The Langlands program and adelic theory

St.Petersburg, Russia, June 10–14, 2013

Anatoliy N. Andrianov. Interaction of symplectic and orthogonal Hecke–Shimura rings and zeta-functions

An automorphic structure on a Lie group consists of a Hecke–Shimura ring of an arithmetic discrete subgroup and a linear representation of this ring on an invariant space of automorphic forms given by Hecke operators. The talk is devoted to interactions, i.e., transfer homomorphisms, of Hecke–Shimura rings of integral symplectic groups and integral orthogonal groups of integral positive definite quadratic forms operating on theta-series of the quadratic forms and also to explicit relations between zeta-functions corresponding to common eigenfunctions of Hecke operators in the spaces of theta-series.

Nikolay V. Durov. On absolute tensor products of finite fields

Absolute tensor products of classical rings in the context of new generalized rings turn out to have an important universal property: if such a product is trivial in this theory, it will be automatically trivial in a large class of existing or speculative theories of "absolute tensor products" and "algebras over the field with one element" satisfying a natural set of axioms. After explaining this result, we are going to discuss examples related to absolute tensor products of finite fields. For example, $\mathbb{F}_3 \otimes \mathbb{F}_5$ turns out to be trivial, thus the computation of this absolute tensor product is an important test for any suggested theories of absolute tensor products.

Sergey O. Gorchinskiy. Adelic resolution for homology sheaves

We discuss a new type of resolutions, called adelic resolutions, for a certain class of abelian sheaves on algebraic varieties. This class includes sheaves of K-groups. Adelic resolutions are multiplicative and contravariant (in contrast with the Gersten resolution). There is an explicit quasiisomorphism between the adelic and Gersten resolutions. In particular, this allows to describe (higher) products on Chow groups and biextensions over Chow groups in terms of the adelic resolution.

Valery Gritsenko. Brumer's conjecture on abelian surfaces over $\mathbb{Q}$ and Borcherds products

The Siegel modular forms with respect to the paramodular group of level N (the integral symplectic group of the skew-symmetric form with the elementary divisors $(1, N)$) appear in the following conjecture of A. Brumer which predicts an explicit relation in the Langlands correspondence between L-functions of modular forms on groups of rank more than one and Hasse–Weil L-functions of some algebraic varieties.

The Brumer Paramodular Conjecture: There is a one-to-one correspondence between isogeny classes of rational abelian surfaces A of conductor N with $\mathrm{End}_{\mathbb{Q}}(A) = \mathbb{Z}$ and weight 2 newforms F on the paramodular group of level N with rational eigenvalues, not in the span of the Gritsenko lifts, such that $L(A, s) = L(F, s, \mathrm{spin})$.

In the talk we propose a method of construction of Siegel modular forms of weights 2 and 3 (weight 3 is the canonical weight) using the Borcherds automorphic products based on the so-called theta-blocks. We give a new general theorem about everywhere holomorphic Borcherds products, a first infinite series of identities between Borcherds products and Gritsenko's liftings for paramodular forms of weight 3 and the first example of anti-symmetric (i.e., very new) Hecke eigenforms of weight 2 for $p = 587$. This modular form is a good experimental object in order to check a particular case of the paramodular Brumer conjecture. The talk is based on my new joint results with C. Poor and D. Yuen.

Guy Henniart. On epipelagic representations and the Langlands correspondence

(joint work with C.J. Bushnell) Let F be a locally compact non-Archimedean field. Epipelagic representations of $\mathrm{GL}(n, F)$ are cuspidal representations of that group which have conductor exponent $n+1$: so they are wildly ramified, but minimally so. They are very easy to construct, but the Weil group representations corresponding to them via the Langlands correspondence are very hard to determine.

Jean-Pierre Labesse. The Morning Seminar revisited

The trace formula is one of the main tools to study automorphic forms. In particular, its twisted version is the key to establishing some important instances of Langlands functoriality principle. The extension to the twisted case of the work of Jim Arthur on the trace formula was the subject of the Friday Morning Seminar during academic year 1983–1984 at the IAS in Princeton. The informal notes of this seminar had to be rewritten. An account of this rewriting, that was not as straightforward as expected, is the subject of this talk.

Denis V. Osipov. Some aspects of the possible 2-dimensional Langlands correspondence.

In 1993, M. Kapranov asked a question: what should be a possible generalization of the Langlands correspondence for two-dimesional local fields and for two-dimensional arithmetic schemes. Recently, in 2012, A.N Parshin made a direct image conjecture on the connection between abelian two-dimensional Langlands correspondence and the classical one-dimensional Langlands correspondence.

In my talk, following an idea of Kapranov, I will explain the abelian case of the local two-dimensional Langlands correspondence. I will speak about my recent results: how to extend the construction from the above local case to the case of a global ring of two-dimensional Parshin–Beilinson adeles. I will prove non-commutative reciprocity laws on two-dimensional arithmetic schemes. These reciprocity laws correspond to unramified and tamely ramified extensions. I will also discuss the categorical construction of analogs of unramified principal series representations (for general linear groups over two-dimensional local fields) and its properties.

Alexey N. Parshin. Base change and automorphic induction in relative dimension 1

We consider a generalization of the Langlands program to fields of dimension 2 and define a corresponding version for 1-dimensional unramified Galois representations. A conjecture on direct image of automorphic forms is stated which links the Langlands correspondences in dimensions 2 and 1. In the geometric case of surfaces smoothly fibred over a curve the conjecture is shown to follow from Lafforgue's theorem on the existence of a global Langlands correspondence for curves defined over a finite field. The direct image conjecture also implies the classical Hasse–Weil conjecture on the analytic behaviour of the zeta- and L-functions of curves defined over a global field of dimension 1.

Nikolay V. Proskurin. On distribution of zeros of L-functions (numerical observation)

We report on a numerical computation of the zeros of some L-functions which admit no Euler product. We are interested mainly in the distribution of real parts of the zeros. The L-functions considered are the cubic L-function (i.e., the one attached to the Kubota–Patterson cubic theta function), the zeta-function of the ternary quadratic form $x^2 + y^2 + z^2$, the zeta-function of the Leech lattice.

Alexander L. Smirnov. The Arithmetic Hurwitz Formula, the Artin Conjecture and Big Zeta-functions.

There will be explained interrelations between the Artin conjecture on primitive roots and conjectural Hurwitz formula for maps from arithmetical curves to the projective line over the field with one element. It is planned to discuss big zeta-functions, related to these problems.

Informal discussion of the moduli problem in representation theory

Alexey N. Parshin. Introduction and examples (Rudakov–Shafarevich's theorem for the Lie algebras in positive characteristic, finite dimensional representations of discrete groups, infinite-dimensional representations of discrete Heisenberg groups)

Sergey O. Gorchinskiy. Stacks of irreducible smooth representations of reductive groups over local fields (moduli problem, existence of moduli stacks, relation to Bernstein components, expression of smooth representations in terms of quasi-coherent sheaves)