Irrational anyons under an elastic membrane

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Collaborators:

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Ami Katz

Euler Symposium on Theoretical and Mathematical Physics
D.I. Diakonov Memorial Symposium

BOSTON UNIVERSITY

Fractionalization in the Quantum Hall Effect: Chern-Simons perspective

$$\mathcal{L} = -\frac{p}{4\pi}\epsilon^{\mu\nu\lambda}\, a_\mu \partial_\nu a_\lambda + \frac{e}{2\pi}\epsilon^{\mu\nu\lambda}\, A_\mu \partial_\nu a_\lambda$$

Fractionalization in the Quantum Hall Effect: Chern-Simons perspective

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$$J^\mu = \frac{\delta S}{\delta A_\mu} = \frac{e}{2\pi}\,\epsilon^{\mu\nu\lambda}\,\partial_\nu a_\lambda$$

$$0 = \frac{\delta S}{\delta a_\mu} = -\frac{p}{2\pi}\epsilon^{\mu\nu\lambda}\,\partial_\nu a_\lambda + \frac{e}{2\pi}\epsilon^{\mu\nu\lambda}\,\partial_\nu A_\lambda$$

$$J^\mu = \frac{1}{p}\,\frac{e^2}{h}\,\epsilon^{\mu\nu\lambda}\,\partial_\nu A_\lambda$$

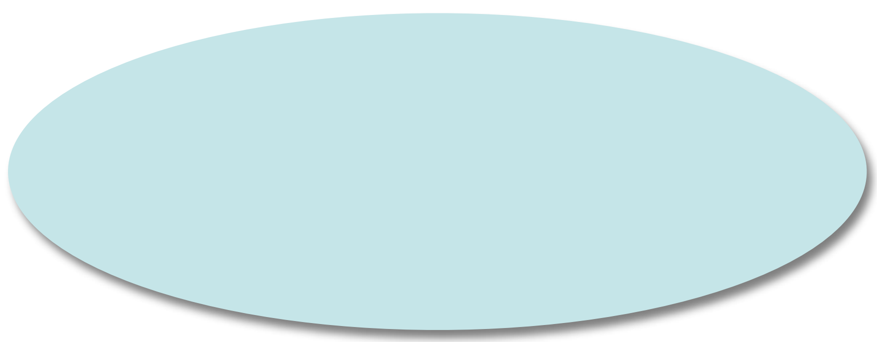
Fractionalization in the Quantum Hall Effect: Chern-Simons perspective

$$J^{\mu} = \frac{1}{p}\,\frac{e^2}{h}\,\epsilon^{\mu\nu\lambda}\,\partial_{\nu} A_{\lambda}$$

$$J_i = \frac{1}{p}\,\frac{e^2}{h}\,\epsilon_{ij}\,E_j$$

$$\rho = \frac{1}{p}\,\frac{e^2}{h}\,\hat{z}\cdot(\vec{\nabla}\times\vec{A})$$

$$Q = \frac{1}{p}\,e\,\frac{\Phi}{\phi_0}$$



Fractionalization in the Quantum Hall Effect: Chern-Simons perspective

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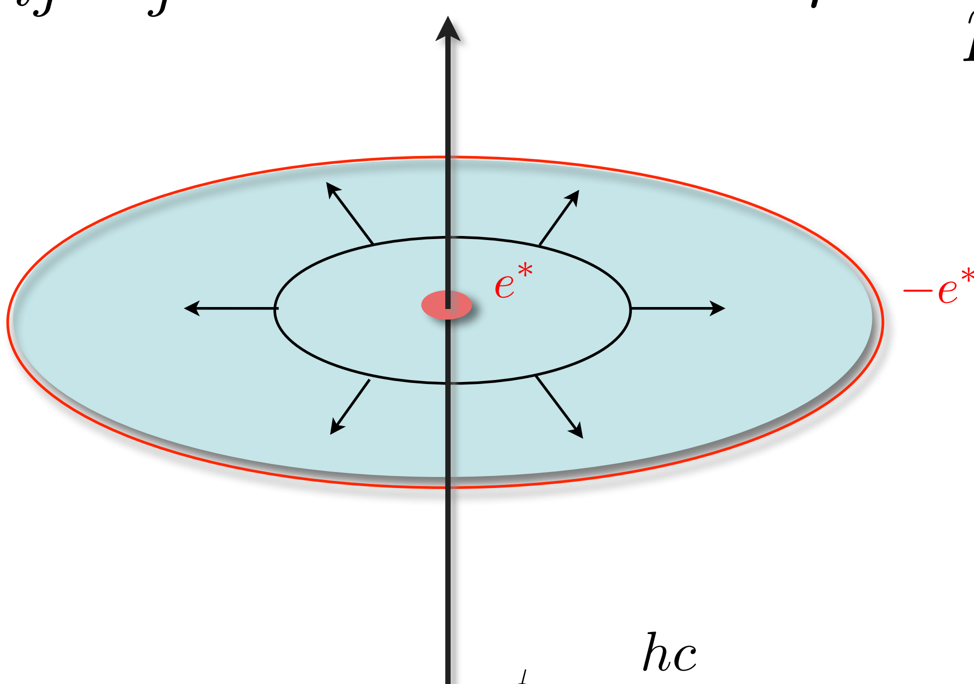

$$\phi_0 = \frac{hc}{e}$$

Laughlin (1983)

Does statistical angle need to be rational?

counter-examples:

Wilczek, PRL (1982)

Jackiw and Nair, Phys. Lett. B (2000)

. . .

Fitzpatrick, Kachru, Kaplan, Katz, and Wacker, arXiv:1205.6816

counter-examples with realization on a lattice:

Chamon, Hou, Jackiw, Mudry, Pi, and Schnyder, PRL (2008)

Ryu, Mudry, Hou, and Chamon, PRB (2009)

Perspective
Departure from Chern-Simons Theory

$$a_i = \alpha \, \epsilon_{ij} \nabla_j \phi \qquad H = \frac{1}{2m} \, \psi^\dagger \left(p_i - \alpha \, \epsilon_{ij} \nabla_j \phi \right)^2 \psi$$

$$b = \epsilon_{ij} \nabla_i a_j = -\alpha \, \nabla^2 \phi$$

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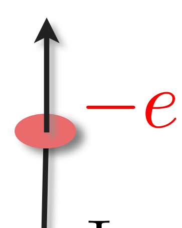

$$\Phi = -\alpha\, g/\tau$$

Wilczek, PRL (1982)
Fitzpatrick, Kachru, Kaplan, Katz, and Wacker, arXiv:1205.6816

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Departure from Chern-Simons Theory

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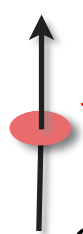
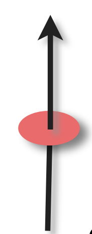


$-e$

$\Phi = -\alpha\, g/\tau$

vs



$e^* = \nu\, e$

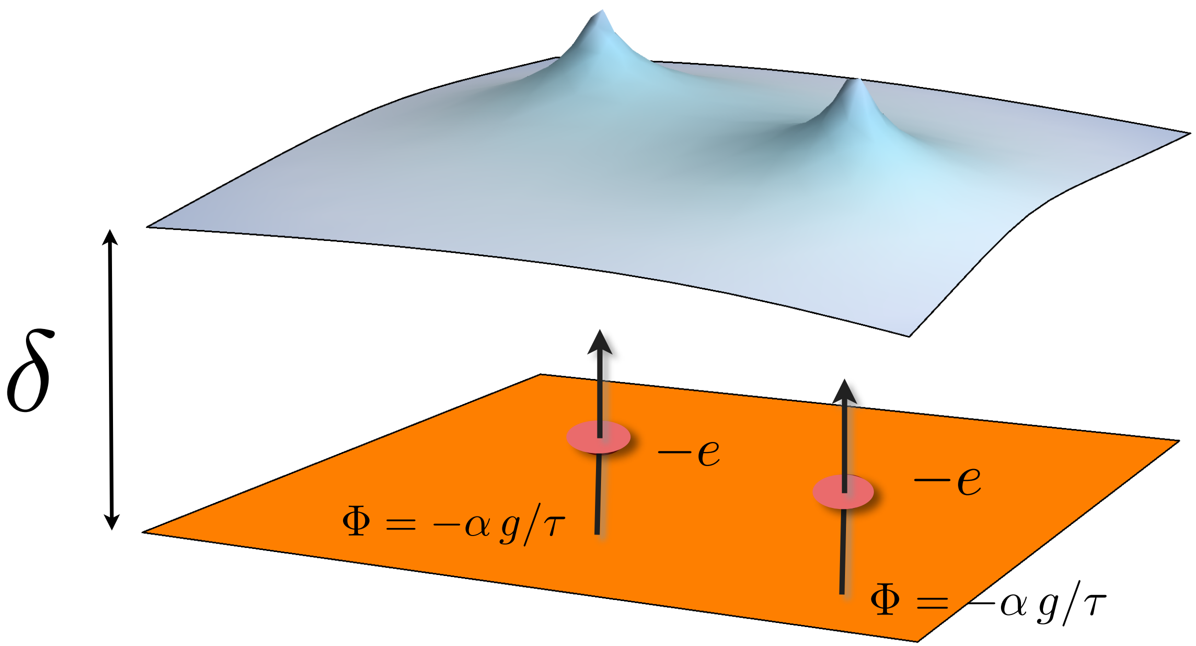
$\Phi_0 = hc/e$

Wilczek, PRL (1982)

Fitzpatrick, Kachru, Kaplan, Katz, and Wacker, arXiv:1205.6816

Flux attachment under an elastic, electric and magnetic membrane

Behbahani, Chamon, and Katz, arXiv:1307.



$$\theta_{\rm stat} = \pi + \theta$$

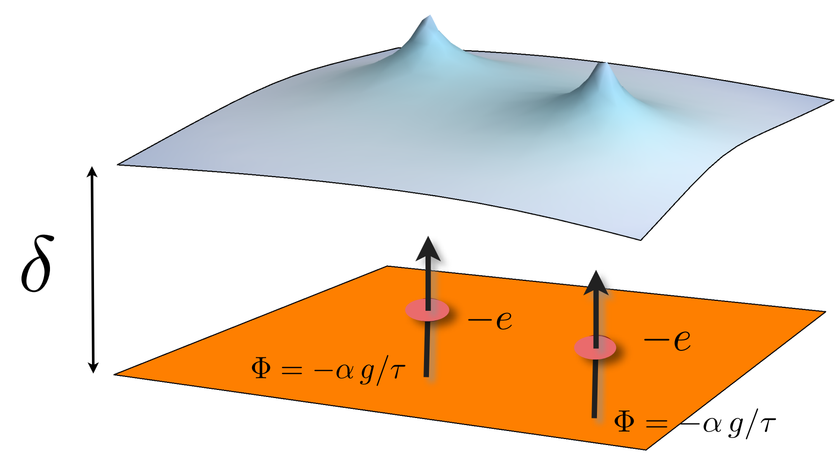
$$\boxed{\theta = -2\pi^2 e^2\ \frac{\sigma_e \sigma_m}{\tau}}$$

Flux attachment under an elastic, electric and magnetic membrane

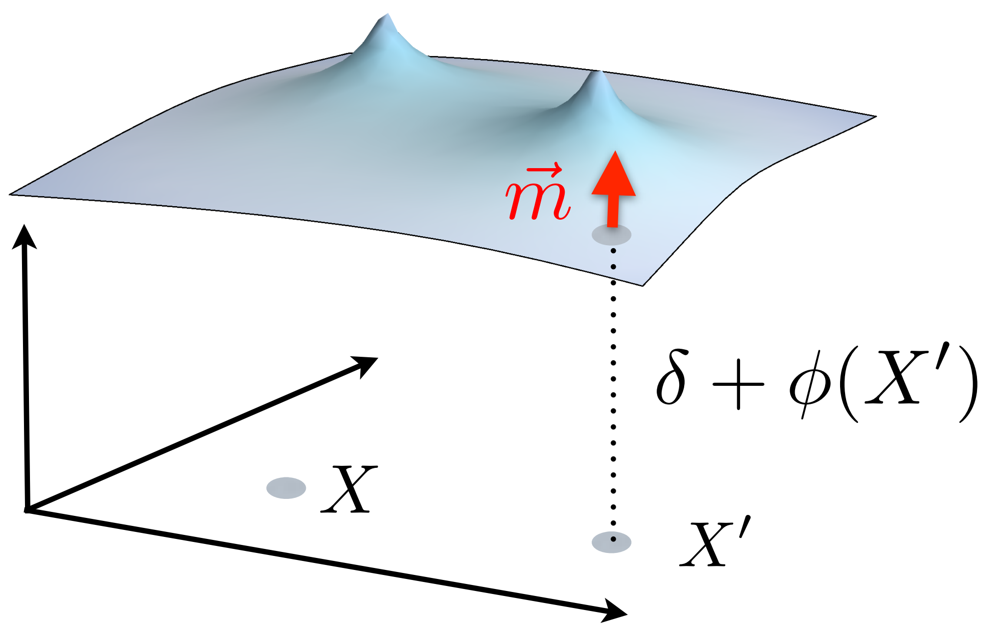
$$S = \int dt\, d^2x \left[\frac{\rho}{2}\, \dot{\phi}^2 - \frac{\tau}{2}\, (\nabla\phi)^2 - \frac{\kappa}{2}\, (\nabla^2\phi)^2 \right.$$

$$\left. + \psi^\dagger i\partial_t \psi - \frac{1}{2m} \left| \left(\nabla_i - i\alpha\, \epsilon_{ij} \nabla_j \phi \right) \psi \right|^2 - g\, \phi\, \psi^\dagger \psi \right]$$

$$H_\psi = \frac{1}{2m} \left(\vec{p} + e\vec{A} \right)^2 - eA_0$$



Flux attachment under an elastic, electric and magnetic membrane



$$A_i(X) = \int d^2X' \, \frac{\sigma_m \, \epsilon_{ij} \, (X - X')_j}{\left[(X - X')^2 + (\delta + \phi\,(X'))^2\right]^{3/2}},$$

Flux attachment under an elastic, electric and magnetic membrane

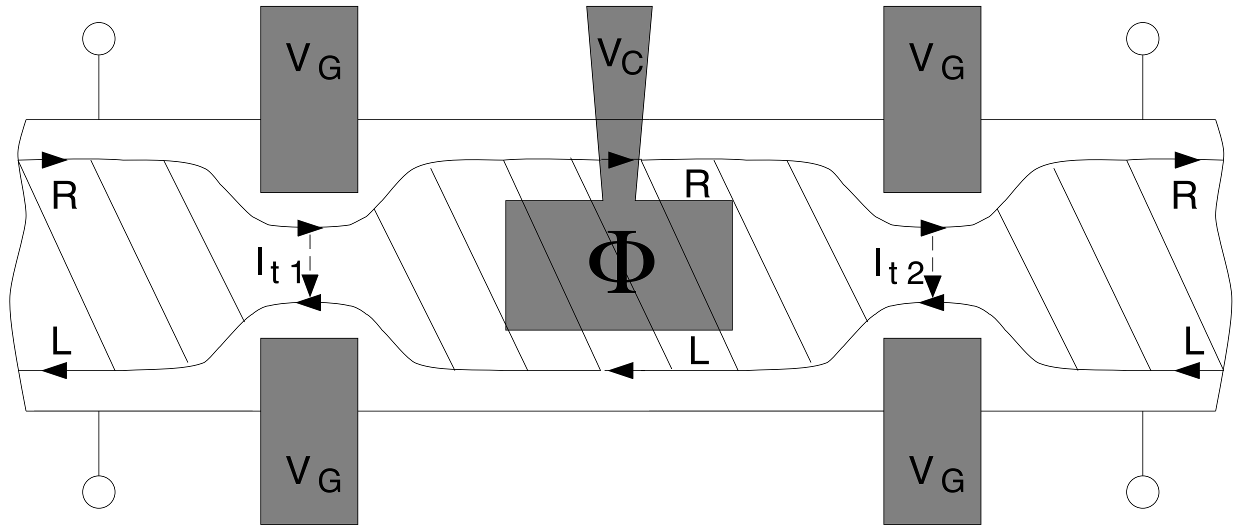
$$\vec{A} = \vec{A}^{BG} + \vec{A}^{\phi}$$

$$A_i^{\phi}(X) \approx 2\pi\,\sigma_m\,\epsilon_{ij}\nabla_j\phi(X)\,\mathrm{sgn}(\delta) \;\longrightarrow\; \alpha = \;-2\pi\,\sigma_m\;e\;\mathrm{sgn}(\delta)$$

$$A_0^{\phi}(X) \approx -2\pi\,\sigma_e\,\phi(X)\,\mathrm{sgn}(\delta) \;\longrightarrow\; g = 2\pi\,\sigma_e\;e\;\mathrm{sgn}(\delta)$$

$$\boxed{\theta = -2\pi^2 e^2\;\frac{\sigma_e\sigma_m}{\tau}}$$

Experimentally probing fractional statistics?



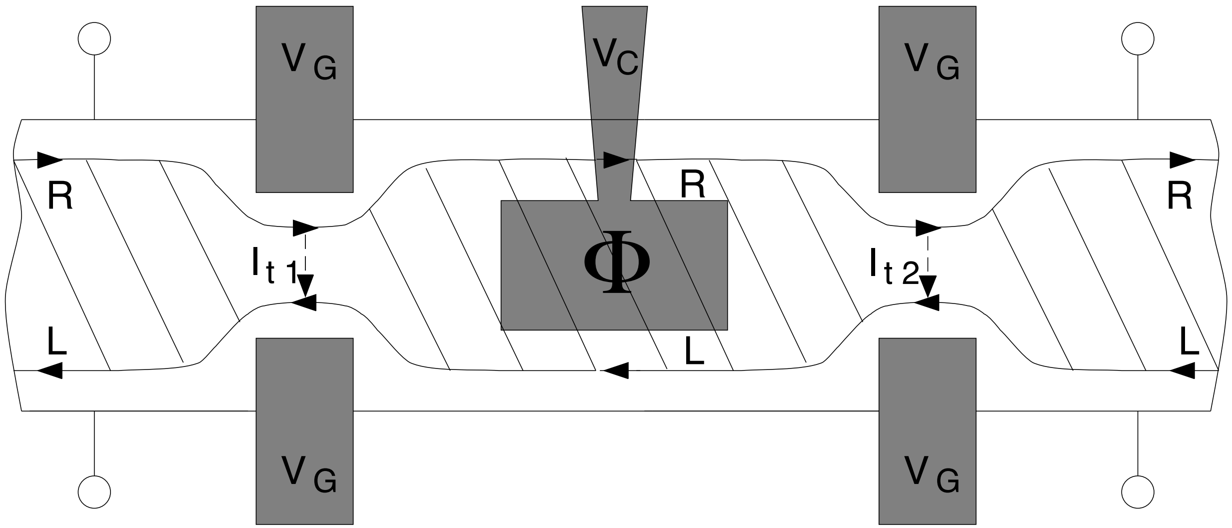
Chamon, Freed, Kivelson, Sondhi, Wen, PRB (1997)

$$\Delta B^* = \Phi^*/A$$

$$\Delta B = \Phi_0/A$$

$$\Phi^* = \frac{e}{e^*}\Phi_0$$

Experimentally probing fractional statistics?



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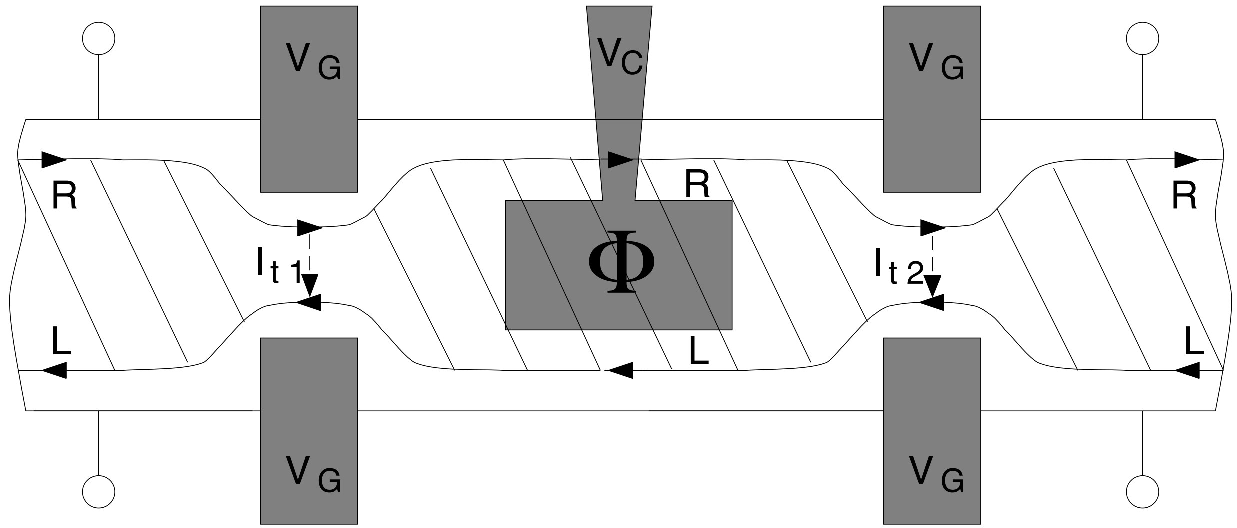

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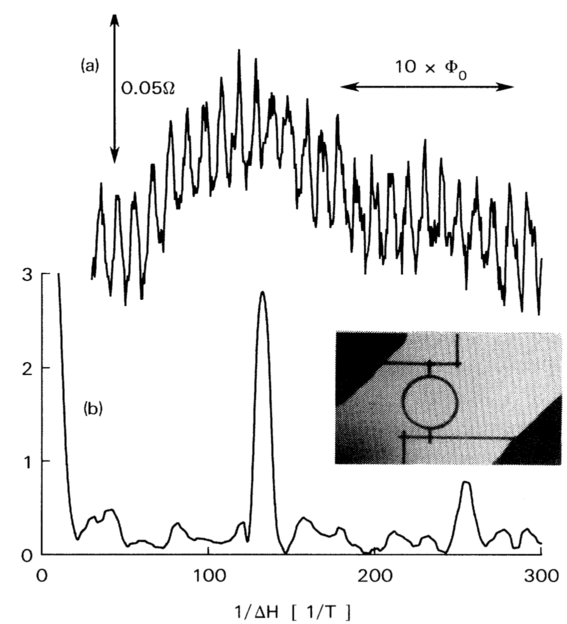

Webb et al., PRL (1985)

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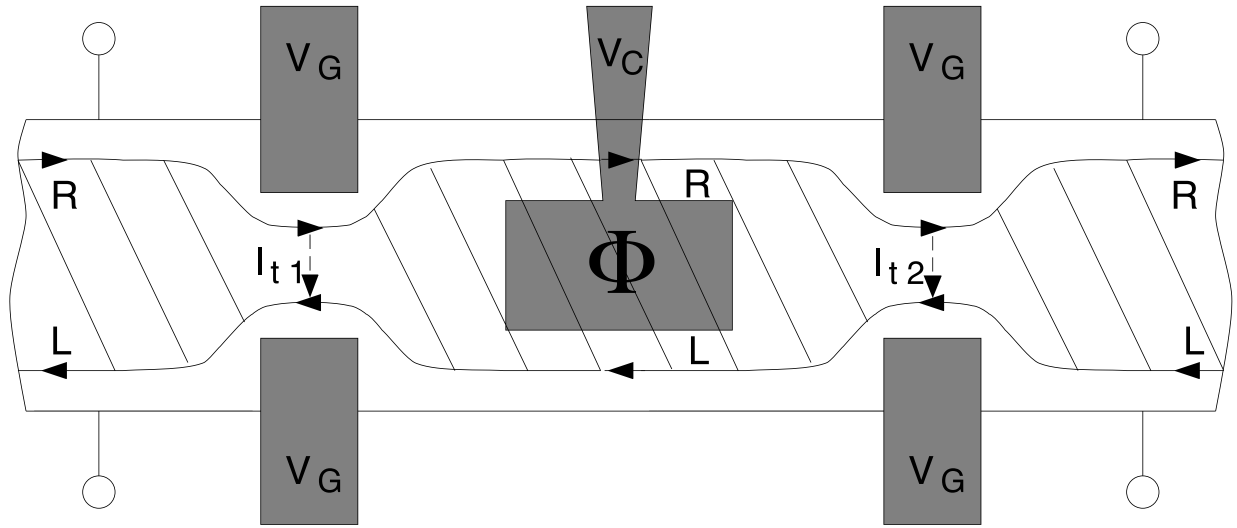

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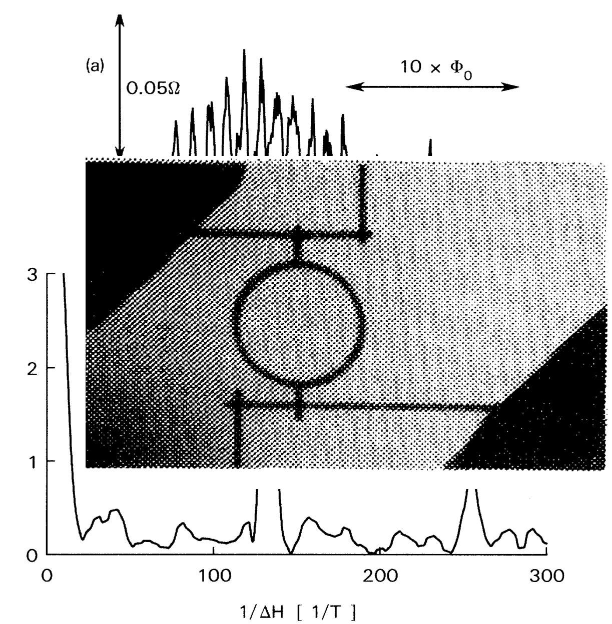

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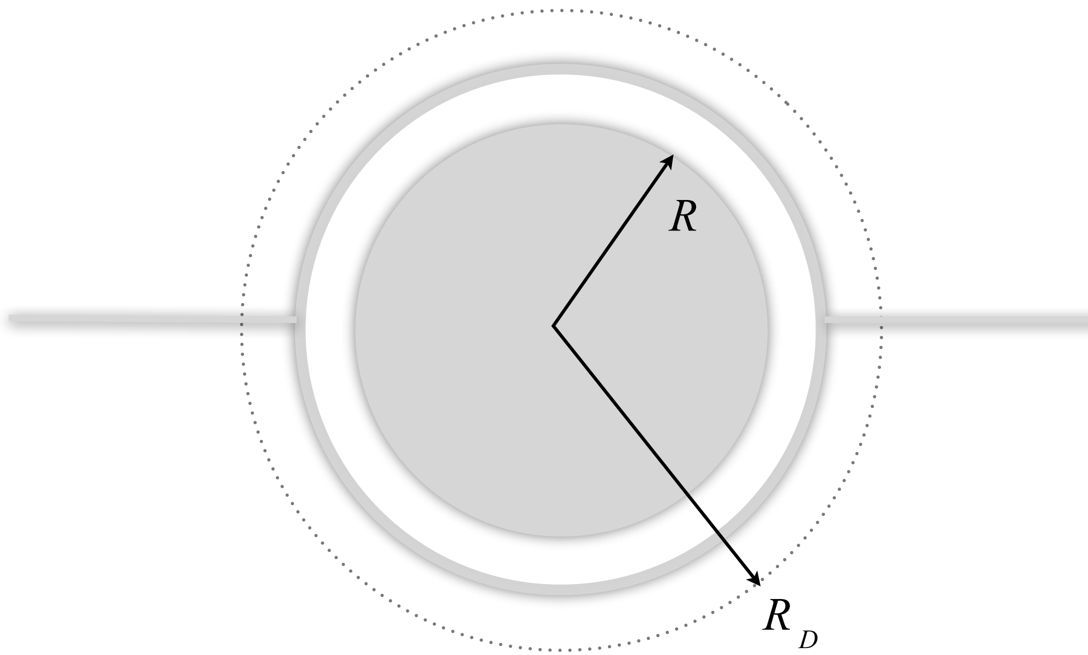
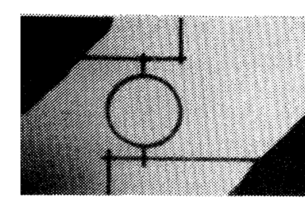
Proposed experimental set-up to probe anyons under a membrane

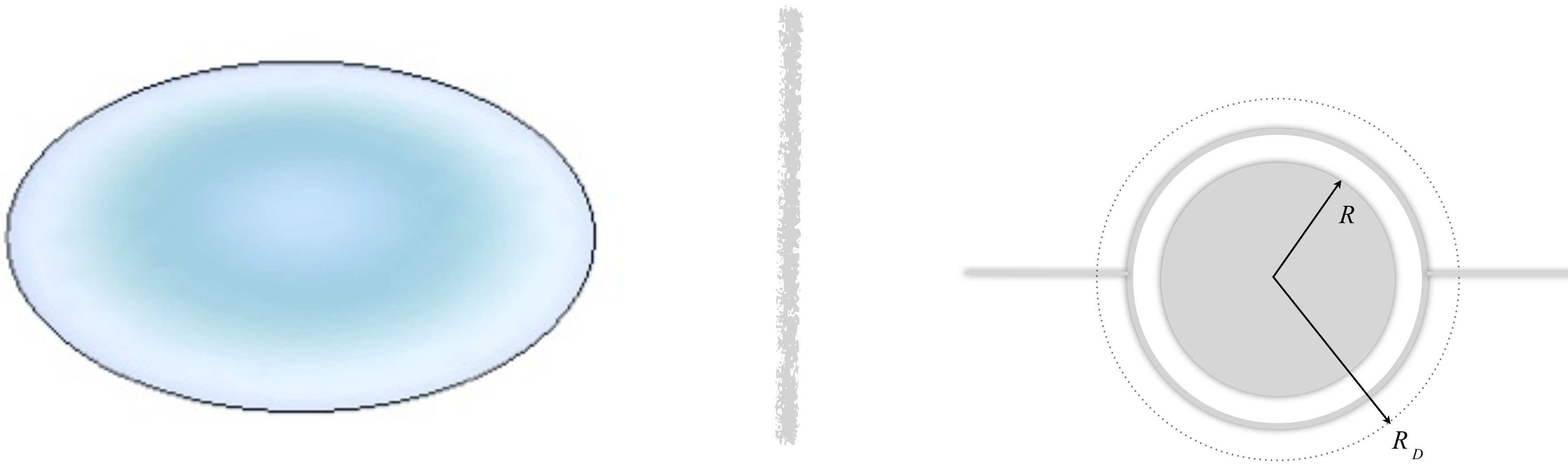




Proposed experimental set-up to probe anyons under a membrane

membrane layer

electron gas layer



$$\nabla^2 \phi(\vec{r}) = \frac{g}{\tau}\,\rho(\vec{r}) - \frac{\alpha}{\tau}\,\hat{z}\cdot\vec{\nabla}\times\vec{J}(\vec{r})$$

$$B = \frac{\alpha}{e}\,\nabla^2\phi$$

Proposed experimental set-up to probe anyons under a membrane

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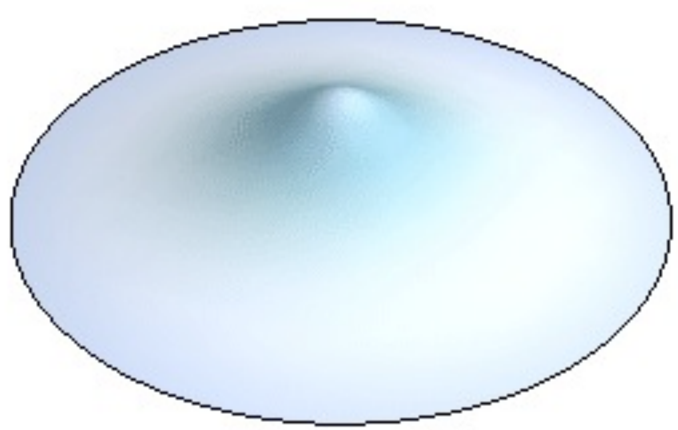

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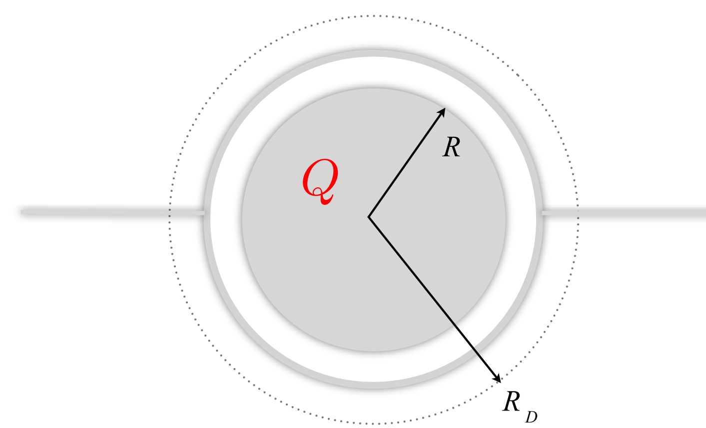

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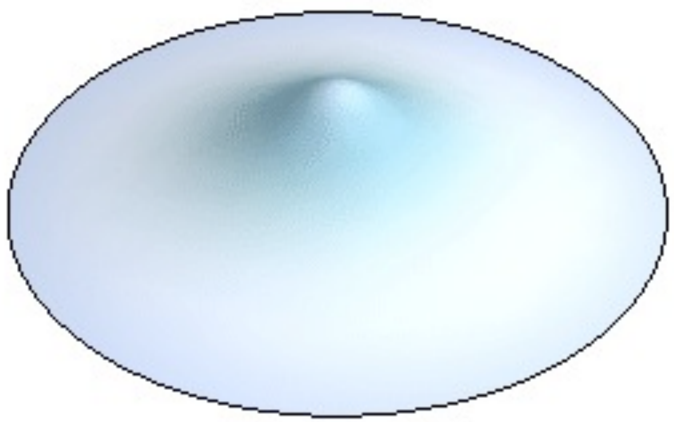

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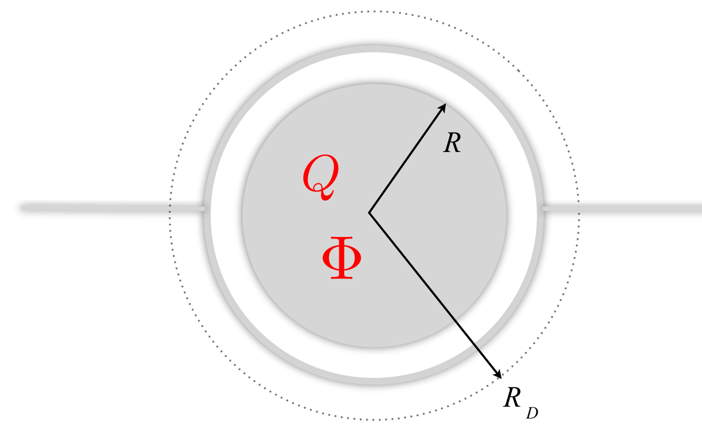

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$$\Phi = -\frac{\alpha g}{\tau}\,\frac{Q}{e^2}$$

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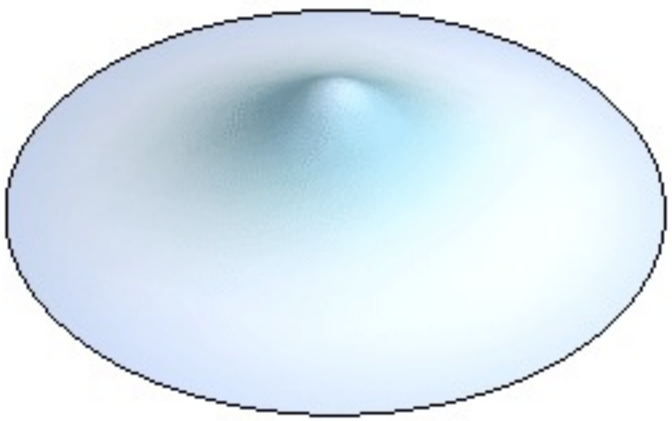
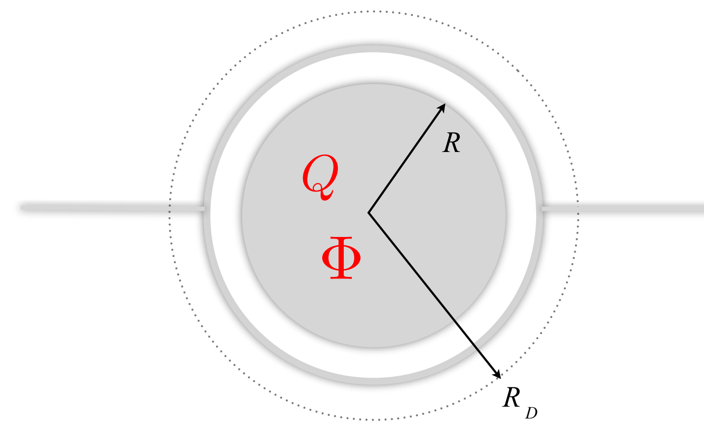


electron gas layer



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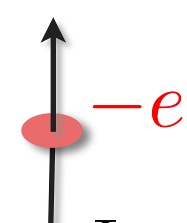


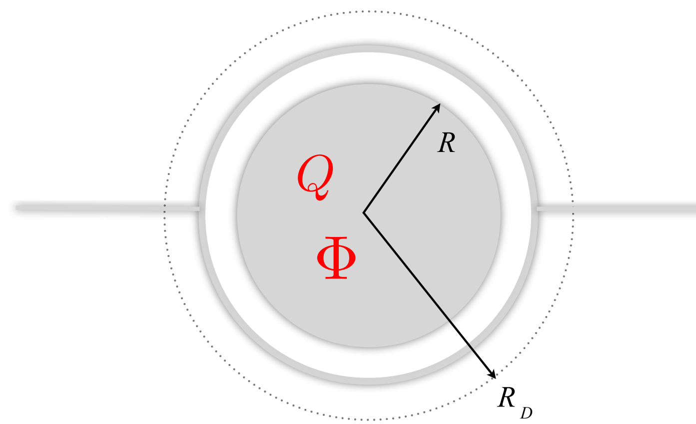
$$\Phi = -\alpha\,g/\tau$$

Proposed experimental set-up to probe anyons under a membrane

$$G(\Phi_{\mathrm{total}}/\Phi_0)$$

$$\Phi_{\mathrm{total}} = \Phi + \Phi_{\mathrm{ext}}$$

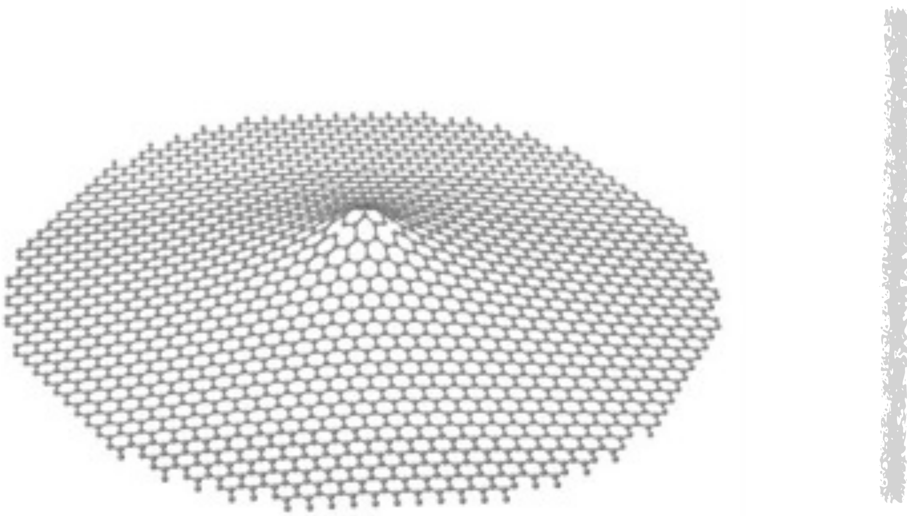
$$2\pi\,\Delta\Phi/\Phi_0 = -(\alpha g/\tau)\,\Delta Q/e$$



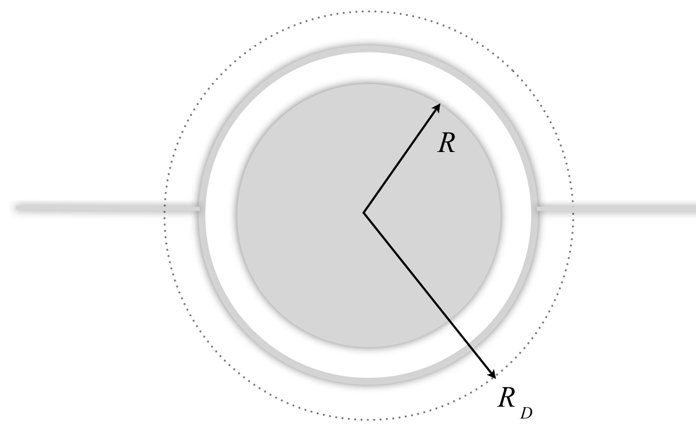
$$\theta = \frac{\alpha g}{2\tau} = \pi\,\frac{\Delta\Phi_{\mathrm{ext}}/\Phi_0}{\Delta Q/e} = \frac{1}{2}\,\frac{e^2}{C}\,\frac{\Delta\Phi_{\mathrm{ext}}}{\Delta V}$$

Possible experimental realization

membrane layer: graphene



electron gas layer: metal



Magnetic impurities spaced about 1 nm away, with each impurity having a magnetic moment of order μ_B, yields a magnetic density of order $\sigma_m \sim 3 \times 10^{-3}\, eV$.

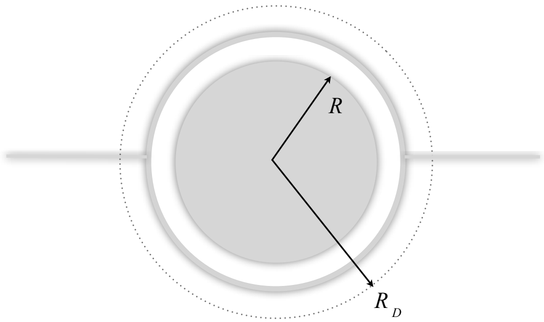
$$\phi_{\max} \sim 300\ nm$$

$R \sim 3 \mu m$

$\Delta R = R_D - R \sim 0.1\, R \sim 300\ nm$

Possible experimental realization

$$
\begin{aligned}
-\frac{Q}{e}\frac{\theta}{\pi} &= 2\pi\sigma_m\, e\, \phi_{\max} \frac{1}{\ln R_D/R} \\
&\approx 2\pi\sigma_m\, e\, \phi_{\max} \frac{R}{\Delta R} \\
&\sim 3 \times 10^{-2}
\end{aligned}
$$



$$N_e \sim 1$$

Comments on anyon interaction

log potential

$$S = \int dt\, d^2x \left[\frac{\rho}{2}\,\dot{\phi}^2 - \frac{\tau}{2}\,(\nabla\phi)^2 - \frac{\kappa}{2}\,(\nabla^2\phi)^2 \right.$$

$$\left. + \psi^\dagger i\partial_t\psi - \frac{1}{2m}\left|(\nabla_i - i\alpha\,\epsilon_{ij}\nabla_j\phi)\,\psi\right|^2 \;-\; g\,\phi\;\psi^\dagger\psi \right]$$

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no log potential

$$\nabla^2 \phi\ \psi^\dagger \sigma_z \psi \qquad\qquad \nabla^2 \phi\ \psi^\dagger \psi$$

Zeeman quadrupole

Comments on anyon interaction

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no log potential

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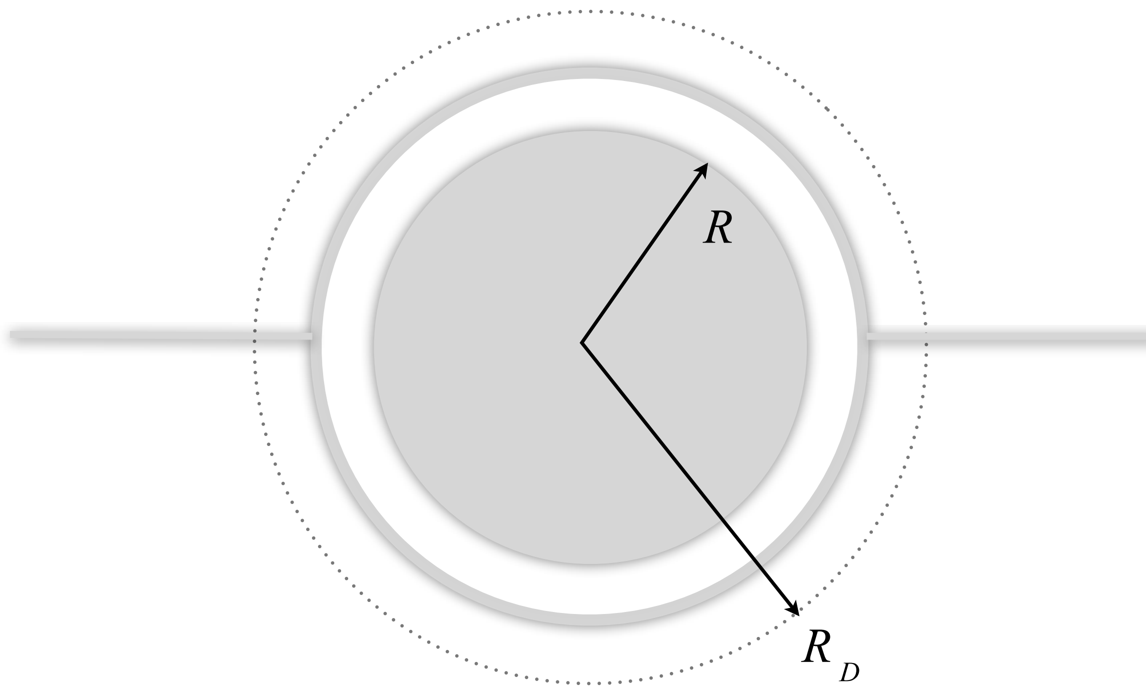
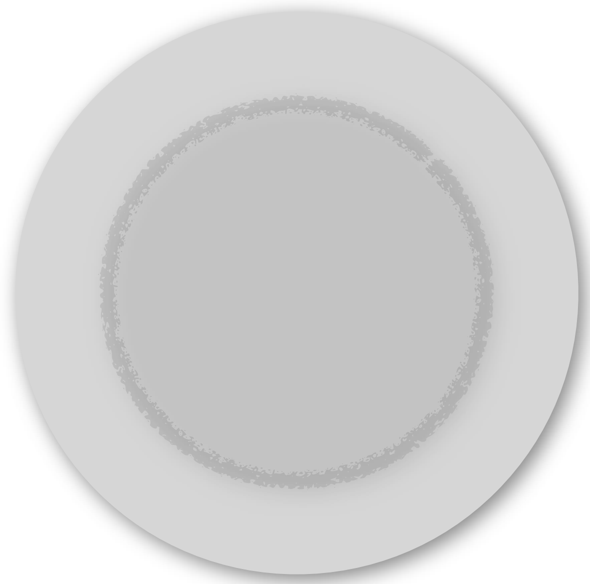


metal: screening of interactions!

Amusement: log-interaction allows the mesoscopic study of "anyon stars"



vary θ

Conclusions

Anyon statistics mediated by electromagnetism and phonons

Statistical angle is a ***continuous function*** of membrane properties

- magnetic dipolar density
- electric charge density
- tension

Proposed physical realization with graphene as membrane

Proposed experiment that does not suffer from “area” problem