Russian Academy of Sciences
St.Petersburg Department of Steklov Mathematical Institute
The Euler International Mathematical Institute

St.Petersburg State University
Chebyshev Laboratory

Dynasty Foundation

Russian–Chinese Seminar on Asymptotic Methods in Probability Theory and Mathematical Statistics

St.Petersburg, 10-14 June, 2013

Programme and Abstracts

Welcome to St.Petersburg!

We are pleased to welcome you to the

Russian–Chinese Seminar on Asymptotic Methods in Probability Theory and Mathematical Statistics

in St.Petersburg from 10th to 14th June, 2013. The seminar is organized under the auspices of the **Euler International Mathematical Institute** at St.Petersburg Department of Steklov Mathematical Institute (Russian Academy of Sciences) and **Chebyshev Research Laboratory** at St.Petersburg State University. It was also co-sponsored by **Dynasty Foundation**.

We are very grateful for the generous support provided by these institutions.

The aim of the seminar is to bring together the scientists from Russian Federation and People Republic of China working in the domains of Probability and Mathematical Statistics, for joint work, exchange of ideas, and establishing long-lasting cooperation. This is the first meeting of this kind.

We hope you will enjoy this meeting and your time in St.Petersburg!

Organizing Committee

`http://www.pdmi.ras.ru/EIMI/2013/RChS/index.html`

Seminar Programme

Monday, June 10

09.00–10.00. Registration.
10.00–10.30. Opening of the seminar.
10.30–11.00. T1. **Ma Zhi-Ming.** Web Markov skeleton processes and their applications.
11.00–11.30. T2. **Ibragimov I.A.** On the Darmois – Skitovich and Ghurye – Olkin theorems.
11.30–11.45. *Coffee.*
11.45–12.15. T3. **Lin Zhengyan.** On convergence to stochastic integrals.
12.15–12.45. T4. **Chelkak D.S.** Double-sided estimates of hitting probabilities in discrete planar domains.
12.45–15.00. *Lunch.*
15.00–15.30. T5. **Su Zhonggen.** On fluctuations of deformed Wigner random matrices.
15.30–16.00. T6. **Tikhomirov A.N.** On the rate of convergence to the semi-circular law and Marchenko–Pastur law.
16.00–16.15. *Coffee.*
16.15–16.45. T7. **Li Yingying.** Statistical properties of microstructure noise.
16.45–17.15. T8. **Mogulskii A.A.** Large and moderate large deviations for general renewal processes.
17.15–17.30. *Break.*

17.30–18.00. T9. **Gushchin A.A.** Translation invariant statistical experiments with independent increments.
18.00–18.30. T10. **Jing Bing-Yi.** Spline LASSO in high-dimensional linear regression.

Tuesday, June 11

10.00–10.30. T11. **Nikitin Ya.Yu.** and **Volkova K.Yu.** Goodness-of-fit tests based on characterizations, and their efficiencies.
10.30–11.00. T12. **Vatutin V.A.** Decomposable branching processes in a Markovian random environment.
11.00–11.15. *Coffee.*
11.15–11.45. T13. **Wang Wensheng.** Invariance principles for the generalized domains of operator semistable attraction.
11.45–12.15. T14. **Shevtsova I.G.** Some moment inequalities and moment estimates for characteristic functions.
12.15–12.25. *Break.*
12.25–12.55. T15. **Eliseeva Yu.S.** and **Zaitsev A.Yu.** Estimates for the concentration functions in the Littlewood–Offord problem.
12.55–15.00. *Lunch.*
15.00–15.30. T16. **Wang Qihua.** Dimension reduction based linear surrogate variable approach for model free variable selection.
15.30–16.00. T17. **Bulinski A.V.** CLT for statistics employed in selection of the most significant factors.
16.00–16.15. *Coffee.*
16.15–16.45. T18. **Luo Shunlong.** Random walks: from classical to quantum.
16.45–17.15. T19. **Vysotsky V.V.** Random walks that avoid a bounded set.
17.15–17.30. *Break.*

Wenbo V. Li memorial session.

17.30–17.45. **Lifshits M.A.** Wenbo Li and St.Petersburg.
17.45–18.15. T20. **Shao Qi-Man.** Lower tail probabilities – In memory of Wenbo Li's contribution.
18.15–18.45. T21. **Nazarov A.I.** and **Pusev R.S.** The Wenbo Li comparison principle and its elaboration.

Wednesday, June 12

10.00–10.30. T22. **Li Xiang-Dong.** Entropy, from Boltzmann H-theorem to Perelman's W-formula for Ricci flow.
10.30–11.00. T23. **Gordin M.I.** Some limit theorems for von Mises statistics of a measure preserving transformation.
11.00–11.15. *Coffee.*
11.15–11.45. T24. **Bulinskaya E.Vl.** Hitting times under taboo for Markov chains.
11.45–12.15. T25. **Ulyanov V.V.** High-dimensional and large-sample approximations in statistics.
12.15–12.30 *Break.*
12.30–13.00. T26. **Mazalov V.V.** Detection of a change point in Bernoulli distribution and applications in computer networks.
13.00–15.00. *Lunch.*
15.00–15.30. T27. **Smorodina N.B.** The probabilistic approximation of the one-dimensional initial boundary value problem solution.
15.30–16.00. T28. **Dong Zhao.** Malliavin matrix of degenerate PDE and gradient estimates.
16.00–16.15. *Coffee.*
16.15–16.45. T29. **Morozov E.V.** Stability analysis of regenerative queues: recent results.
16.45–17.15. T30. **Wang Xue-Ren.** Statistical prediction on the coalbed methane resources in the EnHong basin area of Yunnan, China.
17.15–17.45. T31. **Gong Fuzhou.** Spectral gaps of Schrödinger operators and diffusion operators on abstract Wiener spaces.
17.45–18.00. *Break.*
18.00–18.30. T32. **Shashkin A.P.** Limit theorems for the measure of level sets of Gaussian random fields.
18.30–19.00. T33. **Zheng Xinghua.** Discrete fractal dimensions of the ranges of random walks in $\mathbb{Z}^d$ associated with random conductances.

Thursday, June 13

10.00–10.30. T34. **Borisov I.S.** Limit theorems for quantile empirical processes.
10.30–11.00. T35. **Wachtel V.** Potential analysis for positive recurrent Markov chains with asymptotically zero drift: power-type asymptotics.
11.00–11.15. *Coffee.*
11.15–11.45. T36. **Borodin A.N.** Diffusions with jumps.
11.45–12.15. T37. **Zhang Xinsheng.** Asymptotic properties for multipower variation of semimartingales and Gaussian integral processes with jumps.
12.15–12.25. *Break.*
12.25–12.55. T38. **Wang Hanchao.** Euler scheme for stochastic differential equations driven by semimartingales.
12.55–15.00. *Lunch.*
15.00–15.30. T39. **Alexeev N.V.** Distribution of the genomic distance.
15.30–16.00. T40. **Zhang Qiang.** The asymptotic behavior of Heston model for option prices with stochastic volatility.
16.00–16.15. *Coffee.*
16.15–16.45. T41. **Zhang Li-Xin.** Asymptotically efficient randomized adaptive designs with minimum selection bias.
16.45–17.15. T42. **Zaporozhets D.N.** Gaussian processes and intrinsic volumes.
TBA. T43. **Mao Yong-Hua**. On geometric and algebraic transience for discrete-time Markov chains.
18.00–19.30. *Boat Promenade.*
20.00– ... *Conference Dinner.*

Friday, June 14
Free Discussions

ABSTRACTS

Malliavin matrix of degenerate PDE and gradient estimates

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In this talk, we present the bound of the inverse for Malliavin matrix of degenerate PDE under a new condition, which is equivalent to the Hoermander condition as the coefficients are smooth. Also, the gradient estimates for the semigroup are given.

Spectral Gaps of Schrödinger Operators and Diffusion Operators on Abstract Wiener Spaces

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How to use the information of coefficients in partial differential operators to get the information of spectrum of the operators? There exists a long literature of studying this problem from theory of diffusion processes and partial differential equations, and there are a lot of interesting problems need to answer.

In this talk we extend the fundamental gap comparison theorem of Andrews and Clutterbuck to the infinite dimensional setting. More precisely, we proved that the fundamental gap of the Schrödinger operator $-\mathcal{L}_* + V$ ($\mathcal{L}_*$ is the Ornstein–Uhlenbeck operator) on the abstract Wiener space is greater than that of the one dimensional operator $-\frac{d^2}{ds^2} + s\frac{d}{ds} + \tilde{V}(s)$, provided that $\tilde{V}$ is a modulus of convexity for V. Similar result is established for the diffusion operator $-\mathcal{L}_* + \nabla F \cdot \nabla$. The main results are as follows.

Let (W, H, μ) be an abstract Wiener space and $\mathcal{L}_*$ the Ornstein–Uhlenbeck operator on W associated to the symmetric Dirichlet form $\mathcal{E}_*(f,f) = (f, -\mathcal{L}_* f)$ with domain $\mathcal{D}[\mathcal{E}_*] = D_1^2(W,\mu)$ (i.e. $f \in L^2(W,\mu)$ with its Malliavin derivative $\nabla f \in L^2(W,H)$). Let $V \in D_1^p(W,\mu)$ for some $p > 1$ be a potential satisfying the *KLMN condition*, then one can define $-\mathcal{L} = -\mathcal{L}_* + V$ to be a self-adjoint Schrödinger operator bounded from below.

Correspondingly, let $\tilde{\mathcal{L}}_* = \frac{d^2}{ds^2} - s\frac{d}{ds}$ be the one-dimensional Ornstein–Uhlenbeck operator on R^1 with respect to the Gaussian measure $d\gamma_1 = (4\pi)^{-\frac{1}{2}} \exp(-\frac{s^2}{4})\, ds$. Let $\tilde{V} \in C^1(R^1) \cap L^1(R^1, \gamma_1)$ be a symmetric potential satisfying the KLMN condition too. Then $-\tilde{\mathcal{L}} = -\tilde{\mathcal{L}}_* + \tilde{V}$ is bounded from below. For convenience, a tilde will be added to all notations relative to $\tilde{\mathcal{L}}_*$ and $\tilde{V}$.

Let $<.,.>_H$ denote the inner product in the Cameron–Martin space H, and $|.|_H$ the norm.

Theorem A: Suppose for almost all $w \in W$ and every $h \in H$ with $h \neq 0$,

$$\left\langle \nabla V(w+h) - \nabla V(w), \frac{h}{|h|_H} \right\rangle_H \geq 2\tilde{V}'\left(\frac{|h|_H}{2}\right).$$

Then there exists a comparison

$$\lambda_1 - \lambda_0 \geq \tilde{\lambda}_1 - \tilde{\lambda}_0.$$

Hence, the existence of the spectral gap of $-\mathcal{L}$ on Wiener space can sometimes be reduced to one dimensional case. According to Andrews and Clutterbuck's notion, $\tilde{V}$ is a modulus of convexity for V. However, V doesn't need to be convex at all.

The next result gives the modulus of log-concavity for the ground state ϕ_0 of $-\mathcal{L}$.

Theorem B: Assume the same condition as in Theorem A and the gap $\tilde{\lambda}_1 - \tilde{\lambda}_0 > 0$. Then $-\mathcal{L}$ and $-\tilde{\mathcal{L}}$ have a unique ground state ϕ_0 and $\tilde{\phi}_0$ respectively. Moreover, for almost all $w \in W$ and every $h \in H$ with $h \neq 0$,

$$\left\langle \nabla \log\phi_0(w+h) - \nabla \log\phi_0(w), \frac{h}{|h|_H} \right\rangle_H \leq 2(\log\tilde{\phi}_0)'\left(\frac{|h|_H}{2}\right).$$

We also consider the diffusion operator $-\mathcal{L} = -\mathcal{L}_* + \nabla F \cdot \nabla$ on the Wiener space and we want to compare its spectral gap with the one dimensional operator
$-\tilde{\mathcal{L}} = -\frac{d^2}{ds^2} + (s+\omega'(s))\frac{d}{ds}$. Although this kind of diffusion operator can be transformed to the Schrödinger type operator and their spectrum coincide with each other, the expression for the potential function V is a little complicated, hence it seems inappropriate to derive the gap comparison of diffusion operators from that of the transformed Schrödinger type operators. We shall directly establish the comparison theorem for spectral gaps of diffusion operators, and the main result is as follows.

Theorem C: Assume that $F \in D_1^p(W, R^1)$ satisfies $\int_W e^{-F}\, d\mu = 1$ and two functions F and ω are related by the following inequality: for all $h \in H$ and μ-a.e. $w \in W$,

$$\left\langle \nabla F(w+h) - \nabla F(w), \frac{h}{|h|_H} \right\rangle_H \geq 2\omega'\left(\frac{|h|_H}{2}\right).$$

Suppose also that $\omega \in C^1(R^1)$ is even, satisfying $\int_{R^1} e^{-\omega}\, d\gamma_1 = 1$ and $\lim_{t\to\infty}(\omega'(t)+t) = +\infty$. Then we have

$$\lambda_1 \geq \tilde{\lambda}_1.$$

Furthermore, the probabilistic proofs of fundamental gap conjecture and fundamental gap comparison theorem of Andrews and Clutterbuck were given by Fuzhou Gong and Dejun Luo most recently via the coupling by reflection of the diffusion processes.

Spline LASSO in high-dimensional linear regression

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We consider a linear regression problem in a high dimensional setting where covariates are ordered and the number of covariates p can be much larger than the sample size n. Under sparsity assumptions, we propose a Spline-LASSO approach. It has a number of advantages over its competitor, i.e., the fussed LASSO. First, it can preserve the shape of the parameter values much better. Secondly, it is computationally efficient, as it can be easily modified to use LARS algorithms. Simulations justify our findings. We also mention some possible applications.

Statistical properties of microstructure noise

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We study the estimation of moments and joint moments of microstructure noise. Estimators of arbitrary order of (joint) moments are provided, for which we establish consistency as well as central limit theorems. In particular, we provide estimators of auto-covariances and auto-correlations of the noise. Simulation studies demonstrate excellent performance of our estimators even in the presence of jumps and irregular observation times. Empirical studies reveal (moderate) positive auto-correlation of the noise for the stocks tested.

Entropy, from Boltzmann H-theorem to Perelman's W-formula for Ricci flow

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In 1872-1877, L. Boltzmann proved the famous H-theorem for the Boltzmann equation in the kinetic theory of gas and gave the statistical interpretation of the thermodynamic entropy. In 2002, G. Perelman introduced the notion of W-entropy and proved the W-entropy formula for the Ricci flow. This plays a crucial role in the proof of the no local collapsing theorem and in the final resolution of the Poincar é conjecture and Thurstons geometrization conjecture. In our previous work in 2007, inspired by Perelman's original work, we gave a probabilistic interpretation of the W-entropy using the Boltzmann–Shannon–Nash entropy. In this talk, we make some further efforts for a better understanding of the mysterious W-entropy by comparing the H-theorem for the Boltzmann equation and the Perelman W-entropy formula for the Ricci flow. We also suggest a way to construct the density of states measure for which the Boltzmann H-entropy is exactly the W-entropy for Ricci flow.

On convergence to stochastic integrals

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Weak convergence of various general functionals of partial sums of dependent random variables (statistics) to stochastic integrals now plays an important role in the modern statistical theory. In this work, we obtain the weak convergence of various general functionals of partial sums of causal processes to stochastic integrals driven by both the Brownian motion and Lévy α-stable process.

Random walks: from classical to quantum

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Random walks are fundamental and ubiquitous objects in probability theory, with important applications in many fields such as statistical physics, computer science, and information theory. While classical random walks have been long studied and constitute a well established subject, their quantum extensions, although dated at least back to Feynman [1] and Riazanov [2] for the initial ideas, are attracting wide interests only in the last decade [3, 4, 5]. The theory of quantum walks exhibits a much richer structure, due to quantum coherence and superposition, and the interplay between classical and quantum. In this talk, we present a concise review of quantum walks, including their basic features, spreading and decoherent effects, asymptotic properties, and in particular certain limit theorems.

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Web Markov skeleton processes and their applications

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Recently a new class of stochastic processes, Web Markov Skeleton Processes (WMSP), has been found to be very useful in the study of information retrieval on the Web. The framework of WMSP provides us a unified mathematical instrument applicable to most of the known algorithms for ranking web pages and websites. Moreover, it is essential in designing new algorithms to handle more complex problems. For example, mirror semi-Markov processes, a new class of processes in WMSP family, plays an essential role in designing MobileRank for computing page importance of mobile web. In our research we found that the framework of WMSP enjoys also many interesting theoretical properties by its own. In this talk I shall briefly review some of our work in this research direction. I shall introduce the notion of WMSP, compare it with the previous notion of Markov skeleton processes introduced and studied by Hou et.al, discuss the relation between WMSP and multivariate point processes, discuss in detail the properties of mirror semi-Markov processes. At the end of my talk I shall briefly explore the applications of WMSP in modeling user browsing behaviour on the web, including the roles of WMSP in the algorithms of PageRank, BrowseRank, and MobileRank.

On geometric and algebraic transience for discrete-time Markov chains

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General characterizations of ergodic Markov chains have been developed in considerable detail. In this paper, we study the transience for discrete-time Markov chains on general state spaces, including the geometric transience and algebraic transience.Criteria are presented through establishing the drift condition and considering the first return time. As an application, we give explicit criteria for the random walk on the half line and the skip-free chain on nonnegative integers.

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Lower tail probabilities - In memory of Wenbo Li's contribution

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This talk will give a brief survey on Wenbo Li's contribution on lower tail probabilities for Gaussian processes. Some recent developments and open questions will also be discussed.

On fluctuations of deformed Wigner random matrices

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Let X_n be a standard real symmetric (complex Hermitian) Wigner matrix, $y_1, y_2, \cdots, y_n$ a sequence of independent real random variables independent of X_n. Consider the deformed Wigner matrix $H_{n,\alpha} = \frac{X_n}{\sqrt{n}} + \frac{1}{n^{\alpha/2}} \mathrm{diag}(y_1, \cdots, y_n)$ where $0 < \alpha < 1$. It is well-known that the average spectral distribution is the classical Wigner semicircle law, i.e., the Stieltjes transform $m_{n,\alpha}(z)$ converges in probability to the corresponding Stieltjes transform $m(z)$. In this talk we shall report a recent work on the asymptotic estimate for the expectation $\mathbb{E}m_{n,\alpha}(z)$ and variance $Var(m_{n,\alpha}(z))$, and the central limit theorem for linear statistics with sufficiently regular test function. The focus is upon the understanding of effect of perturbation matrix on precision of estimation and normalizing factor. A basic tool in the study is Stein's equation and its generalization which naturally leads to a certain recursive equation.

Euler scheme for stochastic differential equations driven by semimartingales

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We propose the asymptotic error distributions of the Euler scheme for a stochastic differential equation driven by semimartingales. We extend Jacod's results on Lévy process to general semimartingale.

Dimension reduction based linear surrogate variable approach for model free variable selection

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Most of variable selection methods depend on the model assumptions, while sufficient dimension reduction is a nonparametric method to deal with high dimensional data. In this paper we aim at integrating sufficient dimension reduction into variable selection. A two stage procedure is proposed. First, we obtain dimension reduction directions and integrate them to construct a variable which is linearly dependent on predictors. Then by treating this constructed variable as a new response, we use the traditional variable selection methods such as adaptive LASSO to conduct variable selection. We call such a procedure as dimension reduction based linear surrogate variable (LSV) method. To illustrate that it has wide application, we also apply it to variable selection for the problem of missing responses. Extensive simulation studies show that it is more robust than the variable selection methods depending on model assumptions, and more efficient than the other model-free variable selection methods. Another advantage of the LSV is that it can be easily implemented. A real example is given to illustrate the proposed method.

References

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Invariance principles for the generalized domains of operator semistable attraction

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Let $X, X_1, X_2, ...$ be independent and identically distributed $\mathbb{R}^d$-valued random vectors and assume X belongs to the generalized domain of attraction of some operator semistable law without normal component. Then without changing its distribution, one can redefine the sequence on a new probability space such that the properly affine normalized partial sums converge in probability and consequently even in L^p (for some $p > 0$) to the corresponding operator semistable Lévy motion.

Statistical prediction on the coalbed methane resources in the EnHong basin area of Yunnan, China

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Development and utility of coalbed methane (CBM) in China and abroad were described. In this study, about 784 kilometer square and the available resource of CBM approximately occupied 10of three CBM basin. The CBM spatial database was created and many geological statistical methods were used to simulate and predict the distribution of CBM and optimize synergetic gas-enrichment fields. Characteristic results of CBM concentrations from eight regions in Enhong mining area were achieved.

Asymptotically efficient randomized adaptive designs with minimum selection bias

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Response-adaptive randomized designs for allocating patients in clinical trials have recently attracted a lot of attention in the literature. The adoption of them has proved to be beneficial to researchers, by providing more efficient clinical trials, and to patients, by increasing the likelihood of receiving the better treatment. In this talk, we will introduce several kinds of them from the intuition-driven one to the asymptotically sufficient and optimal one. Main important components of response-adaptive designs including efficiency (power), variability and selection bias will be discussed. At last, a new family of designs will be proposed. This kind of designs can be used to target any specified allocation proportion of patients in clinical trials and have both the lowest asymptotical variability and the lowest selection bias.

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The asymptotic behavior of Heston model for option prices with stochastic volatility

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The well-known Heston model for stochastic volatility captures the reality of the movement of stock prices in our financial market. It models not only the stock price, but also the volatility as stochastic processes. However, it has several difficulties in numerical evaluation of the option prices for this stochastic process with stochastic volatility. We have performed asymptotic analysis of this stochastic process and derived a closed-form asymptotic solution for option prices under the Heston model of stochastic volatility. Our theoretical predictions are in surprisingly good agreement with the numerical results of the Heston model of stochastic volatility.

Asymptotic properties for multipower variation of semimartingales and Gaussian integral processes with jumps

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In this talk, we present some limit theorems of realized multipower variation for semimartingales and Gaussian integral processes with jumps observed in high frequency. In particular, we obtain a central limit theorem of realized multipower variation for semimartingale where some of the powers equal one and the others are less one. Convergence in probability and central limit theorems of realized threshold bipower variation for Gaussian integral processes with jumps are also obtained. These results provide new statistical tools to analyze and test the long memory effect in high frequency situation.

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Discrete fractal dimensions of the ranges of random walks in $\mathbb{Z}^d$ associated with random conductances

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Let $X = \{X_t, t \geq 0\}$ be a continuous time random walk in an environment of i.i.d. random conductances $\{\mu_e \in [1, \infty), e \in E_d\}$, where E_d is the set of non-oriented nearest neighbor bonds on the Euclidean lattice $\mathbb{Z}^d$ and $d \geq 3$. Let $\mathrm{R} = \{x \in \mathbb{Z}^d : X_t = x \text{ for some } t \geq 0\}$ be the range of X. It is proved that, for almost every realization of the environment, $\dim_{\mathrm{H}} \mathrm{R} = \dim_{\mathrm{P}} \mathrm{R} = 2$ almost surely, where $\dim_{\mathrm{H}}$ and $\dim_{\mathrm{P}}$ denote respectively the discrete Hausdorff and packing dimension. Furthermore, given any set $A \subseteq \mathbb{Z}^d$, a criterion for A to be hit by X_t for arbitrarily large $t > 0$ is given in terms of $\dim_{\mathrm{H}} A$. Similar results for Bouchoud's trap model in $\mathbb{Z}^d$ $(d \geq 3)$ are also proven.

Distribution of the genomic distance

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Let us consider two genomes with the same set of n genes, and the only difference between them is the order of genes in the genomes. We are interested in the number of rearrangements, which are needed to get the first genome from the second one. We call the minimal number of such rearrangements *a distance* on a set of genomes. To estimate the distance between two genomes one has to represent these genomes as permutations. Namely, if the first genom is *abcd* and the second is *bcad*, then it is coded by the permutation $\pi = (2314)$. So, instead of computing the distance between two genomes one can compute the distance between a permutation π and the *id* permutation. We compute the number of permutations of length n on the distance k from the *id* permutation and obtain the recurrence equation for this sequence. We also prove, that if one chooses a permutation randomly and n goes to infinity, then the the distance k has an expectation of order $n - \ln(n)$, the variance of order $\ln(n)$ and the random variable $\frac{k-(n-\ln(n))}{\sqrt{\ln(n)}}$ weakly converges to a standard Gaussian random variable.

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Limit theorems for quantile empirical processes

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Let $X_1, X_2, \ldots$ be a stationary sequence of weakly dependent random variables. Let $X_{1:n} \le \ldots \le X_{n:n}$ be the corresponding order statistics. Consider the following two empirical distribution functions:

$$F_n(t) := \frac{1}{n}\sum_{i=1}^{n} \mathbb{I}(X_i < t),$$

$$E_n(t) := \frac{1}{n}\sum_{i=1}^{n} \mathbb{I}(\mathbb{E}X_{i:n} < t) \ \text{ if } \ \mathbb{E}|X_1| < \infty.$$

In the iid case the function $E_n(t)$ was introduced by W. Hoeffding in [1] where it was proved that the distributions corresponding to $\{E_n(\cdot)\}$ weakly converge to the distribution of X_1.

We study limit behavior of the quantile processes $F_n^{-1}(t)$ and $E_n^{-1}(t)$ defined on the unit interval, as well as of some functionals of the processes under consideration.

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Diffusions with jumps

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In the talk we consider a wide class of homogeneous diffusions with jumps. The extreme points of this class are homogeneous diffusion processes and the Poisson processes with variable intensity. The diffusions with jumps have many good properties inherited both from classical diffusion processes and from Poisson ones. This class is closed with respect to a composition with the invertible twice continuously differentiable functions. A special random time change gives us again a diffusion with jumps. A result on transformation of the measure of the process analogous to Girsanov's transformation is valid for this class. There are an effective results for the computation of distributions of certain functionals of diffusions with jumps.

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Hitting times under taboo for Markov chains

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We consider an irreducible Markov chain $\xi = \{\xi(t), t \geq 0\}$ generated by Q-matrix $A = (a(x,y))_{x,y\in S}$ where S is a finite or denumerable set. For $x \in S$, let $\tau_x := \inf\{t \geq 0 : \xi(t) \neq x\}$ on the set $\{\xi(0) = x\}$ and $\tau_x = 0$ otherwise. The stopping time τ_x (w.r.t. the natural filtration of the process ξ) is *the first exit time from* x and $\mathbb{P}(\tau_x \leq t|\xi(0) = x) = 1 - e^{a(x,x)t}$, $t \geq 0$, where $a(x,x) \in (-\infty, 0)$. Following Chung's notation in [1], Ch. 2, Sec. 11, for an arbitrary, possibly empty set $H \subseteq S$ called henceforth the *taboo set* and for $t \geq 0$, denote by

$$_Hp_{x,y}(t) := \mathbb{P}(\xi(t) = y,\ \xi(u) \notin H,\ \min[\tau_x, t] < u < t|\xi(0) = x), \quad x, y \in S,$$

the *transition probability from* x *to* y *in time* t *under the taboo* H. In the case $H = \varnothing$ the function $p_{x,y}(\cdot) =_\varnothing p_{x,y}(\cdot)$ is an ordinary transition probability. Note that $_Hp_{x,y}(\cdot) \equiv 0$ for $x \neq y$ and $y \in H$ whereas $_Hp_{x,x}(t) = e^{a(x,x)t}$ for $x \in H$ and $_Hp_{x,x}(t) \geq e^{a(x,x)t}$ for $x \notin H$, $t \geq 0$. Set

$$_H\tau_{x,y} := \mathbb{I}\{\xi(0) = x\} \inf\{t \geq \tau_x : \xi(t) = y,\ \xi(u) \notin H,\ \tau_x < u < t\}, \quad x, y \in S,$$

where, as usual, we assume that $\inf\{t \in \varnothing\} = \infty$. The stopping time $_H\tau_{x,y}$ is *the first entrance time from* x *to* y *under the taboo* H whenever $x \neq y$ and is *the first return time to* x *under the taboo* H when $x = y$. For $H = \varnothing$ the random variable $\tau_{x,y} =_\varnothing \tau_{x,y}$ is just *first entrance time from* x *to* y (or *first return time to* x *whenever* $x = y$). The moments $_H\tau_{x,y}$, $x, y \in S$, are also called *hitting times* or *first passage times under the taboo* H. Let $_HF_{x,y}(t) := \mathbb{P}(_H\tau_{x,y} \leq t|\xi(0) = x)$, $t \geq 0$, be (improper) c.d.f. of $_H\tau_{x,y}$.

Introduction of taboo probabilities and hitting times under taboo is a powerful tool for establishing results relating to study of functionals of Markov chains (see, e.g., [1], Ch. 2, Sec. 14), potential theory of Markov chains (see, e.g., [2], Ch. 4, Sec. 6), matrix analytic methods in stochastic modeling (see, e.g., [3], Ch. 3, Sec. 5) and many other domains of Markov chains research. Our interest in hitting times under taboo was motivated by their application to analysis of catalytic branching processes with finitely many catalysts. For a single catalyst, the model was proposed in [4]. We extend the results of [4] to the case of any finite number of catalysts. To this end we introduce an auxiliary multi-type Bellman-Harris branching process with the help of hitting times under taboo. Then we may reduce the study of catalytic branching processes to the investigation of such Bellman-Harris processes.

On this way we obtain and afterwards employ an explicit formula for $_HF_{x,y}(\infty)$, $x, y \in S$, $H \subseteq S$, via taboo probabilities $\int_0^\infty {_Hp_{x,y}(t)}\,dt \in (0,\infty)$ for any nonempty set H. We also express $_HF_{x,y}(\infty)$ in terms of $_{H'}F_{x',y'}(\infty)$ with appropriate choice of a collection of states $x', y' \in S$ and a certain set H' such that $H' \subset H$ or $H \subset H'$. Thus, for a finite nonempty set H, the evaluation of $_HF_{x,y}(\infty)$ can be reduced to the case when H consists of a single point. At last, we derive another useful representation of $_zF_{x,y}(\infty)$, $z \in S$, for transient Markov chains and compare it with that established in [5] for a certain class of recurrent Markov chains.

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CLT for statistics employed in selection of the most significant factors

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The problem of identification of the most significant factors which determine the behavior of a response variable is not only of theoretical interest, being motivated by applications. Actually, such problem arises, e.g., in medical and biological studies. There the response variable can characterize the health state of a patient and factors represent the genetic (a collection of single-nucleotide polymorphisms) and non-genetic (or environmental) data, see, for example [1] and references therein. A widely used approach to the mentioned problem is based on the multifactor dimensionality reduction (MDR) method introduced by M.Rithchie et al. [2]. This method was developed in a number of papers, in this regard we refer, e.g., to [3]. Recently in [4] the basis for employment of the MDR method with arbitrary penalty function was provided. Namely, the necessary and sufficient conditions were found to guarantee the strong consistency of statistics used to analyze the relevant prediction error for response variable. Moreover, the choice of the penalty function proposed in [5] is clarified. In this talk we consider the regularization of statistics introduced in [4] which permits to prove for them the central limit theorem. The main difficulty here is due to the cross-validation procedure. We also discuss the importance measures for various collections of factors, see, e.g., [6].

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Double-sided estimates of hitting probabilities in discrete planar domains

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In [1], a number of uniform double-sided estimates relating discrete counterparts of several classical conformal invariants of a quadrilateral (cross-ratios, extremal lengths and random walk partition functions) were presented. These estimates hold true for any simply connected discrete domain (subset of a “roughly regular” planar graph, e.g., the standard square grid) with four marked boundary vertices, and are completely independent of the domain geometry which can be very rough, having many fiords and bottlenecks of various widths. This allows one to use classical methods of geometric complex analysis for discrete domains “staying on the microscopic level”. In this talk we focus on a uniform discrete version of the classical Ahlfors-Beurling-Carleman estimate: the harmonic measure (hitting probability) of a boundary arc is exponentially small if the discrete extremal length to this arc is big.

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Estimates for the concentration functions in the Littlewood–Offord problem

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Let $X_1, \ldots, X_n$ be independent identically distributed random variables. This talk deals with the behavior of the concentration functions of the weighted sums $\sum_{k=1}^{n} a_k X_k$ with respect to the arithmetic structure of coefficients a_k. Such concentration results recently became important in connection with investigations about singular values of random matrices. We formulate some refinements of results of [1–3] which are proved in the recent papers [4–6]. In [1] and [2] we consider the one-dimensional case. The paper [3] is devoted to the multivariate situation with $a_k \in \mathbf{R}^d$.

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Some limit theorems for von Mises statistics of a measure preserving transformation

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For a measure preserving transformation T of a probability space $(X, \mathcal{F}, \mu)$ we investigate almost sure and distributional convergence of random variables of the form

$$x \to \frac{1}{C_n} \sum_{i_1<n,\dots,i_d<n} f(T^{i_1}x, ..., T^{i_d}x),\ n = 1, 2, \dots,$$

where f (called the *kernel*) is a function from X^d to $\mathbb{R}$ and $C_1, C_2, \dots$ are appropriate normalizing constants. It was observed by Manfred Denker and the speaker in the paper [1] that the above random variables are well defined and belong to $L_r(\mu)$ provided that the kernel is chosen from the projective tensor product

$$L_p(X_1, \mathcal{F}_1, \mu_1) \otimes_\pi ... \otimes_\pi L_p(X_d, \mathcal{F}_d, \mu_d) \subset L_p(\mu^d)$$

with $p = d\,r$, $r \in [1, \infty)$. Starting with this observation, a form of the individual ergodic theorem for such sequences is established in [1]. Next, assuming that the space is furnished with a filtration compatible with the transformation T, a martingale approximation argument is given in the paper to derive a central limit theorem in the non-degenerate case (in the sense of classical Hoeffding's decomposition). Furthermore, for $d = 2$ and a wide class of canonical kernels f we also show that the convergence holds in distribution towards a quadratic form $\sum_{m=1}^{\infty} \lambda_m \eta_m^2$ in independent standard Gaussian variables $\eta_1, \eta_2, \dots$.
In our talk we are going to briefly review the results in [1]. Some further development and related open problems will be also discussed.

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Translation invariant statistical experiments with independent increments

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Translation invariance is a typical property of limiting statistical experiments. The notion of a statistical experiment with independent increments was introduced by Strasser [2]. Such models often appear as limits in non-regular cases. Examples of statistical models where limiting experiments are translation invariant and have independent increments are numerous. For instance, one can mention i.i.d. observations with densities having jumps along some smooth lines and smooth elsewhere, different change-point type models, threshold autoregressive processes. In this talk we provide a full description of translation invariant models with independent increments and a large deviation result for the posterior distribution (and, as a consequence, for Pitman estimators) in these models. A special attention is paid to weak convergence inside this class of models. In particular, we extend recent results by Dachian and Negri [1] to a larger class of models.

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On the Darmois – Skitovich and Ghurye – Olkin theorems.

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1. Consider independent random variables $X_1, \ldots X_n$ and two linear statistics

$$L_1 = \sum_1^n a_j X_j, \quad L_2 = \sum_1^n b_j X_j,$$

where $a_j, b_j \neq 0$ are real coefficients. The Darmois (1953), Skitovich (1953) theorem says that if the statistics L_1, L_2 are independent, the random variables X_j have normal distributions.

The theorem has been extended by Mamai (1960) and Ramachandran (1967) on the case of an infinite number of variables X_j. The Ramachandran theorem says that if the forms L_1, L_2 are independent and both sequences $\{a_j b_j^{-1}\}, \{a_j^{-1} b_j\}$ are bounded, the variables X_j are normal.

Theorem 1 *Let $X_1, \ldots X_n, \ldots$ be independent random variables. If the forms*

$$L_1 = \sum_1^\infty a_j X_j, \quad L_2 = \sum_1^\infty b_j X_j, \quad a_j, b_j \neq 0$$

are independent and at least one of the sequences $\{a_j b_j^{-1}\}, \{a_j^{-1} b_j\}$ is bounded, the random variables X_j are normal.

2. Ghurye and Olkin (1962) proved a multivariate analogue of the Darmois - Skitovich theorem. They considered linear statistics

$$L_1 = \sum_1^n A_j X_j, \quad L_2 = \sum_1^n B_j X_j,$$

where now X_j are d-dimensional independent random vectors and A_j, B_j are non–singular $d \times d$ real matrices. Ghurye and Olkin proved that the independence of L_1, L_2 implies the normality of the vectors X_j. Later A. Zinger answering a question rased in the Kagan - Linnik -Rao book generalized the Ghurye - Olkin theorem. He considered the forms of infinite numbers of random vectors and proved that if the forms are independent and both sequences $\{A_i B_i^{-1}\}, \{B_i A_i^{-1}\}$ are bounded, the random vectors X_n are independent.

Theorem 2 *Let $X_1, \ldots, X_n, \ldots$ be independent d-dimensional random vectors. Consider two linear statistics*

$$L_1 = \sum_1^\infty A_j X_j, \quad L_2 = \sum_1^\infty B_j X_j,$$

where A_j, B_j are non-singular $d \times d$ matrices. If the forms L_1, L_2 are independent and
1. at least one of the sequences $\{A_j^{-1} B_j\}, \{A_j B_j^{-1}\}$ is bounded;
2. the sequence $||A_i B_i^{-1}|| \cdot ||B_i A_i^{-1}||$ is bounded,
then the random vectors X_n are normal.

Detection of a change point in Bernoulli distribution and applications in computer networks

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We consider a problem of detection of a change point in Bernoulli distribution of a stochastic sequence describing the flow of jobs arriving to a server. A change point may indicate a change in significant characteristics of the job flow, such as occurrence of a large number of artificially generated jobs aimed at compromising server's availability (denial-of-service attack). We apply the CUSUM method to detect the change and present the explicit solutions for the mean number of observations between false alarms and the mean delay before detecting a change. We propose new protocol of service in the environment of regular and additional flows.

Large and moderate large deviations for general renewal processes

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Let $\{(\tau_i, \xi_i);\ i = 1, 2, \cdots\}$ be a sequence of i.i.d. random vectors, $\mathbf{P}(\tau_1 > 0) = 1$. Put

$$T_n := \tau_1 + \cdots + \tau_n, \quad S_n := \xi_1 + \cdots + \xi_n \quad \text{for} \quad n \geq 1.$$

We study large deviations and moderate large deviations for the general renewal process

$$Z(t) := S_{\eta(t)}, \quad t \geq 0,$$

where $\eta(t) := \min\{m \geq 0 :\ T_{m+1} \geq t\}$. The formula

$$\lim_{\varepsilon \to 0} \lim_{T \to \infty} \frac{1}{T} \ln \mathbf{P}(\frac{1}{T} Z(T) \in (\alpha - \varepsilon,\ \alpha + \varepsilon)) = -G(\alpha),$$

where the function $G(\alpha)$ is known in an explicit form, is proposed. Put

$$z_T(t) := \frac{1}{x} Z(tT), \quad 0 \leq t \leq 1,$$

where a function $x = x(T) > 0$ is such that $x \sim T$ as $T \to \infty$. Large deviation principle for $\{z_T(\cdot);\ T > 0\}$ was obtained:

$$\ln \mathbf{P}(z_T(\cdot) \in B) \sim -T \inf_{f \in B} I(f),$$

where $I(f) := \int_0^1 G(f'(t))dt$ for absolutely continuous functions, $f(0) = 0$,and B is a sufficiently wide class of functions $f = f(t);\ 0 \leq t \leq 1$.

In the lattice case $\mathbf{P}((\tau_1, \xi_1) \in \mathbb{Z}^2) = 1$, the sharp asymptotics of large deviation probabilities for $Z(n)$ is proposed:

$$\mathbf{P}(Z(n) = k) \sim \frac{C(\alpha)}{\sqrt{n}} e^{-nG(\frac{k}{n})}$$

for $k = k_n \in \mathbb{Z}$, $k \sim n\alpha$ as $n \to \infty$, where the function $C(\alpha)$ is known in an explicit form.

Similar results in the domain of moderate large deviations for $Z(t)$ are obtained.

Stability analysis of regenerative queues: recent results

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We present a general method of stability analysis of the regenerative queueing processes. This method is based on a characterization of the limiting behavior of the forward regeneration time in the renewal process generated by regenerations of a basic queueing process $X := \{X(t),\, t \geq 0\}$ [1]. (For simplicity, we assume that $X(t) \in [0, \infty)$.) This characterization gives a straightforward way to establish positive recurrence (finiteness of the mean regeneration period).

A key feature of the approach is that, instead of direct proof of the positive recurrence, we prove that the forward regeneration time does not go to infinity (in probability). The latter condition is typically much easier to verify. To do it, we use predefined *negative drift* and *regeneration* conditions. The proof consists of two main steps: i) using negative drift condition, we show that $X(t) \not\to \infty$ in probability; ii) using regeneration condition, we show that the process X, starting into a compact set B at instant t, hits a regeneration point within a finite time interval $[t,\, t+T]$ with a positive probability ε, where constants ε, T depend on the set B only. Then the mentioned characterization implies the positive recurrence of the process X.

An advantage of the approach is that it can be applied to non-Markov process X, too. In a general form this approach is presented in [2].

In the talk, we first describe the main steps of the analysis and illustrate them by some known queueing models. Then we present some recent results including stability conditions of optical systems, a cascade system and a non-conventional multi-orbit retrial system with constant retrial rates. Some of these results are presented in [3]–[5].

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The Wenbo Li comparison principle and its elaboration

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In memory of Professor Wenbo Li

We discuss the celebrated Li comparison principle [1] for the small ball probabilities, its elaboration and applications in last two decades.

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Goodness-of-fit tests based on characterizations, and their efficiencies

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The idea of building goodness-of-fit tests based on characterizations belongs to Yu. V. Linnik [1].

Suppose we have a sample $X_1, \dots, X_n$ of i.i.d. observations with the df F, and we are testing the composite hypothesis $H_0 : F \in \mathcal{F}$, where $\mathcal{F}$ is some family of distributions, against the alternative $H_1 : F \notin \mathcal{F}$. Consider the characterization of $\mathcal{F}$ by the equidistribution of two statistics $g_1(X_1, \dots, X_r)$ and $g_2(X_1, \dots, X_s)$. We introduce two U-empirical df's

$$L_n^1(t) = \binom{n}{r}^{-1} \sum_{1 \le i_1 < \dots < i_r \le n} \mathbf{1}\{g_1(X_{i_1}, \dots, X_{i_r}) < t\}, \quad t \in R^1,$$
$$L_n^2(t) = \binom{n}{s}^{-1} \sum_{1 \le i_1 < \dots < i_s \le n} \mathbf{1}\{g_2(X_{i_1}, \dots, X_{i_s}) < t\}, \quad t \in R^1.$$

According to Glivenko-Cantelli theorem for U-empirical df's, $L_n^j(t)$ converge uniformly and a.s. to the df's $L^j(t) = P\{g_j < t\}, j = 1, 2$, as $n \to \infty$. As under H_0 we have $L^1(t) \equiv L^2(t)$, it follows that a.s. under H_0 we have $\Delta_n := \sup_{t \in R^1} \mid L_n^1(t) - L_n^2(t) \mid \longrightarrow 0,\ n \to \infty$. Hence the Kolmogorov-type statistic Δ_n can be used for testing H_0 against H_1. We can also use some U-empirical integral statistics, e.g. $I_n = \int_R (L_n^1(t) - L_n^2(t))\, dF_n(t)$, where F_n is the usual empirical df.

Consider as an illustration of this idea the Desu's characterization of exponential law [2]: *Let X and Y be non-negative i.i.d. rv's with df differentiable in zero. Then X and $2\min(X, Y)$ have the same distribution iff X and Y are exponentially distributed.* Here $g_1(x_1) = x_1$, while $g_2(x_1, x_2) = 2\min(x_1, x_2)$. We can construct the statistics I_n and Δ_n, and then one can study their limiting properties and calculate their efficiencies. Another example is the famous Polya's characterization [3] of normality with zero mean by the equal distribution of rv's X and $X + Y/\sqrt{2}$.

Due to technical difficulties, Linnik's idea was developed only in last 10-15 years when the theory of $U-$empirical measures was sufficiently elaborated. We present some recent results on testing normality and exponentiality, and describe further results concerning the Cauchy, arcsine and power function distributions.

We calculate local efficiencies of our statistics in Pitman and Bahadur sense for common alternatives and describe for them the conditions of local asymptotic optimality. Integral statistics like I_n are U-statistics, so their asymptotic theory is known. On the contrary, large sample properties of U-empirical Kolmogorov-type statistics and, in particular, their large deviations, were studied only recently [4]. The research of the authors was supported by the grants RFBR 13-01-00172, Nsh. 4472.2010.1, and SPbGU 6.38.672.2013.

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Limit theorems for the measure of level sets of Gaussian random fields

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Excursion and level sets of continuous random fields have attracted much attention in recent decades due to their applications to modeling complex stochastic structures in tomography and astrophysics. The main difficulty arising while dealing with such objects is in the fact that their geometric characteristics (e.g. the level set area, the Euler characteristics of the excursion set etc.) do not depend smoothly on the realizations of the random process or field. For example, in the one-parameter case, i.e. for the smooth random processes, the study of level sets amounts to the level crossings behavior analysis. For multiparameter random fields, the analogue of level crossings number is the Hausdorff measure of the level set (considered in a bounded measurable observation window). The study of such functionals requires a complicated analysis based on the stochastic geometry as well as on the modern theory of random fields. The celebrated formula by S.Rice gives the expectation of the level crossings number for a continuously differentiable random process. The Rice formula has been generalized both to higher moments and to random fields by H.Cramer, M.Leadbetter, Yu.K.Belyaev, M.Wschebor and other scientists. There are also a number of central limit theorems due to T.Malevich, J.Cuzick, V.I.Piterbarg etc. Books by R.Adler, J.Taylor, J.-M.Azais and M.Wschebor contain detailed account on main achievements in this domain. We will discuss main results and methods that are used to prove limit theorems for the level sets measure of Gaussian random fields. Special attention will be paid to new setup when the level sets corresponding to all real values are considered simultaneously. Thus, for any given observation window, one obtains a random process on the real line consisting of their Hausdorff measures. Several functional limit theorems proved recently for the families of such processes will be presented.

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Some moment inequalities and moment estimates for characteristic functions

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A series of precise moment inequalities and moment estimates for characteristic functions is presented.

Let X be a random variable (r.v.) with the characteristic function $f(t) = \mathsf{E}e^{itX}$, $t \in \mathbf{R}$, and such that $\mathsf{E}|X|^r < \infty$ for some real and positive r. Denote $\alpha_k = \mathsf{E}X^k$, $k = 1, 2, \ldots, [r]$, $\beta_s = \mathsf{E}|X|^s$, $0 < s \leqslant r$, $\alpha_0 \equiv \beta_0 \equiv 1$.

Theorem 1. *If $\beta_3 < \infty$, then*

$$\mathsf{E}|X-\alpha_1|^3 \leqslant \frac{17+7\sqrt{7}}{27}\,\beta_3 < 1.3156\cdot\beta_3$$

with equality attained at the special two-point distribution.

If $\alpha_1 = 0$, $\beta_3 < \infty$, then

$$|\alpha_3| \leqslant c\big(\beta_3/\beta_2^{3/2}\big)\cdot\beta_3, \quad \textit{where } c(b) = \sqrt{\frac{1}{2}\sqrt{1+8b^{-2}}+\frac{1}{2}-2b^{-2}} < 1, \quad b \geqslant 1,$$

with equality attained for each value of $\beta_3/\beta_2^{3/2} \geqslant 1$ at the special two-point distribution.

If the r.v. X is lattice with span $h > 0$ and $\alpha_1 = 0$, $\beta_{2+\delta} < \infty$ for some $0 < \delta \leqslant 1$, then

$$h \leqslant (\beta_{2+\delta}/\beta_2 + \beta_\delta)^{1/\delta},$$

if, in addition, the r.v. X has a symmetric distribution, then

$$h \leqslant \max\big\{(\beta_{2+\delta}/\beta_2)^{1/\delta},\, 2\sqrt{\beta_2}\,\big\}$$

with equalities attained at the special two- and three-point distributions.

Theorem 2. *If $\beta_n < \infty$ for some integer $n \geqslant 1$, then the following estimates hold for all $t \in \mathbf{R}$:*

$$\begin{aligned}
|f^{(n-1)}(t) - i^{n-1}\alpha_{n-1}| &\leqslant 2\beta_{n-1}\sin\Big(\frac{\beta_n|t|}{2\beta_{n-1}} \wedge \frac{\pi}{2}\Big),\\
|f^{(n-1)}(t)| &\leqslant \sqrt{\beta_{n-2}\beta_n}\sin\Big(\Big(\frac{\beta_n}{\beta_{n-2}}\Big)^{1/2}|t| \wedge \frac{\pi}{2}\Big), \ \textit{if } n \textit{ is even and } \alpha_{n-1} = 0,\\
\Big|f(t) - \sum_{k=0}^{n-1}\alpha_k\frac{(it)^k}{k!}\Big| &\leqslant 2\beta_{n-1}\int_0^{|t|}\int_0^{t_{n-1}}\cdots\int_0^{t_2}\sin\Big(\frac{\beta_n t_1}{2\beta_{n-1}} \wedge \frac{\pi}{2}\Big)dt_1\cdots dt_{n-2}\,dt_{n-1}.
\end{aligned}$$

If $n \geqslant 3$ is odd and $\alpha_{n-2} = 0$, then the following estimates hold for all $t \in \mathbf{R}$:

$$\begin{aligned}
\Big|f^{(n-1)}(t) + \frac{\alpha_{n-1}}{\alpha_{n-3}}f^{(n-3)}(t)\Big| &\leqslant 2\beta_{n-1}\sin\Big(\frac{\beta_n|t|}{2\beta_{n-1}} \wedge \frac{\pi}{2}\Big),\\
\Big|f^{(n-2)}(t) + \frac{\alpha_{n-1}}{\alpha_{n-3}}\,tf^{(n-3)}(t)\Big| &\leqslant 2|t|\beta_{n-1}\sin\Big(\frac{\beta_n|t|}{4\beta_{n-1}} \wedge \frac{\pi}{2}\Big).
\end{aligned}$$

If $\alpha_1 = 0$, $\beta_3 < \infty$, *then the following estimates hold for all* $t \in \mathbf{R}:$

$$\Big|\frac{d^k}{dt^k}\Big(f(t) - 1 + \frac{\alpha_2 t^2}{2}\Big)\Big| \leqslant \gamma_{n-k}\big(\beta_3/\beta_2^{3/2}\big)\cdot\beta_3 t^2, \quad k = 0, 1, 2,$$

where

$$\gamma_n(b) = \inf_{\lambda\geqslant 0}\frac{\lambda c(b) + q_n(\lambda)}{n!}, \quad q_n(\lambda) = \sup_{x>0}\frac{n!}{x^n}\Big|e^{ix} - \sum_{k=0}^{n-1}\frac{(ix)^k}{k!} - \lambda\frac{(ix)^n}{n!}\Big|, \ \lambda \geqslant 0.$$

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The probabilistic approximation of the one-dimensional initial boundary value problem solution.

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We consider the equation

$$\frac{\partial u}{\partial t} = \frac{\sigma^2}{2}\frac{\partial^2 u}{\partial x^2} + f(x)u,$$

where σ is a complex-valued parameter, such that $\mathrm{Re}\sigma^2 \geq 0$. When σ is a real number this equation corresponds to the Heat equation while when $\mathrm{Re}\sigma^2 = 0$ it corresponds to the Schrödinger equation. For this equation we consider the initial boundary value problem with Dirichlet condition $u(0,x) = \varphi(x),\ u(a,t) = 0,\ u(t,b) = 0$. In the case when σ is a real number there exists probabilistic representation of the solution in a form of the mathematical expectation (so called Feynman-Kac formula), namely

$$u(t,x) = \mathrm{E}\left\{\varphi(\widetilde{\xi}_x(t\wedge\tau))e^{\int_0^{t\wedge\tau} f(\widetilde{\xi}_x(v))dv}\right\},$$

where $\widetilde{\xi}_x(t)$ is a Brownian motion with a parameter σ, killed at the exit time τ from the interval $[a,b]$. On the base of this representation one can approximate the solution using some suitable approximation of the Wiener process. This approach doesn't work if $\mathrm{Im}\sigma \neq 0$. It is known that when σ is not a real number there exists no analogue of the Wiener measure and hence one can not present the Feynman-Kac formula as an integral with respect to a σ-additive measure in a trajectory space. When $\mathrm{Re}\sigma^2 = 0$ (that corresponds to the Schrödinger equation) one can apply an integral with respect to the so called Feynman measure that is a finitely-additive complex measure in the trajectory space which is defined as a limit over a sequence of partitions of an interval $[0,T]$. It should be mentioned that this approach is not a probabilistic approach in the usual sense since the very notion of a probability space does not appear in it. To get the stochastic approximation of the solution we use another approach based on the theory of generalized function. On a special probability space we define the sequence of probability measures $\{\mathrm{P}_n\}$ and limit object $\mathrm{L} = \lim_{n\to\infty}\mathrm{P}_n$ but this limit object is not a measure, it is only generalized function. That means that the convergence $\int f d\mathrm{P}_n \to (\mathrm{L}, f)$ is valid only if f belongs to the class of test functions. On this probability space we define a complex-valued process so that the mathematical expectation with respect to the measure P_n of some functional of this process converges to the value of the generalized function applied to this test function that leads to a solution of the initial boundary value problem.

On the rate of convergence to the semi-circular law and Marchenko–Pastur law

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Let $\mathbf{X} = (X_{jk})_{j,k=1}^n$ denote a Hermitian random matrix with entries X_{jk}, which are independent for $1 \le j \le k$. We consider the rate of convergence of the empirical spectral distribution function of the matrix $\mathbf{X}$ to the semi-circular law assuming that $\mathbb{E}X_{jk} = 0$, $\mathbb{E}X_{jk}^2 = 1$ and that the distributions of the matrix elements X_{jk} have a uniform subexponential decay in the sense that there exists a constant $\varkappa > 0$ such that for any $1 \le j \le k \le n$ and any $t \ge 1$ we have

$$\Pr\{|X_{jk}| > t\} \le \varkappa^{-1} \exp\{-t^{\varkappa}\}.$$

By means of a short recursion argument it is shown that the Kolmogorov distance between the empirical spectral distribution of the Wigner matrix $\mathbf{W} = \frac{1}{\sqrt{n}}\mathbf{X}$ and the semicircular law is of order
$O(n^{-1} \log^b n)$ with some positive constant $b > 0$.

Furthermore, let $\mathbf{X} = (X_{jk})$ denote $n \times p$ random matrix with entries X_{jk}, which are independent for $1 \le j \le n, 1 \le k \le p$. We consider the rate of convergence of empirical spectral distribution function of the matrix $\mathbf{W} = \frac{1}{p}\mathbf{X}\mathbf{X}^*$ to the Marchenko–Pastur law. We assume that $\mathbb{E}X_{jk} = 0$, $\mathbb{E}X_{jk}^2 = 1$ and that the distributions of the matrix elements X_{jk} have a uniformly subexponential decay. Using the similar as for the semi-circular law argument it is shown that the Kolmogorov distance between the empirical spectral distribution of the sample covariance matrix $\mathbf{W}$ and the Marchenko–Pastur distribution is of order $O(n^{-1} \log^b n)$ for some positive constant $b > 0$ with high probability.

These results, obtained jointly with Professor F. Götze, were recently published in [1] and [2].

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High-dimensional and large-sample approximations in statistics

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We discuss new techniques developed to get error bounds in approximation problems in statistics when either n (sample size) or both parameters n (sample size) and p (dimension of observations) could be large (see [1]). We consider recent results (see [2,3]) and the following theorem for Bartlett-Nanda-Pillai statistic $T_{BNP} = n\,tr\big(S_h\,(S_e + S_h)^{-1}\big)$, where S_h and S_e are independent random matrices with Wishart distributions $W_p(q, I_p)$ and $W_p(n, I_p)$ respectively, here I_p is the identity $p \times p$ matrix.

Theorem. *There exist constants $N = N(p,q)$ and $c = c(p,q)$ depending on p and q such that for all $n > N$ and $z > 0$ we have*

$$|\mathbb{P}\left(T_{BNP} \le z\right) - G_{pq}\left(z\right)| \le \frac{c}{n},$$

where G_{pq} is the distribution function of chi-square distribution with pq degrees of freedom.

The Theorem is a joint result with A.Lipatiev.

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Decomposable branching processes in a Markovian random environment

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We consider a decomposable two-type Bienaymé-Galton-Watson process in a Markovian random environment with particles livingone unit of time and replaced by random numbers of offspring which are conditionally independent given the current state of the environment. It is assumed that particles of type 1 may produce individuals of both types while the type 2 particles can give birth only to type 2 particles. The subject can be viewed as a stochastic model for the sizes of a geographically structured population occupying two islands. Time is assumed discrete, so that one unit of time represents a generation of individuals, some living on island 1 and others on island 2. Those on island 1 give birth under influence of a randomly changing environment. They may migrate to island 2 immediately after birth, with a probability again depending upon the current environmental state. Individuals on island 2 do not migrate and their reproduction law is not influenced by any changing environment.

Our main assumptions are:

- particles of type 1 form (by their own) a critical or subcritical branching process in a random environment,
- the environment is generated by an irreducible aperiodic positive recurrent Markov chain with countably many states,
- particles of type 2 form a critical branching process which is independent of the environment.

Let X_n and Z_n be the numbers of particles of type 1 and of type 2, respectively, present at time n. Assuming that the process is initiated at time $n = 0$ by a single particle of type 1 we investigate asymptotics of the survival probability $\mathbb{P}[X_n + Z_n > 0]$ as $n \to \infty$.

The talk is mainly based on the results published in a recent paper [1].

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Random walks that avoid a bounded set

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Let $S_n = S_0 + X_1 + \cdots + X_n$ be a centered random walk, and let $\tau_B := \min\{k \geq 0 : S_k \in B\}$ be the hitting time of a bounded Borel set B. We are interested in the asymptotic of $\mathbb{P}_x(\tau_B > n)$ as $n \to \infty$, where $\mathbb{P}_x$ is the distribution of the walk conditioned to start at $S_0 = x$. This problem was completely solved in dimension one and two in the 1960s (see [1]) for integer-valued random walk, essentially, under no additional assumptions. The proof of [1] is by induction in the number of lattice points in B and it uses the methods of potential theory. The problem then simplifies to the study of $\mathbb{P}_x(\tau_{\{0\}} > n)$ which may be done with the use of theory of recurrent events.

Of course this does not work in the general case, and we use an entirely different approach that forces us to work in one dimension and assume finiteness of variance. The main idea is that a typical trajectory upcrosses and downcrosses B only for a few times as this is exponentially costly in the number of steps. These crossings are performed at the very first steps and then the walks stays either to the right or to the left of B all the time up to n.

Our main result is as follows.

Theorem. For any non-lattice random walk S_n with $\mathbb{E}S_1 = 0$ and $0 < Var(S_1) := \sigma^2 < \infty$ and any bounded open set B, there exists a function $V_B(x)$ such that the relation

$$\mathbb{P}_x(\tau_B > n) \sim \frac{\sqrt{2}V(x)}{\sigma\sqrt{\pi n}}, \quad n \to \infty$$

holds uniformly as $|x|/\sqrt{n} \to 0$, that is over $|x| \leq a_n\sqrt{n}$ for any fixed sequence $a_n \to 0$. The function $V_B(x)$ is for any x and strictly positive for $x \notin Conv(B)$.

We will also discuss an immediate application of the theorem to get a tight estimate of the size of the largest gap G_n in the range of a random walk S_n, which is defined as

$$G_n := \max\Big\{h > 0 : \text{ there exists an } x \in [m_n, M_n - h] \text{ such that } S_k \notin (x, x+h) \text{ for any } 1 \leq k \leq n\Big\},$$

where $m_n := \min\limits_{1\leq k\leq n} S_k$ and $M_n := \max\limits_{1\leq k\leq n} S_k$.

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Potential analysis for positive recurrent Markov chains with asymptotically zero drift: power-type asymptotics

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We consider a positive recurrent Markov chain on $\mathbb{R}^+$ with asymptotically zero drift which behaves like $-c_1/x$ at infinity; this model was first considered by Lamperti. We are interested in tail asymptotics for the stationary measure. Our analysis is based on construction of a harmonic function which turns out to be regularly varying at infinity. This harmonic function allows us to perform non-exponential change of measure. Under this new measure Markov chain is transient with drift like c_2/x at infinity and we compute the asymptotics for its Green function. Applying further the inverse transform of measure we deduce a power-like asymptotic behaviour of the stationary tail distribution.

Gaussian processes and intrinsic volumes

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This is a joint work with Zakhar Kabluchko (Ulm University).

We start with an example. Let $\xi_1, \xi_2, \dots, \xi_n$ be independent standard Gaussian random variables. Put

$$M_n = \max_{k=1,\dots,n} \xi_k.$$

It is well known from the theory of extreme values that

$$\mathbf{E}\, M_n = \sqrt{2 \log n} \cdot (1 + o(1)), \quad n \to \infty.$$

On the other hand, $\sqrt{2\pi}\mathbf{E}\, M_n$ coincides with a mean width of a regular n-dimensional simplex, which is a simple corollary from much more general result due to Sudakov [1] and Tsirelson [2]. This gives an asymptotic of a mean width of a regular n-dimensional simplex as $n \to \infty$, which is known from the direct geometrical reasoning.

Based on this simple example, we will discuss a general approach which can be deduced from the results of Sudakov and Tsirelson.

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