On the effective hydrodynamics of Quantum Hall fluids

Alexander Abanov

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Acknowledgments: Paul Wiegmann

Main Result

Collective behavior of the simplest FQHE fluid (Laughlin's state with $\nu = 1/\beta$) is captured by an effective theory of two scalar fields ρ and π

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\begin{aligned}
L &= -\rho\,\partial_t\pi - H \\
H &= \rho\left[A_0 + \frac{\tilde{\boldsymbol{v}}^2}{2} + \frac{1}{2}(\boldsymbol{\nabla}\times\boldsymbol{A})\right]
\end{aligned}
$$

with

$$
\begin{aligned}
\tilde{\boldsymbol{v}} &= \boldsymbol{\nabla}\pi + \boldsymbol{A} - \boldsymbol{\nabla}^*\left(\phi + \left(\frac{\beta}{4} - \frac{1}{2}\right)\ln\rho\right) \\
\rho &= \frac{1}{2\pi\beta}\Delta\phi
\end{aligned}
$$

1989, Read; Zhang, Hansson and Kivelson

$$
L = \Phi^*\left(i\partial_t + a_0 + A_0 - \frac{1}{2m^*}\left(-i\boldsymbol{\nabla} - \boldsymbol{a} - \boldsymbol{A}\right)^2\right)\Phi + \frac{1}{4\pi\beta}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda + V(|\Phi|^2)
$$

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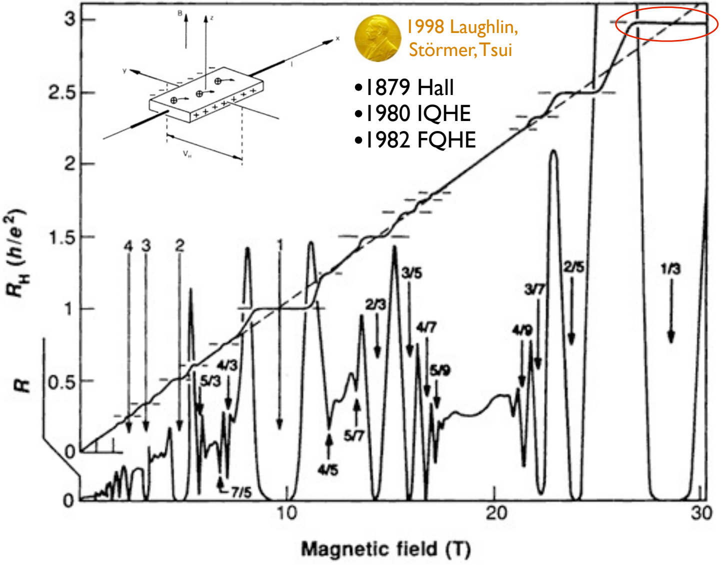
$$

Recent Related Works

- Paul Wiegmann
  - arXiv:1305.6893, *Anomalous Hydrodynamics of Fractional Quantum Hall States*
  - arXiv:1211.5132, *Quantum Hydrodynamics of Fractional Hall Effect: Quantum Kirchhoff Equations*
  - Phys. Rev. Lett. 108, 206810 (2012), *Non-Linear hydrodynamics and Fractionally Quantized Solitons at Fractional Quantum Hall Edge*
- Dam Son
  - arXiv:1306.0638, *Newton-Cartan Geometry and the Quantum Hall Effect*
- Eldad Bettelheim
  - arXiv:1306.3782, *Integrable Quantum Hydrodynamics in Two Dimensional Phase Space*

Fractional Quantum Hall Effect (FQHE)



FQHE

- Two-dimensional electron gas in magnetic field forms a *new type of quantum fluid*
- Quantum condensation of *electrons coupled to vortices* (“flux quanta”)
- Incompressible fluid
- Quasiparticles are *gapped*, have *fractional charge and statistics*
- Low energy dynamics is at the *boundary*
- Microscopics is captured by Laughlin’s wave function

$$\Psi_{1/3} = \prod_{j<k}(z_j - z_k)^3 \, e^{-\frac{1}{4}\sum_j |z_j|^2}, \qquad z \equiv x + iy$$

Hydrodynamics of FQHE

Goal: effective classical hydrodynamic description of FQHE.

- Ideal 2D fluid (no dissipation, Hamiltonian formulation)
- Density-vorticity relation (FQHE constraint)
- Incompressibility
- Hall viscosity
- Linear response (Hall conductivity, etc)
- Nonlinear physics
- Chirality and dynamics of the boundary

Very brief (and incomplete) history

- 1983, Laughlin
  - Laughlin's wave function and plasma analogy
- 1986, Girvin, MacDonald, Platzman
  - Magnetoplasmon, structure factor, single mode approximation
- 1989, Read; Zhang, Hansson, Kivelson; 1990, Stone; 1991, Lee and Zhang
  - Chern-Simons-Ginzburg-Landau theory of FQHE
  - Hydrodynamics of FQHE and FQHE constraint
- 1996, Simon, Stern, Halperin
  - Magnetization current
- 1992, Wen, Zee; 1995, Avron, Seiler, Zograf; 2007, Tokatly, Vignale; 2009, Read; Haldane
  - Hall viscosity
- 2006, Zabrodin, Wiegmann; 2005-9, Bettelheim, Wiegmann, AA
  - 2D Dyson gas
  - 1D hydrodynamics of Calogero-Sutherland model

Density, current, velocity

Microscopic fields: $m = e = c = 1$

$$\rho(\boldsymbol{r}) = \sum_\alpha \delta(\boldsymbol{r} - \boldsymbol{r}_\alpha) \tag{1}$$

$$\boldsymbol{j}(\boldsymbol{r}) = \sum_\alpha (\boldsymbol{p}_\alpha - \boldsymbol{A}_\alpha)\delta(\boldsymbol{r} - \boldsymbol{r}_\alpha) \tag{2}$$

Poisson brackets (commutators), Landau, 1941

$$\{\rho, v'\} = \boldsymbol{\nabla}\delta(\boldsymbol{r} - \boldsymbol{r}') \tag{3}$$

$$\{v_i, v_j'\} = \epsilon_{ij}\frac{\boldsymbol{\nabla}\times\boldsymbol{v} + B}{\rho}\delta(\boldsymbol{r} - \boldsymbol{r}') \tag{4}$$

Here the velocity is defined as $\boldsymbol{j} = \rho\boldsymbol{v}$.

Hamiltonian dynamics of ideal fluid

Hamiltonian: $$H = \int d^2r\, \frac{\rho \boldsymbol{v}^2}{2} + U[\rho]$$

$U[\rho] = \int d^2r\ \left[\rho\epsilon(\rho) + A(\boldsymbol{\nabla}\rho)^2\right]$

- the Boussinesq approximation.

Hamitlonian +Poisson brackets $\longrightarrow$ Equations of motion

$$\begin{aligned} \dot{\rho} &\ + \boldsymbol{\nabla}(\rho \boldsymbol{v}) = 0, && \text{continuity} \\ \dot{\boldsymbol{v}} &\ + (\boldsymbol{v}\boldsymbol{\nabla})\boldsymbol{v} + \boldsymbol{\nabla} w = \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}. && \text{Euler} \end{aligned}$$

where $w = \delta U/\delta\rho$ - chemical potential

Dynamics does not have to be vorticity free.

Vorticity and FQHE constraint

$\omega = \boldsymbol{\nabla} \times \boldsymbol{v}$ – vorticity

$$\partial_t(\omega + B) + \boldsymbol{\nabla}\left((\omega + B)\boldsymbol{v}\right) = 0$$

$$\partial_t \rho + \boldsymbol{\nabla}(\rho \boldsymbol{v}) = 0$$

Constraint $\omega + B = \zeta\rho$ is respected by equations of motion.

$$\boxed{\boldsymbol{\nabla} \times \boldsymbol{v} = \zeta(\rho - \rho_0)}$$

FQHE constraint; Stone, 1990; incompressibility! dimension: $[\zeta] = \left[\frac{\hbar}{m}\right]$

– might emerge in quantum physics! $\zeta = \frac{\hbar}{m}\frac{2\pi}{\nu}$.

Stress tensor T_{ik}

Dynamics of conserved quantities

$$\begin{aligned} \partial_t \rho + \partial_k j_k &= 0, \\ \partial_t j_i + \partial_k T_{ik} &= \rho F_i. \end{aligned}$$

$$\begin{aligned} j_k &= \rho v_k, \\ T_{ik} &= P(\rho)\delta_{ik} + \rho v_i v_k, \\ F_i &= E_i + \epsilon_{ik} v_k B. \end{aligned}$$

Linear expansion around $\rho = \rho_0$, $\boldsymbol{v} = 0$

Isotropic fluid: no shear $\delta T_{ik} \sim u_{ik}$.

Ideal fluid: no viscosity $\delta T_{ik} \sim v_{ik}$

$$u_{ik} = \frac{1}{2}(\partial_i u_k + \partial_k u_i)$$

$$v_{ik} = \frac{1}{2}(\partial_i v_k + \partial_k v_i)$$

Hall viscosity $\delta T_{ik} \sim (\epsilon_{in} v_{ik} + \epsilon_{kn} v_{ni})$

strain and strain rate

Hall viscosity (Lorentz shear modulus)

Isotropic non-dissipative fluid
"Viscous" stress tensor (known in plasma physics)

$$\delta T_{ik} = \Lambda(\epsilon_{in} v_{nk} + \epsilon_{kn} v_{ni}).$$

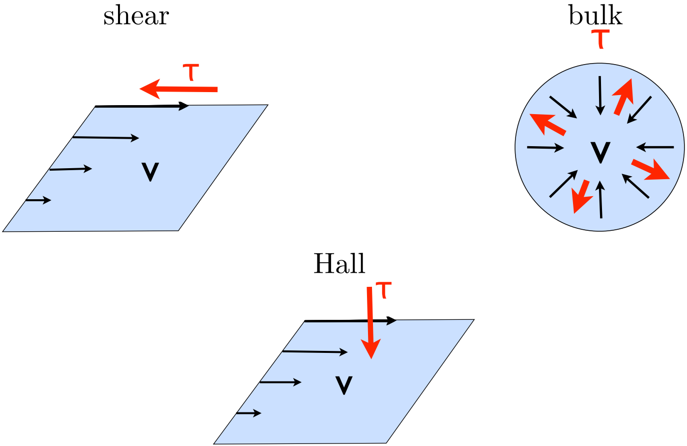
Time-reversal invariance is broken!
For Laughlin's states

$$\Lambda = \frac{\hbar\rho_0}{4}\nu^{-1} = \frac{\hbar}{8\pi l_B^2} = \frac{1}{8\pi}\frac{eB}{c} \qquad \text{– Hall viscosity}$$

Generalization (Read, 2009)

$$\Lambda = \frac{\hbar\rho_0}{2}\left(\frac{\nu^{-1}}{2} + h_\psi\right)$$

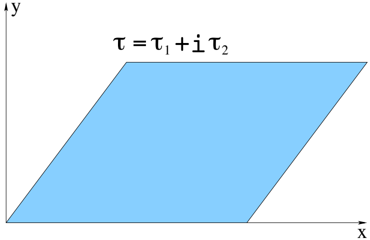
Hall viscosity



Stress forces are orthogonal to the motion → no dissipation!

Hall viscosity from adiabatic metric deformation

Hall viscosity → Berry's phase with respect to variations of the metric $ds^2 = \frac{1}{\tau_2}|dx + \tau dy|^2$. Avron, Seiler, Zograf, 1995
Read, 2009



Ψ_β - Laughlin's function on torus $\tau = \tau_1 + i\tau_2$.

Λ – adiabatic curvature!

$$\Lambda = \frac{2\hbar}{L^2}\,\mathrm{Im}\left\langle \frac{\partial\Psi_\beta}{\partial\tau_1}\middle|\frac{\partial\Psi_\beta}{\partial\tau_2}\right\rangle$$

Hall viscosity and Hall conductivity

Hall conductivity – non-dissipative part of conductivity tensor σ_{ik}
Hall viscosity – non-dissipative part of viscosity tensor η_{ikmn}
(Odd viscosity)
(Lorentz shear modulus)

$$\Psi_\beta(\tau_1, \tau_2; \Phi_1, \Phi_2) = \frac{1}{\mathcal{N}} \prod_{i<j} \theta_1(z_{ij}/L_x|\tau)\, e^{-\frac{1}{2}\sum_j (\operatorname{Im} z_j)^2}$$

Laughlin's function on torus $\tau = \tau_1 + i\tau_2$ with fluxes $\Phi_{1,2}$ through torus' cycles. $(z = x - \Phi_2 + \tau(y + \Phi_1))$
σ_{xy}, Λ – adiabatic curvatures!

$$\sigma_{xy} \sim \operatorname{Im}\left\langle \frac{\partial \Psi_\beta}{\partial \Phi_1} \middle| \frac{\partial \Psi_\beta}{\partial \Phi_2} \right\rangle \qquad \Lambda \sim \operatorname{Im}\left\langle \frac{\partial \Psi_\beta}{\partial \tau_1} \middle| \frac{\partial \Psi_\beta}{\partial \tau_2} \right\rangle$$

Avron, Seiler, Zograf, 1995
Read, 2009

Hall viscosity (Hamiltonian)

What is the Hamiltonian generating dynamics with Hall viscosity?

$$H = \int d^2r\,\rho\left[\frac{1}{2}\boldsymbol{v}^2 + \eta\,\boldsymbol{\nabla}\times\boldsymbol{v}\right] + U[\rho].$$

Hall viscosity $\Lambda = \eta\rho_0$.

$$H = \int d^2r\,\rho\,\frac{1}{2}\Big(\boldsymbol{v} + \eta\boldsymbol{\nabla}^*\log\rho\Big)^2 + U'[\rho].$$

Here $\boldsymbol{\nabla}^* = -\hat{z}\times\boldsymbol{\nabla}$.

η-term can be absorbed into $\boldsymbol{v}$ but Poisson's brackets will change.

$$\int d^2x\,(\boldsymbol{r}\times\rho\boldsymbol{v} + \mu\rho) = \int d^2x\,\boldsymbol{r}\times(\rho\boldsymbol{v} + \frac{\mu}{2}\boldsymbol{\nabla}^*\rho)$$

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FQHE constraint + Hall viscosity

$$H = \int d^2r \, \frac{\rho(\boldsymbol{v} + \eta \boldsymbol{\nabla}^* \log \rho)^2}{2} + U[\rho] \qquad a_i^* \equiv \epsilon^{ij} a_j$$

with $(\boldsymbol{\nabla}^* W = -\boldsymbol{A})$

$$\rho = \frac{1}{2\pi\beta} \Delta\phi, \qquad \boldsymbol{v} = \boldsymbol{\nabla}\pi - \boldsymbol{\nabla}^*(\phi - W), \qquad \{\pi, \rho'\} = \delta(\boldsymbol{r} - \boldsymbol{r}')$$

gives equations of motion with Hall viscosity

$$\Lambda = \eta\rho_0.$$

FQHE constraint is automatically satisfied

$$\boldsymbol{\nabla} \times \boldsymbol{v} = 2\pi\beta(\rho - \rho_0).$$

Classical Hydrodynamics I

Classical Hamiltonian

$$H_{cl} = \int d^2 z \, \rho \, \frac{1}{2m} \left(\boldsymbol{\nabla} \pi - \boldsymbol{\nabla}^* \left(\phi - W + \frac{\beta - 2}{4} \ln \rho \right) \right)^2$$

produces correct static structure factor $s(k) = \frac{1}{2} k^2 \left(1 + \frac{\beta - 2}{4} \, k^2 \right)$ and FQHE constraint but wrong Hall viscosity.

Quantum fluctuations should be incorporated into classical (effective) Hamiltonian!

Quantum zero motion $\to$ Landau diamagnetism!

Classical Hydrodynamics II

diamagnetic current $\boldsymbol{j}_B = -\frac{1}{2}\boldsymbol{\nabla}^*\rho$

$$H_{cl} = \int d^2z\,\rho\,\left[\frac{1}{2}\left(\boldsymbol{v} - \boxed{\frac{\beta-2}{4}}\boldsymbol{\nabla}^*\ln\rho\right)^2 + \boxed{\frac{1}{2}}\hbar\omega_B\right]$$

$\rho = \frac{1}{2\pi\beta}\Delta\phi$ $\qquad\qquad$ $\{\pi, \rho'\} = \delta(\boldsymbol{r} - \boldsymbol{r}'),$

$\boldsymbol{v} \equiv \boldsymbol{\nabla}\pi - \boldsymbol{\nabla}^*(\phi - W) \rightarrow$ FQHE constraint: $\boldsymbol{\nabla}\times\boldsymbol{v} = 2\pi\beta(\rho - \rho_0)$

Static structure factor $\quad + \quad$ Hall viscosity

$$s(k) = \frac{1}{2}k^2\left(1 + \boxed{\frac{\beta-2}{4}}k^2\right) \qquad \Lambda = \left(\boxed{\frac{\beta-2}{4}} + \boxed{\frac{1}{2}}\right)\hbar\rho_0 = \frac{\beta}{4}\hbar\rho_0$$

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$$\rho = \frac{1}{2\pi\beta}\Delta\phi \qquad\qquad \{\pi,\rho'\} = \delta(\boldsymbol{r}-\boldsymbol{r}'),$$

$\boldsymbol{v} \equiv \boldsymbol{\nabla}\pi - \boldsymbol{\nabla}^*(\phi - W) \rightarrow$ FQHE constraint: $\boldsymbol{\nabla}\times\boldsymbol{v} = 2\pi\beta(\rho-\rho_0)$

Static structure factor + Hall viscosity

$$s(k) = \frac{1}{2}k^2\left(1 + \boxed{\frac{\beta-2}{4}}k^2\right) \qquad \Lambda = \left(\boxed{\frac{\beta-2}{4}} + \boxed{\frac{1}{2}}\right)\hbar\rho_0 = \frac{\beta}{4}\hbar\rho_0$$

Hydrodynamic Lagrangian

$$L_{cl} \;=\; -\int d^2x\,\rho\,\left[\dot{\pi} + \frac{1}{2}\boldsymbol{v}_\alpha^2 + \frac{1}{2}(\boldsymbol{\nabla}\times\boldsymbol{A})\right] \qquad \text{Main Result}$$

$$\boldsymbol{v}_\alpha \equiv \boldsymbol{\nabla}\pi + \boldsymbol{A} - \boldsymbol{\nabla}^*\left(\phi + \alpha\ln\rho\right).$$

$$\alpha = \frac{\beta-2}{4}, \quad \eta = \frac{\beta}{4}.$$

True e/m current: $\boldsymbol{j} = \rho\boldsymbol{v}_\eta$

FQHE constraint: $\boldsymbol{\nabla}\times\boldsymbol{v}_\alpha = 2\pi\beta(\rho-\rho_0) + \alpha\Delta\ln\rho.$

Linear response

- Static structure factor $s(k) = \frac{1}{2}k^2\left(1 + \frac{\beta-2}{4}\,k^2\right)$
  1986 Girvin, MacDonald, Platzman

- Hall viscosity $\Lambda = \frac{\beta}{4}\,\hbar\rho_0$
  1995 Avron, Seiler, Zograf;
  2007 Tokatly, Vignale;
  2009 Read

- Electromagnetic response, e.g., $\sigma_H = \frac{1}{2\pi\beta}\left(1 + \frac{\beta-4}{4}k^2\right)$
  2011 Hoyos, Son

- other E&M and dynamic response
  2006 Tokatly

Linear response

Change of density under small variations of $\boldsymbol{E}$ and B

$$\frac{\delta\rho}{\rho_0} = \frac{\omega_0^2}{\omega^2 - \Omega_k^2}\left[\frac{e}{m\omega_0^2}(\boldsymbol{\nabla E}) - \left(1 - \frac{\eta k^2}{m\omega_0}\right)\frac{\delta\omega_c}{\omega_0}\right]$$

with magnetoplasmon dispersion

$$\frac{\Omega_k^2}{\omega_0^2} = 1 - \frac{\beta - 2}{2}\frac{k^2}{m\omega_0} + \dots$$

Chern-Simons-Ginzburg-Landau theory

1989, Read; Zhang, Hansson and Kivelson

$$L = \Phi^* \Big(i\partial_t + a_0 + A_0 - \frac{1}{2m^*} \left(-i\boldsymbol{\nabla} - \boldsymbol{a} - \boldsymbol{A}\right)^2 \Big) \Phi + \frac{1}{4\pi\beta} \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + V(|\Phi|^2)$$

Hydrodynamic form:

Change $\Phi = \sqrt{\rho}\, e^{i\theta}$ and solve for constraints

$\boldsymbol{a} = \boldsymbol{\nabla}(\pi - \theta) + \boldsymbol{\nabla}^* \phi, \qquad \Delta\phi = 2\pi\beta\,\rho$

$$L = -\rho \left[\dot{\pi} + \frac{1}{2} \left(\boldsymbol{\nabla}\pi - \boldsymbol{\nabla}^* \left(\phi - W - \frac{1}{2} \ln \rho \right) \right)^2 + A_0 + \epsilon(\rho) \right]$$

UV regularization is missing ($-\frac{1}{2} \to \frac{\beta-2}{4}$).

Extra term is needed: $\sim \frac{\beta}{4} |\Phi|^2 \boldsymbol{\nabla} \times (\boldsymbol{a} + \boldsymbol{A})$.

Quantum Hydrodynamics

$$H = \int d^2x\, \frac{1}{2}\bar{V}\rho V \qquad \text{acts on } \Psi[\rho]$$

where

$$V = \bar{\partial}\left[\pi + i\left(\phi - W + \frac{\beta - 2}{4}\ln\rho\right)\right], \qquad \pi = -i\frac{\delta}{\delta\rho}$$

Ground state: $V\Psi_\beta = 0$ for

$$\Psi_\beta[\rho] = e^{-\frac{1}{2}E_\beta[\rho]}$$

with

$$E_\beta[\rho] \quad = \quad -\beta\int d^2z\, d^2z'\;\rho(z)\ln|z - z'|\rho(z') + 2\int d^2z\;\rho W + \frac{2-\beta}{2}\int d^2z\;\rho\;\ln\rho.$$

The norm of the GS wave function: $||\Psi_\beta||^2 = \int[D\rho]\; e^{-E_\beta[\rho]}$

Dyson's argument (2006 Zabrodin, Wiegmann)

Ground state: Laughlin's wave function for filling fraction $\nu = \frac{1}{\beta}$:

$$\Psi_\beta = \prod_{j<k} (z_j - z_k)^\beta \, e^{-\sum_j W(z_j, \bar{z}_j)}, \qquad W = \frac{1}{4}|z|^2 + \boxed{W_1(z)}$$

Laughlin's plasma in collective variables

$$\sum_{j \neq k} \ln|z_j - z_k| \quad \to \quad \int d^2z \, d^2z' \; \rho(z) \ln|z - z'| \rho(z') - \int d^2z \; \rho \; \ln \frac{1}{\sqrt{\rho}}$$

$$\prod_j d^2 z_j \quad \to \quad [D\rho] \prod_j \frac{1}{\rho(z_j)} \to [D\rho] \; \exp\left[- \int d^2 z \; \rho \ln \rho \right]$$

Norm - partition function for plasma $||\Psi_\beta||^2 = \int [D\rho] \; e^{-E_\beta[\rho]}$ with

2d Coulomb plasma

$$\begin{aligned} E_\beta[\rho] &= -\beta \int d^2 z \, d^2 z' \; \rho(z) \ln|z - z'| \rho(z') \\ &+ 2 \int d^2 z \; \rho \, W + \frac{2 - \beta}{2} \int d^2 z \; \rho \; \ln \rho . \end{aligned}$$

Equilibrium density

Electrostatic energy

$$E_\beta[\rho] \quad = \quad -\beta \int d^2z\, d^2z'\ \rho(z) \ln|z-z'|\rho(z') + 2\int d^2z\ \rho W + \frac{2-\beta}{2}\int d^2z\ \rho\ \ln\rho.$$

Electrostatic potential

$$\phi(z) = \beta \int d^2z'\ \ln|z-z'|\,\rho(z') \qquad\qquad \Delta\phi = 2\pi\beta\ \rho$$

Equilibrium

$$\frac{\delta E_\beta}{\delta\rho} = 0 \qquad\qquad \phi = W + \frac{2-\beta}{4}\ln\rho \qquad\qquad \Delta W = B = 2\pi\beta\,\rho_0$$

$$\rho = \rho_0 + \tfrac{2-\beta}{8\pi\beta}\,\Delta\ln\rho$$

$$\begin{aligned} \rho &= \rho_0 = \frac{B}{2\pi\beta} = \frac{1}{\beta}\frac{1}{2\pi l_0^2} \\ \nu &= 1/\beta\text{– filling fraction} \end{aligned}$$

QFT wave function

Identity for Laughlin's function:

$$\left[\pi + i\left(\phi - W - \frac{2-\beta}{4}\ln\rho\right)\right]\Psi_\beta = 0$$

$$\pi = -i\frac{\delta}{\delta\rho} \qquad\qquad [\pi(x), \phi(x')] = -i\delta^{(2)}(x - x')$$

More general solution for $V\Psi = 0$ gives Chiral constraint

$$\bar{\partial}\left[\pi + i\left(\phi - W - \tfrac{2-\beta}{4}\ln\rho\right)\right] = 0$$

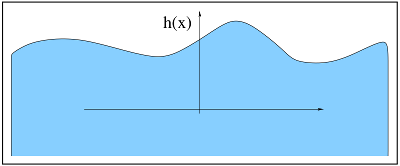
This is valid for any "holomorphic" wave function

Boundary dynamics

(dispersionless case)

$$\bar\partial\left[\pi + i\left(\phi - W - \frac{2-\beta}{4}\ln\rho\right)\right] = 0 \qquad \text{– chiral constraint}$$

$$\pi + i\left(\phi - W - \frac{2-\beta}{4}\ln\rho\right) = V(z,t) \qquad \text{– analytic}$$



$V(z,t) \to h(x,t)$ – boundary

$H = \int d^2r\,\rho\,A_0$ – dynamics

$h_t + A_0'\,h_x + A_0''\,hh_x = 0$ – incompressible droplet Iso, Rey, 1995

Hall viscosity → boundary profile → dispersive corrections!

1D Calogero-Sutherland model

Calogero model in harmonic potential

$$H = \frac{1}{2}\sum_j (p_j^2 + x_j^2) + \frac{1}{2}\sum_{j<k} \frac{\lambda(\lambda-1)}{(x_j - x_k)^2}$$

The ground state wave function

$$\Psi_0 = \prod_{j<k}(x_j - x_k)^\lambda \, e^{-\frac{1}{2}\sum_j x_j^2}$$

Collective field theory

$$H = \int dx \, \left(\frac{\rho v^2}{2} + \rho\epsilon(\rho)\right)$$

$\epsilon(\rho) = \frac{1}{2}\left[\pi\lambda\rho^H - (\lambda-1)\partial_x \ln\sqrt{\rho} + x\right]^2$

Hydrodynamics of 1D Calogero-Sutherland model

Hydrodynamics for Calogero model in harmonic potential

$$H = \int dx\; \rho\, \frac{1}{2}|\partial_x \Phi|^2$$

$$\Phi = \pi - i\left(\phi - W + \frac{\lambda - 1}{2}\ln\rho\right)$$

Here $\phi = \int dx' \,\log|x - x'|\rho(x')$, $W = x^2/2$ and λ is Calogero coupling constant ($\lambda = 1$ for free fermions).

FQHE

$$\Phi = \pi - i\left(\phi - W + \frac{\beta - 2}{4}\ln\rho\right)$$

Conclusions

1. Hamiltonian formulation for FQHE hydrodynamics is constructed
   - FQHE constraint
   - Hall viscosity
   - Linear response
   - Correspondence to Chern-Simons-Ginzburg-Landau
   - Connection to Laughlin's function and quantum hydro
2. Chiral constraint and boundary dynamics of FQHE droplet
3. Analogies with Calogero-Sutherland model

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