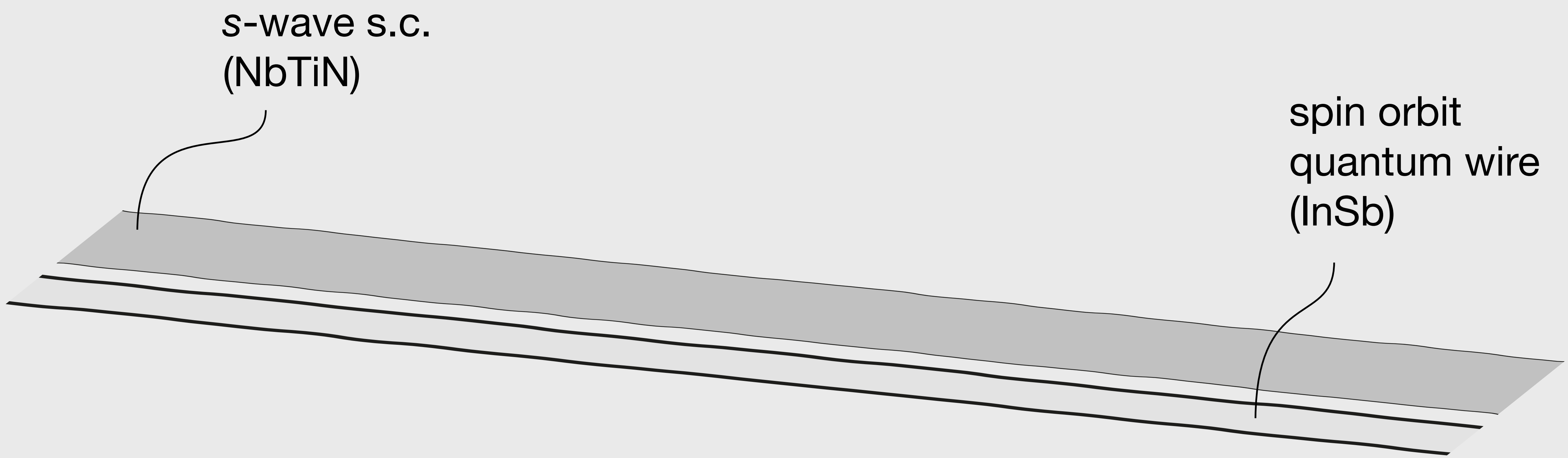
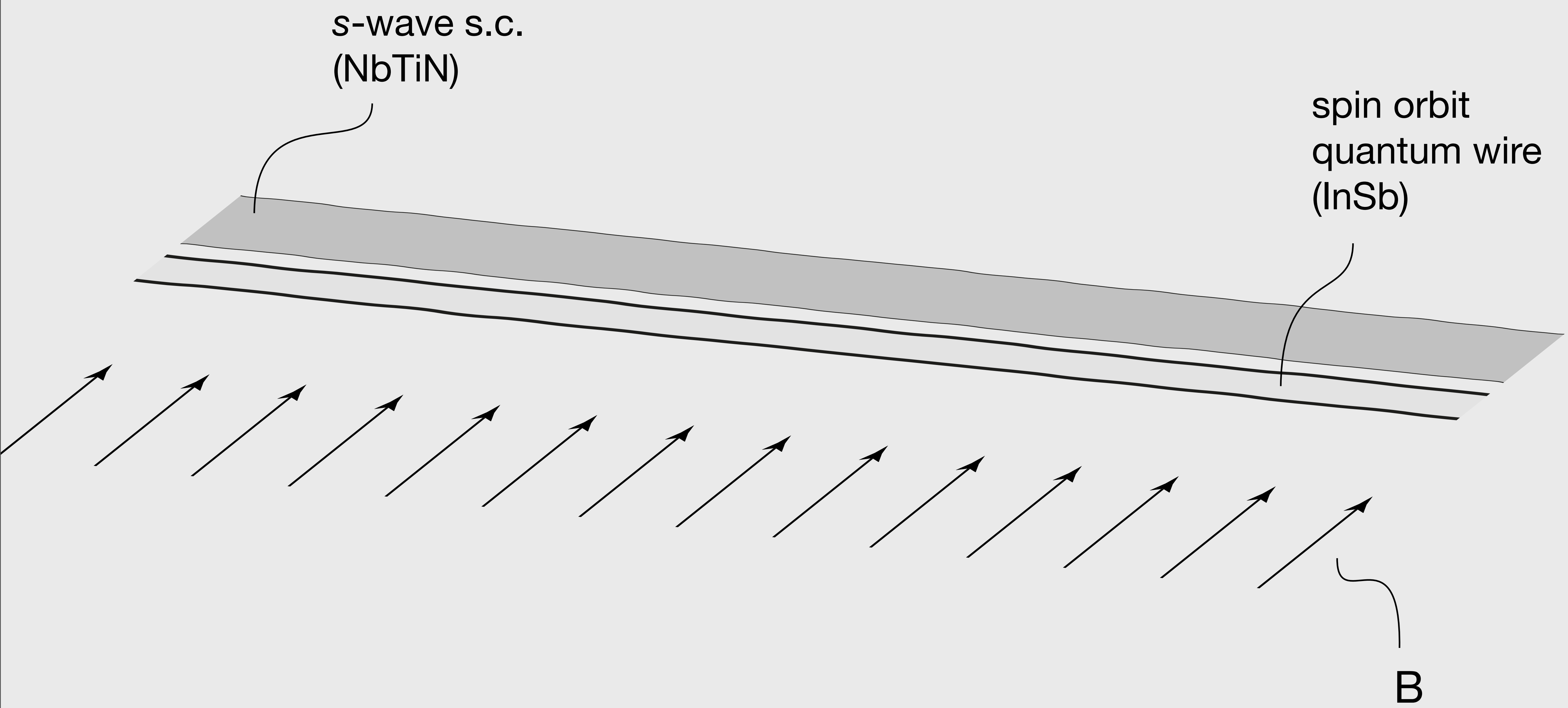
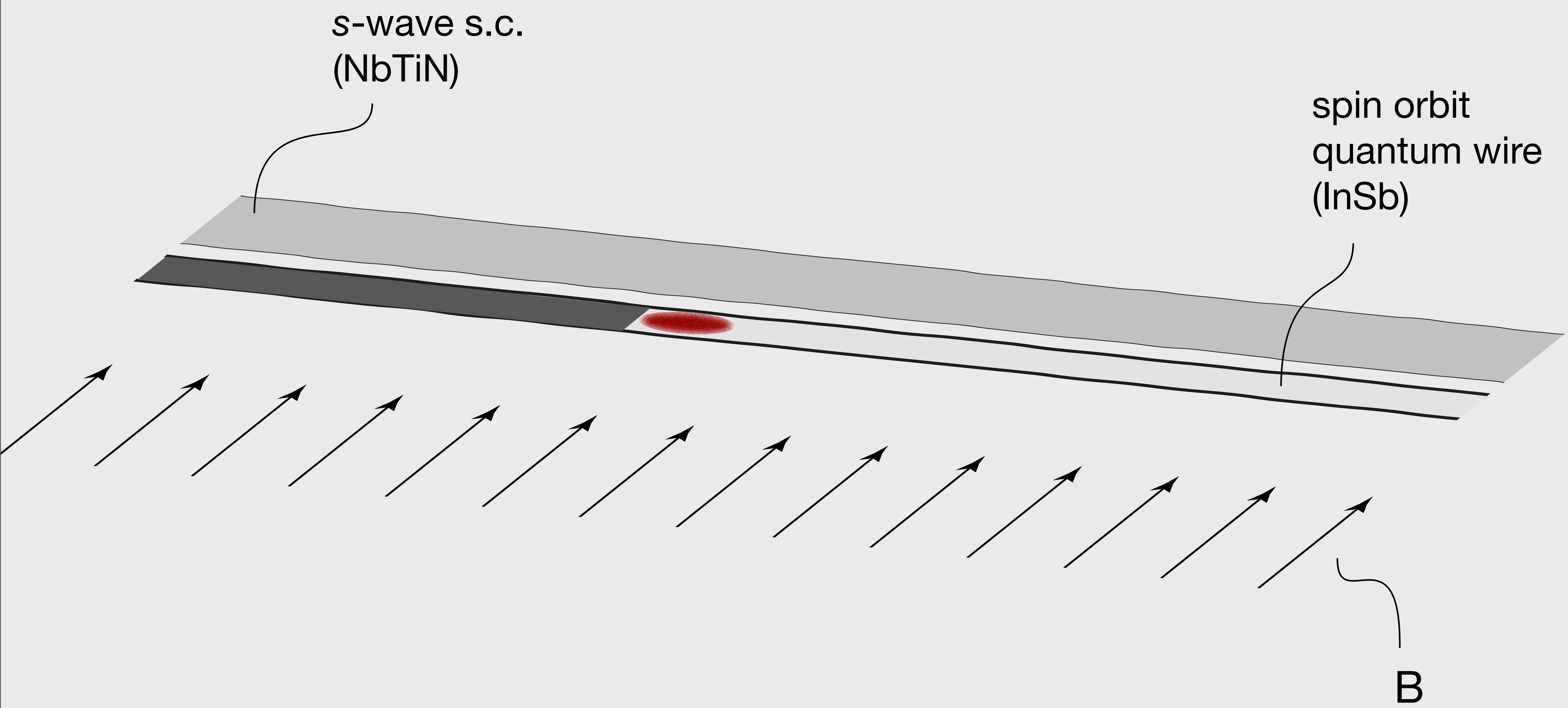
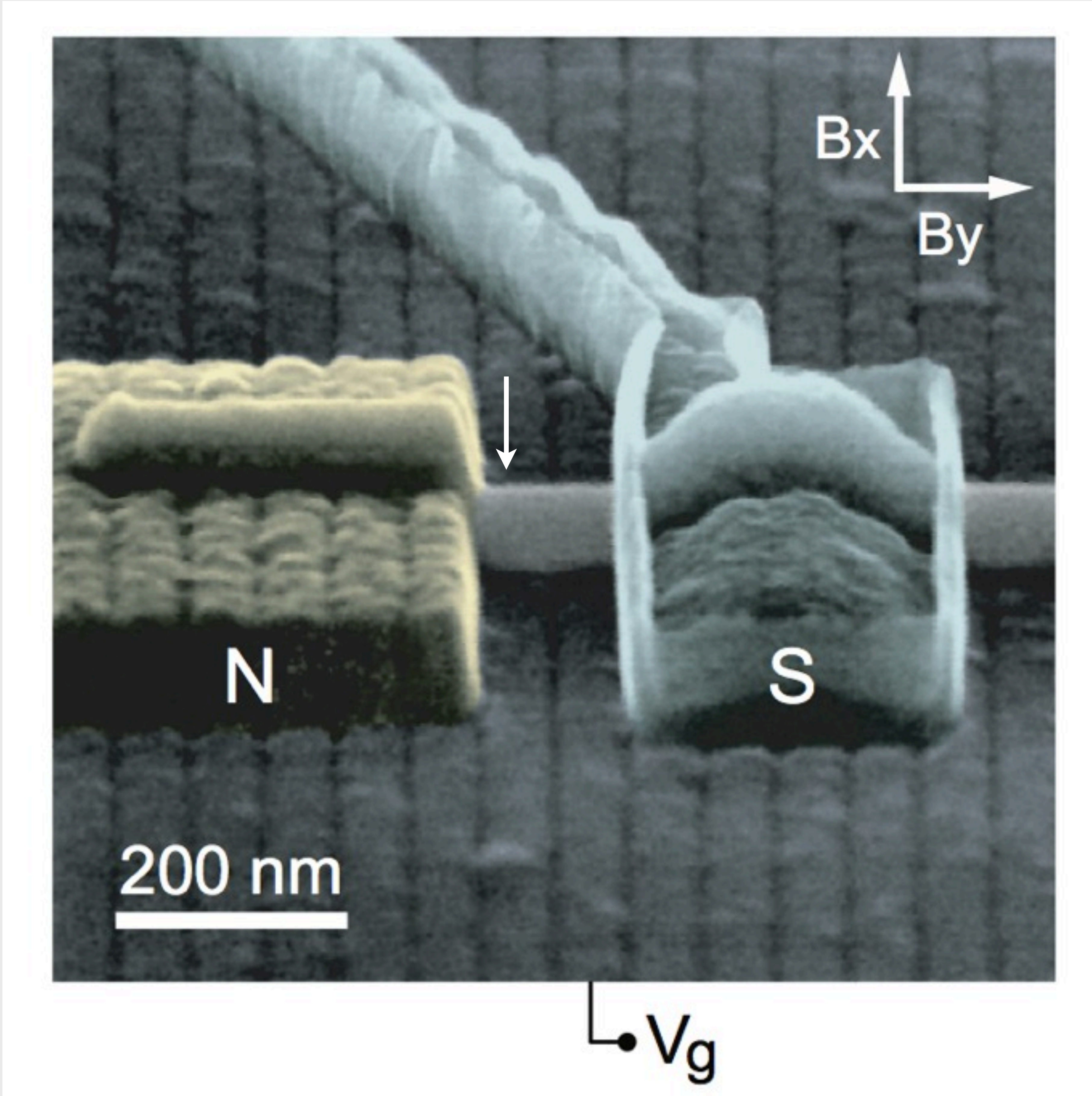

s-wave s.c.
(NbTiN)
spin orbit
quantum wire
(InSb)


s-wave s.c.
(NbTiN)
spin orbit
quantum wire
(InSb)
B

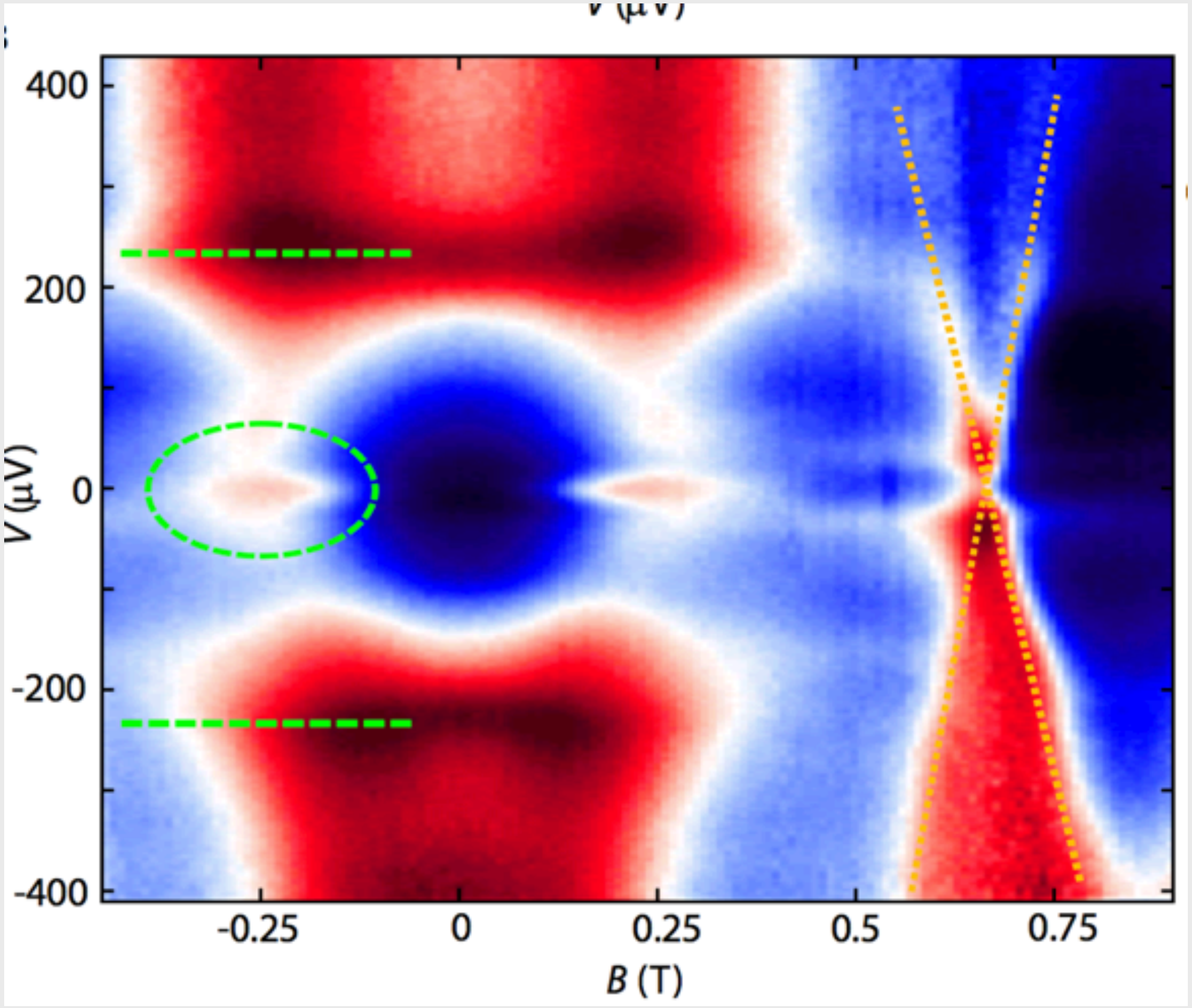

s-wave s.c.
(NbTiN)
spin orbit
quantum wire
(InSb)
B

experiment



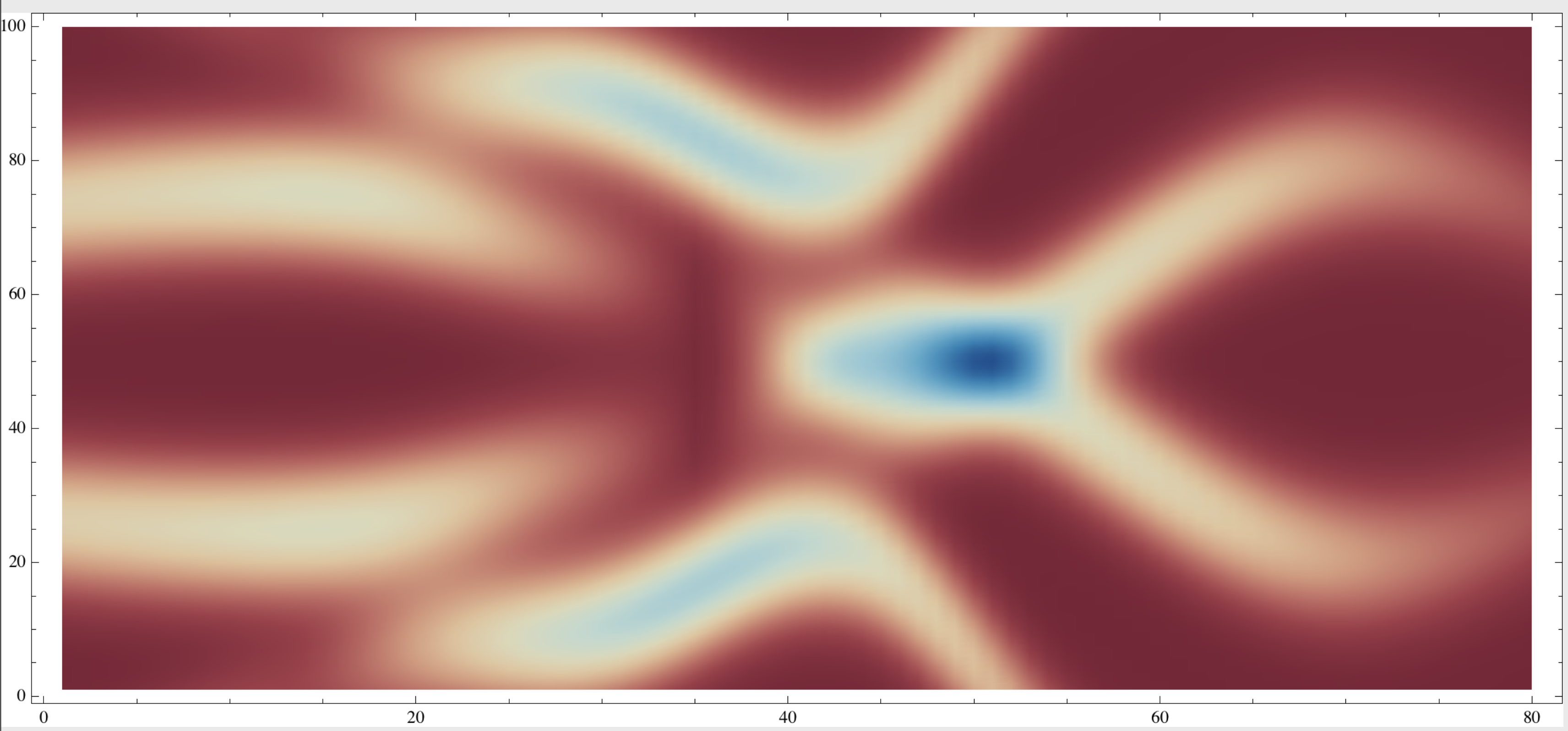
Churchill et al 2013

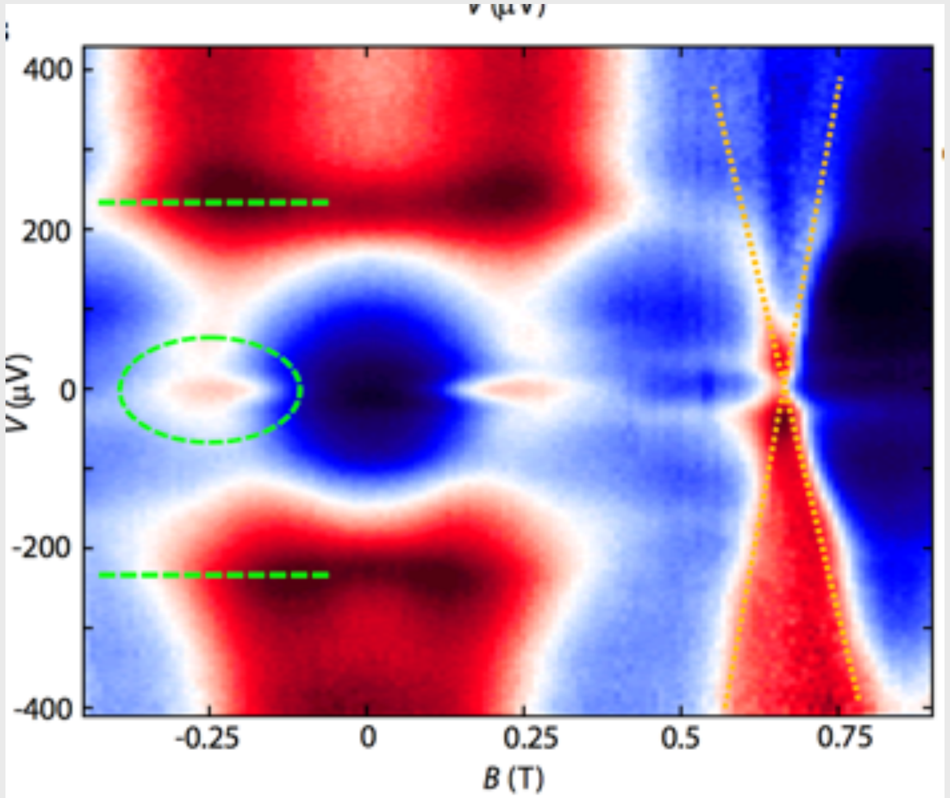
experiment



Mournik et al., 2012

experiment



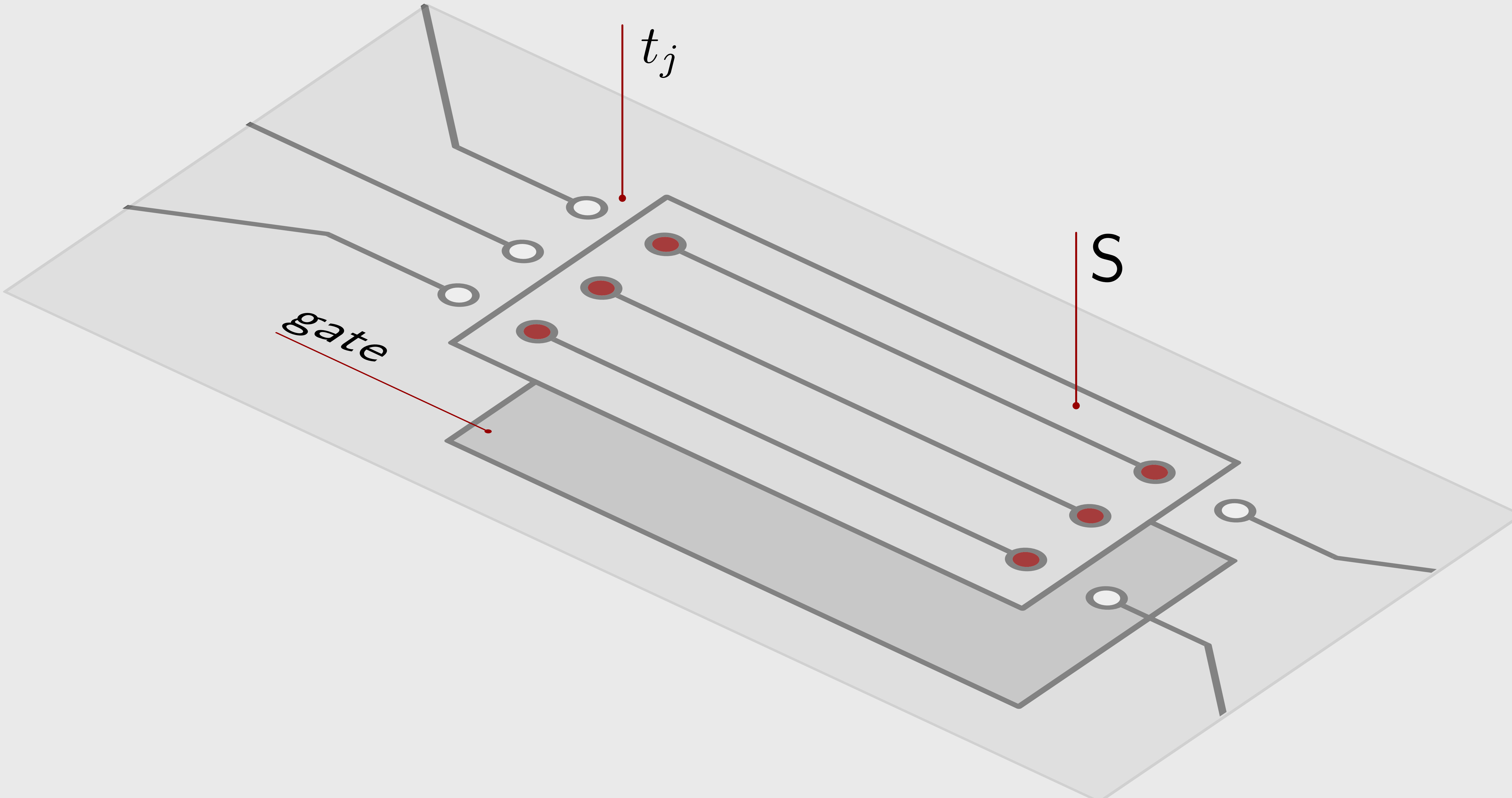


class D RMT (no Majoranas)

cf. A.A. & Bagrets, 12; Beenakker, 12; Lee & Parker, 12

directions of the field

- explore Majorana based quantum information
- explore quantum transport properties of Majorana devices


t_j
S
gate

B = 175 mT

Transport physics of the Coulomb-Majorana junction

St. Peterburg, July 2013

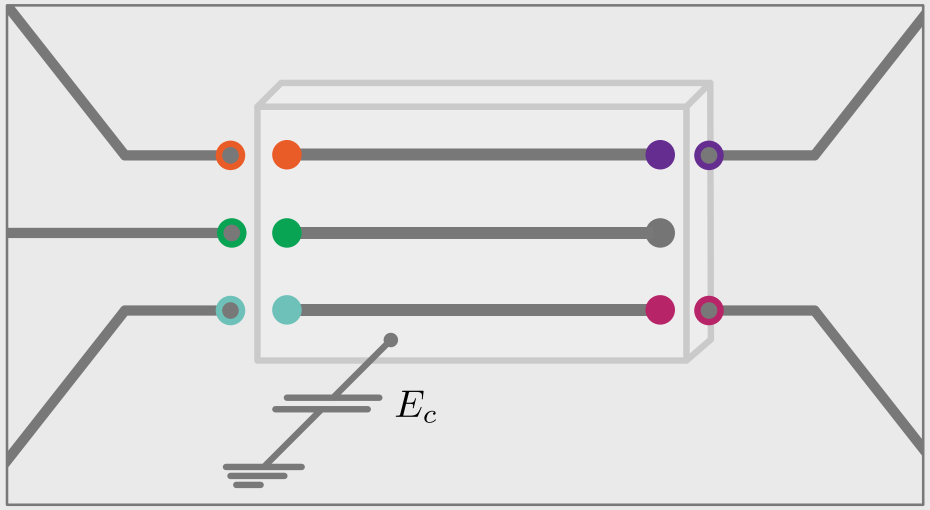
Alexander Altland (Cologne), Reinhold Egger (Düsseldorf)

- ▷ the system & its Hamiltonian
- ▷ (Keldysh) phase action
- ▷ transport

PRL **110**, 196401 (2013), arXiv 1307.0210

system

The system



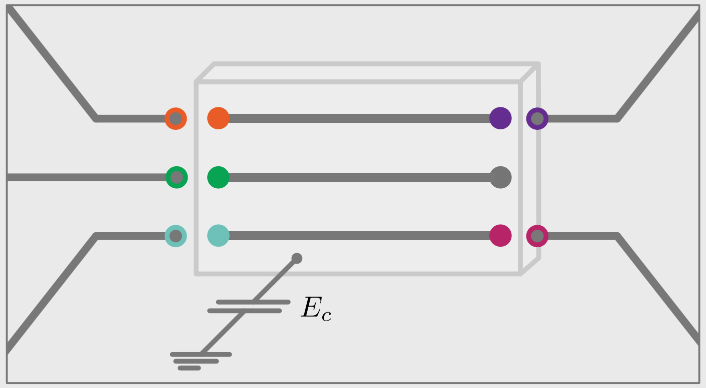
▷ finite superconductor with charging energy

▷ N wires with $2N$ Majorana end states. (No direct tunnel coupling between Majoranas)

▷ tunnel coupled to M single channel quantum wires modeled as (interacting) Luttinger liquids.

▷ goal: understand conductance tensor, noise characteristics at low excitation energies.

degrees of freedom



- *2N* Majorana operators:

$$\{\gamma_i | i = 1, \ldots, 2N\}, \qquad \gamma_i^\dagger = \gamma_i, \quad [\gamma_i, \gamma_j]_+ = \delta_{ij}$$

$$d_i = \frac{1}{\sqrt{2}}(\gamma_{i-1} + i\gamma_i)$$

- superconductor:

$$(\hat{N}, \hat{\phi}), \qquad [\hat{N}, \hat{\phi}] = -\frac{i}{2}$$

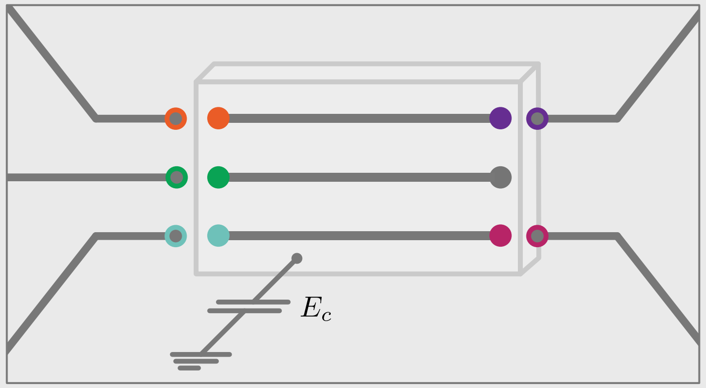
- *M* attached Luttinger liquids:

$$(\theta_i, \phi_i), \qquad [\phi_i(x), \theta_j(x')] = i\frac{\pi}{2}\delta_{ij}\delta(x - x')$$

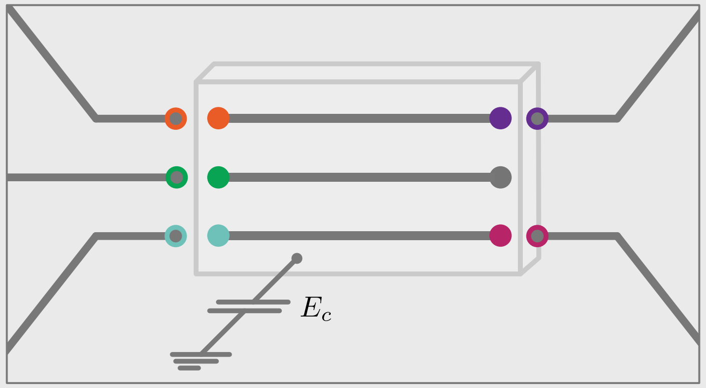
Hamiltonian

- charging

$$\hat{H}_c = E_c \left(2\hat{N}_c + \sum_i d_i^\dagger d_i - n_g \right)^2$$



Hamiltonian

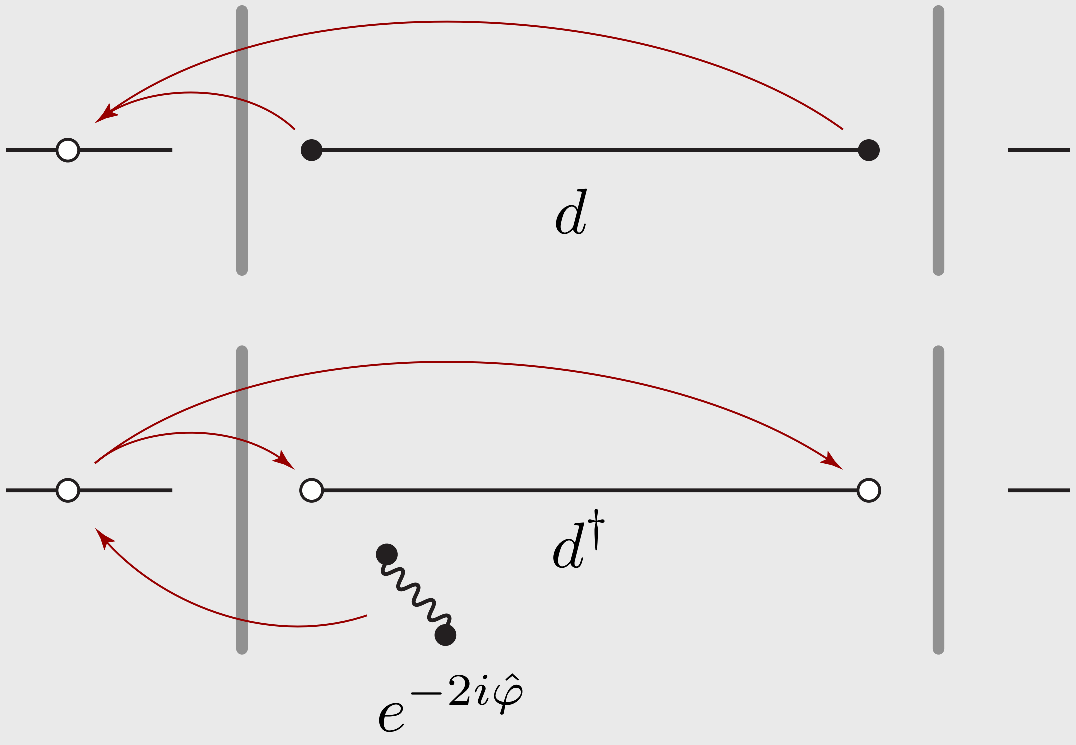


- charging

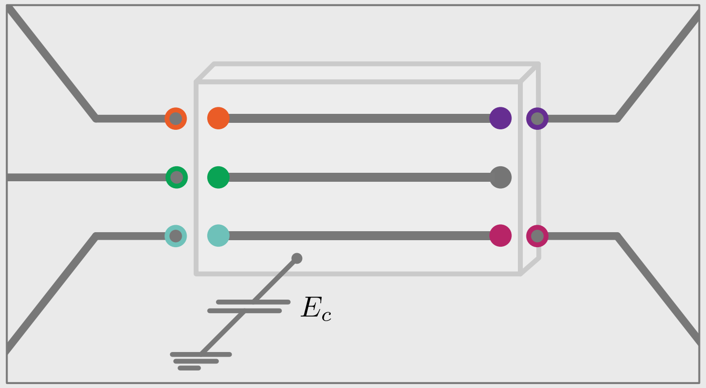
$$\hat{H}_c = E_c\left(2\hat{N}_c + \sum_i d_i^\dagger d_i - n_g\right)^2$$

- lead/dot tunneling

$$\hat{H}_t = \sqrt{a/2}\sum_j t_j \Psi_j^\dagger\left(d_{\alpha_j} + (-)^{j-1} e^{-2i\varphi} d_{\alpha_j}^\dagger\right) + \text{h.c.}$$



Hamiltonian



- charging

$$\hat{H}_c = E_c \left(2\hat{N}_c + \sum_i d_i^\dagger d_i - n_g \right)^2$$

- lead/dot tunneling

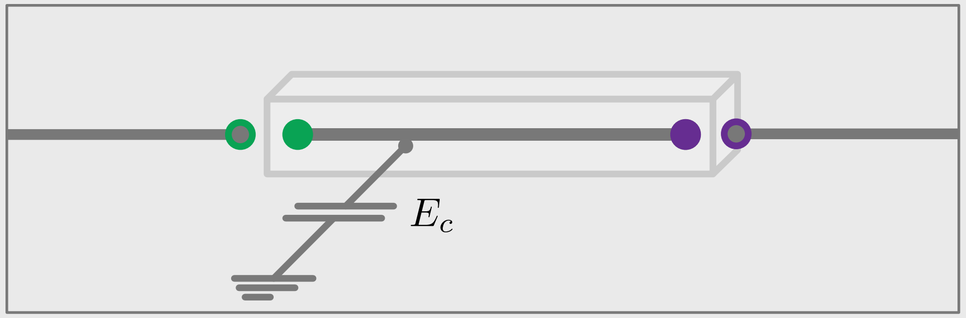
$$\hat{H}_t = \sqrt{a/2} \sum_j t_j \Psi_j^\dagger \left(d_{\alpha_j} + (-)^{j-1} e^{-2i\varphi} d_{\alpha_j}^\dagger \right) + \text{h.c.}$$

- lead Hamiltonian (bosonized)

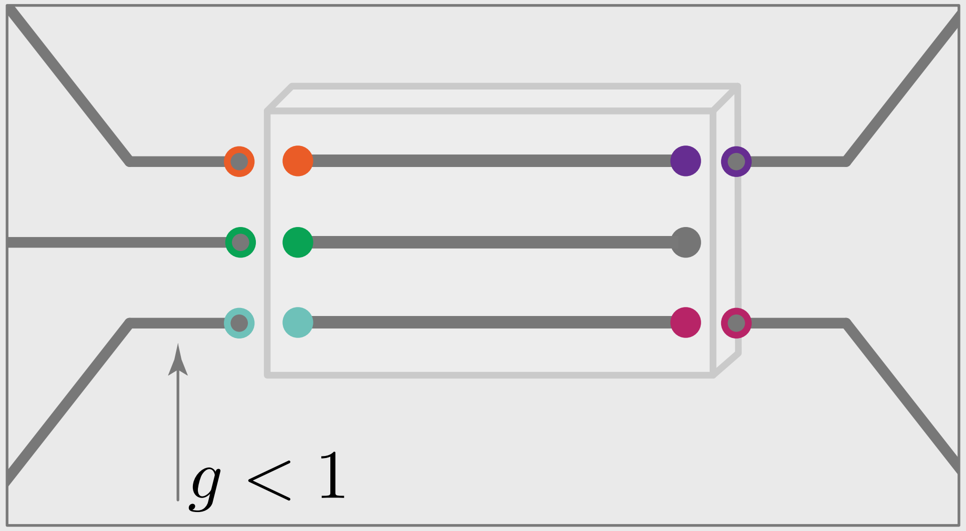
$$\hat{H}_l = \frac{v}{2\pi} \sum_{j=1}^{M} \int_0^\infty dx \left[g(\partial_x \phi_j)^2 + g^{-1} (\partial_x \theta_j)^2 \right]$$

previous work

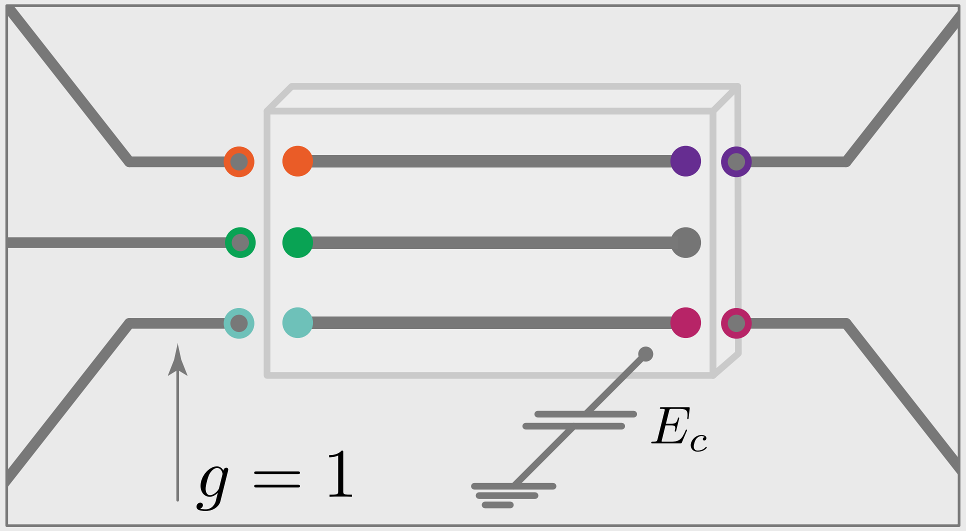
▷ Fu, 09: "quantum teleportation"



▷ Fidkovski et al., 12: "resonant Andreev reflection"



▷ Beri & Cooper, 12: topological Kondo effect



reduction of variables

- Majorana fusion in the tunneling operator

$$\hat{H}_t = \sqrt{a/2}\sum_j t_j \Psi_j^\dagger \left(d_{\alpha_j} + (-)^{j-1} e^{-2i\varphi} d_{\alpha_j}^\dagger \right) + \text{h.c.}$$

$$\Psi_j \sim \eta_j e^{i\Phi_j}, \qquad \Phi_j(t) \equiv \phi(0,t)$$

$$\hat{H}_t = \sum_j t_j \sigma_j \sin(\Phi_j + \varphi)$$

$$\sigma_j = \eta_j \gamma_j, \qquad [\sigma_j, \hat{H}] = 0, \quad \sigma_j \in \{-1, 1\}$$

effective action

$$S[\Phi, \varphi] = S_c[\varphi] + S_{\text{diss}}[\Phi] + S_t[\Phi, \varphi],$$

$$S_l[\Phi] = \frac{Tg}{\pi} \sum_j |\omega_n||\Phi_{j,n}|^2,$$

$$S_c[\varphi] = \int d\tau \, \frac{\dot{\varphi}^2}{4E_c},$$

$$S_t[\Phi, \varphi] = \frac{1}{2} \sum_j t_j \int d\tau \, \sin(\Phi_j + \varphi)$$

- scaling — three distinct regimes

scaling I: resonant Andreev

effective action

▷ action at high frequencies $\omega > E_c$

$$S[\Phi,\varphi] = S_c[\varphi] + S_{\text{diss}}[\Phi] + S_t[\Phi,\varphi],$$

$$S_l[\Phi] = \frac{Tg}{\pi}\sum_j |\omega_n||\Phi_{j,n}|^2,$$

$$S_c[\varphi] = \int d\tau\, \frac{\dot{\varphi}^2}{4E_c},$$

$$S_t[\Phi,\varphi] = \frac{1}{2}\sum_j t_j \int d\tau\, \sin(\Phi_j + \varphi)$$

effective action

- action at high frequencies $\omega > E_c$

$$S[\Phi] = S_{\text{diss}}[\Phi] + S_t[\Phi],$$

$$S_l[\Phi] = \frac{Tg}{\pi} \sum_j |\omega_n| |\Phi_{j,n}|^2,$$

$$S_t[\Phi] = \frac{1}{2} \sum_j t_j \int d\tau \, \sin(\Phi_j)$$

- scaling $t_j \sim t_j b^{1-\frac{1}{2g}}$ (resonant Andreev reflection)
- stops at

$$t_{j,\text{eff}} \sim t_j \left(\frac{\Lambda}{E_c} \right)^{1-\frac{1}{2g}}$$

scaling II: dipole gas

effective action

$$S[\Phi,\varphi] = S_c[\varphi] + S_{\rm diss}[\Phi] + S_t[\Phi,\varphi],$$

$$S_l[\Phi] = \frac{Tg}{\pi}\sum_j |\omega_n||\Phi_{j,n}|^2,$$

$$S_c[\varphi] = \int d\tau\, \frac{\dot{\varphi}^2}{4E_c},$$

$$S_t[\Phi,\varphi] = \frac{1}{2}\sum_j t_j \int d\tau\, \sin(\Phi_j + \varphi)$$

effective action

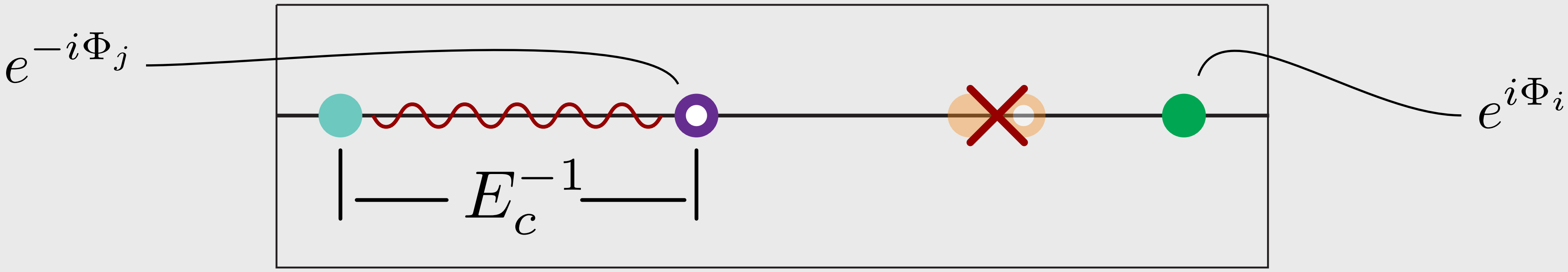
$$S[\Phi,\varphi] = S_c[\varphi] + S_{\text{diss}}[\Phi] + S_t[\Phi,\varphi],$$

$$S_l[\Phi] = \frac{Tg}{\pi}\sum_j |\omega_n||\Phi_{j,n}|^2,$$

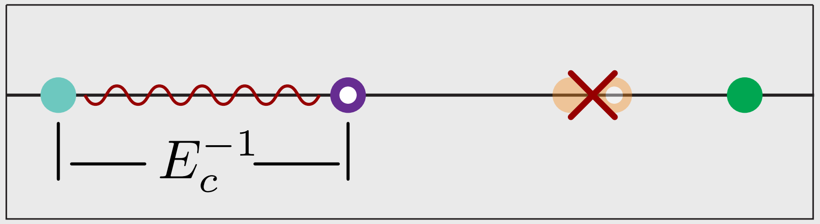
$$S_c[\varphi] = \int d\tau\,\frac{\dot{\varphi}^2}{4E_c},$$

$$S_t[\Phi,\varphi] = \frac{1}{2}\sum_j t_j \int d\tau\,\sin(\Phi_j+\varphi)$$

▷ at frequencies $\omega \sim E_c$: linear confinement between tunneling events



dipole gas



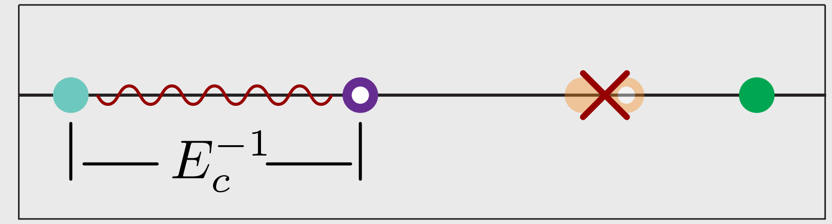
$$S[\Phi,\varphi] = S_c[\varphi] + S_{\mathrm{diss}}[\Phi] + S_t[\Phi,\varphi],$$

$$S_l[\Phi] = \frac{Tg}{\pi}\sum_j |\omega_n||\Phi_{j,n}|^2,$$

$$S_c[\varphi] = \int d\tau\, \frac{\dot{\varphi}^2}{4E_c},$$

$$S_t[\Phi,\varphi] = \frac{1}{2}\sum_j t_j \int d\tau\, \sin(\Phi_j + \varphi)$$

dipole gas

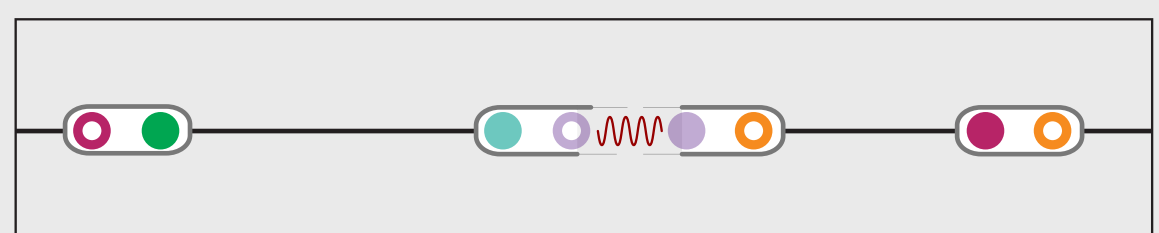


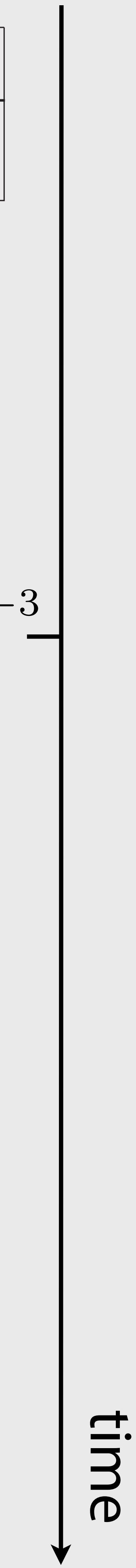
$$S[\Phi] = S_l[\Phi] + S_t[\Phi],$$

$$S_l[\Phi] = \frac{Tg}{\pi} \sum_{j,n} |\omega_n||\Phi_{j,n}|^2,$$

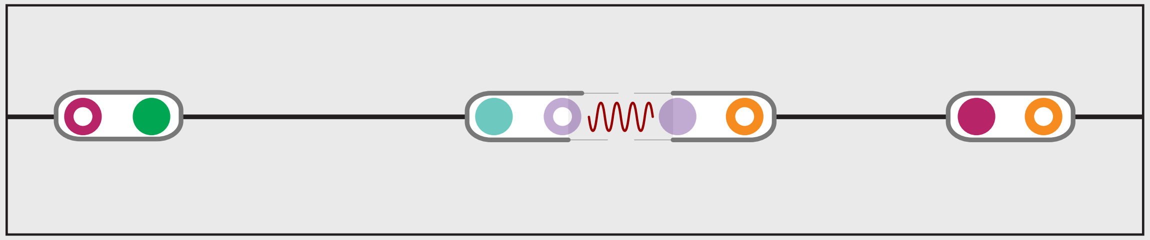
$$S_t[\Phi] = \sum_{j,k} \lambda_{jk} \int dt \cos\left(\Phi_j - \Phi_k\right)$$

$$\lambda_{jk}^{(0)} \sim \frac{t_j t_k}{E_c} \sim E_c^{\frac{1}{g}-3}$$





dipole gas renormalization



▷ RG equations

nonuniversal O(1)

$$\frac{d\lambda_{jk}}{d\ln b} = -(g^{-1} - 1)\lambda_{jk} + \frac{\kappa}{E_c} \sum_{m\neq(j,k)} \lambda_{jm}\lambda_{mk}$$

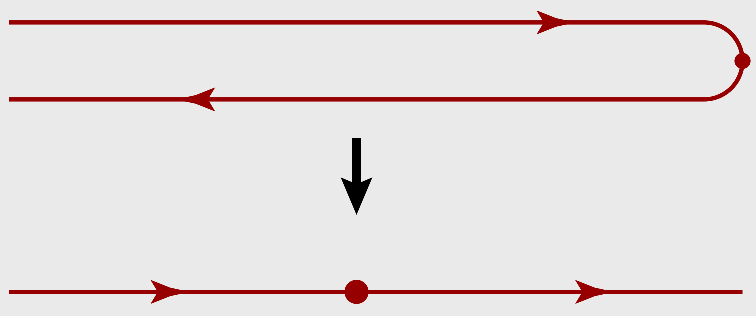
ZBA

dipole merging

$$\lambda_{jk}^{(0)} \sim \frac{t_j t_k}{E_c} \sim E_c^{\frac{1}{g}-3}$$

Kondo symmetry made manifest

▷ consider g=1. Represent leads in terms of chiral fermions



▷ refermionize tunnel boson operators $e^{i\Phi_j} \to \sqrt{a}\Psi_j(0)\eta_j$

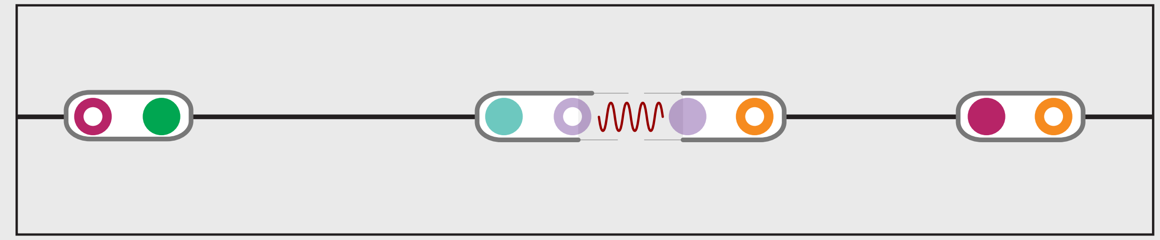
▷ isotropic limit $\lambda_{ij}a \to J$

$$\hat{H}_f = -iv\int dx \sum_j \psi_j^\dagger \partial_x \psi_j + J\sum_{j\neq k} \eta_k\eta_j\psi_j^\dagger(0)\psi_k(0)$$

▷ $\eta_j\eta_k \equiv A_{jk}$ generates so(M). Canonical transformation to real fermions

$$\boxed{\hat{H}_f = -iv\int dx\ \mu^T\partial_x\mu - J\mu^T(0)\hat{A}\mu(0) + (\mu \leftrightarrow \nu)}$$

dipole gas renormalization



- RG equations

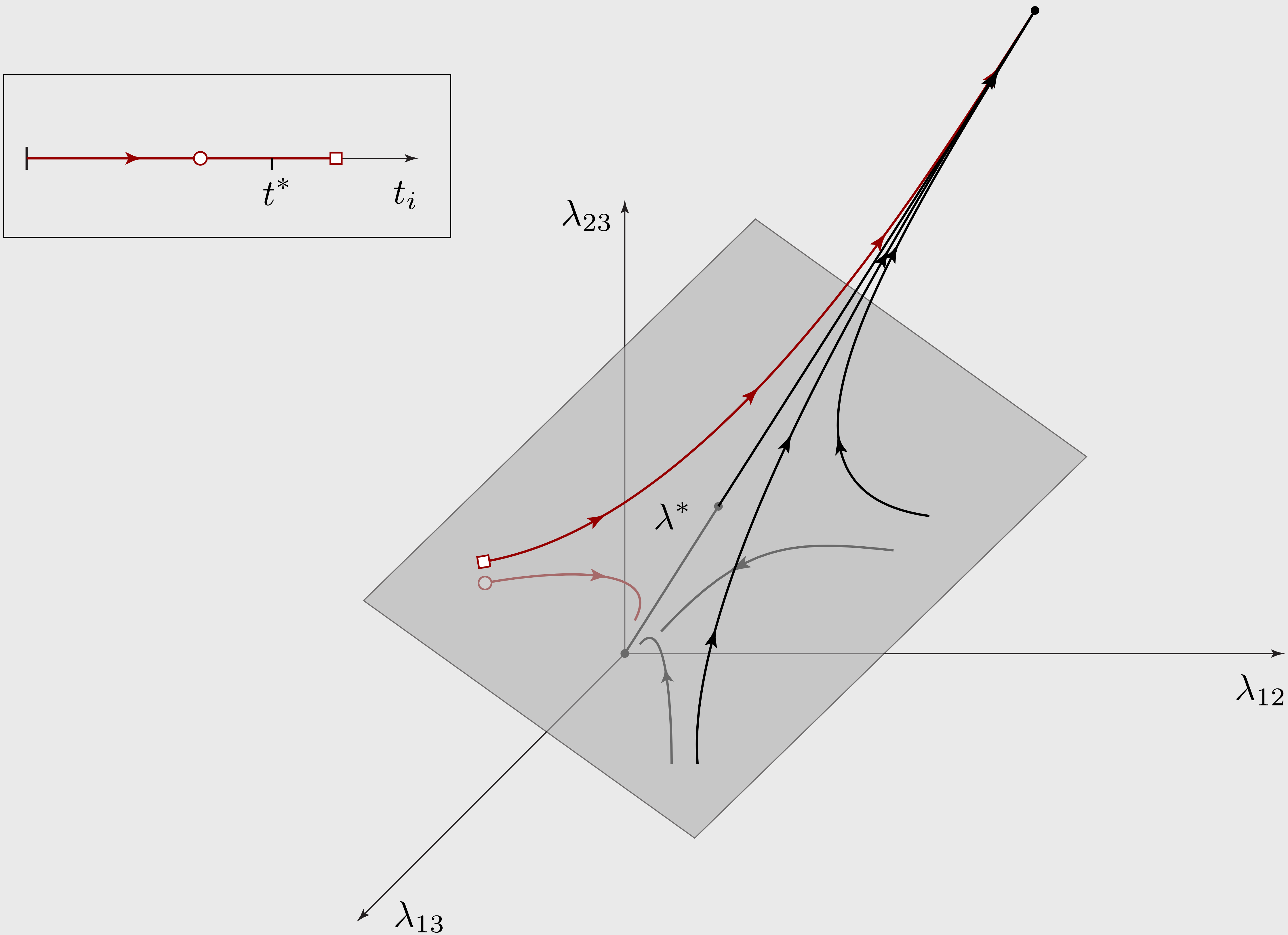
$$\frac{d\lambda_{jk}}{d\ln b} = -(g^{-1}-1)\lambda_{jk} + \frac{\kappa}{E_c}\sum_{m\neq(j,k)}\lambda_{jm}\lambda_{mk}$$

- fixed point at

$$\lambda_{jk} = \lambda^*(1-\delta_{jk}), \qquad \lambda^* = \frac{g^{-1}-1}{\kappa(M-2)}E_c$$

- isotropic deviations $\lambda^* \to \lambda^* + \delta\lambda$ unstable, anisotropic deviations stable

schematic phase diagram



scaling III: strong coupling

strong coupling

- ▷ coupling constants diverge at Kondo temperature

$$T_K \approx E_c \exp\left(-\frac{1}{\kappa(M-2)}\frac{E_c}{\langle\lambda^{(1)}\rangle_{\mathrm{av}}}\right)$$

- ▷ at lower energies ...

$$\boxed{\begin{aligned} &S[\Phi] = S_l[\Phi] + S_t[\Phi], \\ &\quad S_l[\Phi] = \frac{Tg}{\pi}\sum_{j,n}|\omega_n||\Phi_{j,n}|^2, \\ &\quad S_t[\Phi] = \sum_{j,k}\lambda_{jk}\int dt\cos\left(\Phi_j - \Phi_k\right) \end{aligned}}$$

strong coupling

- coupling constants diverge at Kondo temperature

$$T_K \approx E_c \exp\left(-\frac{1}{\kappa(M-2)}\frac{E_c}{\langle\lambda^{(1)}\rangle_{\mathrm{av}}}\right)$$

- at lower energies tunneling between minima of hypertriangular lattice structure (cf. Affleck et al, 05)

- dual action

$$\boxed{\begin{aligned} &S[\beta] = S_{\mathrm{diss}}[\beta] + S_{\mathrm{nlin}}[\beta], \\ &S_{\mathrm{diss}}[\beta] = \frac{T}{\pi g}\sum_{n,j,k} |\omega_n| \beta^T_{n,j}\,(\Delta)_{jk}\,\beta_{-n,k}, \\ &S_{\mathrm{nlin}}[\beta] = y\int d\tau \sum_{j=1}^{M-1}\cos(\beta_j - \beta_{j-1}), \qquad y \sim \exp(-S_{\mathrm{inst}}) \end{aligned}}$$

RG at strong coupling

▷ perturbative RG around strong coupling fixed point

$$y \sim \left(\frac{T}{T_K}\right)^{\Delta_M - 1}, \qquad \Delta_M = 2g\left(1 - \frac{1}{M}\right)$$

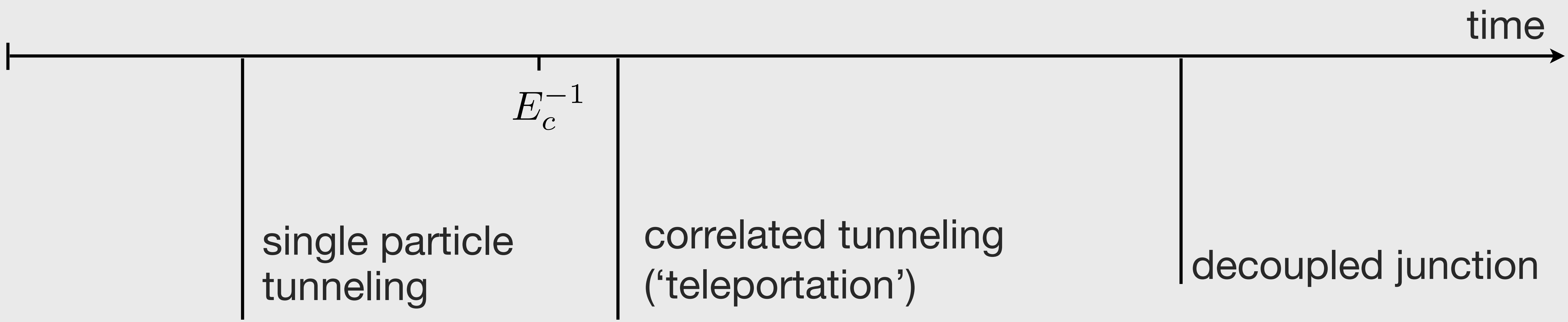
▷ system flows towards infinite coupling. Residual dynamics generated by (symmetry protected) mode

$$\Phi_0 = \frac{1}{M}\sum_j \Phi_j$$

of original theory.

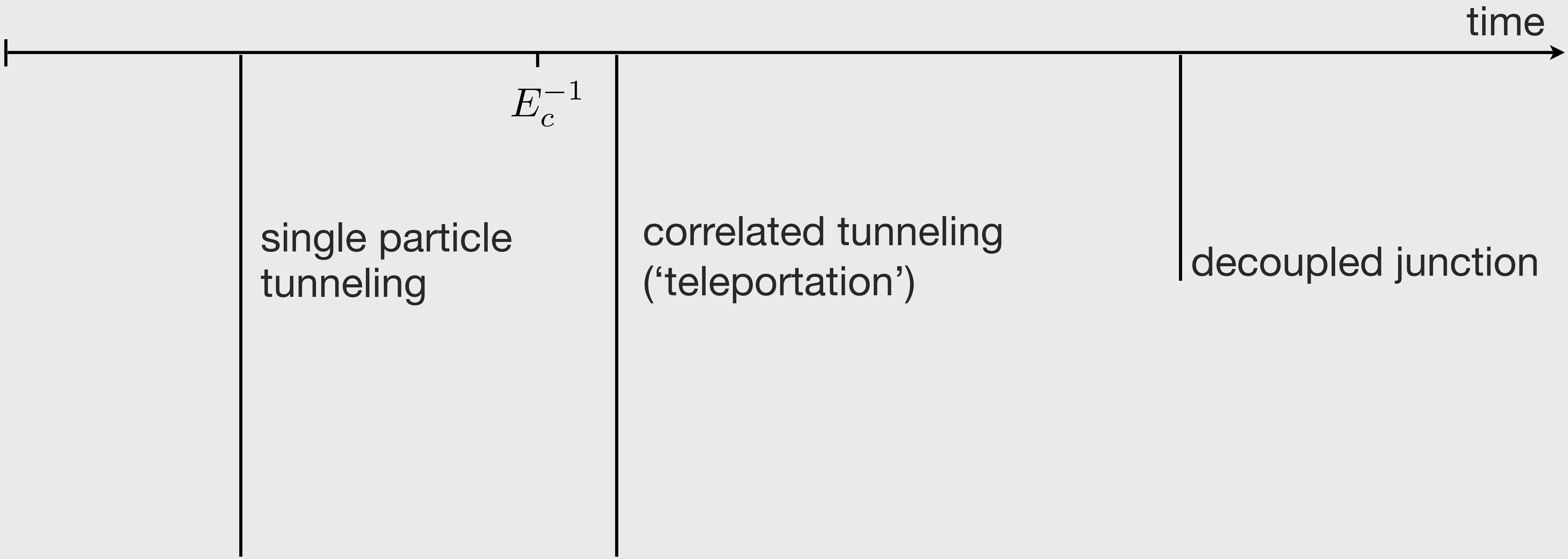
scaling summary

strong charging/weak initial coupling



scaling summary

strong charging/weak initial coupling



weak charging/strong initial coupling

transport

Keldysh/counting fields

- goal: compute average conductance and noise tensor

$$\boxed{\begin{aligned} G_{jk}(\{\mu_i\}) &\equiv -\frac{\partial I_j}{\partial \mu_k} \\ S_{jk}(t-t') &\equiv \frac{1}{2}\left\langle \left[\Delta \hat{I}_j(t), \Delta \hat{I}_k(t')\right]_+ \right\rangle \end{aligned}}$$

- counting fields

$$Z[\chi] = \frac{\mathrm{Tr}\left(\mathcal{T}_K e^{+iH_{-\chi}t_0}\rho_0 e^{-iH_{+\chi}t_0}\right)}{\mathrm{Tr}\rho_0}$$

$$I_j(t) \equiv \langle \hat{I}_j(t)\rangle = -i\left.\frac{\delta \ln Z[\chi]}{\delta \chi_j(t)}\right|_{\chi=0}$$

$$S_{jk}(t-t') = -\left.\frac{\delta^2 \ln Z[\chi]}{\delta\chi_j(t)\delta\chi_k(t')}\right|_{\chi=0}$$

results

- ▷ weak coupling $G_{jk} \sim T^{2/g-2}$ (ZBA suppression of tunneling)

- ▷ strong coupling

$$G_{jk}(T) \overset{T \ll T_K}{=} \frac{2ge^2}{h} \left(\delta_{jk} - \frac{1}{M} \right) \left[1 - c_0 (T/T_K)^{2\Delta_M - 2} + \cdots \right]$$

$$T_K = \left(\frac{\Gamma(2\Delta_M) E_c^2}{2\pi g^2 y^2} \right)^{1/2(\Delta_M - 1)} \frac{E_c}{2g}, \qquad \Delta_M = 2g \left(1 - \frac{1}{M} \right)$$

flow towards isotropic conductance tensor — a perfect ‘beam splitter’

- ▷ noise

$$S_{jk} = -\frac{2ge^2}{\hbar} \sum_{l=1}^{M} \left(\delta_{jl} - \frac{1}{M} \right) \left(\delta_{kl} - \frac{1}{M} \right) \left| \frac{\tilde{\mu}_l}{T_K} \right|^{2\Delta_M - 2} |\tilde{\mu}_l|, \qquad \tilde{\mu}_l = \mu_l - \frac{1}{M} \sum_k \mu_k$$

no ‘genuine shot’ (~V) noise at strong coupling.

summary

- Coulomb-Majorana junction generates universal transport fixed points.
- native Majoranas + ‘auxiliary’ Majoranas -> simple.
- low temperature regimes define unique signatures of Majoranas.