Euler International Mathematical Institute
St.Petersburg Department of Steklov Mathematical Institute of Russian Academy of Sciences

The Fourth St.Petersburg Conference in Spectral Theory

2 – 6 July 2012

Dedicated to the memory of M. Sh. Birman (1928–2009)

Supported by

Russian Foundation of Basic Research, grant 12-01-06044-g
Chebyshev Laboratory under the grant 11.G34.31.0026 of the Government of the Russian Federation

Program

Abstracts

St.Petersburg, 2012

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Alexandre Fedotov, Nikolai Filonov, Alexander Pushnitski

Organizing committee:

Alexandre Fedotov, Nikolai Filonov, Alexander Pushnitski, Tatayna Vinogradova, Nadia Zalesskaya

International Conference in Spectral Theory.
Proceedings of the conference. Euler International Mathematical Institute, St.Petersburg, 2012.

The conference was organized at the Euler International Mathematical Institute that is a part of the St.Peterburg Department of Steklov Institute of Mathematics. The organizers are grateful to the staff of the Euler and Steklov Institutes for helping with the organization of the conference.

The International Conference in Spectral Theory is supported by RFBR and Chebyshev laboratory, Russia.

The conference website: http://www.pdmi.ras.ru/EIMI/2012/ST

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Scientific programme

MONDAY 2 July:

9:30–10:00: REGISTRATION

10:00–11:00: Nikolai Nikolski (Bordeaux 1)
The spectrum of Fourier-Muckenhoupt multipliers

COFFEE BREAK

11:30–12:30: Anatoly Vershik (Steklov Institute, St.Petersburg)
Quasisimilarity and invertibility of the dynamical systems

LUNCH

14:30–15:30: Vladimir Peller (Michigan)
Functions of perturbed collections of commuting self-adjoint operators

15:35–16:35: Igor Krasovsky (Imperial College, London)
Asymptotic properties of Toeplitz matrices

COFFEE BREAK

17:00–18:00: Sergei Levin (St.Petersburg)
On the wave functions coordinate asymptotics for the three 3d charged quantum particles scattering problem

18:30 (TBC): BOAT TRIP

TUESDAY 3 July:

10:00–11:00: Igor Rodnianski (MIT)
L^2 curvature conjecture in General Relativity

COFFEE BREAK

11:30–12:30: Mihalis Dafermos (Cambridge)
Construction of dynamical vacuum black holes parametrized by scattering data

LUNCH

14:30–15:30: Hideo Tamura (Okayama)
Aharonov-Bohm effect in resonances of magnetic Schrödinger operators in two dimensions

15:35–16:35: Stephane Nonnenmacher (CEA Saclay)
Normally hyperbolic trapping: resonances and time evolution

COFFEE BREAK

17:05–17:30: Leander Geisinger (Princeton)
Semiclassical analysis of the Laplace operator with variable boundary conditions

17:35–18:00: Nikita Senik (St.Petersburg)
Homogenization for periodic elliptic second order differential operators in a strip

18:05–18:30: Andrei Smirnov (St.Petersburg)
Adiabatic dynamics generated by a Schrödinger operator

WEDNESDAY 4 July:

10:00–11:00: Tatiana Suslina (St.Petersburg)
Homogenization of the Dirichlet problem for elliptic systems with periodic coefficients

COFFEE BREAK

11:30–12:30: Rainer Hempel (Braunschweig)
Dislocation problems

LUNCH

14:30–15:30: Alexander Kiselev (Univ. of Wisconsin, Madison)
Biomixing by chemotaxis and enhancement of biological reactions

15:35–16:35: Stanislav Molchanov (UNC Charlotte)
On the spectral theory of Schrödinger operator on Dyson hierarchical lattice

COFFEE BREAK

17:05–17:30: Semyon Dyatlov (Berkeley)
Semiclassical limits of plane waves

17:35–18:00: Amru Hussein (Mainz)
Indefinite operators on bounded domains: Spectral asymptotics

18:05–18:30: Alexander Poretckii (St.Petersburg)
The Maxwell system in waveguides with several cylindrical ends

THURSDAY 5 July:

10:00–11:00: Svetlana Jitomirskaya (UC Irvine)
Duality and absolutely continuous spectrum for discrete multidimensional quasiperiodic operators at small couplings

COFFEE BREAK

11:30–12:30: Michael Aizenman (Princeton)
Resonant delocalization for random Schrödinger operators on tree graphs

LUNCH

14:30–15:30: Abel Klein (UC Irvine)
Bounds on the density of states for Schrödinger operators

15:35–16:35: Frederic Klopp (Paris, Jussieu)
Level statistics at the spectral edges

COFFEE BREAK

17:00–18:00: Robert Sims (Tuscon, Arizona)
Dynamical Localization in Disordered Quantum Spin Systems

18:30–22:00 (TBC): CONFERENCE DINNER

FRIDAY 6 July:

10:00–11:00: Nikolai Filonov (St.Petersburg)
Maxwell operator in the twentieth century

COFFEE BREAK

11:30–12:30: Shu Nakamura (Tokyo)
Microlocal properties of scattering matrices for Schrödinger equations on scattering manifolds

LUNCH

14:30–15:30: Alexander Volberg (Michigan)
Free boundary problems with arbitrary sign and David–Semmes problem from Geometric Measure Theory

15:35–16:35: Mouez Dimassi (Bordeaux)
Asymptotic expansions of the spectral shift function for Stark Hamiltonian

COFFEE BREAK

17:00–18:00: Mikhail Lyalinov (St.Petersburg)
Scattering of waves by a circular penetrable cone

Abstracts

Resonant Delocalization for Schrdinger Operators with Random Potential on Tree Graphs

Michael Aizenman

Princeton University

Joint work with Simone Warzel

We consider self adjoint operators of the form $H = T + V$ acting in the Hilbert space of square integrable functions on regular tree graphs, with T the graph incidence operator and V a random potential. Of particular interest is the existence and location of a “mobility edge”, which marks a transition between spectral regimes of pure point versus absolutely continuous spectra, where the unitary evolution generated by H exhibits different conductive and dynamical properties. We find that a mechanism of relevance is the formation of extended states through disorder enabled resonances, for which the exponential increase of the volume plays an essential role. For unbounded random potentials at weak disorder the extended states appear even well beyond the spectrum of the operators hopping term, including in a “Lifshitz tail regime” of very low density of states. For bounded potentials we show that at weak disorder there is no mobility edge in the form that was envisioned before. Throughout the ac spectrum the evolution is ballistic, a behavior which on hyperbolic graphs is also consistent with diffusion.

Construction of dynamical vacuum black holes parametrized by scattering data

Mihalis Dafermos

University of Cambridge

Joint work with Gustav Holzegel and Igor Rodnianski

I will discuss a construction of dynamic vacuum black holes (with smooth event horizon and complete null infinity) which asymptotes in time to the Kerr metric. The spacetimes are constructed by solving backwards an asymptotic characteristic initial value problem for the Einstein vacuum equations with prescribed ”scattering” data at the event horizon and null infinity. These are in fact the first examples of vacuum black hole spacetimes which do not coincide with the exactly stationary Kerr family.

Asymptotic expansions of the spectral shift function for Stark Hamiltonian

Mouez Dimassi

Universit Bordeaux 1

Joint work with S. Fuji

We study the asymptotic expansion of the spectral shift function for the perturbed Stark Hamiltonian. We give a weak and pointwise asymptotic expansions in powers of h of the derivative of the spectral shift function corresponding to the pair $\big(P = P_0 + V(x), P_0 = -h^2\Delta + x_1\big), x = (x_1, \cdots, x_n)$ where $V(x) \in \mathcal{C}^\infty(\mathbb{R}^n, \mathbb{R})$ is a decreasing function, $\mathcal{O}(|x|^{-\delta})$ for some $\delta > n$ and h is a small positive parameter. To prove the pointwise asymptotic expansion of the spectral shift function, we establish a new method based on a stationary approach.

Semiclassical limits of plane waves

Semyon Dyatlov

University of California

Joint work with Colin Guillarmou

On a complete non-compact Riemannian manifold which is either Euclidean or hyperbolic near infinity, we study microlocal convergence of distorted plane waves $E(z, \xi)$ as $z \to \infty$. Here z is the spectral parameter and ξ indicates the direction of the wave at infinity. The functions $E(z, \xi)$ are generalized eigenfunctions of the Laplacian, they are also known as Eisenstein functions in the hyperbolic setting. We show that if the trapped set has zero Liouville measure, then plane waves converge to a limiting measure, if we average in ξ and in $z \in [R, R+1]$. The rate of convergence is estimated in terms of the maximal expansion rate and classical escape rate of the geodesic flow, giving a negative power of z when the flow is Axiom A. As an application, we obtain expansions of local traces and of the scattering phase with fractal remainders.

Maxwell operator in the twentieth century

Nikolai Filonov

Steklov Institute, St.Peterburg

In this talk we describe briefly the theory of the Maxwell operator on compact manifolds. Maxwell operator can be defined in three following situations:

- in a domain in $\mathbb{R}^3$ (the "classical" case);
- in a domain in $\mathbb{R}^n$ in the framework of the theory of differential forms (Weyl);
- or on a manifold (Weck).

We discuss a correct definition of the Maxwell operator M for non-smooth cases. We consider the domain $\operatorname{Dom} M$ of the operator M and describe possible singularities of functions belonging to $\operatorname{Dom} M$ (Birman-Solomyak). We show that the imbedding $\operatorname{Dom} M \subset L_2$ is compact (Picard).

Finally, we give the results on asymptotics of the spectrum of M in the most general case.

Semi-classical Analysis of the Laplace Operator with variable Boundary Conditions

Leander Geisinger

Princeton University

Joint work with Rupert L. Frank

We consider the Laplacian $-\Delta$ and the fractional Laplacian $(-\Delta)^s$, $0 < s < 1$, on a domain and investigate the asymptotic behavior of the eigenvalues. Extending methods from semi-classical analysis we prove a two-term formula for the sum of the eigenvalues with the leading (Weyl) term given by the volume and the subleading term depending on properties of the boundary. These results are valid under very weak assumptions on the regularity of the boundary.

It is remarkable that, despite the fact that the fractional Laplacian is a non-local operator, even the second term has a local form depending only on the surface area of the boundary.

In the local case, $s = 1$, we show how the second term depends on different boundary conditions, including Dirichlet, Neumann and varying Robin conditions.

Dislocation problems in two dimensions

Rainer Hempel

TU Braunschweig

Joint work with M. Kohlmann, M. Stautz, and J. Voigt

We consider a simple quantum mechanical model in $\mathbf{R}^2$ for a grain boundary in a crystal or an alloy. Here the potential $V = V(x, y)$ is derived from two periodic potentials V_1 and V_2 on $\mathbf{R}^2$ by setting $V(x, y) := V_1(x, y)$, for $x > 0$, and $V(x, y) := V_2(x, y)$, for $x < 0$. We assume that there is a (non-trivial) interval (a, b) which is free of spectrum of the periodic Schrödinger operators $H_1 = -\Delta + V_1$ and $H_2 = -\Delta + V_2$, both acting in $L_2(\mathbf{R}^2)$. We are then interested in the spectral properties of $H = -\Delta + V$ inside (a, b). Special attention is given to the cases where V_2 is obtained from V_1 by a translation or by a rotation. We also study related problems on an infinite strip $\mathbf{R} \times (0, 1) \subset \mathbf{R}^2$ where the potentials V_1 and V_2 are not necessarily assumed to be periodic.

Indefinite operators on bounded domains

Amru Hussein

Johannes Gutenberg-Universität Mainz

Joint work with Vadim Kostrykin, David Krejcirik and Stephan Schmitz

For a not necessarily sign-definite coefficient matrix $A(x)$, a self-adjoint operator, formally given by the differential expression $-\operatorname{div} A(x)\nabla$ with Dirichlet boundary conditions, is constructed. The Weyl-type asymptotics for eigenvalues of this operator is discussed.

Duality and absolutely continuous spectrum for discrete multidimensional quasiperiodic operators at small couplings

Svetlana Jitomirskaya

UC Irvine

Report on a joint work in progress with J. Bourgain and L. Parnovsky

We prove a version of Aubry duality that works in multi-dimensional and multi-frequency case, and couple it with a result that, in certain situations, localization type estimates imply existence of absolutely continuous component in the integrated density of states. This allows to prove, in particular, the existence of an absolutely continuous component in the spectrum of a class of discrete multidimensional quasiperiodic operators at small coupling.

Biomixing by chemotaxis and enhancement of biological reactions

Alexander Kiselev

University of Wisconsin

It is well known that reaction rates can be strongly affected by the ambient fluid flow. The phenomenon where reaction rates in biology can be influenced by chemotaxis has been much less studied. I will describe a simple model motivated by studies of coral life cycle. The results suggest that chemotaxis can play a crucial role in some marine ecosystems.

Bounds on the density of states for Schrödinger operators

Abel Klein

University of California

Joint work with J. Bourgain

We establish bounds on the density of states of Schrödinger operators. These are deterministic results that do not require the existence of the integrated density of states. The results are stated in terms of a "density of states outer-measure" that always exists. We prove log-Hölder continuity for the density of states in one, two, and three dimensions for Schrödinger operators, and in any dimension for discrete Schrödinger operators.

Level statistics at the spectral edges

Frédéric Klopp

Universit Pierre et Marie Curie

The talk will be devoted to the description of recent results on the statistics of the eigenlevels and eigenfunctions of certain random operators near the edges, mainly the bottom, of the almost sure spectrum.

Asymptotic properties of Toeplitz matrices

Igor Krasovsky

Imperial College, London

Based on the joint work with P.Deift and A.Its

We review results on asymptotic properties of Toeplitz and Hankel determinants and Toeplitz matrices of large dimension. We discuss some applications of these results, in particular, to correlation functions in the 2D Ising model.

On the wave functions coordinate asymptotics for the three 3d charged quantum particles scattering problem

Sergei Levin

St.Petersburg State University

Joint work with V.S.Buslaev

The quantum system of two interacting charged particles is one of the most known explicitly solvable models of the Quantum Mechanics. On contrary, the mathematical understanding of the system of three quantum particles with the Coulomb pair interactions is relatively poor. The system of three particles with short-range pair interactions was successfully studied by L.D. Faddeev [1]. In the case of the Coulomb pair interactions, the direct generalization of his approach was found impossible. However, one knows the qualitative nature of the spectrum and the large time asymptotic behavior of solutions of the non-stationary Schrödinger equation. These results were obtained in the framework of a non-stationary approach, see [2-3]. Nevertheless, a mathematically consistent stationary approach was not developed yet. Such an approach is needed to analyze numerical parameters of many important physical processes, i.g., the dissociative recombination in the atomic and molecular physics, and also for applications to the astrophysics, to the medicine, to the chemical physics...

There are specific difficulties that are characteristic for the systems with Coulomb type interactions. They are naturally related to the fact that the long-range interactions crucially affect the asymptotic behavior at infinity (in the configuration space) of the eigenfunctions, Green's functions and other similar objects. Up to now, the consequences of the influence of the long-range interactions have not been understood in a correct mathematical manner.

These difficulties were discovered in the known work of John D. Dollard [6] who also proposed a way to regularize the wave operators in the case of the Coulomb type interaction. More general long-range potentials were considered later in [7], where also the elements of the stationary regularization approach were proposed.

The asymptotic behavior of the wave functions for the systems of few charged particles has been studied only in some sectors of the configuration space. Let us shortly list some known results. In [8-9], the asymptotic behavior of three charged particles wave function was studied in the case of large distances between all three particles. In [10], the authors considered another special case corresponding to the configurations where one of the Jacobi coordinates is much larger than the other.

Speaking about the analysis of theoretical aspects of the problem we mention also [11-17]. Computational aspects of the problem were discussed in [18-24].

Here, following [25], we propose explicit asymptotic formulas describing the behavior of the generalized eigenfunctions at infinity in the configuration space (up to the simple diverging waves with smooth amplitudes). Our anzatz satisfies the Schrödinger equation up to error terms quickly decaying at infinity uniformly in the angles. It can be used, say, for systematic numerical computations of the eigenfunctions.

References

[1] Faddeev L. D. *Trudy Matematicheskogo Instituta,* 69(1),1963 (in Russian); *Israel Program for Scientific Translations, Jerusalem,* 1965 (in English).

[2] Enss V. *Ann.Phys.,* 119:117-132, 1979.

[3] Derezinski J. and Gerard C. *J.Math.Phys.,* 38(8):3925-3942, 1997.

[4] Yafaev D. R. *Mathematical scattering theory. General theory.* (Amer.Math.Soc.,) 431, 1992.

[5] Alt E.O., Graßberger P. and Sandhas W. *Nucl. Phys. B* 2:167, 1967.

[6] Dollard J. *J.Math.Phys.,* 5:729, 1964.

[7] Buslaev V.S. and Matveev V.B. *Theor.Math.Phys.,* 2(3):367-376, 1970.

[8] Peterkop R.K. *Sov. Phys. JETP,* 14:1377, 1962.

[9] Merkuriev S.P. *Theor. Math. Phys.,* 32:680, 1977; Brauner M., Briggs J.S. and Klar H.J. *J. Phys. B,* 22:2265, 1989.

[10] Alt E.O. and Mukhamedzhanov A.M. *JETP Lett.,* 56: 435, 1992; *Phys. Rev. A,* 47:2004, 1994; Kim Y.E. and Zubarev A.L. *Phys. Rev. A,* 56:521, 1997.

[11] Macek J.H. and Ovchinnikov S.Yu. *Phys. Rev. A,* 54:1, 1996.

[12] Rudge M.R.H. *Rev. Mod. Phys.,* 40:564, 1968.

[13] Peterkop R.K. *Theory of Ionization of Atoms by Electron-Impact.* Colorado Associated University Press, Boulder, 1977.

[14] Faddeev L.D. and Merkuriev S.P. *Quantum Scattering Theory for Several Particle Systems.* Kluwer, Dordrecht, 1993.

[15] Merkuriev S.P. *Ann. Phys. (NY),* 130:395, 1980.

[16] Alt E.O., Sandhas W. and Ziegelmann H. *Phys. Rev. C,* 17:1981, 1978.

[17] Alt E.O., Levin S.B. and Yakovlev S.L. *Phys. Rev. C,* 69:034002, 2004.

[18] Alt E.O. and Sandhas W. In *Coulomb Interactions in Nuclear and Atomic Few-Body Collisions.* Plenum, New York, 1996.

[19] Oryu S., Nishinohara S., Shiiki N. and Chiba S. *Phys. Rev. C,* 75:021001(R), 2007.

[20] Deltuva A., Fonseca A.C. and Sauer P.U. *Phys. Rev. C,* 71:054005, 2005.

[21] Kievsky A., Viviani M. and Rosati S. *Phys. Rev. C,* 64:024002, 2001.

[22] Belyaev V.B., Levin S.B. and Yakovlev S.L. *J.Phys. B,* 37:1369-1380, 2004.

[23] Suslov V.M. and Vlahovic B. *Phys.Rev. C,* 69:044003, 2004.

[24] Rescigno T.N., Baertschy M., Isaacs W.A. and McCurdy C.W. *Science,* 286:2474, 1999; Baertschy M., Rescigno T.N. and McCurdy C.W. *Phys. Rev. A,* 64:022709, 2001; Bray I. *Phys. Rev. Lett.,* 89:273201, 2002.

[25] Buslaev V.S. and Levin S.B. *Functional analysis and its applications*, to appear.

Scattering of waves by a circular penetrable cone

Mikhail Lyalinov

St.Petersburg State University

The work is devoted to the diffraction of a plane wave by an acoustically transparent semi-infinite cone. The problem of diffraction is reduced to a singular integral equation in the framework of the incomplete separation of variables. The Fredholm property and regularization of the integral equation are discussed. Some important integral representations of the wave field are considered. On the basis of the detailed study of singularities of the integrands the far field asymptotics is then given by means of the asymptotic evaluation of the integrals. Expressions for the diffraction coefficient of the spherical wave scattered from the vertex of the cone are developed. The reflected from and transmitted across the conical surface waves are also discussed.

On the spectral theory of the Schrödinger operator on Dyson hierarchical lattice

Stanislav Molchanov

UNC Charlotte

Based on our joint work with B. Vainberg

A Dyson hierarchical lattice and the corresponding hierarchical Laplacian $-\triangle_h$ represent the simplest and in many ways typical fractal. The spectral properties of $-\triangle_h$ preserve all major features of the spectral theory of the nested fractals (similar to the Sierpinski gasket): presence of the eigenvalues of the infinite multiplicity, compactly supported eigenfunctions etc. The spectral dimension S_h, which depends on the two parameters of the model, can be an arbitrary positive number. If $S_h > 2$, the random walk with the generator $\triangle_h$ is transient, otherwise ($S_h \leq 2$) it is recurrent. The spectral theory of the negative spectrum for the Schrödinger operator of the form $H = -\triangle_h - V,\ V \geq 0,\ V \to 0$ as $x \to \infty$, for $S_h > 2$ contains the results similar to the classical estimates by Cwikel-Lieb-Rozeblum and Lieb-Tirring. The theory of $S_h \leq 2$ (recurrent case) and especially the study of the spectral bifurcations near $S_h \asymp 2$ require other methods.

The talk will present the spectral theory of $\triangle_h$ and several results concerning classical fractals (Sierpinski lattice, infinite Sierpinski gasket).

Microlocal properties of scattering matrices for Schrödinger equations on scattering manifolds

Shu Nakamura

Tokyo University

Joint work with K. Ito

We consider Schrödinger operators on a non-compact Riemannian manifold with asymptotically conic metric at infinity. We construct a time-dependent scattering theory for this system, and show that the scattering matrix is a Fourier integral operator associated to a geodesic flow on the boundary manifold (at infinity). This is a joint work with K. Ito [1], and the main result is a generalization of a result by Melrose and Zworski. I will also discuss recent generalization by S. Itozaki to more general manifolds.

References

[1] Ito, K., Nakamura, S. *Microlocal properties of scattering matrices for Schrödinger equations on scattering manifolds.* To appear in Analysis and PDE (arxiv.org/abs/1102.2009)

The spectrum of Fourier-Muckenhoupt multipliers

Nikolai Nikolski

Université Bordeaux 1

We discuss the problem of spectral localizations for Fourier multipliers on weighted L^2 spaces. Examples of weights in both senses are given (with- and without- localization). A part of the talk is based on the joint project with I.Verbitsky (Missouri-Columbia).

Normally hyperbolic trapping: resonances and time evolution

Stéphane Nonnenmacher

Institut de physique théorique, CEA-Saclay, France

Joint work with Maciej Zworski

We consider a semiclassical scattering system on the Euclidean space or more general smooth Riemannian manifolds, for which the corresponding classical Hamiltonian flow has the following dynamical property. In some positive energy range, the set K of trapped trajectories (trapped set) is a smooth symplectic submanifold of the phase space, and the flow transversely to K is *normally hyperbolic* (this means that the transverse directions split between stable and unstable directions).

Under this dynamical condition, Wunsch and Zworski had proved the presence of a *gap* in the resonance spectrum of the Hamiltonian operator $P(h)$. Using a refined analysis in the neighborhood of K, we provide an explicit lower bound for the gap, determined by the transverse Lyapunov exponents of the classical flow. This result generalizes a similar bound obtained in the '80s by Gérard and Sjöstrand in the analytic setting.

This formalism can be applied to more general settings than the Schrödinger equation. One of them is the linear wave equation on the Kerr-de Sitter spacetime; there resonances are replaced by quasinormal modes, as described recently by Dyatlov. Another application is to consider for $P(h)$ the (rescaled) vector field generating a contact Anosov flow on a compact manifold, as was done by Faure and Sjöstrand; the resonances are then equivalent with the Ruelle-Pollicott resonances of the classical flow.

In these two applications, the presence of this gap induces a splitting of the long time evolution into a constant term plus a remainder decaying exponentially in time, where the rate of decay is governed by the gap. In the Anosov case, we thus recover the recent results of Tsujii on the explicit rate of exponential decay of the correlations.

References

[1] Dyatlov S. *Quasi-normal modes and exponential energy decay for the Kerr–de Sitter black hole*, Commun. Math. Phys., 306:119-163, 2011.

[2] Faure F. and Sjöstrand J. *Upper Bound on the Density of Ruelle Resonances for Anosov Flows*, Commun. Math. Phys., 308:325-364, 2011.

[3] Gérard Ch. and Sjöstrand J. *Résonances en limite semiclassique*, Commun. Math. Phys. 116: 193–213, 1988.

[4] Tsujii M. *Contact Anosov flows and the FBI transform*, to appear in Ergod. Th. Dyn. Syst.

[5] Wunsch J. and Zworski M. *Resolvent estimates for normally hyperbolic trapped sets*, Ann. Inst. Henri Poincaré, 12: 1349–1385, 2011.

Functions of perturbed collections of commuting self-adjoint operators

Vladimir Peller

Michigan State University

Based on joint results with F. Nazarov

I am going to speak about properties of functions of perturbed tuples of commuting self-adjoint operators. In particular, I will discuss sharp conditions on a function f on the Euclidean space $\mathbb{R}^n$ to be operator Lipschitz, i.e., satisfy the inequality

$$\|f(A_1, \cdots, A_n) - f(B_1, \cdots, B_n)\| \leq \text{const} \max \|A_j - B_j\|.$$

I will also speak about operator Hölder functions of order α, $\alpha \in (0,1)$, and consider the case of perturbations of Schatten–von Neumann class $\boldsymbol{S}_p$.

The Maxwell system in waveguides with several cylindrical ends

Boris Plamenevskii and Aleksandr Poretckii

St. Petersburg State University

A waveguide occupies a domain G in $\mathbf{R}^3$ with smooth boundary ∂G having several cylindrical outlets to infinity ("cylindrical ends"). This means that outside a large ball centered at the origin the domain G coincides with the union of non-overlapping semi-cylinders $\Pi_+^1, \dots, \Pi_+^P$ and the cross-section Ω^p of Π_+^p is a bounded 2-dimensional domain. In G we consider the Maxwell system

$$\begin{aligned} i \operatorname{rot} u^2(x) - k u^1(x) &= f^1(x), \\ -i \operatorname{div} u^2(x) &= h^1(x), \\ -i \operatorname{rot} u^1(x) - k u^2(x) &= f^2(x), \\ i \operatorname{div} u^1(x) &= h^2(x) \end{aligned} \tag{1}$$

with boundary conditions

$$\nu(x) \times u^1(x) = 0, \ \langle u^2(x), \nu(x) \rangle = 0, \ x \in \partial G; \tag{2}$$

here u^1 and u^2 are electric and magnetic vectors, $k \in \mathbf{R}$ is a spectral parameter, and ν is the outward normal to ∂G. Let ρ_δ be a smooth positive function on $\bar{G}$ such that $\rho_\delta(x) = \exp \delta(|x|)$ on $\Pi_+^p \cap G$. Introduce the space $W_\delta^l(G)$ with norm

$$\|u; W_\delta^l(G)\| = \sum_{|\alpha|=0}^{l} \int_G |D^\alpha(\rho_\delta(x) u(x))|^2 \, dx$$

for $l = 0, 1, \dots$. Given k, denote by $kerM(k)$ the space of eigenfunctions in $L_2(G)$ of problem (1), (2) (the equality $kerM(k) = 0$ is not excluded); then $kerM(k) \subset W_\delta^l(G)$ with sufficiently small positive δ. The eigenvalues of problem (1), (2) are real, isolated, of finite multiplicity and can accumulate only at infinity.

Assume that k is not an eigenvalue, $\mathcal{F} = (f^1, h^1, f^2, h^2) \in W_\delta^{l-1}(G; \mathbf{C}^8)$, and the compatibility conditions

$$\begin{aligned} \operatorname{div} f^1(x) - ikh^2(x) = 0, \ x \in G, \\ \operatorname{div} f^2(x) + ikh^1(x) = 0, \ x \in G, \\ \langle f^2(x), \nu(x) \rangle = 0, \ x \in \partial G, \end{aligned}$$

are fulfilled. Then there exists a unique solution $\mathcal{U} = (u^1, u^2)$ of problem (1), (2) satisfying the radiation condition

$$\mathcal{V} := \mathcal{U} - c_1 w_1^- - \dots - c_N w_N^- \in W_\delta^l(G; \mathbf{C}^{\mathbf{6}}),$$

where w_j^- is an "outgoing wave", $c_j = c_j(\mathcal{F})$, $j = 1, \dots,$ are some functionals, the number $N = N(k)$ depends on k being constant on every interval of continuous spectrum between neighboring thresholds. Moreover, there holds the inequality

$$\|\mathcal{V}; W_\delta^l(G; \mathbf{C^6})\| + |c_1| + \cdots + |c_N| \le \text{const}\|\mathcal{F}; W_\delta^{l-1}(G; \mathbf{C^8})\|.$$

We also present a version of the above assertion for the case that k is an eigenvalue, describe the eigenfunctions of continuous spectrum, and introduce a unitary scattering matrix.

The study of problem (1), (2) begins with extension of the overdetermined Maxwell system to an elliptic one. As a result, there arises an elliptic boundary value problem self-adjoint with respect to a Green formula. Using general theory of elliptic boundary problems in waveguides we analyze the obtained problem and clarify its specific properties coming from the Maxwell system. Then we derive the information on the Maxwell system from that obtained about the elliptic one.

L^2 curvature conjecture in General Relativity

Igor Rodnianski

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Joint work with S. Klainerman and J. Szeftel

The talk will discuss a recent work concerning a local existence and uniqueness result or, alternatively a breakdown criterion, for the Einstein vacuum equations. The conjecture asserted that the equations are well-posed with the data of bounded L^2 curvature. Its proof requires a combination of geometric and analytic techniques including a choice of Cartan formalism and Yang-Mills gauge reducing the problem to a system of quasilinear hyperbolic equations with a hidden null structure, construction of a geometric parametrix, bilinear and trilinear estimates.

Homogenization for periodic elliptic second order differential operators in a strip

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Let $\Pi = \mathbb{R} \times (0, a)$ stand for a strip on $\mathbb{R}^2$. Given $\varepsilon > 0$ and $\lambda \in \mathbb{R}$, consider the differential expression

$$\mathcal{B}_\lambda^\varepsilon = \sum_{j=1}^{2} \mathrm{D}_j g_j\left(\frac{x_1}{\varepsilon}, x_2\right)\mathrm{D}_j + \sum_{j=1}^{2}\left(h_j\left(\frac{x_1}{\varepsilon}, x_2\right)\mathrm{D}_j + \mathrm{D}_j h_j^*\left(\frac{x_1}{\varepsilon}, x_2\right)\right) + Q\left(\frac{x_1}{\varepsilon}, x_2\right) + \lambda Q_*\left(\frac{x_1}{\varepsilon}, x_2\right). \tag{3}$$

Let ${}^P\mathcal{B}_\lambda^\varepsilon$, ${}^N\mathcal{B}_\lambda^\varepsilon$, ${}^D\mathcal{B}_\lambda^\varepsilon$ denote the operators in $L_2(\Pi)$ generated by the expression (3) with periodic, Neumann and Dirichlet boundary conditions, respectively. Below, ${}^\#\mathcal{B}_\lambda^\varepsilon$ is written instead of these operators.

All the coefficients in (3) are supposed to be periodic of period 1 with respect to the first variable. It is assumed that g_1, g_2 and Q_* are uniformly positive definite, bounded and belong to $\mathrm{Lip}\left([0,\, a]\,;\, L_\infty\left(0,\, 1\right)\right)$. Next, $h_j \in \mathrm{Lip}\left([0,\, a]\,;\, L_2\left(0,\, 1\right)\right)$, $j \in \{1,\, 2\}$, and $Q = \overline{Q} \in \mathrm{Lip}\left([0,\, a]\,;\, L_1\left(0,\, 1\right)\right)$. Besides, in the case of periodic boundary conditions, the traces of each coefficient on $\mathbb{R} \times \{0\}$ and $\mathbb{R} \times \{a\}$ coincide, and in the case of Neumann or Dirichlet boundary conditions, $h_2\left(x_1,\, 0\right) = h_2\left(x_1,\, a\right) = 0$ for a. e. $x_1 \in \mathbb{R}$. The parameter λ in (3) obeys the restriction $\lambda \geq \lambda_0$ which insures positive definiteness of the corresponding operator.

The aim is to approximate the operator $\left({}^{\#}\mathcal{B}_\lambda^\varepsilon\right)^{-1}$ for $\varepsilon \to 0$ in the sense of $\mathbf{B}\left(L_2\left(\Pi\right)\right)$- and $\mathbf{B}\left(L_2\left(\Pi\right),\, H^1\left(\Pi\right)\right)$-convergence. To formulate the results, introduce the effective operator ${}^{\#}\mathcal{B}_\lambda^0$, given by the expression

$$\mathcal{B}_\lambda^0 = \sum_{j=1}^{2} \mathrm{D}_j g_j^0\left(x_2\right) \mathrm{D}_j + \sum_{j=1}^{2} \left(h_j^0\left(x_2\right) \mathrm{D}_j + \mathrm{D}_j h_j^0\left(x_2\right)\right) + Q^0\left(x_2\right) + \lambda Q_*^0\left(x_2\right) \tag{4}$$

with the same boundary conditions as for ${}^{\#}\mathcal{B}_\lambda^\varepsilon$, and the corrector

$${}^{\#}\mathcal{K}_\lambda^\varepsilon = \left(\Lambda^\varepsilon \mathrm{D}_1 + \widetilde{\Lambda}^\varepsilon\right)\left({}^{\#}\mathcal{B}_\lambda^0\right)^{-1}. \tag{5}$$

The effective coefficients in (4) are defined as follows

$$\begin{gathered} g_1^0\left(x_2\right) = \left\langle g_1^{-1}\left(\cdot,\, x_2\right)\right\rangle^{-1}, \quad g_2^0\left(x_2\right) = \left\langle g_2\left(\cdot,\, x_2\right)\right\rangle, \\ h_1^0\left(x_2\right) = g_1^0\left(x_2\right) \Re \left\langle \frac{h_1\left(\cdot,\, x_2\right)}{g_1\left(\cdot,\, x_2\right)}\right\rangle, \quad h_2^0\left(x_2\right) = \left\langle h_2\left(\cdot,\, x_2\right)\right\rangle, \\ Q^0\left(x_2\right) = \left\langle Q\left(\cdot,\, x_2\right)\right\rangle - \left\langle \frac{\left|h_1\left(\cdot,\, x_2\right)\right|^2}{g_1\left(\cdot,\, x_2\right)}\right\rangle + g_1^0\left(x_2\right)\left|\left\langle \frac{h_1\left(\cdot,\, x_2\right)}{g_1\left(\cdot,\, x_2\right)}\right\rangle\right|^2, \\ Q_*^0\left(x_2\right) = \left\langle Q_*\left(\cdot,\, x_2\right)\right\rangle, \end{gathered}$$

and the mappings Λ and $\widetilde{\Lambda}$ are given by:

$$i\Lambda\left(x_1,\, x_2\right) = -\frac{1}{2} + x_1 - g_1^0\left(x_2\right) \int\limits_{(0,\,1)} \left\langle \chi_{\left(y_1,\, x_1\right)}\left(\cdot\right) g_1^{-1}\left(\cdot,\, x_2\right)\right\rangle dy_1,$$

$$i\widetilde{\Lambda}\left(x_1, x_2\right) = \int\limits_{(0,1)} \left\langle \chi_{\left(y_1, x_1\right)}\left(\cdot\right) \frac{\overline{h_1\left(\cdot, x_2\right)}}{g_1\left(\cdot, x_2\right)}\right\rangle dy_1 - g_1^0\left(x_2\right)\left\langle \frac{\overline{h_1\left(\cdot, x_2\right)}}{g_1\left(\cdot, x_2\right)}\right\rangle \int\limits_{(0,1)} \left\langle \chi_{\left(y_1, x_1\right)}\left(\cdot\right) g_1^{-1}\left(\cdot, x_2\right)\right\rangle dy_1.$$

Here, $\langle f\rangle$ denotes the mean value of a function f over $(0,\, 1)$.

Theorem. *Let ${}^{\#}\mathcal{B}_\lambda^\varepsilon$ stand for ${}^{P}\mathcal{B}_\lambda^\varepsilon$, ${}^{N}\mathcal{B}_\lambda^\varepsilon$ or ${}^{D}\mathcal{B}_\lambda^\varepsilon$. Let ${}^{\#}\mathcal{B}_\lambda^0$ be the effective operator and ${}^{\#}\mathcal{K}_\lambda^\varepsilon$ be the corrector for ${}^{\#}\mathcal{B}_\lambda^\varepsilon$. Then for any $0 < \varepsilon \leq 1$*

$$\left\|\left({}^{\#}\mathcal{B}_\lambda^\varepsilon\right)^{-1} - \left({}^{\#}\mathcal{B}_\lambda^0\right)^{-1}\right\|_{\mathbf{B}\left(L_2(\Pi)\right)} \leq C_1 \varepsilon, \tag{6}$$

$$\left\|\left({}^{\#}\mathcal{B}_\lambda^\varepsilon\right)^{-1} - \left({}^{\#}\mathcal{B}_\lambda^0\right)^{-1} - \varepsilon\left({}^{\#}\mathcal{K}_\lambda^\varepsilon\right)\right\|_{\mathbf{B}\left(L_2(\Pi),\, H^1(\Pi)\right)} \leq C_2 \varepsilon. \tag{7}$$

The estimates (6) and (7) are order-sharp. The constants C_1, C_2 depend only on the problem data: the norms of g_j, g_j^{-1}, $\partial_2 g_j$ and Q_*, Q_*^{-1}, $\partial_2 Q_*$ in $L_\infty(\Pi)$; the norms of h_j, $\partial_2 h_j$ in $L_\infty((0,\, a)\,;\, L_2(0,\, 1))$; the norms of Q, $\partial_2 Q$ in $L_\infty((0,\, a)\,;\, L_1(0,\, 1))$; and λ.

The inequality (6) for the case where $h_1 = h_2 = 0$, $Q = 0$, $\lambda Q_* = 1$, and boundary conditions are periodic has been proved in [1]. The proof of the current result is based upon the operator-theoretic (spectral) approach presented in [2] and is further development of the scheme suggested in [1].

References

[1] Suslina T. A. *On homogenization for a periodic elliptic operator in a strip*, St.Petersburg Math. J., 16(2):237-257, 2005.

[2] Suslina T. A. *Homogenization in the Sobolev class $H^1(\mathbb{R}^d)$ for periodic elliptic second order differential operators including first order terms*, St. Petersburg Math. J., 22(1):81-162, 2011.

Dynamical Localization in Disordered Quantum Spin Systems

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Joint work with Gunter Stolz and Eman Hamza

We say that a quantum spin system is dynamically localized if the time-evolution of local observables satisfies a zero-velocity Lieb-Robinson bound. In terms of this definition we have the following main results: First, for general systems with short range interactions, dynamical localization implies exponential decay of ground state correlations, up to an explicit correction. Second, the dynamical localization of random xy spin chains can be reduced to dynamical localization of an effective one-particle Hamiltonian. In particular, the isotropic xy chain in random exterior magnetic field is dynamically localized.

Adiabatic dynamics generated by a Schrödinger operator

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Consider the following Cauchy problem for a one-dimensional non-stationary Schrödinger operator:

$$\begin{aligned} &i\varepsilon\partial_t\Psi = A(t)\Psi = -\partial_{xx}\Psi + v(x,t)\Psi, \\ &\Psi\Big|_{t=0} = \Psi_0, \end{aligned} \qquad x\in\mathbb{R}, \quad t\in[0,T]. \tag{8}$$

We are interested in the asymptotic behavior of the solution in the limit $\varepsilon \to 0$.

Let the potential v be a smooth and non-negative function, and let v and all its derivatives with respect to x rapidly decay as $x \to \infty$. Assume in addition that the solution Ψ_{A_0} of the free Schrödinger equation,

$$\begin{aligned} &i\varepsilon\partial_t\Psi_{A_0} = A_0\Psi_{A_0} = -\partial_{xx}\Psi_{A_0}, \\ &\Psi_{A_0}\Big|_{t=0} = \Psi_0, \end{aligned} \qquad (9)$$

and all its derivatives with respect to x grow at infinity not faster, than a polynomial. Under these assumptions, we develop a recurrent procedure to construct the adiabatic asymptotics of the solution Ψ of (8). We prove that

$$\Psi \sim \Psi_{A_0} + \sum_{l=0}^{+\infty} \tilde{\Psi}_l,$$

where $\tilde{\Psi}_l$ are smooth functions of x, and

$$||\tilde{\Psi}_l||_{L_2} = O\big(\varepsilon^{\frac{l}{2}-1}\big), \qquad \big|\big|\Psi - \Psi_{A_0} - \sum_{l=0}^{L} \tilde{\Psi}_l\big|\big|_{L_2} = O\big(\varepsilon^{\frac{L+1}{2}-1}\big).$$

We also discuss two applications of our approach: we study the case where the initial condition is a rapidly decaying function and the case where it is an oscillating exponent. In the first case, the leading term of the asymptotics is determined by the potential only at the initial moment, and it behaves almost as for the solution of the problem (9). The second case is more intricate: at the distances of order of 1, the asymptotics is determined by the potential at the time moment under consideration, "far" from the origin the solution behaves as a free wave with a slowly varying amplitude, "very far" from the origin it behaves as in the case of $v = 0$. The transition between the last two regimes is described in terms of the Fresnel integral.

The author thanks V.Buslaev and A.Fedotov for many helpful discussions.

Homogenization of the Dirichlet problem for elliptic systems with periodic coefficients

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Let $\mathcal{O} \subset \mathbb{R}^d$ be a bounded domain of class $C^{1,1}$. Consider a matrix elliptic differential operator $A_\varepsilon = b(\mathbf{D})^* g(\mathbf{x}/\varepsilon) b(\mathbf{D})$, $\varepsilon > 0$. Here an $(m\times m)$-matrix-valued function $g(\mathbf{x})$ is bounded, uniformly positive definite and periodic with respect to some lattice Γ. The elementary cell of Γ is denoted by Ω. Next, $b(\mathbf{D}) = \sum_{j=1}^d b_j D_j$ is an $(m \times n)$-matrix first order differential operator (b_j are constant matrices). It is assumed that $m \geq n$ and the symbol $b(\boldsymbol{\xi}) = \sum_{j=1}^d b_j \xi_j$ has maximal rank.

We study the Dirichlet problem $A_\varepsilon \mathbf{u}_\varepsilon = \mathbf{F}$ in $\mathcal{O}$, $\mathbf{u}_\varepsilon|_{\partial\mathcal{O}} = 0$, where $\mathbf{F} \in L_2(\mathcal{O};\mathbb{C}^n)$. It turns out that $\mathbf{u}_\varepsilon$ converges in $L_2(\mathcal{O};\mathbb{C}^n)$ to $\mathbf{u}_0$, as $\varepsilon \to 0$. Here $\mathbf{u}_0$ is the solution of the "homogenized" problem $A^0\mathbf{u}_0 = \mathbf{F}$ in $\mathcal{O}$, $\mathbf{u}_0|_{\partial\mathcal{O}} = 0$, where $A^0 = b(\mathbf{D})^* g^0 b(\mathbf{D})$. The effective matrix g^0 is a constant positive $(m\times m)$-matrix defined as follows. Denote by $\Lambda(\mathbf{x})$ the $(n\times m)$-matrix-valued

Γ-periodic solution of the equation $b(\mathbf{D})^* g(\mathbf{x})(b(\mathbf{D})\Lambda(\mathbf{x}) + \mathbf{1}_m) = 0$ such that $\int_\Omega \Lambda(\mathbf{x})\, d\mathbf{x} = 0$. Then $g^0 = |\Omega|^{-1} \int_\Omega g(\mathbf{x})(b(\mathbf{D})\Lambda(\mathbf{x}) + \mathbf{1}_m)\, d\mathbf{x}$.

Theorem 1. (see [2]) *We have the following sharp order error estimate*:

$$\|\mathbf{u}_\varepsilon - \mathbf{u}_0\|_{L_2(\mathcal{O};\mathbb{C}^n)} \leq C\varepsilon \|\mathbf{F}\|_{L_2(\mathcal{O};\mathbb{C}^n)}. \tag{1}$$

Now we give approximation of $\mathbf{u}_\varepsilon$ in the Sobolev space $H^1(\mathcal{O};\mathbb{C}^n)$.

Theorem 2. (see [1]) 1) *Assume that $\Lambda \in L_\infty$. Let $\Lambda^\varepsilon(\mathbf{x}) := \Lambda(\varepsilon^{-1}\mathbf{x})$. Then*

$$\|\mathbf{u}_\varepsilon - \mathbf{u}_0 - \varepsilon\Lambda^\varepsilon b(\mathbf{D})\mathbf{u}_0\|_{H^1(\mathcal{O};\mathbb{C}^n)} \leq C\varepsilon^{1/2} \|\mathbf{F}\|_{L_2(\mathcal{O};\mathbb{C}^n)}. \tag{2}$$

2) *In the general case, we have*

$$\|\mathbf{u}_\varepsilon - \mathbf{u}_0 - \varepsilon\Lambda^\varepsilon b(\mathbf{D})(S_\varepsilon \widetilde{\mathbf{u}}_0)\|_{H^1(\mathcal{O};\mathbb{C}^n)} \leq C\varepsilon^{1/2} \|\mathbf{F}\|_{L_2(\mathcal{O};\mathbb{C}^n)}. \tag{3}$$

Here $\widetilde{\mathbf{u}}_0 = P_\mathcal{O}\mathbf{u}_0$ and $P_\mathcal{O} : H^2(\mathcal{O};\mathbb{C}^n) \to H^2(\mathbb{R}^d;\mathbb{C}^n)$ is a continuous extension operator; $(S_\varepsilon\mathbf{u})(\mathbf{x}) = |\Omega|^{-1}\int_\Omega \mathbf{u}(\mathbf{x} - \varepsilon\mathbf{z})\, d\mathbf{z}$.

We use the results of M. Birman and T. Suslina for homogenization problem in $\mathbb{R}^d$: the analogs of estimates (2), (3) in $\mathbb{R}^d$ are of sharp order ε. The problem is reduced to estimating of the discrepancy $\mathbf{w}_\varepsilon$, which is the solution of the problem $A_\varepsilon \mathbf{w}_\varepsilon = 0$ in $\mathcal{O}$, $\mathbf{w}_\varepsilon|_{\partial\mathcal{O}} = \varepsilon\Lambda^\varepsilon b(\mathbf{D})(S_\varepsilon\widetilde{\mathbf{u}}_0)|_{\partial\mathcal{O}}$ (or $\mathbf{w}_\varepsilon|_{\partial\mathcal{O}} = \varepsilon\Lambda^\varepsilon b(\mathbf{D})\mathbf{u}_0|_{\partial\mathcal{O}}$ in the case $\Lambda \in L_\infty$). We show that $\|\mathbf{w}_\varepsilon\|_{H^1} = O(\varepsilon^{1/2})$. This leads to (2), (3). At the same time, $\|\mathbf{w}_\varepsilon\|_{L_2} = O(\varepsilon)$, this allows us to prove sharp order estimate (1).

References

[1] Pakhnin M.A., Suslina T.A. *Operator error estimates for homogenization of the Dirichlet problem in a bounded domain.* Preprint, 2012, http://arxiv.org/abs/1201.2140.

[2] Suslina T.A. *Homogenization of the elliptic Dirichlet problem: operator error estimates in L_2.* Preprint, 2012, http://arxiv.org/abs/1201.2286.

Aharonov–Bohm effect in resonances of magnetic Schrödinger operators in two dimensions

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In quantum mechanics, a vector potential is said to have a direct significance to particles moving in a magnetic field. This quantum effect is known as the Aharonov–Bohm (AB) effect. We study the AB effect through resonances in a simple scattering system in two dimensions. The system consists of three scatterers, one bounded obstacle and two scalar potentials with compact supports at large separation, where the obstacle is placed between two supports and shields completely the support of a magnetic field. The field does not influence particles from a classical mechanical point of view, but quantum particles are influenced by the corresponding vector potential which does not necessarily vanish outside the obstacle. The resonances are shown to be generated near the real axis by the trajectories oscillating between two supports of the scalar potentials as the distances between the three scatterers go to infinity. The location is described in terms of the backward amplitudes for scattering by the two scalar potentials, and it depends heavily on the magnetic flux of the field. We also discuss what happens in the case of two obstacles. This system yields a two dimensional model of scattering by toroidal solenoids in three dimensions. The result depends on the location of the obstacles as well as on the fluxes.

Some of the obtained results are going to be published in [1].

References

[1] Tamura H. Aharonov–Bohm effect in resonances of magnetic Schrödinger operators in two dimensions, to appear in Kyoto Journal of Mathematics (2012).

Quasisimilarity and invertibility of the dynamical systems

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The notion of quasisimilarity in operator theory can be interpreted in the theory of dynamical systems asa weak form of isomorphisms up to Markov operator (=positive contraction). There problem is: what kind of non-invertible dynamics could be quasisimilar to what kind of invertible. The second part of the question now is known - only so-called Kolmogorov systems. The are lot of similar details in the theory of such questions with Foias-Nagy theory, spectral theory, and perhaps scattering theory.

Free boundary problems with arbitrary sign and David–Semmes problem from Geometric Measure Theory

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Newton capacity is responsible for the existence of non-constant bounded harmonic functions (they exist outside of the sets whose capacity is positive). What quantity is responsible for the existence of non-trivial harmonic function, which is Lipschitz? This is a much harder question called David-Semmes problem. This problem will be described along with related ones. They are all sort of free boundary problems, and I will present the idea of a recent solution obtained by F. Nazarov, X. Tolsa and myself.