Confined Magnetic Monopoles in Dense QCD

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1 Introduction

Nambu, Mandelstam and 't Hooft 1970's:

Condensation of monopoles should lead to confinement of quarks

However, the very notion of the magnetic monopoles remains obscure in QCD

Apply ideas from supersymmetric gauge theories:

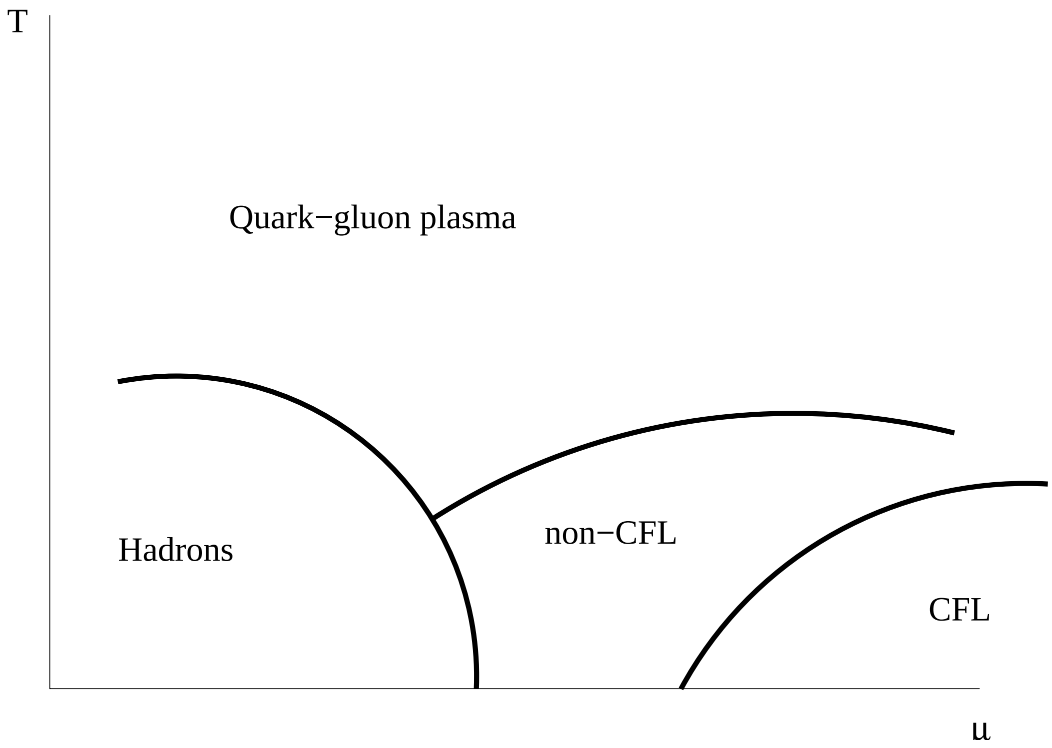
Non-Abelian flux tubes (strings) in SUSY

Hanany, Tong 2003
Auzzi, Bolognesi, Evslin, Konishi, Yung 2003
Shifman, Yung 2004
Hanany, Tong 2004

Similar non-Abelian strings were found in large-μ QCD

Nakano, Nitta, Matsuura 2008
Eto, Nitta, Yamamoto 2009
Gorsky, Mikhailov 2007

QCD phase diagram



CFL phase at large μ. Weak coupling

Alford, Rajagopal, Wilczeck 1998

Ginzburg-Landau effective Lagrangian

Iida, Baym 2001; Giannakis, H. c. Ren 2002

2 Ginzburg–Landau effective description

At large μ QCD is in the CFL phase. Diquark condensate

$$\Phi^{kC} \sim \varepsilon_{ijk}\,\varepsilon_{ABC}\left(\psi^{iA}_{\alpha}\,\psi^{jB\,\alpha} + \bar{\tilde{\psi}}^{iA\,\dot{\alpha}}\,\bar{\tilde{\psi}}^{jB}_{\dot{\alpha}}\right)$$

At $T \to T_c$ gap fluctuations become important.

Chiral fluctuations (π-mesons) are considered less important

$$\begin{aligned} S &= \int d^4x \left\{ \frac{1}{4g^2}\left(F^a_{\mu\nu}\right)^2 + 3\,\mathrm{Tr}\,(\mathcal{D}_0\Phi)^\dagger\,(\mathcal{D}_0\Phi) \right. \\ &+ \left. \mathrm{Tr}\,(\mathcal{D}_i\Phi)^\dagger\,(\mathcal{D}_i\Phi) + V(\Phi) \right\} \end{aligned}$$

with the potential

$$V(\Phi) = -m_0^2\,\mathrm{Tr}\,\left(\Phi^\dagger\Phi\right) + \lambda\left(\left[\mathrm{Tr}\,\left(\Phi^\dagger\Phi\right)\right]^2 + \mathrm{Tr}\,\left[\left(\Phi^\dagger\Phi\right)^2\right]\right)$$

Here

$$m_0^2 = \frac{48\pi^2}{7\zeta(3)} T_c(T_c - T), \qquad \lambda = \frac{18\pi^2}{7\zeta(3)} \frac{T_c^2}{N(\mu)},$$

while $N(\mu) = \mu^2/(2\pi^2)$ is the density of states on the Fermi surface.

The critical temperature T_c is much smaller than μ,

$$T_c \sim \frac{\mu}{(g(\mu))^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g(\mu)}\right) \ll \mu$$

Therefore

$$m_0^2 \sim T_c(T_c - T), \qquad \lambda \sim \frac{T_c^2}{\mu^2} \ll 1,$$

First we assume that quarks are massless: $\qquad m_u = m_d = m_s = 0$

Vacuum

$$\Phi_{\text{vac}} = v \, \text{diag}\, \{1, 1, 1\}$$

where

$$v^2 = \frac{m_0^2}{8\lambda} = \frac{4\pi^2}{3} \, \frac{T_c - T}{T_c} \, \mu^2$$

The symmetry breaking pattern

$$\text{SU}(3)_C \times \text{SU}(3)_F \times \text{U}(1)_B \to \text{SU}(3)_{C+F}$$

9 symmetries are broken.

8 are eaten by Higgs mechanism.

One Goldstone boson associated with broken $\text{U}(1)_B$.

Mass spectrum

Higgsed gluons

$$m_g = gv \sim g\mu\sqrt{\frac{T_c - T}{T_c}}$$

Scalars: Φ^{kA}

8	$+1$	$+8$	$+1$
eaten	Goldstone	octet	singlet

$$m_1 = 2m_8 = \sqrt{2}\, m_0 \sim \sqrt{T_c(T_c - T)}$$

We have type I superconductor $m_g \gg m_1 \sim m_8$

Weak coupling condition

$$m_g \gg \Lambda$$

3 Non-Abelian strings in the CFL phase

Z_3 strings

$$\Phi(r,\alpha) = \mathrm{diag}\left(e^{i\alpha}\phi_1, \phi_2, \phi_2\right),$$

$$A_i(r) = \frac{\varepsilon_{ij}x^j}{r^2}(1-f)\,\mathrm{diag}\left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right),$$

Boundary conditions

$$\phi_1(\infty) = v, \quad \phi_2(\infty) = v\,, \qquad f(\infty) = 0$$
$$\phi_1(0) = 0, \qquad f(0) = 1$$

The profile functions of the non-Abelian strings satisfy the second order equations of motion

$$f'' - \frac{f'}{r} - \frac{g^2}{3}(1+2f)\,\phi_1^2 + \frac{g^2}{3}(1-f)\,\phi_2^2 = 0,$$
$$\phi_1'' + \frac{\phi_1'}{r} - \frac{1}{9}\frac{(1+2f)^2}{r^2}\phi_1 - \frac{1}{2}\frac{\partial V}{\partial \phi_1} = 0,$$
$$\phi_2'' + \frac{\phi_2'}{r} - \frac{1}{9}\frac{(1-f)^2}{r^2}\phi_2 - \frac{1}{4}\frac{\partial V}{\partial \phi_2} = 0,$$

At $r \to \infty$ ($r \gg 1/m_0$) the profile functions behave as (*Eto, Nitta 2010*)

$$\phi_1 \sim \phi_2 \sim v\left(1 - \frac{1}{3m_1^2 r^2} + \dots\right),$$
$$(\phi_1 - \phi_2) \sim v\, e^{-m_0 r}, \qquad f \sim e^{-m_0 r}.$$

String profile functions have a two-scale structure due to smallness of the ratio of the scalar to gluon mass

At $1/m_g \ll r \ll 1/m_0$, the gluons can be considered as heavy, and we can neglect the gauge kinetic term.

$$f \approx \frac{\phi_2^2 - \phi_1^2}{\phi_2^2 + 2\phi_1^2}\,.$$

$$\phi_1 \approx b\, v\,(m_0 r) + \dots,$$
$$\phi_2 \approx v\left[1 + O((m_0 r)^4)\right],$$
$$f \approx 1 - 3b^2\,(m_0 r)^2 + \dots,$$

where b is a number, $b \sim 1$

Non-Abelian strings are constructed from Z_3 much in the same way as in SUSY theories.

Z_3 string solution breaks

$$\mathrm{SU}(3)_{C+F} \to \mathrm{SU}(2) \times \mathrm{U}(1)\,,$$

We rotate the given Z_3 solution inside the unbroken diagonal $\mathrm{SU}(3)_{C+F}$. This costs no energy; therefore orientational moduli appear.

Modular space

$$CP(2) \sim \frac{\mathrm{SU}(3)_{C+F}}{\mathrm{SU}(2) \times \mathrm{U}(1)}$$

Solution

$$\begin{aligned}\Phi &= e^{i\alpha/3}\frac{1}{3}\left[2\phi_2 + \phi_1\right] + e^{i\alpha/3}(\phi_1 - \phi_2)\left(n\,\cdot\bar{n} - \frac{1}{3}\right),\\ A_i &= \left(n\,\cdot\bar{n} - \frac{1}{3}\right)\varepsilon_{ij}\,\frac{x_j}{r^2}\,f(r)\,,\end{aligned}$$

where n^A $(A = 1, 2, 3)$ are complex orientational moduli

$$|n^A|^2 = 1$$

4 On the string world sheet

We have the two translational zero modes and four orientational.

We assume that moduli acquire slow dependence on world-sheet coordinates

$$S_{\rm NG} = T_0 \int d^2x\ \mathcal{L}_{\rm NG}\,, \qquad T_0 = 2\pi\, v^2\ \ln\left(Lm_0\right)\ ,$$

where L is a typical size of the color-flavor locked medium.

The orientational moduli's interaction is governed by CP($N-1$) (with $N=3$ in the case at hand). In the gauged formulation the CP(2) model takes the form

$$S_{\rm CP(2)} = 2\beta \int dt\, dx_3 \left\{3\left|\mathcal{D}_0 n^A\right|^2 + \left|\mathcal{D}_3 n^A\right|^2\right\}\,,$$

where β is the CP(2) coupling constant.

$$\mathcal{D}_\alpha n^A \equiv \left(\partial_\alpha - iA_\alpha\right) n^A \qquad A_\alpha = \frac{i}{2}\left(\bar{n}_A \overset{\leftrightarrow}{\partial}_\alpha n^A\right)$$

Number of degrees of freedom: $2N-1-1 = 2(N-1) = 4$

The coupling constant β is determined by substituting the solution for the non-Abelian string in the kinetic terms of the bulk action. It is expressed in terms of profile functions of the string. The main contribution comes from the region of intermediate r, $1/m_g \ll r \lesssim 1/m_0$.

$$\beta \approx \pi \int_0^\infty r dr \frac{(\phi_2^2 - \phi_1^2)^2}{\phi_1^2 + \phi_2^2} = c\, \frac{v^2}{m_0^2} \sim \frac{\mu^2}{T_c^2} \gg 1\,.$$

β is large

In quantum theory the coupling constant of the CP(2) model runs. The CP$(N-1)$ models are asymptotically free and generate their own scale Λ_{CP}. The estimate above is classical and refers to the scale which determines the inverse thickness of the string given by m_0.

$$4\pi\beta(m_0) = N\, \ln \frac{m_0}{\Lambda_{CP}}\,, \qquad N = 3\,,$$

Thus

$$\Lambda_{CP} = m_0\, \exp\left(-\frac{4\pi c}{N}\,\frac{v^2}{m_0^2}\right) \ll m_0, \qquad N = 3$$

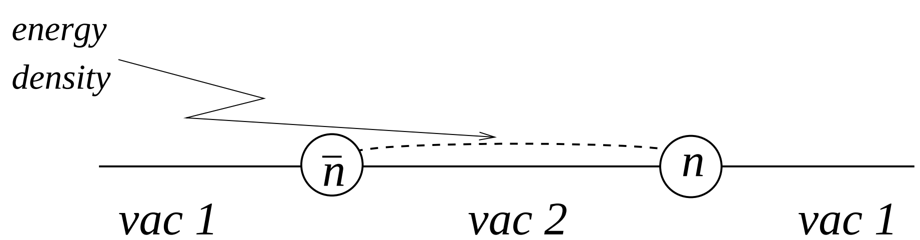
5 Kink-antikink mesons at large N

The CP($N-1$) model at large N was solved by Witten, 1979

The vacuum is unique, but there are N quasivacua *Witten 1998*

$$E_k \sim N\,\Lambda^2_{CP}\left\{1 + \text{const}\left(\frac{2\pi k}{N}\right)^2\right\}, \quad k = 0\ldots, N-1$$

Therefore, kinks are confined on the string



The splitting at large N is

$$\Lambda^2_{CP}/N$$

while the mass of an individual kink is $\quad m_{\text{kink}} \sim \Lambda_{CP}$

Thus, the distance between kinks in the meson is

$$\sim N/\Lambda_{CP}$$

Kinks are well separated inside the meson

kinks= n^A

Form fundamental of $\mathrm{SU}(N)_{C+F} \Rightarrow$ mesons are singlets or in the adjoint representation

Adjoints are stable

Zamolodchikovs 1979 Solved exactly CP(1) model.

Lightest state == triplet of SU(2)

6 Confined monopoles in CFL phase

$\mathcal{N}=2$ supersymmetric U(N) QCD

Z_N strings $\Longleftrightarrow$ N degenerative classical vacua of $CP(N-1)$ model

$$A_i^{\mathrm{SU}(N)} = \left(n \,\cdot n^* - \frac{1}{N}\right) \varepsilon_{ij} \,\frac{x_j}{r^2}\, f_{NA}(r)$$

$CP(N-1)$ classical vacua:

$$n^A = (1,0,0)\,, \quad n^A = (0,1,0)\,, \quad n^A = (0,0,1)\,,$$

$\mathcal{N}=2$ supersymmetric QCD has adjoint scalars $\Rightarrow$
it has 't Hooft-Polyakov monopoles

Higgs phase for quarks $\Longrightarrow$ confinement of monopoles

Elementary monopoles – junctions of two Z_N strings

Consider first two strings

Difference of their fluxes =

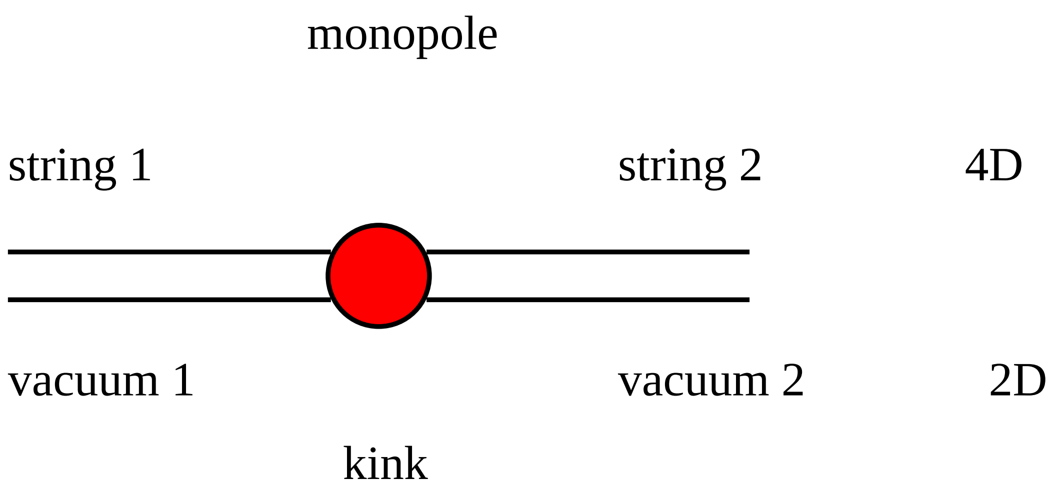
=monopole flux $= 4\pi \times \text{diag}\frac{1}{2}\{1, -1, 0\}$

In 2D $CP(N-1)$ model on the string we have

N vacua $= N$ Z_N strings

and kinks interpolating between these vacua

Kinks = confined monopoles



Now add mass m_{adjoint} to adjoint scalars and tend it to infinity. Fluxes are smeared over the whole group space.

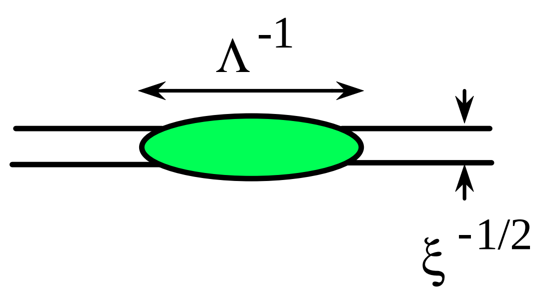
Classically monopole disappear

Confined monopoles = kinks

are stabilized by quantum (non-perturbative) effects in $CP(N-1)$ model on the string worldsheet

$$M_{\text{monopole}} = M_{\text{kink}} \sim \Lambda_{CP}$$

$$\text{monopole size} \sim \Lambda_{CP}^{-1}$$



Lessons from SUSY for dense QCD:

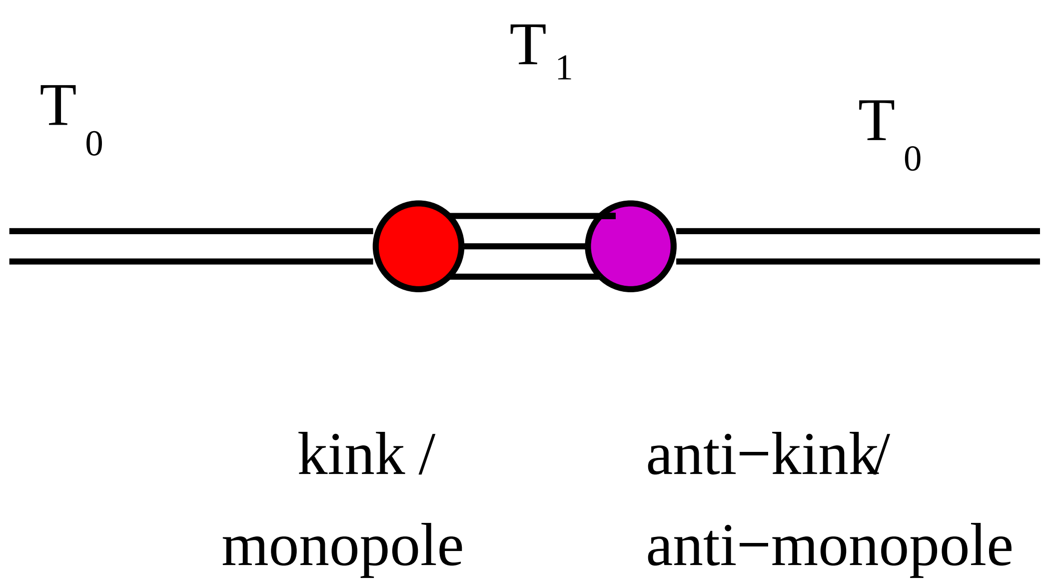
We might think that there is a certain deformation of the GL model which includes adjoint scalar fields. If these fields develop VEVs, the conventional 't Hooft-Polyakov monopoles are formed.

These confined monopoles in CFL phase are seen as kinks in the CP$(N-1)$ model on the string.

Now we give a mass to the adjoint scalars and decouple them.

Still there are confined monopoles = kinks of CP$(N-1)$ model on the string

Quantum splitting of the string tensions:



7 Towards a more realistic setting

7.1 $N = 3$

N is not large, but even for $N = 2$

Zamolodchikovs 1979 Solved exactly CP(1) model.

Lightest state == triplet of SU(2)

Question:

Given a state belonging to the adjoint representation of $\mathrm{SU}(3)_{C+F}$, can we say whether this state is a "perturbative" state, or a kink-antikink meson?

Analogy with SUSY:

In the quasiclassical regime, outside the so-called curves of the marginal stability (CMS), perturbative states are present in the spectrum of the CP(($N-1$) model, while inside CMS, in the strong coupling domain, they just do not exist as stable states. They decay into the kink-antikink pairs.

In the non-supersymmetric CP($N-1$) models CMS are replaced by the phase transition lines.

The analogy with SUSY tells us:

There are no “perturbative” stable states ”inside CMS” in the strong coupling domain.

7.2 Non-zero strange quark mass

$$m_u = m_d = 0, \qquad m_s \neq 0$$

GL potential acquires correction

$$\delta V(\Phi) = \epsilon \left\{ \Phi_u^\dagger \Phi^u + \Phi_d^\dagger \Phi^d \right\},$$

where

$$\epsilon = \frac{48\pi^2}{7\zeta(3)} \frac{m_s^2}{4\mu^2} T_c^2 \ln \frac{\mu}{T_c}$$

Now we have

$$\langle \Phi \rangle = \mathrm{diag}\left(v_u, v_u, v_s\right) \,,$$

where

$$v_u^2 = \frac{m_0^2 - 2\epsilon}{8\lambda}, \qquad v_s^2 = \frac{m_0^2 + 2\epsilon}{8\lambda}$$

The color-flavor global group is broken

$$\mathrm{SU}(3)_{C+F} \to \mathrm{SU}(2)_{C+F} \times \mathrm{U}(1)$$

We calculate the response on the string world sheet to the leading order in m_s. Use the same solution for the non-Abelian string

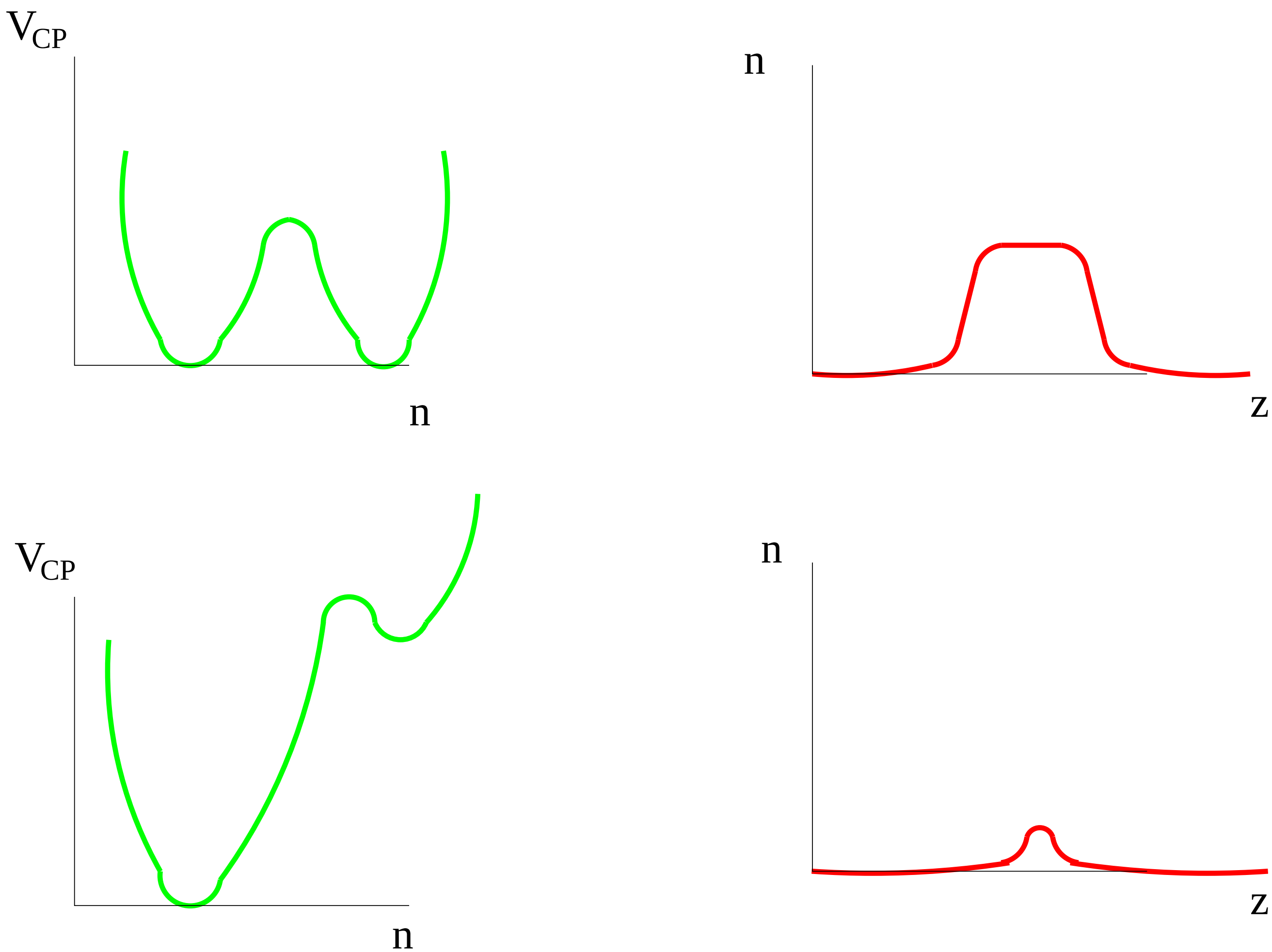
Result:

$$V_{CP} = \omega \int dt\, dx_3 \left(3n_3^2 - 1\right),$$

where

$$\omega = \frac{2\pi}{3}\,\epsilon \int_0^\infty r dr \left(\phi_2^2 - \phi_1^2\right) \sim \epsilon\,\frac{v^2}{m_0^2}$$

The $(0,0,1)$ string has a significantly larger tension than the $(1,0,0)$ and $(0,1,0)$ strings and is, in fact, classically unstable. It is not even a local minimum of the potential (rather, it corresponds to a maximum).


V CP
n
n
z
V CP
n
n
z

This effect is much larger then quantum splitting

$$\epsilon \gg \Lambda_{CP}$$

Two monopoles (out of three) become unstable (annihilate).

n^3 field is heavy and can be integrated out from the $CP(2)$ model. Then we are left with the $CP(1)$ model with no potential.

Confined non-Abelian monopoles (of the third type) are still present in dense QCD. They attached to the non-Abelian strings.

These monopoles correspond to junctions of $(1, 0, 0)$ and $(0, 1, 0)$ strings

Compare our results to those obtained by

Eto, Nitta, Yamamoto 2010

A realistic dense matter inside neutron stars was studied. In particular, the electromagnetic interactions and the presence of electrons were taken into account. This leads to the complete breaking of the non-Abelian color-flavor symmetry $\mathrm{SU}(3)_{C+F} \to \mathrm{U}(1)^3$, All three strings are classically split by the strange quark mass. Two excited strings become classically unstable, and the monopoles effectively disappear from the string. They are annihilated by the would-be antimonopoles.

We do not attempt to study realistic neutron stars. Instead, we focus just on the quark-gluon matter in dense QCD. Then $\mathrm{SU}(2)_{C+F}$ is preserved.

After our paper was published another paper by

Eto, Nitta, Yamamoto 2011

appeared reporting results similar to ours. In this paper they considered massless quarks, while took into account quantum splitting of non-Abelian strings.

8 Conclusions

- Internal dynamics of non-Abelian strings in CFL phase of dense QCD is described by two dimensional CP(2) model on the string world sheet
- Kinks of CP(2) model are interpreted as confined monopoles of the bulk theory.

  This is the first analytic demonstration of the presence of monopoles in QCD
- Non-Abelian strings in CFL phase of dense QCD are split in quantum theory. This splitting induces formation of monopole-antimonopole mesons on the string.
- One out of three types of monopoles survives switching on the strange quark mass.