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Tunneling Into a Luttinger Liquid Revisited

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Outline

- ***Introduction:***
  experimental motivation, why 1D is special

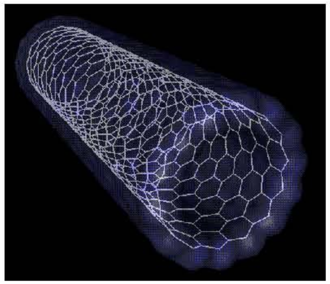
- ***Simplest model of the tunneling junction:***
  point contact, tunneling Hamiltonian

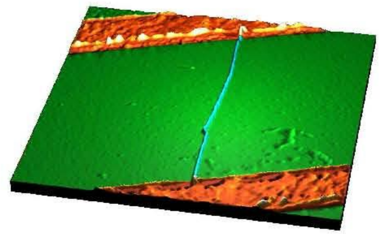
- ***Infinitesimally weak tunneling coupling*:**
  suppression of tunneling density of states
  (conventional zero bias anomaly)

- ***Coupling is not infinitesimally weak → new physics:*** **non-trivial zero bias anomaly, intermediate-coupling fixed point → suppression vs enhancement**

- ***Fermionic S-matrix RG:*** **renormalization of the generic contact**

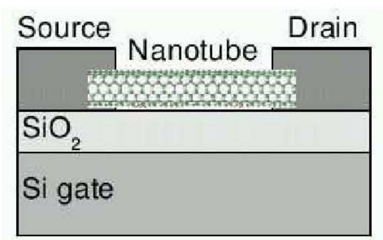
Single-channel quantum wires (aka Nanowires)





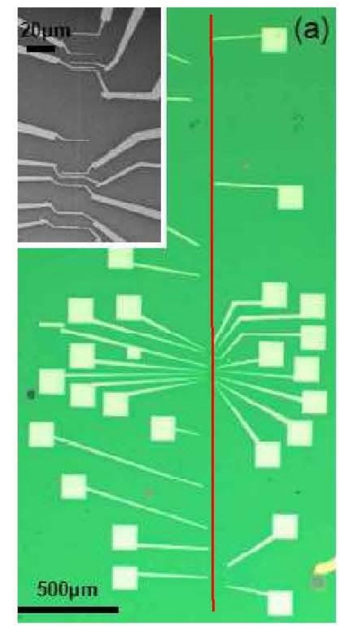
- Carbon nanotubes
- Semiconductor quantum wires
- Quantum Hall edges
- Polymer nanofibers
- Metallic nanowires
- ...

Single-wall carbon nanotube = cylindrical roll of graphene



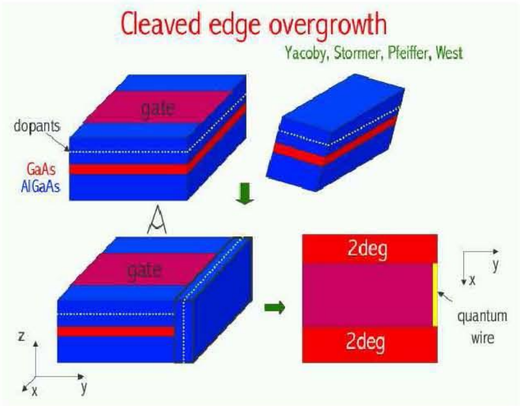
$$R \sim \underline{1\,\mathrm{nm}}\ ,\ \ L \sim 1\,\mu\mathrm{m} - \underline{1\,\mathrm{mm}}$$

$$\text{Metallic nanotubes: Mean free path } l \sim \underline{1\,\mu\mathrm{m}}$$



From Purewal et al., PRL '07

Single-channel quantum wires (*aka* Nanowires)



Atomic-precision "cleaved-edge" single-channel GaAs wires at the intersection of two quantum wells

$R \sim \underline{10\,\mathrm{nm}}$

- Carbon nanotubes
- Semiconductor quantum wires
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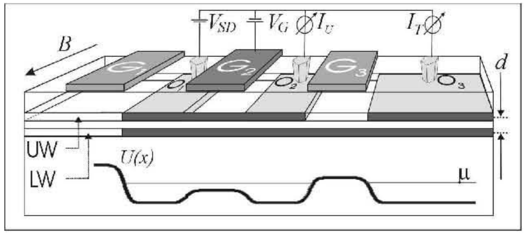
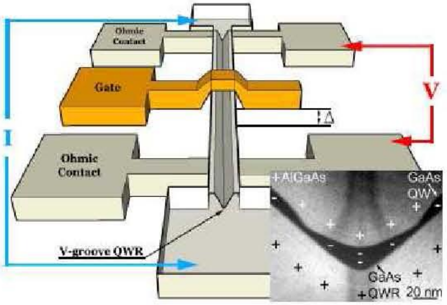
- ...



From Auslaender et al., *Spin-charge separation and localization in one dimension,* Science '05

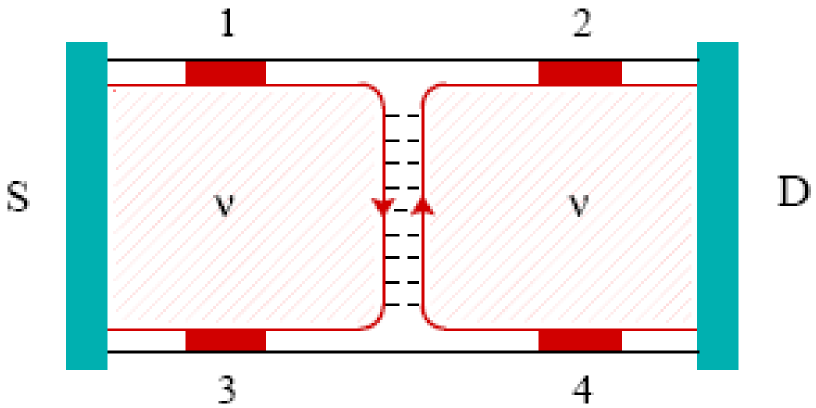
V-groove nanowire



From Levy et al., *Luttinger-liquid behavior in weakly disordered quantum wires,* PRL '06

Semiconductor nanowires :
Mean free path $l \sim \underline{10\,\mu\mathrm{m}}$

Single-channel quantum wires (*aka* Nanowires)



Quantum-Hall line junctions : longest ($L \sim \underline{1\,\mathrm{cm}}$) single-channel GaAs quantum wires

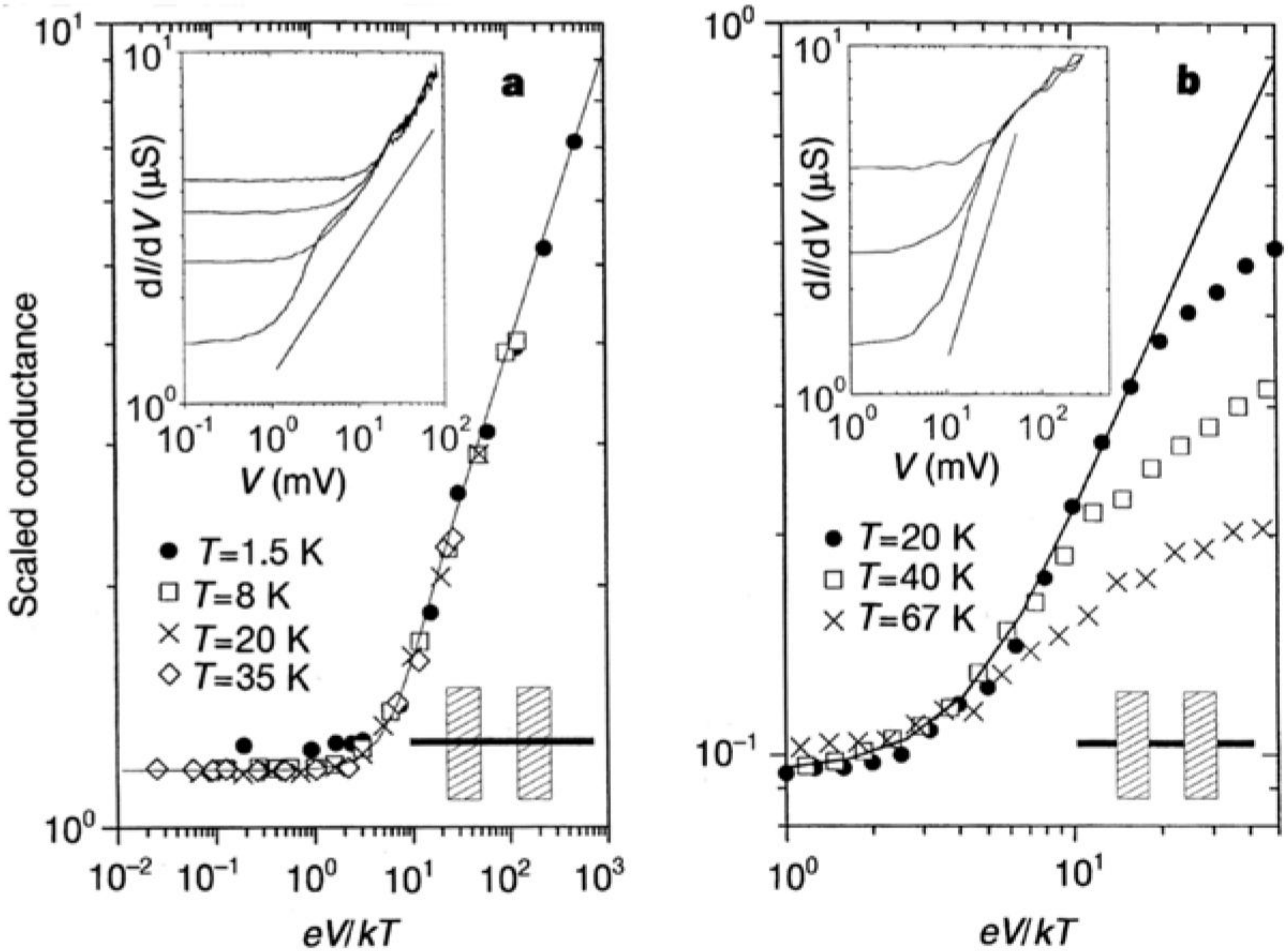
- Carbon nanotubes
- Semiconductor quantum wires
- Quantum Hall edges
- Polymer nanofibers
- Metallic nanowires
- ...

backscattering disorder = random interedge tunneling

1D barrier in 2D : Kang et al., Nature '00; Yang et al., PRL '04
L-shaped quantum wells : Grayson et al., APL '05, PRB '07

Scaling of the tunneling conductance

$$\frac{dI}{dV} = V^{\alpha} f\left(\frac{V}{T}\right) \qquad \alpha_{\text{end}} \simeq 2\alpha_{\text{bulk}}$$



Bockrath *et al* , *Nature*,*1999*

Interacting electrons in 1D ➔ Luttinger liquid

exact excitations: plasmons (+spinons)

Fermionic description:

$$H_F = \sum_\mu \int dx \left(iv\mu\, \psi^\dagger_\mu \partial_x \psi_\mu \;+\; \frac{V_0}{2} n_\mu n_{-\mu} \right)$$

μ = (+,-) right/left movers, $\quad n_\mu = \psi^+_\mu \psi_\mu$

linear dispersion relation, short-range interaction V_0, spinless

dimensionless interaction strength $\quad g = \dfrac{V_0}{2\pi v} \ll 1$

Luttinger parameter $\quad K = \left(\dfrac{1-g}{1+g} \right)^{1/2}$

Bosonic description:

$$H_B = \frac{u}{2} \int dx \left[\frac{1}{K} (\partial_x \phi)^2 + K (\partial_x \theta)^2 \right]$$

$$\partial_x \theta = \pi \Pi, \quad [\phi(x), \Pi(x')] = i\delta(x - x')$$

Electron operators: $\quad \psi_\pm \to e^{i(\theta \pm \phi)}$

$u = v_F / K$ -plasmon velocity

Zero bias anomaly

Dzyaloshinskii& Larkin (1973)

Luther & Peschel; Luther & Emery (1974)

Kane & Fisher (1992)

"bulk" tunneling DOS

$$\rho(\epsilon) \propto |\epsilon|^{\alpha_b}$$

$$\alpha_b = \frac{(1-K)^2}{2K}$$

Scaling of the tunneling conductance

$$G_t \propto \begin{cases} |t_0|^2 \left(\frac{eV}{\Lambda}\right)^{\alpha_b}, & \text{for } eV \gg T \\ |t_0|^2 \left(\frac{T}{\Lambda}\right)^{\alpha_b}, & \text{for } eV \ll T. \end{cases}$$

usual assumption: tunneling matrix element t_0 is infinitesimally small **!!!**

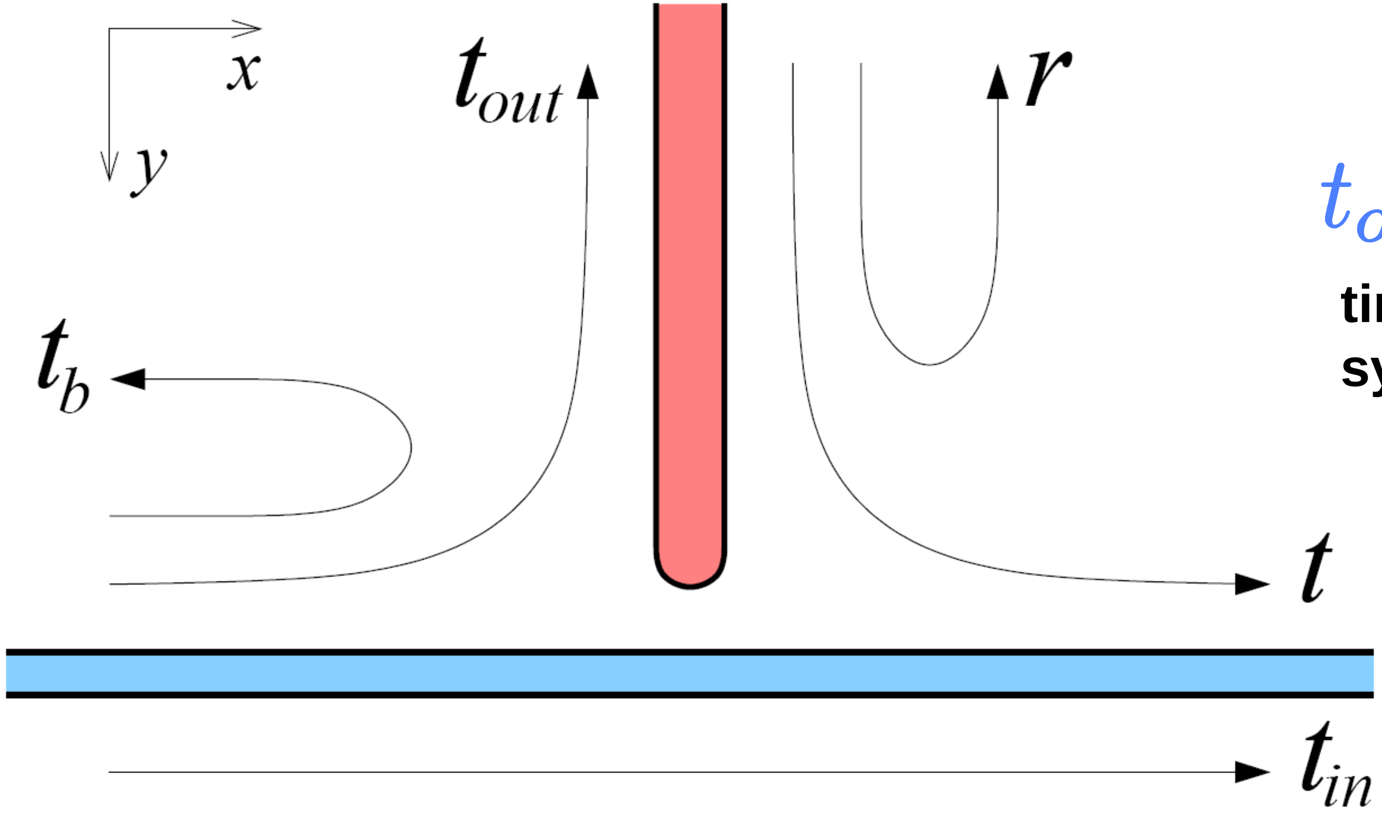
tunneling into the end of a semi-infinite wire

$$\alpha_b \to \alpha_e \qquad \alpha_e = \frac{1-K}{K}$$

$$g \ll 1: \qquad \alpha_b \approx g^2/2 \quad \ll \quad \alpha_e \approx g$$

Setup

tunnel electrode: **noninteracting, single channel**



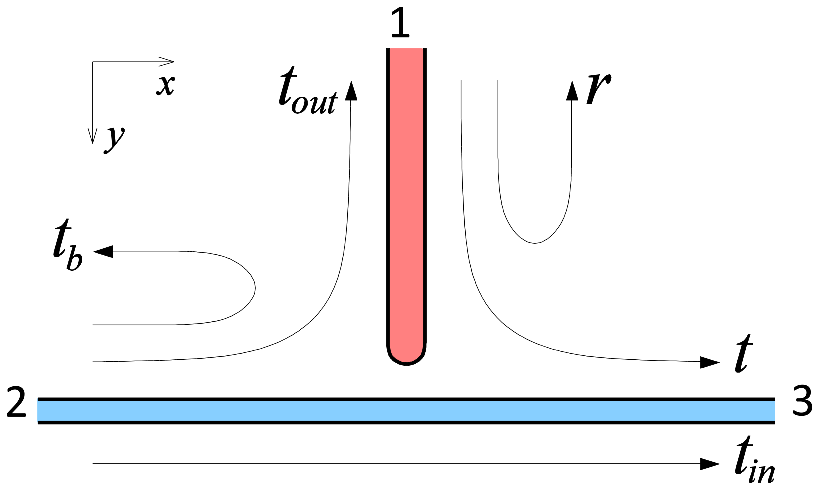
$t_{out} = t$

time reversal symmetry

quantum wire:

single channel, spinless, ballistic, interaction is described by LL model

Contact: 3x3 scattering matrix



$$\hat{S} = \begin{pmatrix} r & t_{\rm out} & t_{\rm out} \\ t & t_{\rm b} & t_{\rm in} \\ t & t_{\rm in} & t_{\rm b} \end{pmatrix}$$

for finite tunneling $t_b \neq 0$!!!

Symmetry: 1) time-reversal
2) 2 $\leftrightarrow$ 3

Most general parametrization

$$t_b = -e^{i\beta}\gamma, \quad t = t_{out} = e^{i(\chi-\beta-\varphi)/2}\sqrt{2\gamma\left(\cos\varphi-\gamma\right)},$$

$$t_{in} = e^{i\beta}\left(e^{i\varphi}-\gamma\right), \quad r = e^{i(\chi-2\beta-\varphi)}\left(2\gamma-e^{-i\varphi}\right),$$

$$-\pi/2 < \varphi < \pi/2, \quad 0 < \gamma < \cos\varphi, \quad -2\pi < \beta, \chi < 2\pi.$$

Hamiltonian

$$H = H_{\mathrm{w}} + H_{\mathrm{e}} + H_{\mathrm{tun}},$$

$$H_{\mathrm{w}} = \sum_\mu \int dx \left(-i\mu v \psi_\mu^\dagger \partial_x \psi_\mu + \frac{1}{2} V_0 n_\mu n_{-\mu}\right)$$

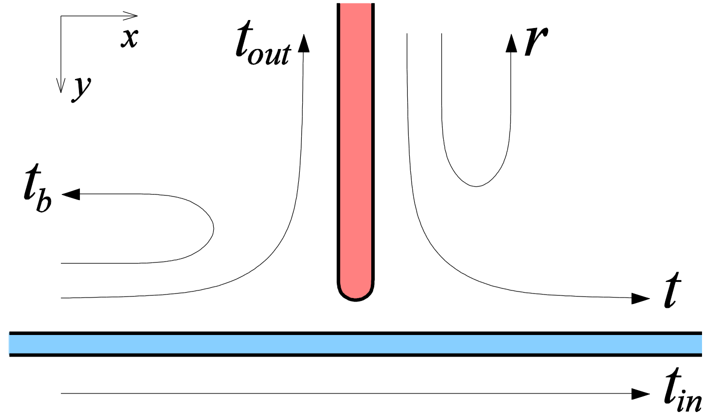
$$H_{\mathrm{e}} = -i v_{\mathrm{e}} \sum_\mu \mu \int_{-\infty}^{0} dy\, \psi_\mu^\dagger \partial_y \psi_\mu$$

Simplest model of the tunneling contact

$$H_{\mathrm{tun}} = t_0 \psi^\dagger(y=-0)\psi(x=0) + h.c.$$

point-like contact described by the tunneling Hamiltonian

Tunneling Hamiltonian: noninteracting case



Specific choice of the S-matrix

$$t = t_{out} = -i\sqrt{2\gamma(1-\gamma)}$$

$$r = 1 - 2\gamma$$

$$t_b = -\gamma, \quad t_{in} = 1 - \gamma$$

$$\gamma = \frac{2t_0^2}{vv_e + 2t_0^2}$$

single parameter

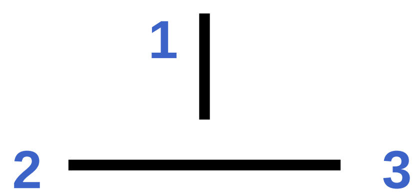
tunnel conductance

$$G_t = 2|t|^2 = 4\gamma(1-\gamma)$$

tunneling transparency of the contact

$$t_0 \to 0 \qquad \gamma \approx 2t_0^2 \, \frac{1}{v} \, \frac{1}{v_e} \to 0, \quad G_t \to 0$$

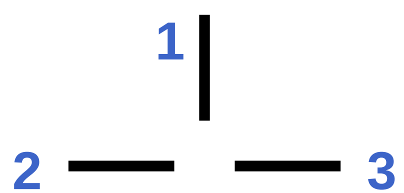
$$t \sim t_0, \; t_b \sim t_0^2 \;\Rightarrow\; |t_b| \ll |t|$$



tunnel contact decouples from the wire

$$t_0 \to \infty \qquad \gamma \to 1, \quad G_t \to 0$$

$$t \sim 1/t_0, \; t_b \to 1$$

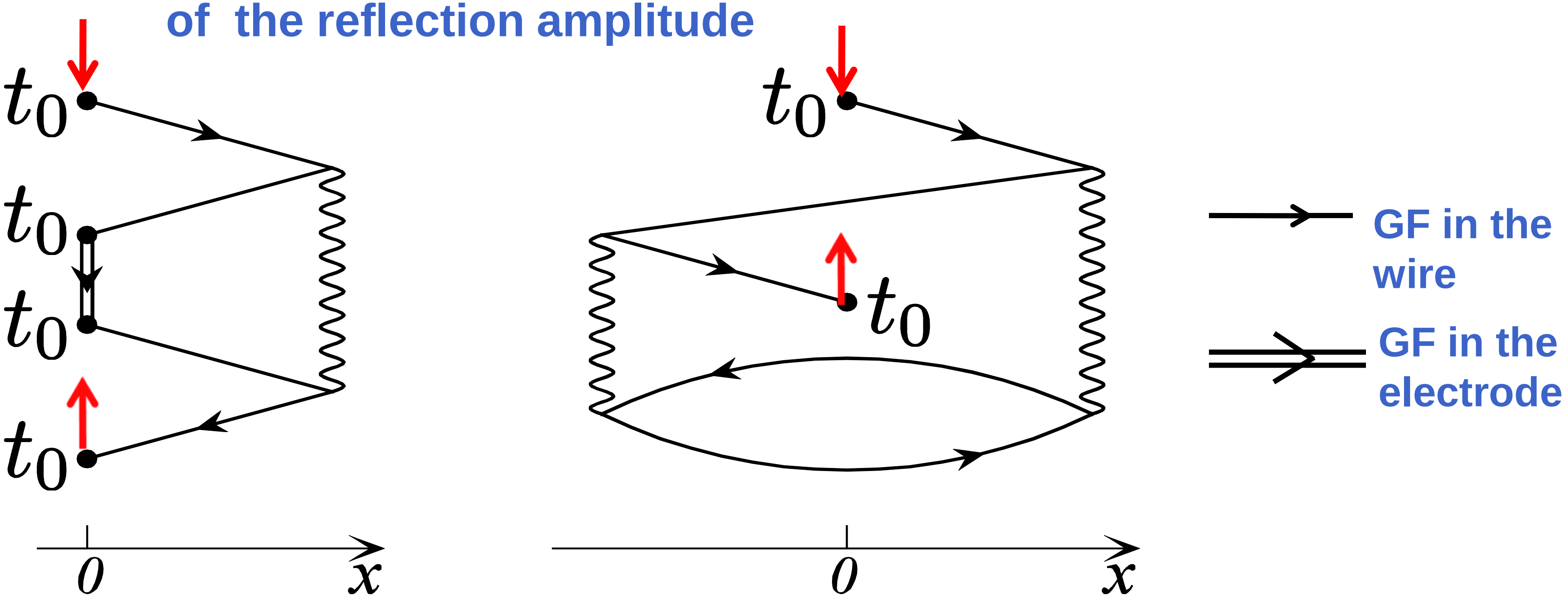


all three channels decouple

Tunneling Hamiltonian (point contact): small γ

$$g \ll 1, \quad t_0 \to 0, \quad \gamma \approx 2t_0^2/vv_e \ll 1$$

Lowest-order processes leading to the renormalization of the reflection amplitude



gt_0^4 $\qquad$ $-g^2t_0^2$ **opposite sign**

$$\beta(\gamma) \simeq g\gamma^2 - g^2\gamma/2, \qquad \gamma \ll 1$$

RG in the weak-tunneling limit

$$\frac{d\gamma}{d\mathcal{L}} = g\gamma^2 - \frac{g^2\gamma}{2}$$

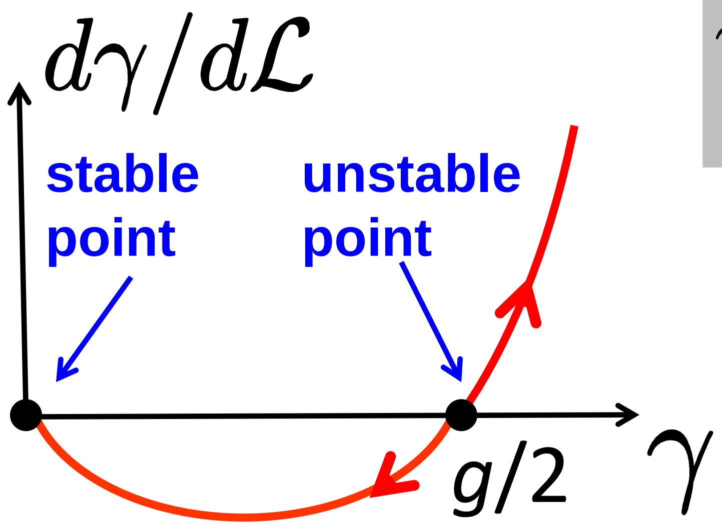
γ - tunneling transparency of the contact

$$\mathcal{L} = \ln\left(\Lambda/|\epsilon|\right)$$

non-trivial geometry, junction of 3 wires

Das, Rao, Sen '02, '04

renormalization of the bulk tunneling density of states



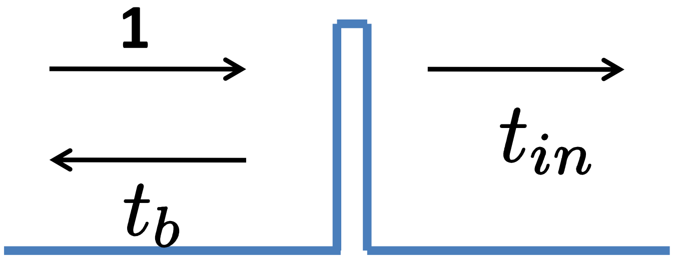
$$\gamma \simeq \frac{\gamma_0}{2\gamma_0/g + (1 - 2\gamma_0/g)(\Lambda/|\epsilon|)^{g^2/2}}$$

$\gamma_0 < g/2$ ➡ renormalization of the density of states dominates

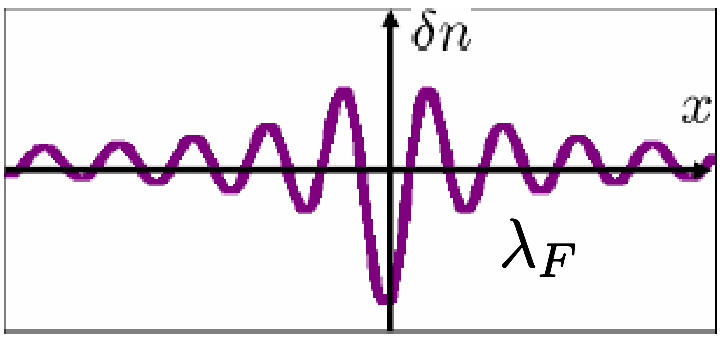
$\gamma_0 > g/2$ ➡ enhancement of the tunneling

Physics behind: scattering off Friedel oscillations vs. bulk ZBA

Friedel oscillations in 1D



$$x<0: \quad \psi(x) = e^{ikx} + t_b e^{-ikx}$$



$$\delta n = \frac{|t_b|}{2\pi|x|}\sin(2k_F|x| + \phi_b)$$

$$|x| \gg \lambda_F \qquad t_b = |t_b|e^{i\phi_b}$$

Hartree potential $V_H(x) = \int dx_1 V(x-x_1)\delta n(x_1)$ **oscillates with the period** $\lambda_F/2$ **at** $x \to \infty$

$$\delta t_b \propto \int dx e^{-2ikx} V_H(x) \quad \propto \; g\mathcal{L} = g \ln\left(\Lambda/|\epsilon|\right)$$

$$\epsilon = E - E_F$$

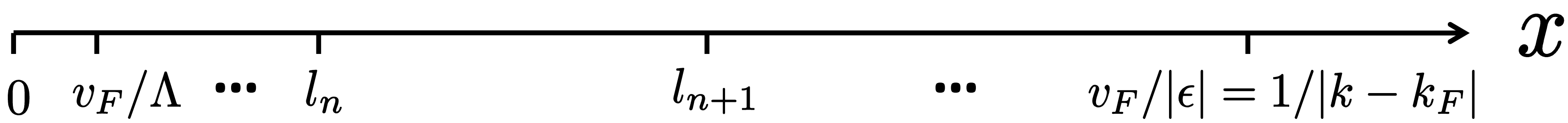
Exchange contribution — similar

Leading-log summation:

sums up the most divergent terms, $[g\ln(\Lambda/|\epsilon|)]^n$, of the perturbation theory

Split the important interval, $[v_F/\Lambda,\ v_F/|\epsilon|]$, on smaller pieces,

so that $l_{n+1}/l_n = const \gg 1$ **but** $\boxed{g\ln\left(l_{n+1}/l_n\right) \ll 1}$

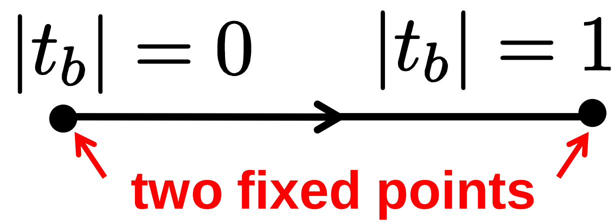


Single impurity: fermionic RG for the backscattering and transmission

$$\delta t_b = t_b^{n+1} - t_b^n = g t_b^n \left(1 - |t_b^n|^2\right) \ln(l_{n+1}/l_n)$$

$$\boxed{\frac{dt_b}{d\mathcal{L}} = g t_b\left(1 - |t_b|^2\right)}$$

β-function in the first order in *g*



Fermionic RG: explicit connection between fixed points for weak interaction, Matveev, Yue, Glazman '93

Bosonic RG: exact exponents at fixed points for arbitrary interaction, Kane , Fisher '92

Single impurity

$$dt_b/d\mathcal{L} = gt_b(1 - |t_b|^2) \Longrightarrow \boxed{\beta(t_b) \simeq gt_b}, \quad |t_b| \ll 1$$

Tunneling contact

$t_b = -\gamma$

$$\boxed{\beta(\gamma) \simeq g\gamma^2} - g^2\gamma/2, \qquad \gamma \ll 1$$

gt_b^2

Key difference:

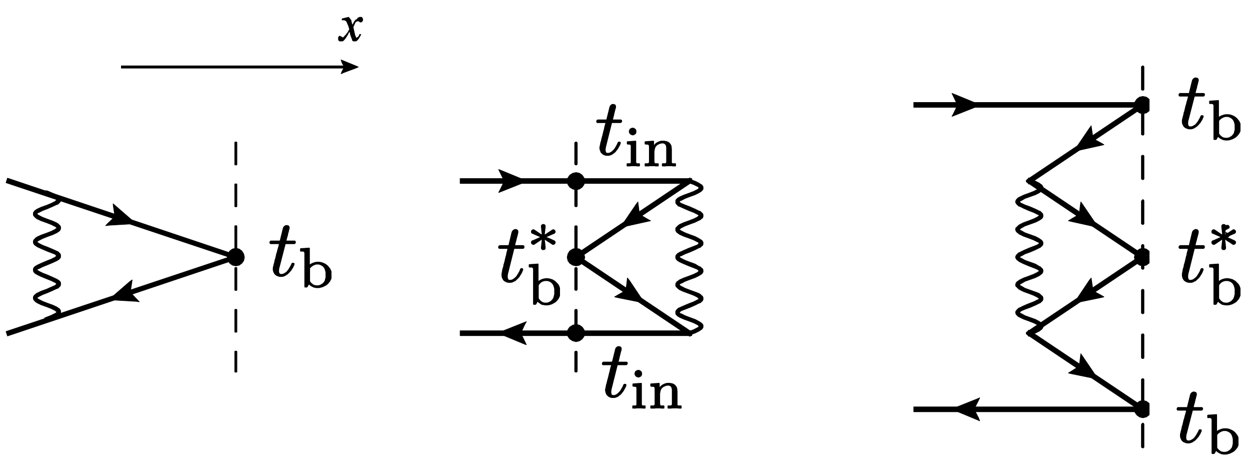
gt_b **absent**

Phase of the Friedel osc's is important

Weak impurity: $\phi_b = \pi/2$

Point tunnel contact: $\phi_b = 0$

Renormalization of backscattering



$$\delta t_{\rm b} = \frac{g}{2}\left[t_{\rm b}\int_0^{\Lambda} \frac{d\epsilon'}{\epsilon'-\epsilon} + t_{\rm b}^*\left(t_{\rm in}^2 + t_{\rm b}^2\right)\int_{-\Lambda}^{0}\frac{d\epsilon'}{\epsilon'-\epsilon}\right]$$

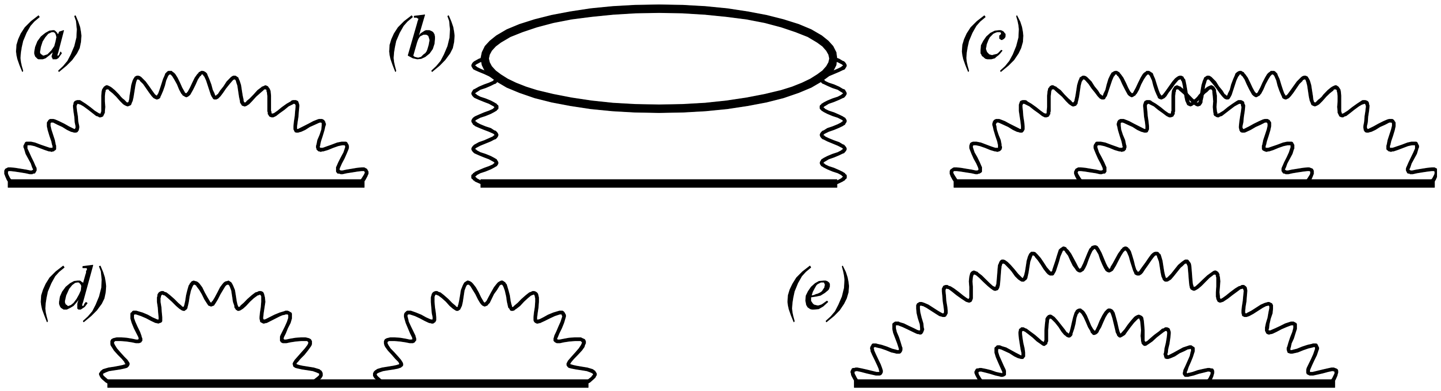
$$\frac{dt_{\rm b}}{d\mathcal{L}} = \frac{g}{2}\left[t_{\rm b} - t_{\rm b}^*(t_{\rm in}^2 + t_{\rm b}^2)\right] \underset{t_{\rm b}\to 0}{\simeq} \frac{g}{2}\left(t_{\rm b} - t_{\rm b}^*\right)$$

Weak impurity: $\phi_b = \pi/2 \;\Rightarrow\; (g/2)(t_{\rm b} - t_{\rm b}^*) = g t_{\rm b}$

Point tunnel contact: $\phi_b = 0 \;\Rightarrow\; (g/2)(t_{\rm b} - t_{\rm b}^*) = 0$

Generic tunnel contact

fermionic S-matrix RG: exact in S-matrix and perturbative in *g*



thick lines: exact noninteracting GF (fully dressed by the tunneling vertices)

One-loop skeletons (a),(b) contribute to the exact-in-γ β–function, diagrams (c)-(e) do not

For tunneling problem the quadratic in *g* terms has to be included into *β*-function

$$\mathbf{A} = \begin{pmatrix} t_{\mathrm{b}} \\ t_{\mathrm{in}} \\ t \\ r \end{pmatrix}, \qquad \delta\mathbf{A} = [\, g\vec{\beta}_1(\mathbf{A}_0) + g^2\vec{\beta}_2(\mathbf{A}_0)\,]\mathcal{L} + \tfrac{1}{2} g^2 \vec{\eta}(\mathbf{A}_0)\mathcal{L}^2$$

$$\to [\, g\vec{\beta}_1(\mathbf{A}) + g^2\vec{\beta}_2(\mathbf{A})\,]\mathcal{L}$$

$$\partial\mathbf{A}/\partial\mathcal{L} = \beta(\mathbf{A})$$

S-matrix of the tunnel contact is renormalizable to order g^2

Four (two)-parameter scaling for the tunnel contact

$$\partial t_{\mathrm{b}}/\partial\mathcal{L} = (g/2)\left[t_{\mathrm{b}} - t_{\mathrm{b}}^*(t_{\mathrm{in}}^2 + t_{\mathrm{b}}^2)\right] - (g^2/4)\left[t_{\mathrm{b}}^*|t_{\mathrm{b}}|^2(t_{\mathrm{in}}^2 + t_{\mathrm{b}}^2) + 2t_{\mathrm{b}}|t_{\mathrm{in}}|^4 - t_{\mathrm{b}}|t_{\mathrm{b}}|^2\right]$$

$$\partial t_{\mathrm{in}}/\partial\mathcal{L} = -g t_{\mathrm{in}}|t_{\mathrm{b}}|^2 + (g^2/4)t_{\mathrm{in}}\left[|t_{\mathrm{in}}|^2 - (t_{\mathrm{in}}^*)^2(t_{\mathrm{in}}^2 + t_{\mathrm{b}}^2) - 2|t_{\mathrm{b}}|^4\right]$$

$$\partial t/\partial\mathcal{L} = -(g/2)tt_{\mathrm{b}}^*(t_{\mathrm{in}} + t_{\mathrm{b}}) - (g^2/4)t(t_{\mathrm{in}} + t_{\mathrm{b}})(t_{\mathrm{b}}^*|t_{\mathrm{b}}|^2 + t_{\mathrm{in}}^*|t_{\mathrm{in}}|^2)$$

$$\partial r/\partial\mathcal{L} = -g t^2 t_{\mathrm{b}}^* - (g^2/2)t^2(t_{\mathrm{in}}^*|t_{\mathrm{in}}|^2 + t_{\mathrm{b}}^*|t_{\mathrm{b}}|^2)$$

Equations for t_b and t_{in} are closed ⟶ two "leading equations"

$$t_b = -e^{i\beta}\gamma, \qquad t = t_{out} = e^{i(\chi-\beta-\varphi)/2}\sqrt{2\gamma\left(\cos\varphi - \gamma\right)},$$

$$t_{in} = e^{i\beta}\left(e^{i\varphi} - \gamma\right), \qquad r = e^{i(\chi-2\beta-\varphi)}(2\gamma - e^{-i\varphi}),$$

$$\frac{\partial\gamma}{\partial\mathcal{L}} = \beta_\gamma(\gamma,\varphi)$$

$$\frac{\partial\varphi}{\partial\mathcal{L}} = \beta_\varphi(\gamma,\varphi)$$

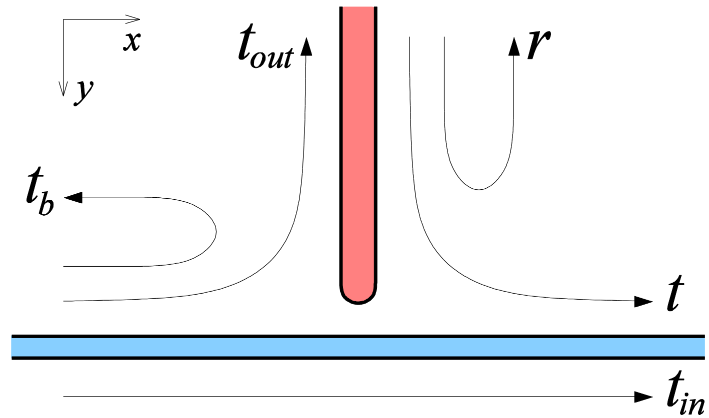
Point tunnel contact: $\beta_\varphi \equiv 0$, four RG equations "collapse" onto a single one

$$d\gamma/d\mathcal{L} = \beta(\gamma), \qquad \varphi = \beta = 0,\ \chi = -\pi$$

Tunneling Hamiltonian (point contact): arbitrary γ

$$t_b = -\gamma, \quad t_{in} = 1-\gamma, \quad r = 1-2\gamma$$

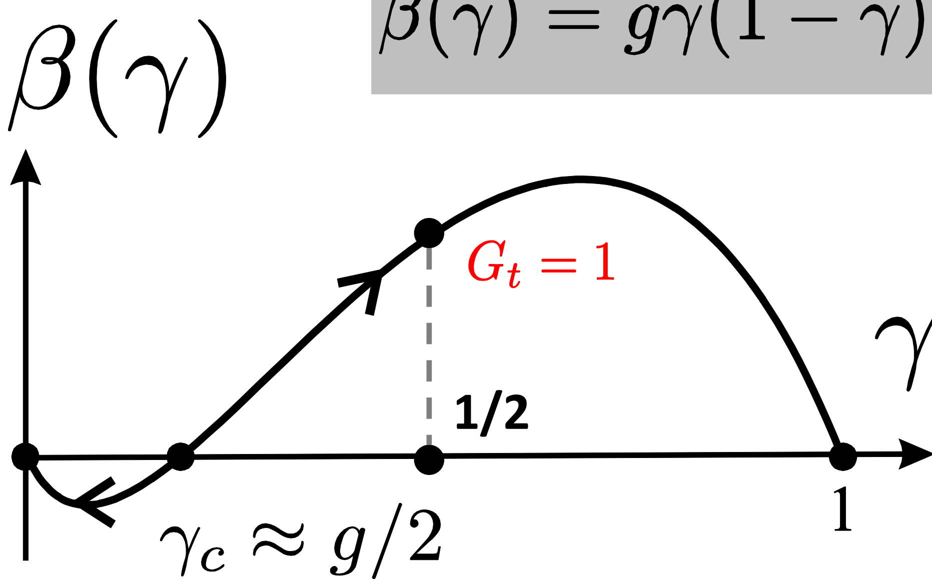
$$t = t_{out} = -i\sqrt{2\gamma(1-\gamma)},\ G_t = 4\gamma(1-\gamma)$$



$$\gamma = \frac{2t_0^2}{vv_e + 2t_0^2}$$

t_0 **– amplitude entering tunn. Ham.**

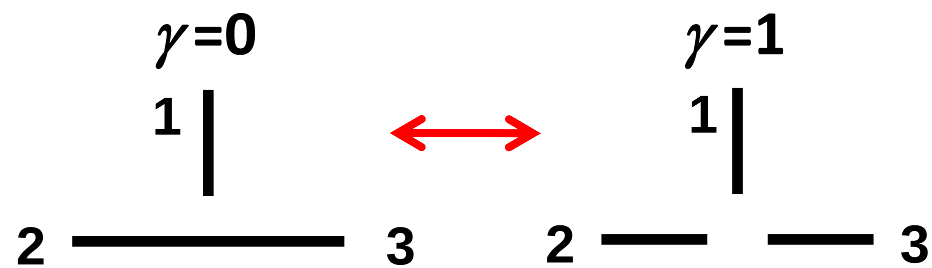
$$\beta(\gamma) = g\gamma(1-\gamma)\left[\gamma - \frac{g}{2}(1-2\gamma)(1-\gamma+\gamma^2)\right]$$



1) unstable point: $\gamma = \gamma_c$

2) stable points: γ=0, γ=1,

$$G_t = 4\gamma(1-\gamma) = 0$$



Phase transition

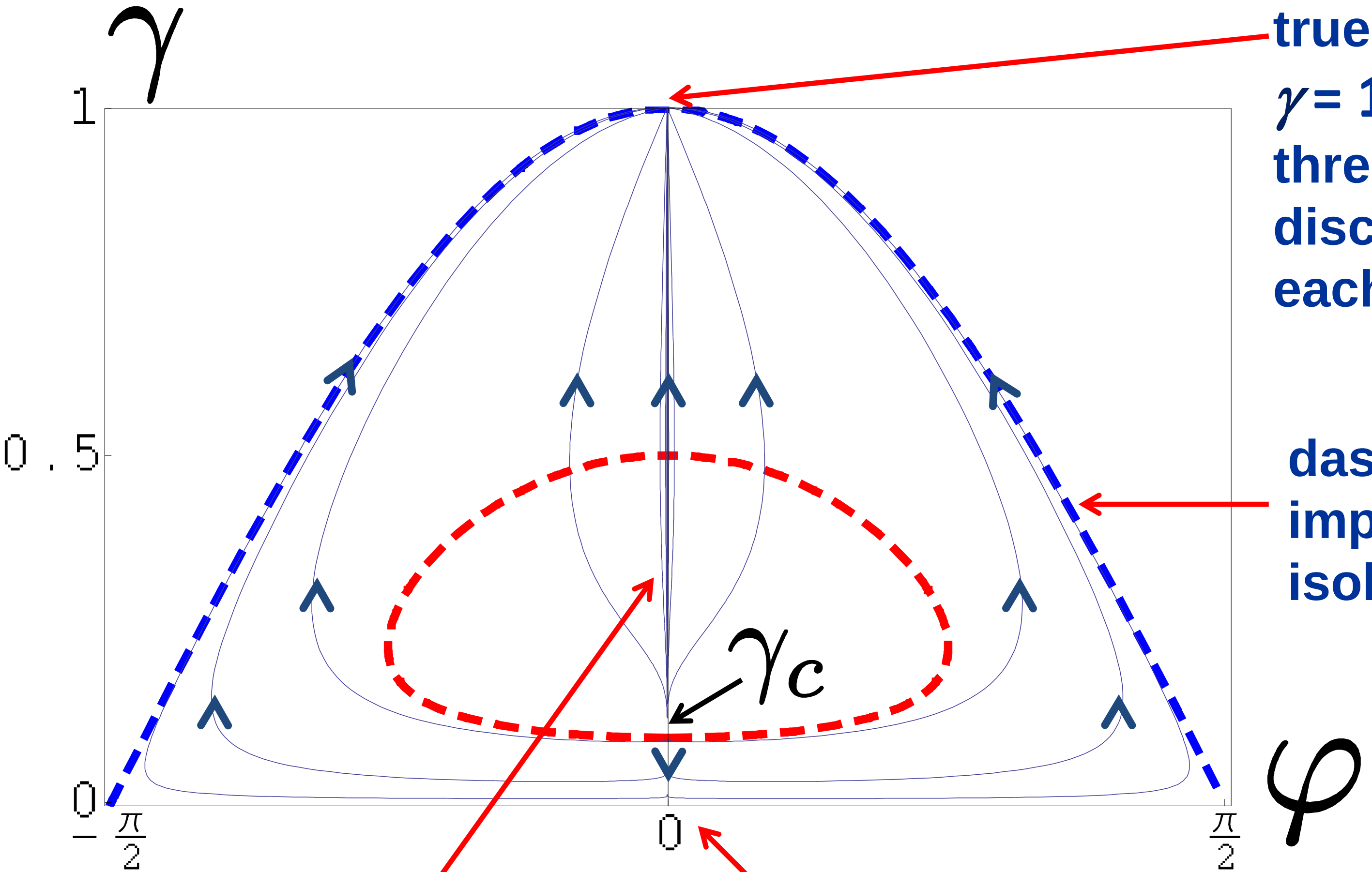
But! **In fact, γ=0 is not a *stable* point,**

it is a *saddle* point ➔ no phase transition!!

true stable point:

γ= 1, φ = 0

three wires are disconnected from each other



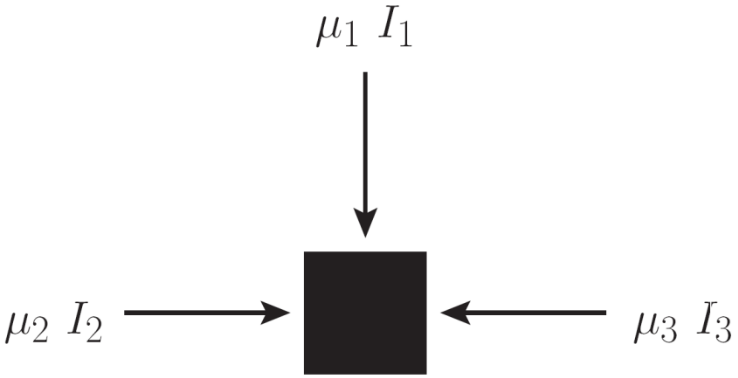
dashed line: impurity in the isolated wire

vertical line: point contact, tunneling hamiltonian

saddle point

$$t_b = -e^{i\beta}\gamma, \quad t = t_{out} = e^{i(\chi-\beta-\varphi)/2}\sqrt{2\gamma\left(\cos\varphi - \gamma\right)},$$
$$t_{in} = e^{i\beta}\left(e^{i\varphi} - \gamma\right), \quad r = e^{i(\chi-2\beta-\varphi)}\left(2\gamma - e^{-i\varphi}\right),$$

Conductances



$$\boldsymbol{I_i = \sum_j G_{ij}\mu_j}$$

time reversal and 2↔3 symmetries ➔ 2 independent conductances:

$$G_{\mathrm{t}} = 2|t|^2, \qquad G_{\mathrm{w}} = |t_{\mathrm{in}}|^2 + |t|^2/2$$

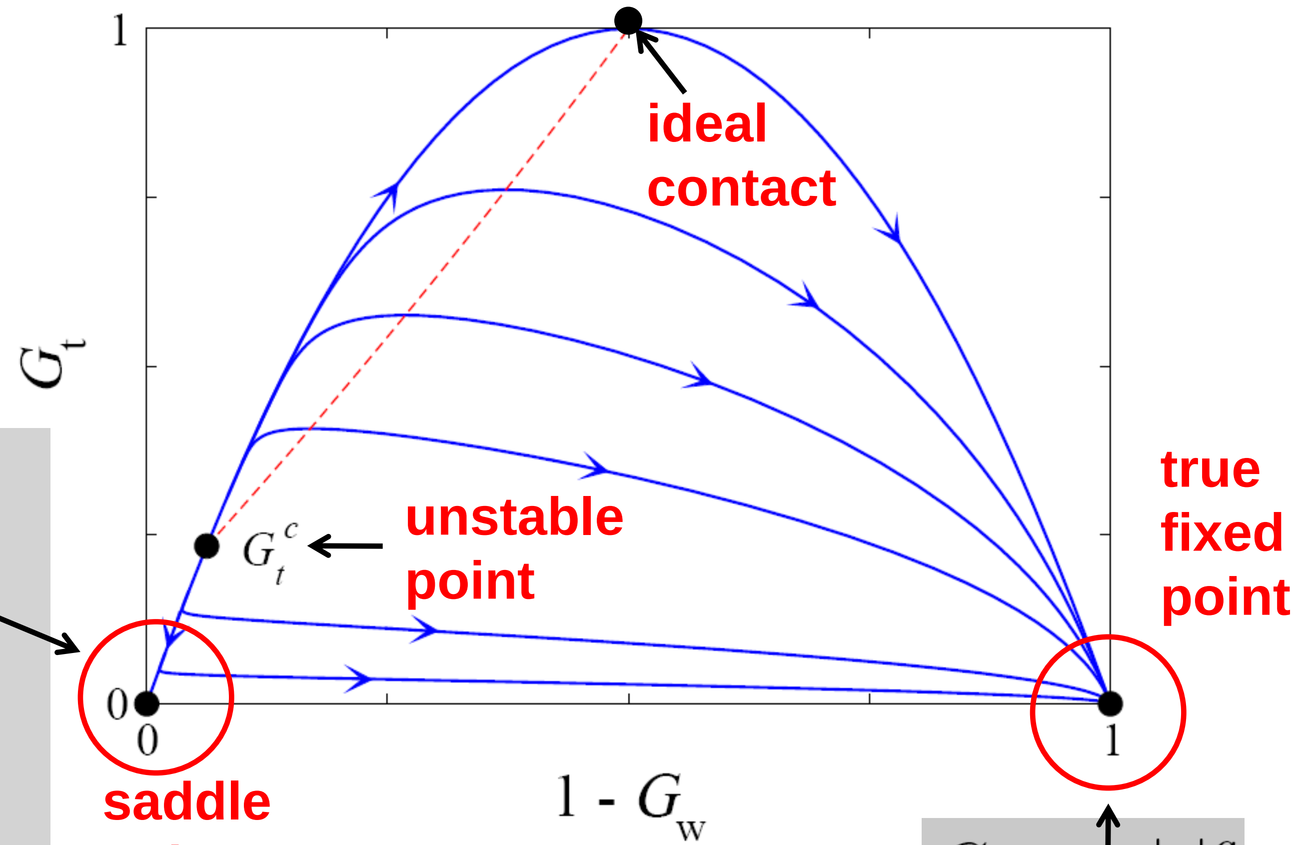
tunneling conductance

current in the wire if no current flows in the tunnel electrode

$$\begin{aligned} I_1 &= G_{\mathrm{t}} V_{\mathrm{t}} \\ I_2 &= -G_{\mathrm{t}} V_{\mathrm{t}}/2 - G_{\mathrm{w}} V \\ I_3 &= -G_{\mathrm{t}} V_{\mathrm{t}}/2 + G_{\mathrm{w}} V \end{aligned}$$

$$\begin{aligned} V_{\mathrm{t}} &= \mu_1 - (\mu_2 + \mu_3)/2 \\ V &= \mu_3 - \mu_2 \end{aligned}$$

RG flows for $G_{\rm t} = 2|t|^2, \;\; G_{\rm w} = |t_{\rm in}|^2 + |t|^2/2$



before the turn:

$G_{\rm t}, 1 - G_{\rm w} \propto |\epsilon|^{g^2/2}$

after the turn:

$G_{\rm t} \propto |\epsilon|^{g^2/2},$

$1 - G_{\rm w} \propto |\epsilon|^{-2g}$

$$\frac{1 - G_{\rm w} - G_{\rm t}/4}{1 - G_{\rm w0} - G_{\rm t0}/4} \simeq \left(\frac{G_{\rm t0}}{G_{\rm t}}\right)^{\kappa},$$

$\kappa \simeq 2(2-g)/g$

$G_{\rm t,w} \propto |\epsilon|^g$

Limiting curves:

1) $G_{\rm t} = 4G_{\rm w}(1 - G_{\rm w})$ - point tunnel contact,

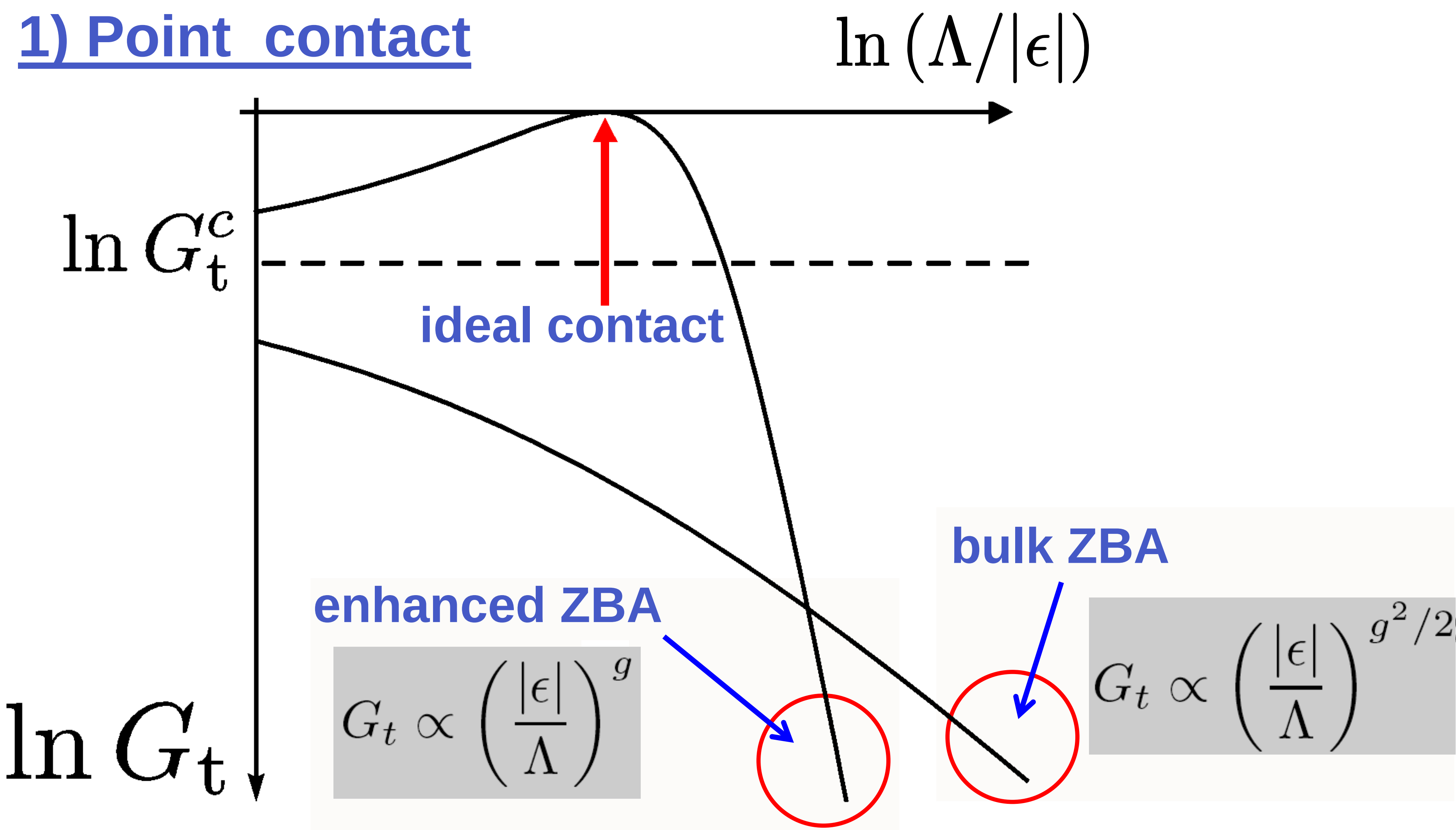
2) $G_{\rm t} = 0$ - impurity in the isolated wire

Fixed points ➔ bosonic RG

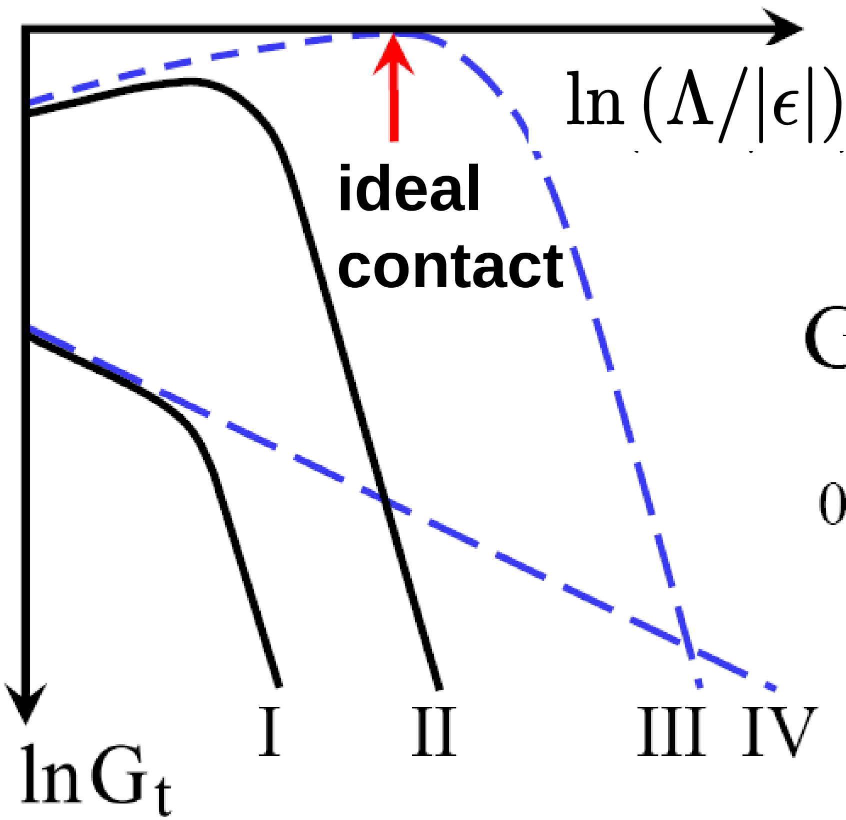
Kane, Fisher'92

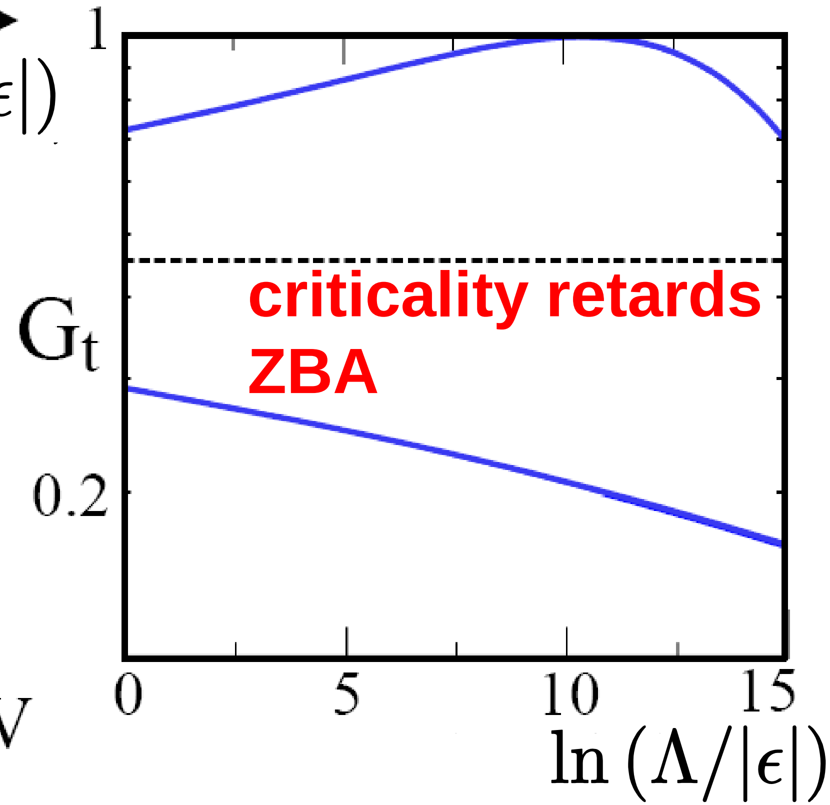
Scaling of the tunneling conductance

1) Point contact



2) Generic contact





generic tunnel contact $\varphi \neq 0$

point tunnel contact $\varphi = 0$

I - enhancement of ZBA

II,III- interaction first make the tunneling contact more transparent

IV- conventional (bulk) ZBA

Summary

- Tunneling into a Luttinger liquid is described by RG flow of 3x3 S-matrix

- Conventional fixed point has a finite basin of attraction only in the point contact model

- Bulk ZBA generically unstable to the breakup of the liquid into two parts

- Interaction-induced enhancement of tunneling

- Nonmonotonic behavior of the tunneling current with temperature or bias voltage

Stability analysis:

point-contact fixed point $\gamma = 0, \varphi = 0$ is a saddle point in (γ, φ) space

RG equations for small γ:

$$\frac{\partial\gamma}{\partial\mathcal{L}} \simeq g\gamma^2\cos\varphi - \frac{g^2\gamma}{2} + g\gamma\sin^2\varphi$$

$$\frac{\partial\varphi}{\partial\mathcal{L}} \simeq (g/2)\sin 2\varphi$$

instability

Compare to point contact (tunneling ham.)

$$\frac{d\gamma}{d\mathcal{L}} = g\gamma^2 - \frac{g^2\gamma}{2}$$

$$\varphi \equiv 0$$

Linear in $g\gamma$ term

$$g\,\mathrm{Re}(t_{\mathrm{b}} - t_{\mathrm{b}}^{*}t_{\mathrm{in}}^{2}) \propto g\gamma\sin^2\varphi + \mathcal{O}(\gamma^2)$$

no exact cancelation at order $\mathcal{O}(g\gamma)$ of the contributions to β-function from the Friedel osc's that screen the junction at x < 0 and x > 0

Impurity in the isolated wire

$$t_b = -e^{i\beta}\gamma, \qquad t = t_{out} = e^{i(\chi-\beta-\varphi)/2}\sqrt{2\gamma\left(\cos\varphi-\gamma\right)},$$

$$t_{in} = e^{i\beta}\left(e^{i\varphi}-\gamma\right), \qquad r = e^{i(\chi-2\beta-\varphi)}(2\gamma - e^{-i\varphi}),$$

$$\gamma = \cos\varphi \;\Rightarrow\; t = 0, G_t = 0, \quad \boxed{t_{\mathrm{b}} = -e^{i\beta}\cos\varphi,\;\; t_{\mathrm{in}} = ie^{i\beta}\sin\varphi}$$

RG equations

$$d\beta/d\mathcal{L} = 0$$

$$d\varphi/d\mathcal{L} = -(g/2)\sin 2\varphi\left[1+(g/2)\cos 2\varphi\right]$$

$$\Updownarrow$$

$$d\gamma/d\mathcal{L} = g\gamma(1-\gamma^2)\left[1+(g/2)(2\gamma^2-1)\right]$$

$$\simeq g\gamma(1-\gamma^2)$$

beyond

Matveev, Yue, Glazman '93

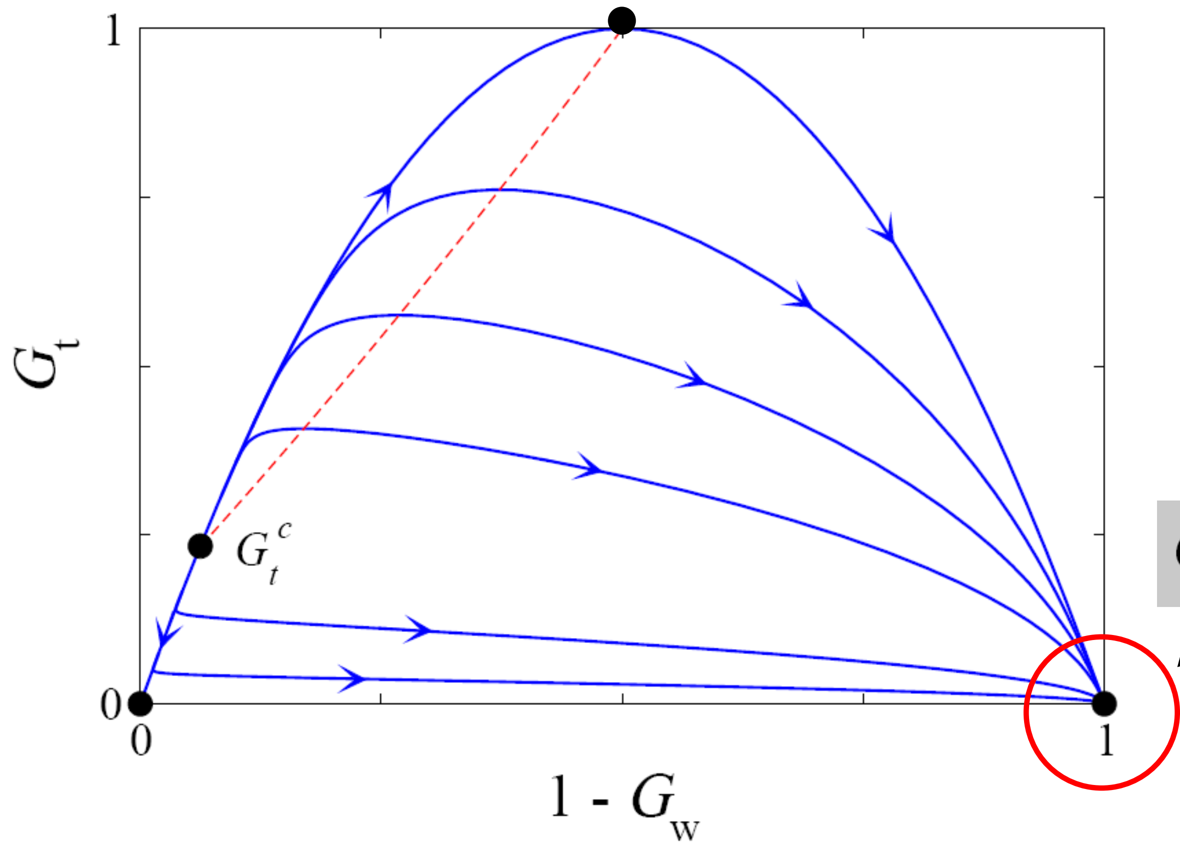
RG equations for the conductances

$$G_{\mathrm{t}} = 2|t|^2, \quad G_{\mathrm{w}} = |t_{\mathrm{in}}|^2 + |t|^2/2$$

$$\partial G_{\mathrm{t}}/\partial \mathcal{L} = g G_{\mathrm{t}}(f_1 + g f_2)$$
$$\partial G_{\mathrm{w}}/\partial \mathcal{L} = g(f_3 + g f_4)$$

$$f_1 = -1 + G_{\mathrm{t}}/2 + G_{\mathrm{w}}$$
$$f_2 = -1/2 + G_{\mathrm{t}}(3 - G_{\mathrm{t}})/8 + G_{\mathrm{w}}(1 - G_{\mathrm{w}})$$
$$f_3 = G_{\mathrm{t}}(1 + G_{\mathrm{w}})/4 - 2G_{\mathrm{w}}(1 - G_{\mathrm{w}})$$
$$f_4 = (1 - 2G_{\mathrm{w}})[\,G_{\mathrm{t}}(4 + G_{\mathrm{t}})/32 - G_{\mathrm{w}}(1 - G_{\mathrm{w}})\,]$$

Tunneling DOS at the site of a weak impurity



$G_{\mathrm{t,w}} \propto |\epsilon|^g$

Linearizing the RG equations in G_t around the stable fixed point tunneling DOS $\rho(\varepsilon)$ at $\varepsilon \to 0$ for an arbitrary bare strength of impurity U_0

$$\boxed{\rho(\epsilon) \propto |\epsilon|^{\alpha_e}} \qquad \alpha_e = g + g^2/2 + \dots$$