83, 115446 (2011)

Y-junction, symmetry 1-2, no flux



$$\begin{aligned} \psi_{j,in}(x) &= \psi_j(x) \\ \psi_{j,out}(x) &= S_{jk}\psi_k(-x) \end{aligned}$$

$$S = \begin{pmatrix} r_1 & t_1 & t_2 \\ t_1 & r_1 & t_2 \\ t_2 & t_2 & r_2 \end{pmatrix}$$

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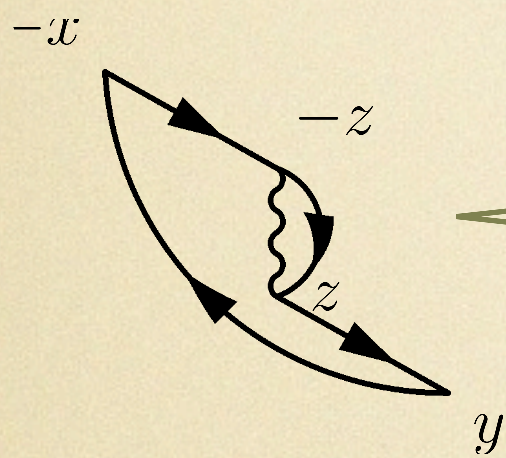
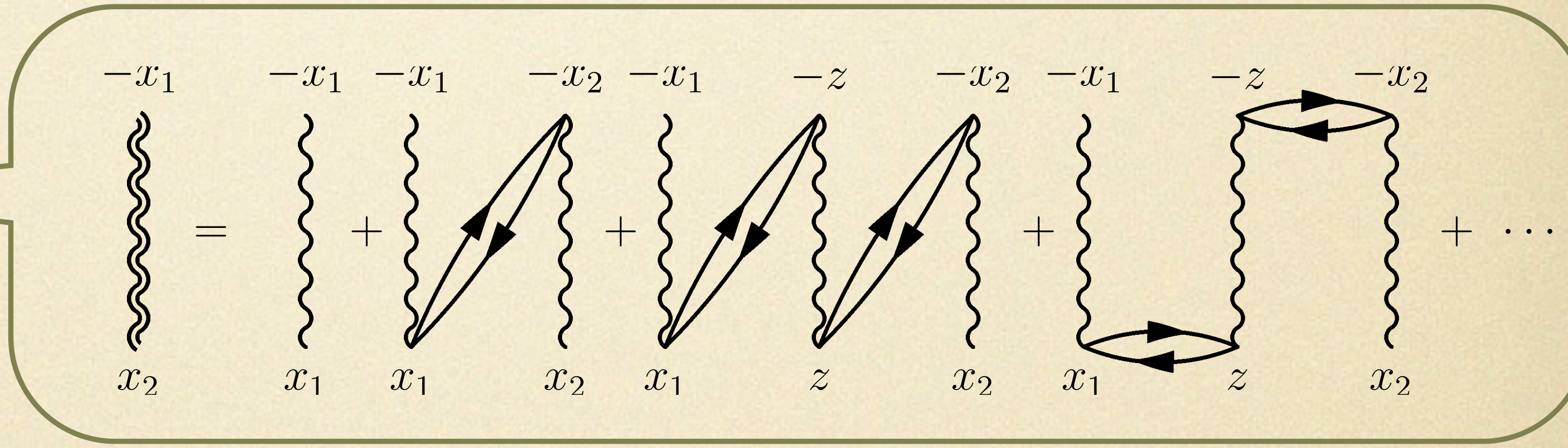
γ drops out of RG eqs., conductance

D.A., Dmitriev, Gornyi, Kachorovskii, Polyakov, Wölfle, (PRL 105, 266404)

renormalization of Y-junction

schematically the same summation of Log corrections, now ladder series for set of two coupled Eqs.





- these are one-loop contributions
- computer symbolic calculations of all (~10^4) diagrams to 3rd order, vertices - 3x3 matrices, dc limit ; from PT to RG eqs.
- absence of two-loop contributions
- three-loop contributions are negligible near fixed points

results in one-loop RG

$$\frac{dG_a}{d\Lambda} = \alpha_3\left(8G_a(1-G_a) - \tfrac{3}{2}G_b\right) + \tfrac{3}{2}\alpha_8 G_b(1-2G_a),$$

$$\frac{dG_b}{d\Lambda} = 2\alpha_3 G_b(1-2G_a) + 6\alpha_8 G_b(1-G_b)$$

$$\alpha_3 = -\frac{1}{2}\frac{1}{q-1+2G_a}, \quad \alpha_8 = -\frac{1}{2}\frac{1}{Q_1-1+\frac{3}{2}G_b},$$

$$Q_1 = \frac{3qq_3 - q - 2q_3}{2q+q_3-3}$$

$$q = \frac{1+K}{1-K}, \quad q_3 = \frac{1+K_3}{1-K_3}$$

$$K = \sqrt{\frac{1-g}{1+g}}, \quad K_3 = \sqrt{\frac{1-g_3}{1+g_3}}$$

RG fixed points and flows

$$\frac{dG_a}{d\Lambda} = \alpha_3\left(8G_a(1-G_a) - \tfrac{3}{2}G_b\right) + \tfrac{3}{2}\alpha_8 G_b(1-2G_a),$$
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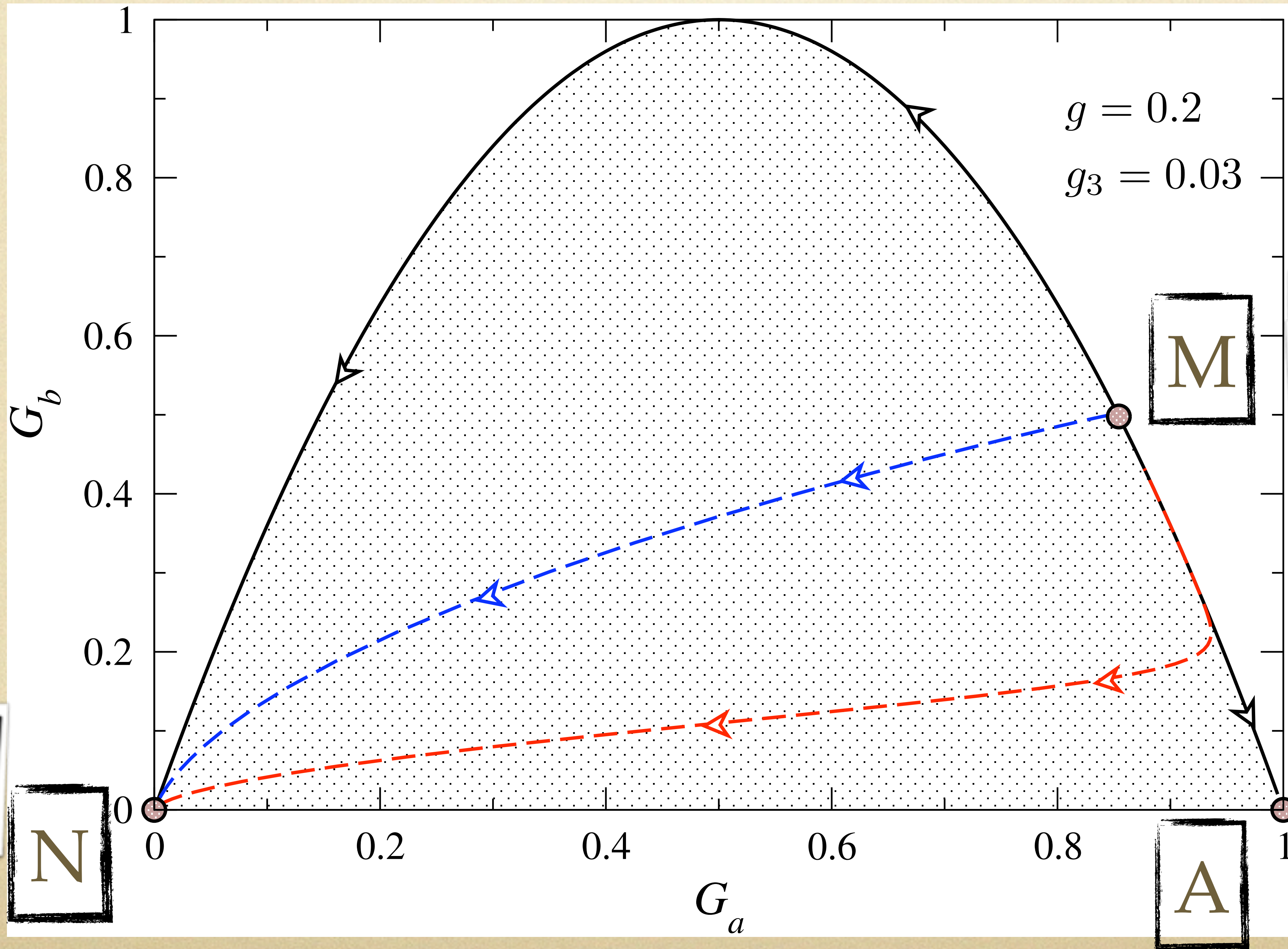


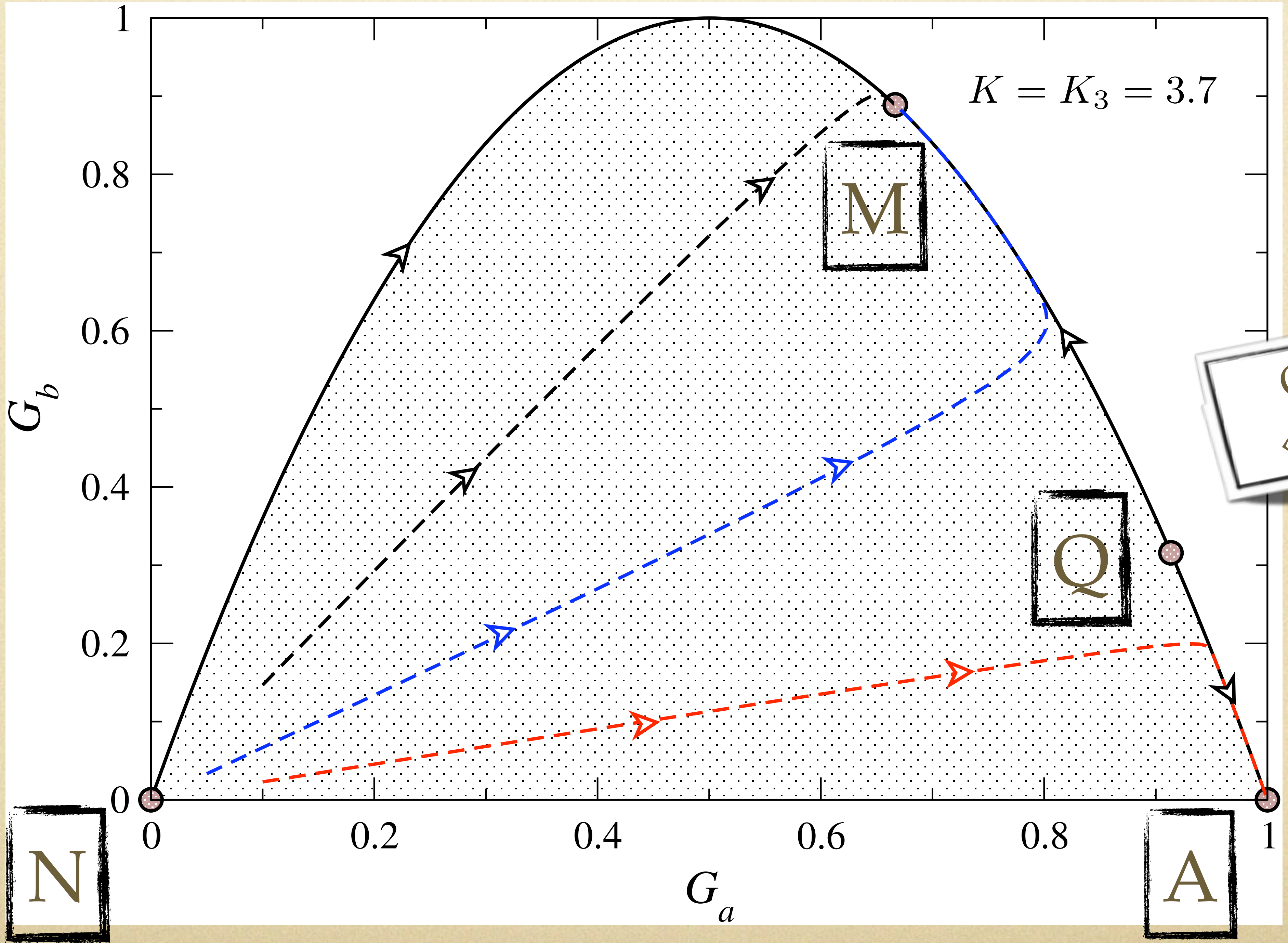
M is for «mystery»

3 wires detached

1 wire detached

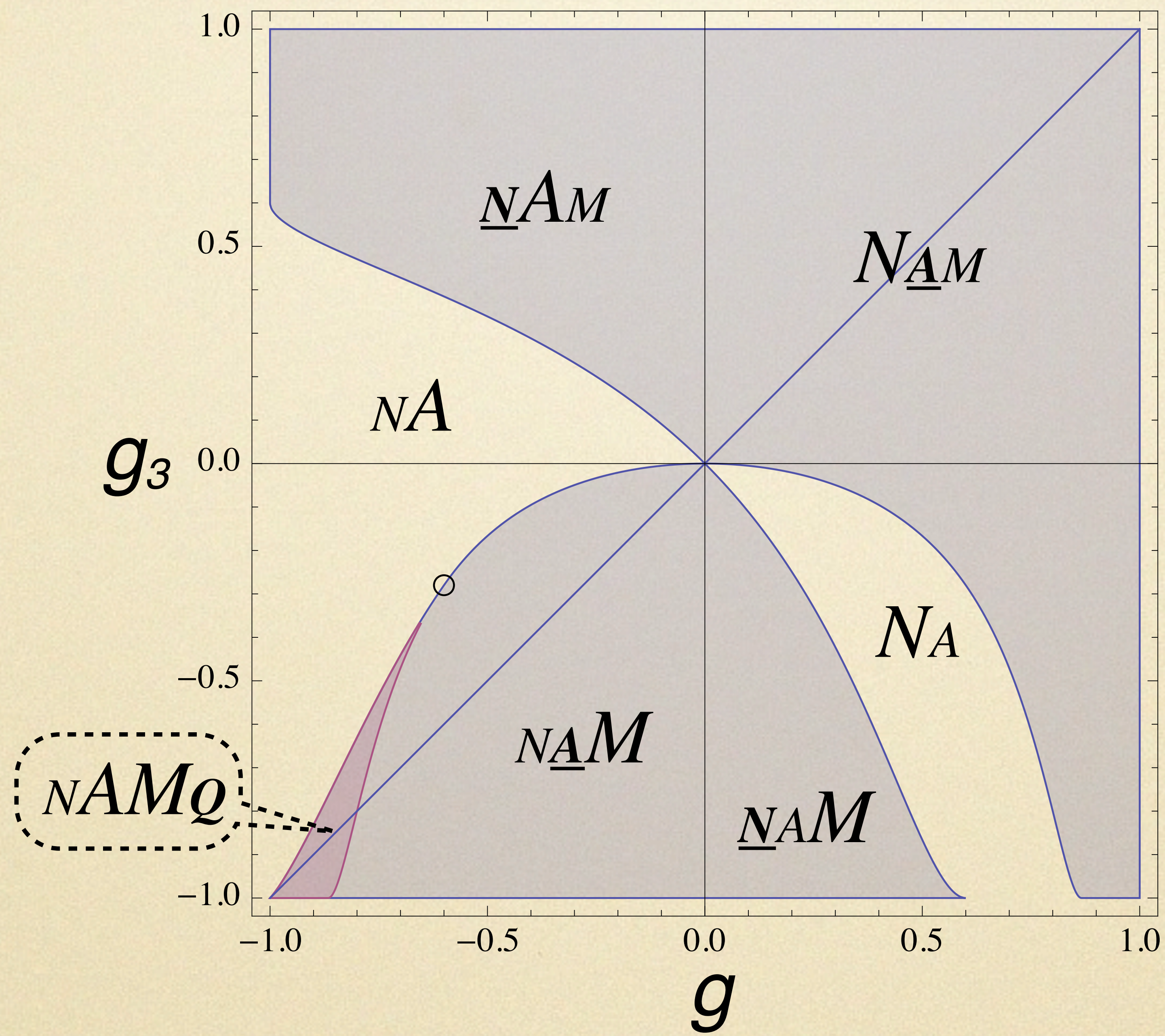
RG fixed points and flows

attraction, $K = K_3 > 3$ new fixed point appears



Q is for «quaint»

number of fixed points, without flux



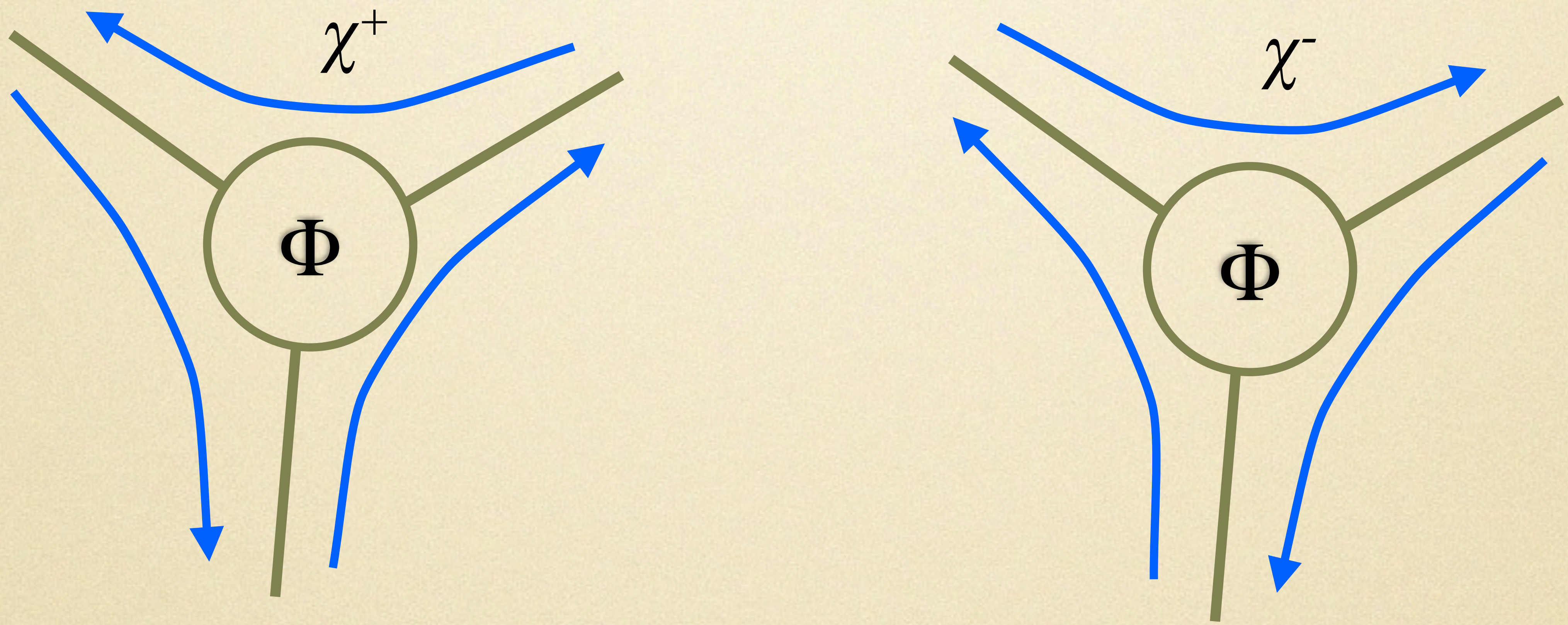
Q first appears at «tricritical» point K=2, K_3=4/3

D.A., Wölfle, arXiv

full symmetry + flux

equal interactions, K= K_3 Oshikawa et al. ('06)

χ^{+-} = chiral FPs in the presence of magnetic flux



$$\begin{pmatrix} G_{aa} & G_{ab} \\ G_{ba} & G_{bb} \end{pmatrix} = 1 - \begin{pmatrix} a, & c \\ -c, & a \end{pmatrix}$$

RG equations, full symmetry + flux

$$\begin{pmatrix} G_{aa} & G_{ab} \\ G_{ba} & G_{bb} \end{pmatrix} \propto \begin{pmatrix} 1-a, & c \\ -c, & 1-a \end{pmatrix}$$

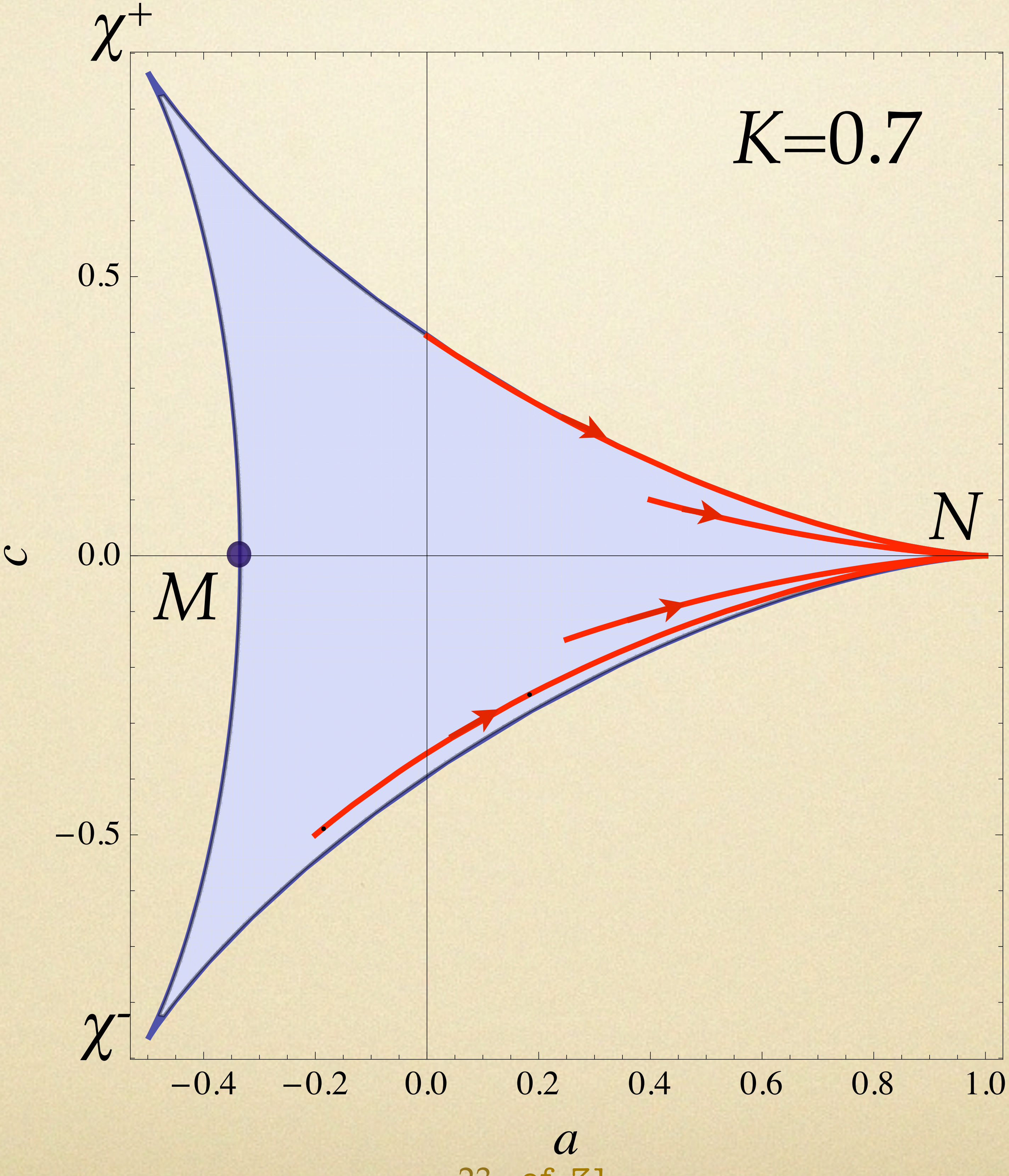
$$\frac{da}{d\Lambda} = \frac{(q-a)\left((1-a)(1+3a)+c^2\right) - 2c^2(1+2a)}{(q-a)^2+c^2}$$

$$\frac{dc}{d\Lambda} = c\frac{2(q-a)(1+2a) - 3c^2 + (1-a)^2}{(q-a)^2+c^2}$$

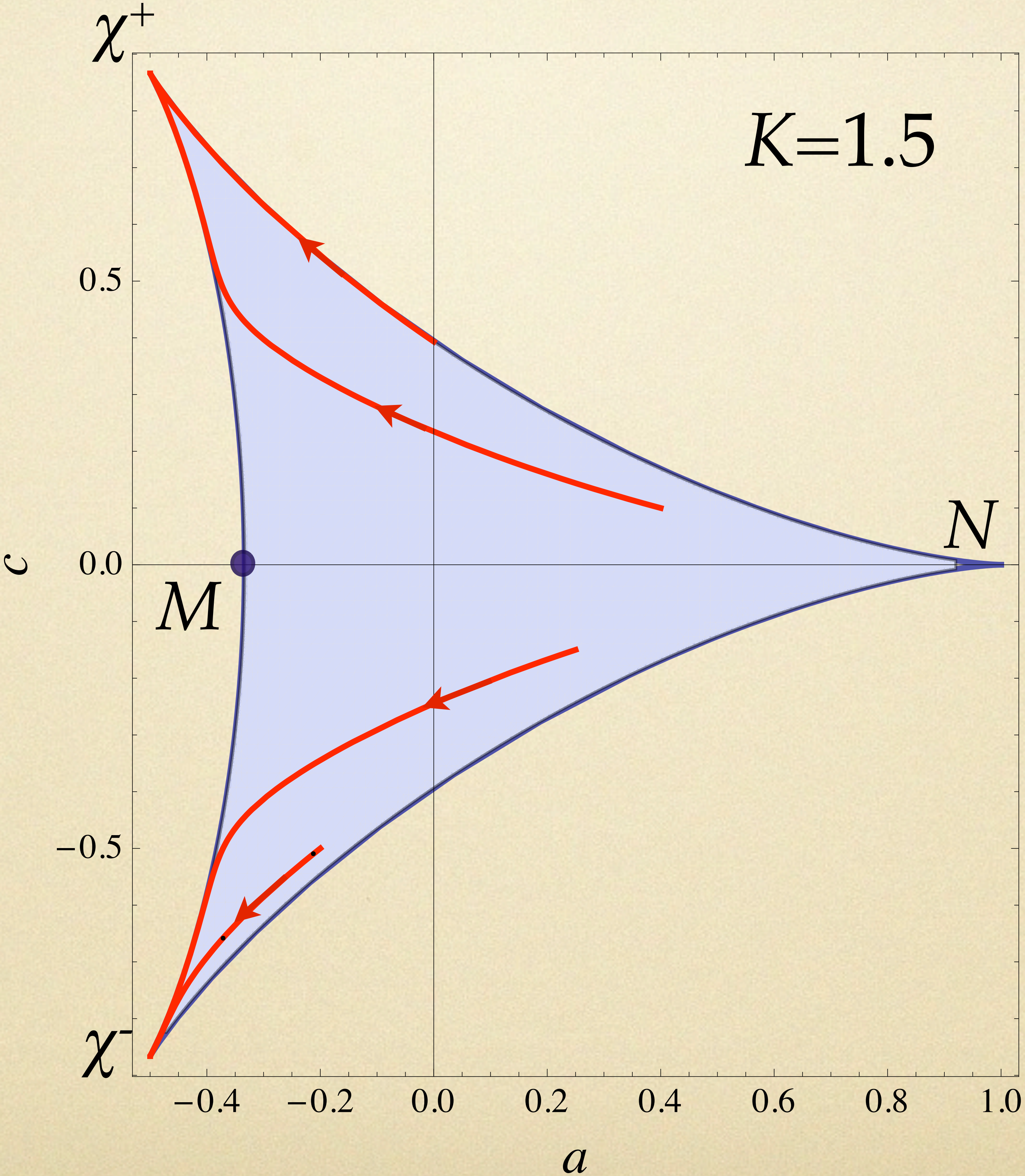
$$q = \frac{1+K}{1-K}$$

D.A., Wölfle, in preparation

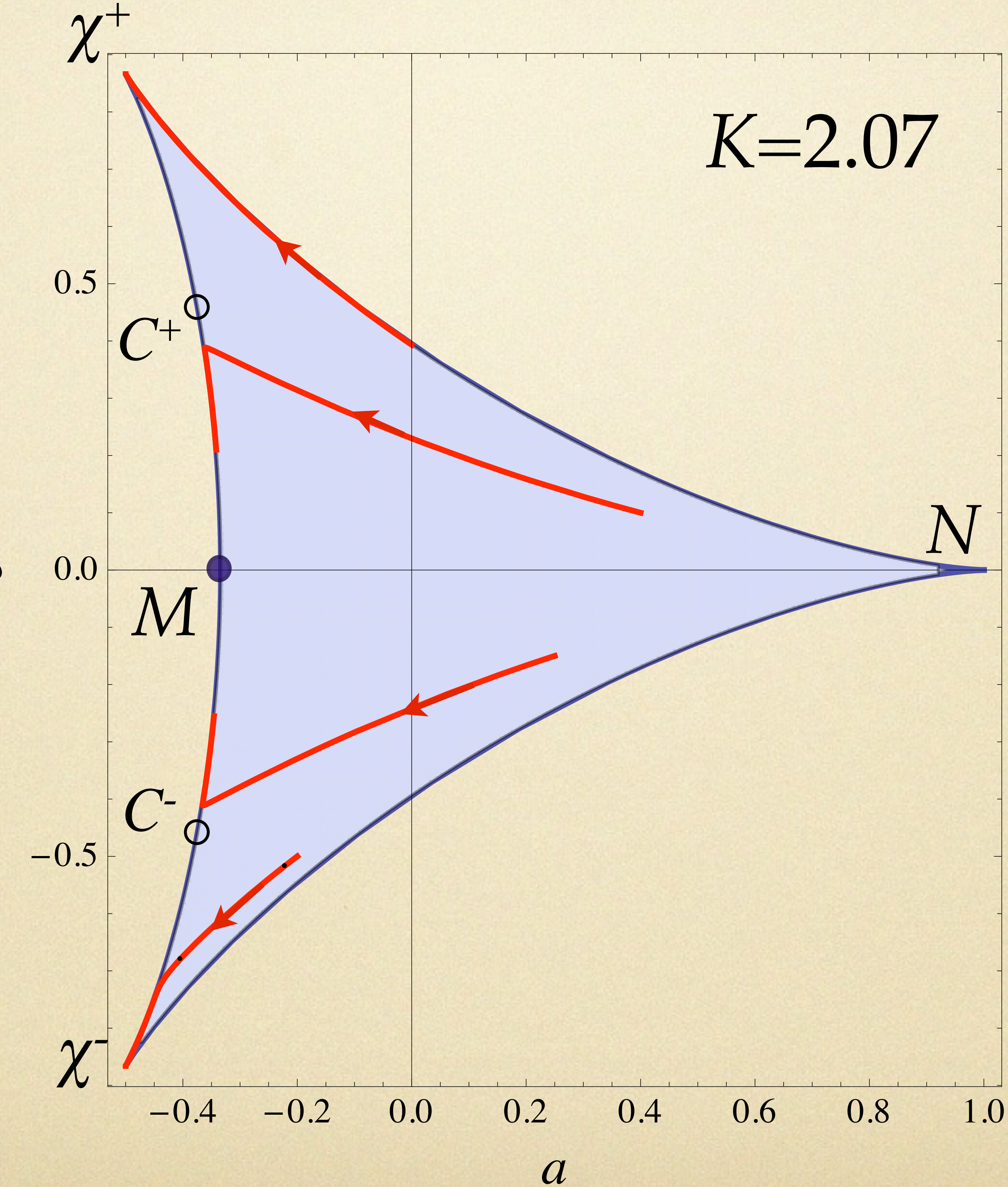
RG flows, full symmetry + flux



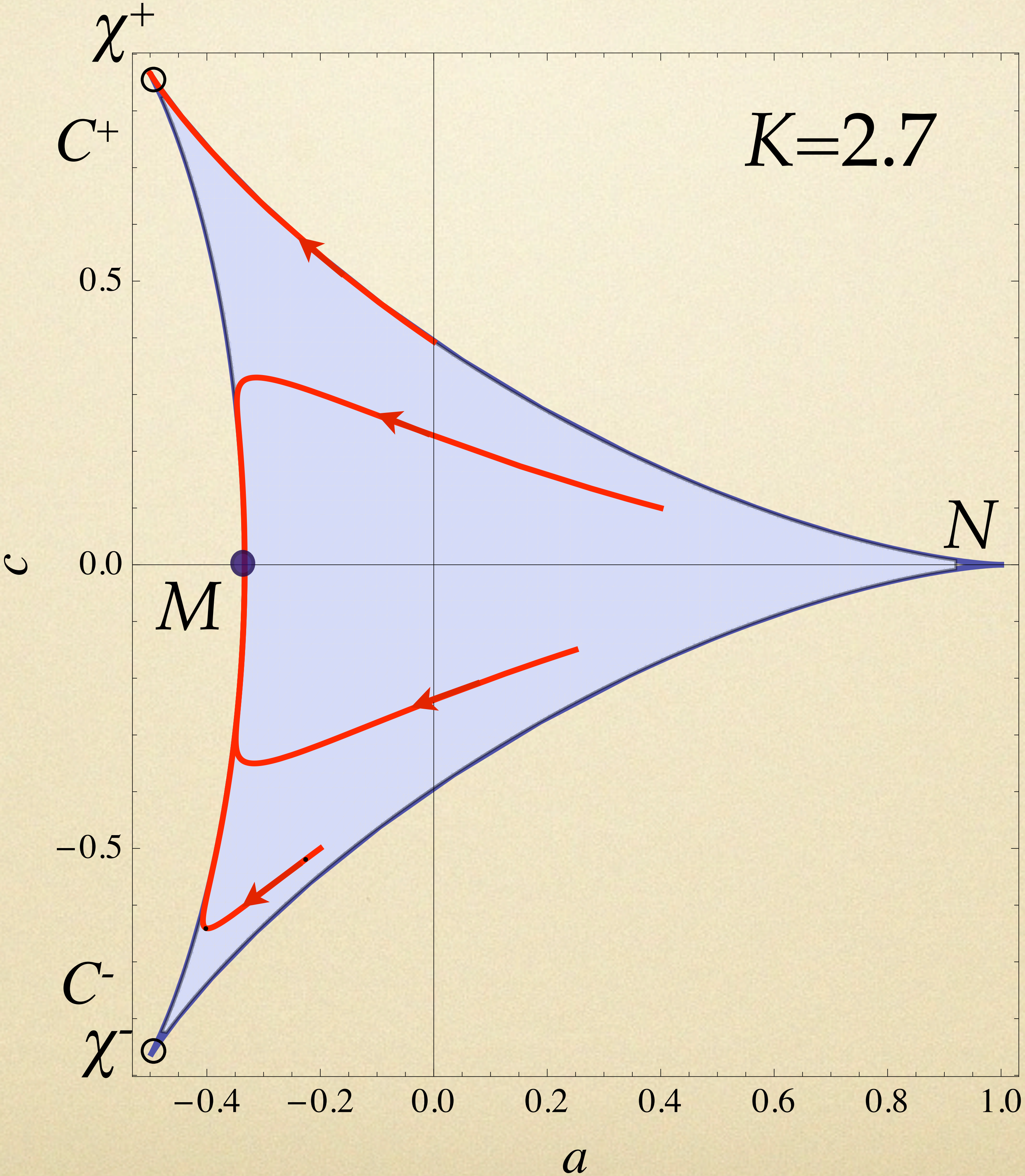
RG flows, full symmetry + flux



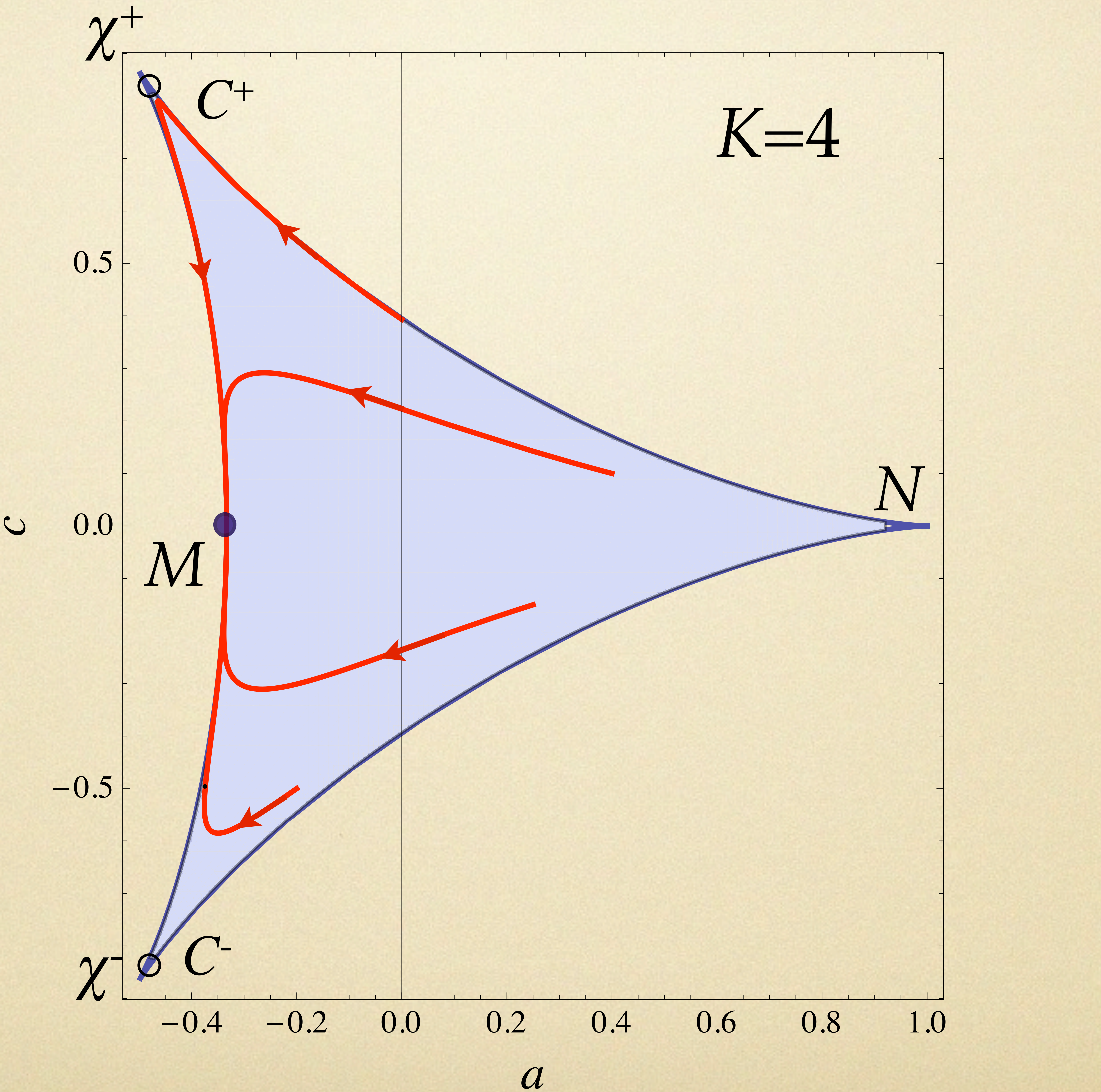
RG flows, full symmetry + flux



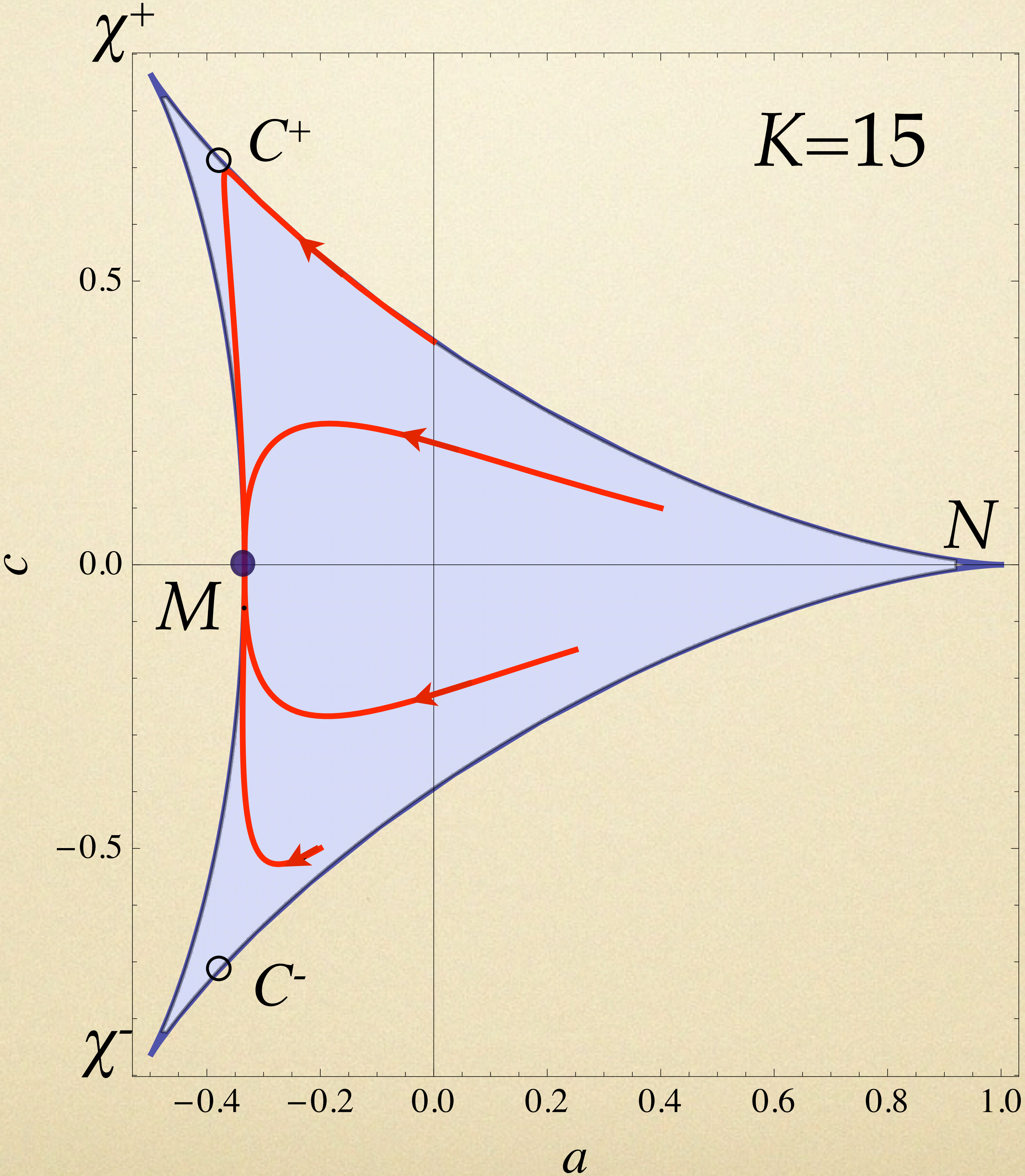
RG flows, full symmetry + flux



RG flows, full symmetry + flux



RG flows, full symmetry + flux



fermions vs. bosonization

fermions => current algebra = non-Abelian bosonization
bosonization = Abelian bosonization

bosonization: (e.g. Oshikawa, Affleck, Chamon, 2006)
boundary condition at Y-junction = incoming and outgoing currents are connected by rotation

$$\begin{aligned} \rho_{j,out}(x=0) &= M_{jk}\,\rho_{k,in}(x=0) \\ M_{jk} &= |S_{jk}|^2 \\ M^\dagger M &= 1 \end{aligned}$$

current-splitting matrix

RG fixed (conformally invariant) point

determine scaling dimensions of leading perturbations around RG FP => directions of RG flows

fermions vs. bosonization

but if RG fixed point does not satisfy «unitarity condition» for currents, then it is missed by A. bosonization

At least for small interaction:
There are only 6 points, available for A. bosonization analysis, corresponding to unitary S-matrix, they are

$$N+3A+\chi^{+}+\chi^{-}$$

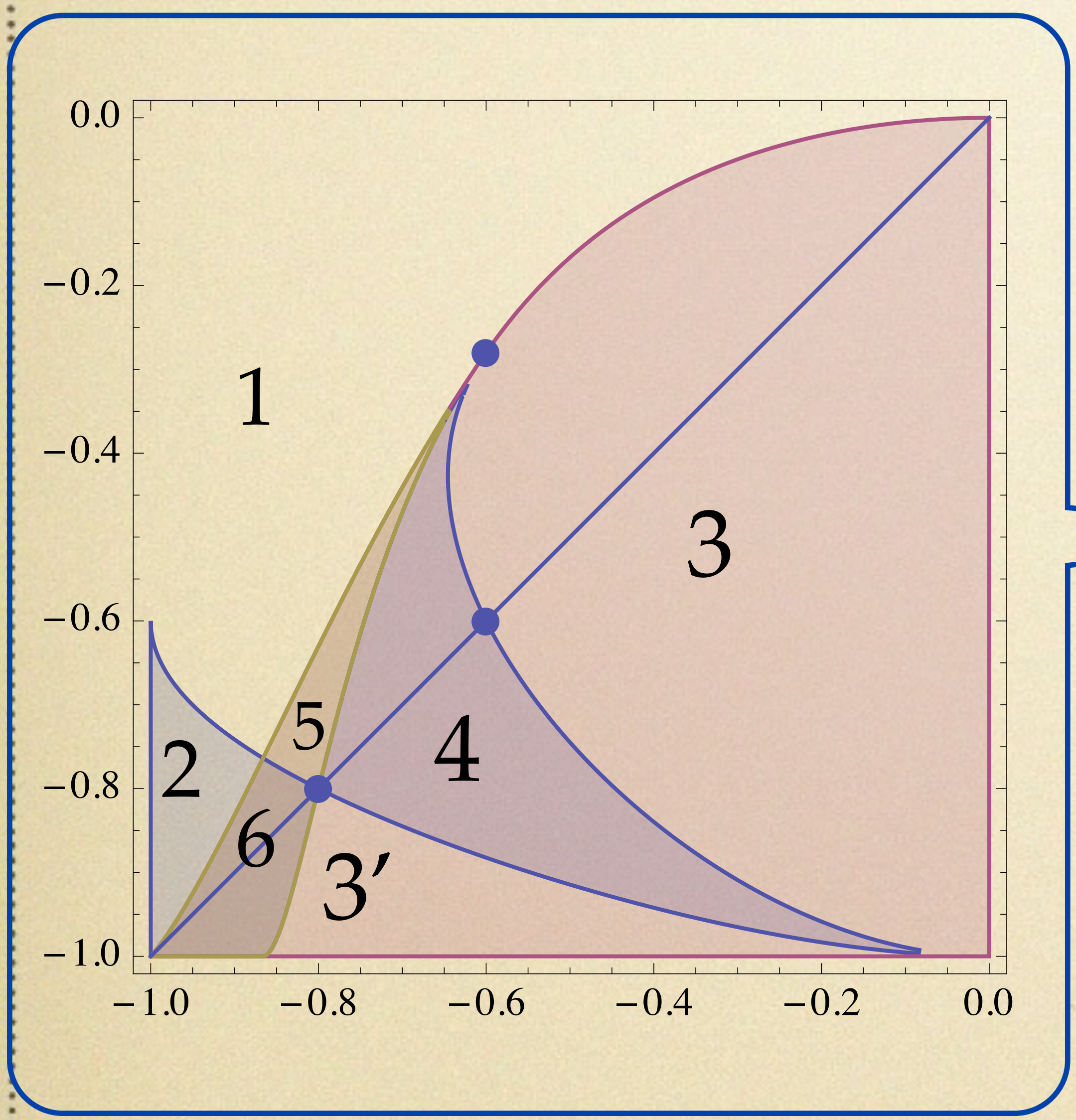
Our results at these points coincide with those in A. bosonization for arbitrary strength of interaction

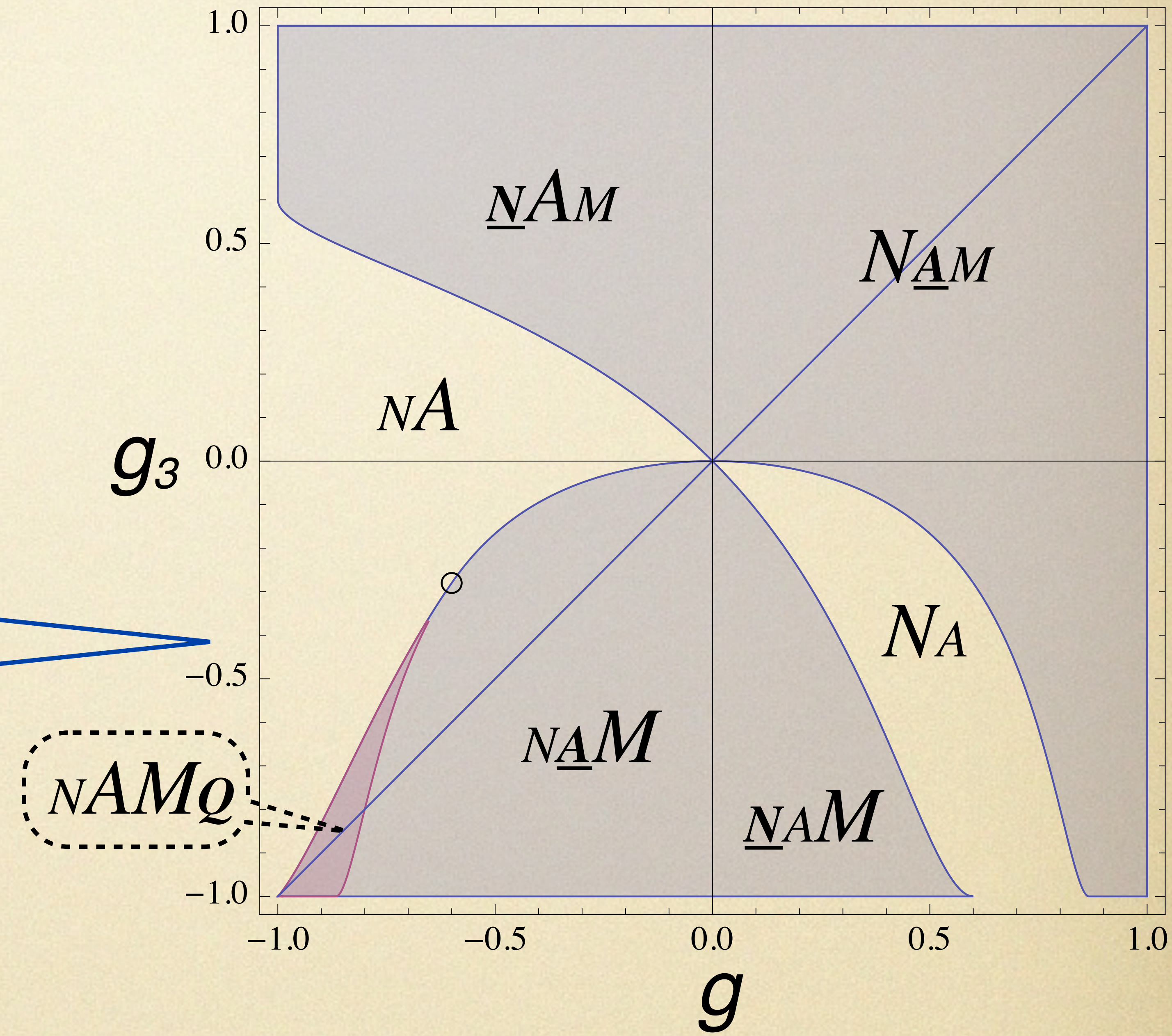
The existence of other FPs is deduced from flow directions (two stable FPs should be separated by unstable one). Hence M point is discussed, but not Q, C^{+-} points («economy principle»).

conclusions

- renormalization of the transparency of Y-junction by e-e interaction is considered
- we study RG equations for fermionic S-matrix
- for symmetric setup all one-loop RG contributions are summed. RG equations can be written in terms of conductances only
- results are checked by direct computer calculation of all fermionic diagrams to third order
- results are compared with previous studies, an advantage over (Abelian) bosonization is shown
- next to be done: non-equilibrium, spinful case, N-wire junctions of higher symmetry

modified phase portrait, with flux





stable FPs:

1=*A*, 2=*AM*, 3=χ
4= χ*M*, 3′=*M*,
5=*AM*, 6=*AM*