Superconductivity and Anderson Localization

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D.I. Diakonov and superconductivity

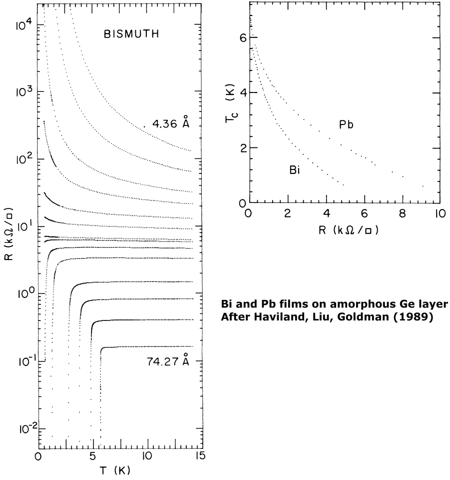
G.W. Carter and D. Diakonov, “Light Quarks in the Instanton Vacuum at Finite Baryon Density”, Phys.Rev. D 60, 016004 (1999).

Gregory W. Carter and Dmitri Diakonov, “Chiral Symmetry Breaking and Color Superconductivity in the Instanton Picture”, arXiv:hep-ph/9905465.

G.W. Carter and D. Diakonov, “The Nonperturbative Color Meissner Effect in a Two-Flavor Color Superconductor”, Nucl. Phys. B 582, 571 (2000).

Color superconductivity in QCD breaks the SU(3) color gauge group down to SU(2), inducing masses in five of the eight gluons. This is a dynamical Higgs effect, in which the diquark condensate acts as the vacuum expectation value of a composite scalar field. In order to analyze this effect at low quark density, when gaps are large and generated nonperturbatively, we use instanton-induced quark interactions augmented with gauge-invariant interactions between quarks and perturbative gluons.

Motivation / Superconductor–Insulator transition



Bi and Pb films on amorphous Ge layer
After Haviland, Liu, Goldman (1989)

recent review: Gantmakher & Dolgopolov (2010)

- Superconductor-insulator transition in homogeneously disordered materials

amorphous Mo-Ge films (thickness $b = 15 - 1000$ Å) Graybeal, Beasley (1984)

Bi and Pb layers on amorphous Ge ($b = 4 - 75$ Å) Strongin, Thompson, Kammerer, Crow (1971); Haviland, Liu, Goldman (1989)

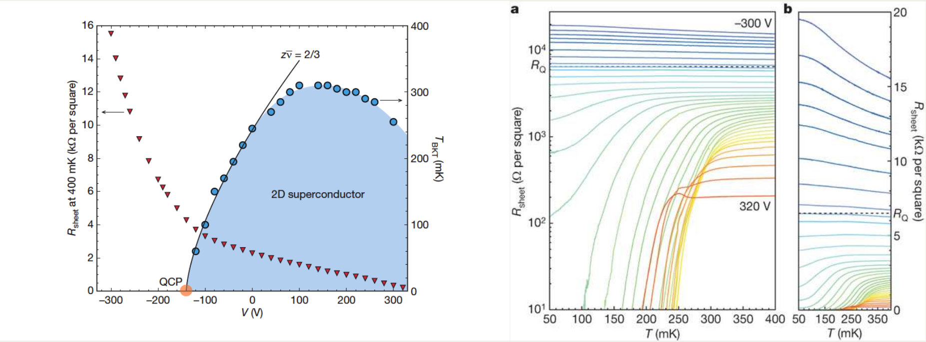
ultrathin Be films ($b = 4 - 15$ Å) Bielejec, Ruan, Wu (2001)

amorphous thick In-O films ($b = 100 - 2000$ Å) Shahar, Ovadyahu (1992); Gantmakher (1998); Gantmakher, Golubkov, Dolgopolov,Tsydynzhapov, Shashkin (1998),(2000); Sambandamurthy, Engel, Johansson, Shahar (2004); Sacépé, Dubouchet, Chapelier, Sanquer, Ovadia, Shahar, Feigel'man, Ioffe (2011)

thin TiN films Baturina, Mironov, Vinokur, Baklanov, Strunk (2007)

$LaAlO_3/SrTiO_3$ interface Caviglia, Gariglio, Reyren, Jaccard, Schneider, Gabay, Thiel, Hammerl, Mannhart, Triscone (2008);

Phase diagram of the $LaAlO_3/SrTiO_3$ interface Caviglia et al. (2008)

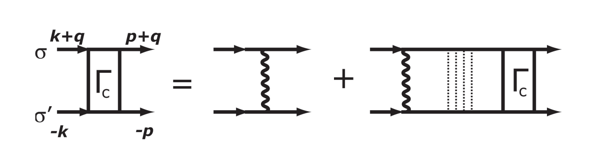


Giant background dielectric constant:
Coulomb interaction strongly screened

Abrikosov & Gor'kov (1959); Anderson (1959)

Anderson Theorem:

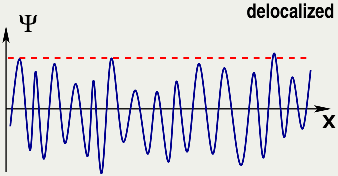
- nonmagnetic impurities do not affect s-wave superconductors
- Cooper-instability is the same for diffusive electrons:



- mean free path does not enter expression for T_c

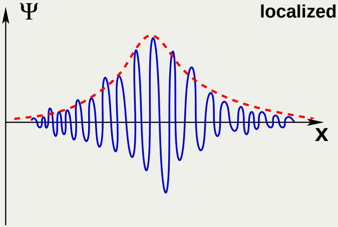
$$T_c^{BCS} \sim \omega_D \exp(-2/\lambda_{\rm e-ph})$$

Anderson transition



quasi-1D, 2D: metallic → localized crossover with increasing L

$d > 2$: metal-insulator transition



Anderson Theorem vs Anderson Localization?

- BCS model in the basis of exact electron states ϕ_ε for given disorder (no Coulomb repulsion):

Bulaevskii, Sadovskii '84; Ma, Lee '85; Kapitulnik, Kotliar '85

superconductivity survives as long as

$$T_c^{BCS} \gtrsim \delta_\xi \propto \xi^{-d}$$

ξ – localization length, d – dimensionality

- Superconductivity at 3D Anderson metal-insulator transition (no Coulomb repulsion)
- Enhancement of T_c as compared to BCS result $T_c^{BCS} \propto \exp(-2/\lambda)$

$$T_c \propto \lambda^{d/|\Delta_2|}$$

Feigelman, Ioffe, Kravtsov, Yuzbashyan (2007); Feigelman, Ioffe, Kravtsov, Cuevas (2010)

where $\Delta_2 < 0$ – multifractal exponent for inverse participation ratio

- Multifractality near Anderson transition (no e-e interactions)

Wegner (1980); Kravtsov, Lerner (1985); Pruisken(1985); Castellani, Peliti (1986); Wegner (1987)

$$\left\langle \int d^d \boldsymbol{r} \, |\phi_\varepsilon(\boldsymbol{r})|^{2q} \right\rangle \sim L^{-\tau_q}$$

perfect metal: $\tau_q = d(q-1)$

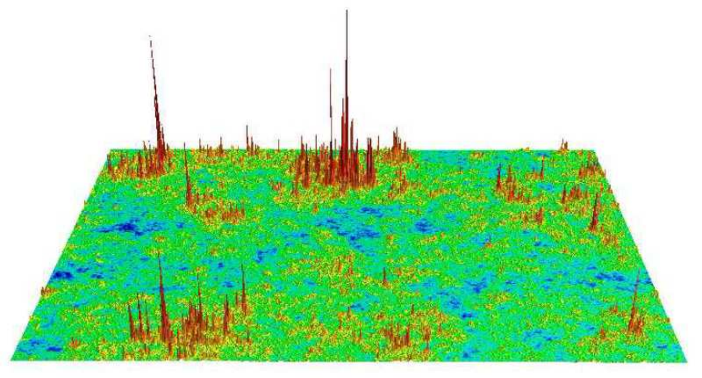
criticality: $\tau_q = d(q-1) + \Delta_q$ with Δ_q being non-trivial function of q

perfect Anderson insulator $\tau_q = 0$

Multifractality

Wegner (1980)

- critical wave function:



Evers, Mildenberger, Mirlin

- enhanced correlations in matrix elements $\sim \psi^4$ of Cooper attraction
- stronger attraction $\Rightarrow$ enhancement of T_c

Feigelman, Ioffe, Kravtsov, Yuzbashyan (2007)

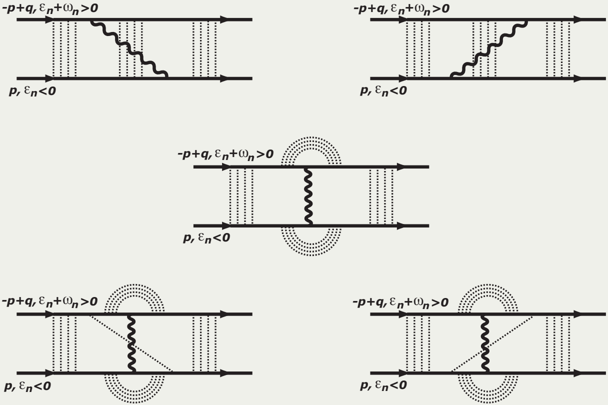
- Suppression of T_c in a film as compared with BCS result

$$\frac{\delta T_c}{T_c^{BCS}} = -\frac{e^2}{6\pi^2\hbar} R_\square \left(\ln \frac{1}{T_c^{BCS}\tau}\right)^3 < 0$$

Ovchinnikov (1973) (wrong sign); Maekawa, Fukuyama (1982); Takagi, Kuroda (1982); Finkelstein (1987)

Film thickness b: $b \lesssim l$

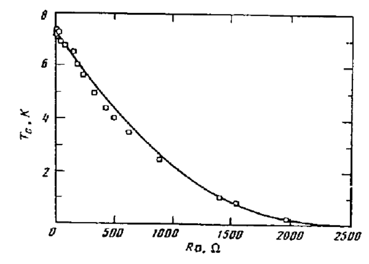
- Disorder, Coulomb (long-ranged) repulsion, (short-ranged) attraction in the Cooper channel



Diagrams for the renormalization of attraction in the Cooper channel

- RG theory for disorder and interactions

Finkelstein (1983,1987); Castellani, Di Castro, Lee, Ma (1984)



Experiments on Mo-Ge films, Graybeal & Beasley (1984)

- 2D: T_c vanishes at the critical resistance

$$R_\square \sim \left(\ln \frac{1}{T_c^{BCS} \tau} \right)^{-2}$$

Finkelstein (1987)

Can suppression of T_c due to Coulomb repulsion and enhancement of T_c due to multifractality be described in a unified way?

Does weak multifractality enhance T_c in 2D systems ?

Does the enhancement of T_c hold if one takes into account short-ranged repulsion in particle-hole channels ?

- Free electrons:

$$H_0 = \int d^d\boldsymbol{r}\, \bar{\psi}_\sigma(\boldsymbol{r}) \left[-\frac{\nabla^2}{2m}\right] \psi_\sigma(\boldsymbol{r})$$

where $\sigma = \pm 1$ is spin projection

- Scattering off random potential :

$$H_{\rm dis} = \int d^d\boldsymbol{r}\, \bar{\psi}_\sigma(\boldsymbol{r})\, V(\boldsymbol{r}) \psi_\sigma(\boldsymbol{r})$$

Gaussian white-noise distribution:

$$\langle V(\boldsymbol{r})\rangle = 0, \qquad \langle V(\boldsymbol{r}_1)\, V(\boldsymbol{r}_2)\rangle = \frac{1}{2\pi\nu_0\tau}\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)$$

ν_0 – thermodynamic density of states

- Electron-electron interaction:

$$H_{\rm int} = \frac{1}{2}\int d^d\boldsymbol{r_1}\, d^d\boldsymbol{r_2}\, \bar{\psi}_\sigma(\boldsymbol{r_1})\psi_\sigma(\boldsymbol{r_1})\, U(\boldsymbol{r_1}-\boldsymbol{r_2})\, \bar{\psi}_{\sigma'}(\boldsymbol{r_2})\psi_{\sigma'}(\boldsymbol{r_2})$$

  - short-ranged repulsion with BCS-type attraction ($\lambda > 0$)

$$U(\boldsymbol{R}) = u_0\frac{a^{2\alpha}}{[a^2+R^2]^\alpha} - \frac{\lambda}{\nu_0}\delta(\boldsymbol{R}), \qquad \alpha > 2d, \qquad u_0 > 0$$

  - Coulomb (long-ranged) repulsion with BCS-type attraction ($\lambda > 0$)

$$U(\boldsymbol{R}) = \frac{e^2}{\epsilon R} - \frac{\lambda}{\nu_0}\delta(\boldsymbol{R})$$

- Particle-hole channel:

$$H_{\rm int}^{\rm p-h} = \frac{1}{2\nu_0} \int\limits_{ql \lesssim 1} \frac{d^d \boldsymbol{q}}{(2\pi)^d} \sum_{a=0}^{3} \mathbb{F}_a(q) m^a(\boldsymbol{q}) m^a(-\boldsymbol{q})$$

where $m^a(\boldsymbol{q}) = \int \frac{d^d \boldsymbol{k}}{(2\pi)^d} \bar{\boldsymbol{\psi}}(\boldsymbol{k}+\boldsymbol{q}) \boldsymbol{\sigma}_a \boldsymbol{\psi}(\boldsymbol{k})$

$$F_0(q) = F_s, \qquad F_1(q) = F_2(q) = F_3(q) = F_t$$

- Particle-particle channel:

$$H_{\rm int}^{\rm p-p} = -\frac{F_c}{\nu_0} \int\limits_{ql \lesssim 1} \frac{d^d \boldsymbol{q}}{(2\pi)^d} \int \frac{d^d \boldsymbol{k}_1 d^d \boldsymbol{k}_2}{(2\pi)^{2d}} \bar{\psi}_\sigma(\boldsymbol{k}_1) \bar{\psi}_\sigma(-\boldsymbol{k}_1+\boldsymbol{q}) \psi_{-\sigma}(\boldsymbol{k}_2+\boldsymbol{q}) \psi_\sigma(-\boldsymbol{k}_2)$$

Field theory for Anderson localization: Non-linear sigma-model

$$S[Q] = \frac{\pi\nu}{4}\int d^d\mathbf{r}\,\mathrm{Tr}\,[-D(\nabla Q)^2 - 2i\omega\Lambda Q], \qquad Q^2(\mathbf{r}) = 1$$

Wegner (1979)

supersymmetry: Efetov (1982)

sigma-model manifold $\mathcal{M} = \{\mathcal{M}_B \times \mathcal{M}_F\}$

"dressed" by anticommuting variables

$\mathcal{M}_B$ — non-compact, $\mathcal{M}_F$ — compact

with electron-electron interaction: fermionic replicas or Keldysh

Finkelstein (1983), Kamenev, Andreev (1999)

Field theory: interacting non-linear sigma-model

Finkelstein (1983)

$$
\begin{aligned}
S &= S_0 + S_{\rm int}^{(s)} + S_{\rm int}^{(t)} + S_{\rm int}^{(c)}, \\
S_0 &= -\frac{g}{32}\int d\boldsymbol{r}\,\mathrm{Tr}(\nabla Q)^2 + 4\pi T z \int d\boldsymbol{r}\,\mathrm{Tr}\,\eta(Q-\Lambda), \\
S_{\rm int}^{(s)} &= -\frac{\pi T}{4}\Gamma_s \sum_{\alpha,n}\sum_{p=0,3}\int d\boldsymbol{r}\,\mathrm{Tr}\left[I_n^\alpha t_{p0} Q\right]\mathrm{Tr}\left[I_{-n}^\alpha t_{p0} Q\right], \\
S_{\rm int}^{(t)} &= -\frac{\pi T}{4}\Gamma_t \sum_{\alpha,n}\sum_{p=0,3}\sum_{j=1}^{3}\int d\boldsymbol{r}\,\mathrm{Tr}\left[I_n^\alpha t_{pj} Q\right]\mathrm{Tr}\left[I_{-n}^\alpha t_{pj} Q\right], \\
S_{\rm int}^{(c)} &= -\frac{\pi T}{2}\Gamma_c \sum_{\alpha,n}\sum_{p=0,3}(-1)^p\int d\boldsymbol{r}\,\mathrm{Tr}\left[I_n^\alpha t_{p0} Q I_n^\alpha t_{p0} Q\right], \\
\Lambda_{nm}^{\alpha\beta} &= \mathrm{sgn}\, n\,\delta_{nm}\delta^{\alpha\beta} t_{00}, \quad \eta_{nm}^{\alpha\beta} = n\delta_{nm}\delta^{\alpha\beta} t_{00}, \quad (I_k^\gamma)_{nm}^{\alpha\beta} = \delta_{n-m,k}\delta^{\alpha\beta}\delta^{\alpha\gamma} t_{00}
\end{aligned}
$$

$$
Q^2 = 1, \qquad \mathrm{Tr}\,Q = 0, \qquad Q^\dagger = C^T Q^T C, \qquad C = i t_{12}, \qquad C^T = -C
$$

n, m – Matsubara, α, β – replicas, p – particle-hole, j – spin; $\quad t_{pj} = \tau_p \otimes s_j$

On short scales $\gamma_{s,t,c} = \Gamma_{s,t,c}/z$ are related to microscopic parameters:

- Particle-hole channel

$$\gamma_s = -\frac{F_s}{1+F_s}, \qquad \gamma_t = -\frac{F_t}{1+F_t}$$

- Cooper channel (provided $\omega_D \tau \gg 1$)

$$\gamma_c = -\frac{F_c}{1 - F_c \ln \omega_D \tau} = \frac{1}{\ln T_c^{BCS} \tau}$$

Coulomb repulsion: $\gamma_s = -1$

2D electrons (orth. symmetry class) / RG equations from σ-model

$$\frac{dt}{dy} = t^2\Big[1 + f(\gamma_s) + 3f(\gamma_t) - \gamma_c\Big]$$

$$\frac{d\gamma_s}{dy} = -\frac{t}{2}\left[1+\gamma_s\right]\left[\gamma_s + 3\gamma_t + 2\gamma_c\right]$$

$$\frac{d\gamma_t}{dy} = -\frac{t}{2}\left[1+\gamma_t\right]\left[\gamma_s - \gamma_t - 2\gamma_c(1+2\gamma_t)\right]$$

$$\frac{d\gamma_c}{dy} = -\frac{t}{2}\left[\gamma_s - 3\gamma_t + \gamma_c(\gamma_s + 3\gamma_t)\right] - 2\gamma_c^2$$

Finkelstein (1983,1984); Castellani, Di Castro, Lee, Ma (1984)

Castellani, DiCastro, Forgacs, Sorella (1984); Ma, Fradkin (1986); Finkelstein (1987)

where $y = \ln L/l$ and $f(x) = 1 - (1 + x^{-1}) \ln(1 + x)$

- lowest order in disorder, $t = 2/\pi g$, g is conductivity in units e^2/h
- exact in γ_s and γ_t
- lowest order in γ_c

$$\frac{dt}{dy} = t^2\Big(1 - \big[\gamma_s + 3\gamma_t + 2\gamma_c\big]/2\Big)$$

$$\frac{d}{dy}\begin{pmatrix}\gamma_s\\ \gamma_t\\ \gamma_c\end{pmatrix} = -\frac{t}{2}\begin{pmatrix}1 & 3 & 2\\ 1 & -1 & -2\\ 1 & -3 & 0\end{pmatrix}\begin{pmatrix}\gamma_s\\ \gamma_t\\ \gamma_c\end{pmatrix} - \begin{pmatrix}0\\ 0\\ 2\gamma_c^2\end{pmatrix}$$

- Weak interaction, $|\gamma_s|, |\gamma_t|, |\gamma_c| \ll 1$
- Weak disorder, $t \ll 1$.
- Initial values $\gamma_s(0) = \gamma_{s0} < 0$, $\gamma_t(0) = \gamma_{t0} > 0$, $\gamma_c(0) = \gamma_{c0} < 0$, $t(0) = t_0$

- Moderately strong disorder: $|\gamma_{c0}| \ll t_0 \ll 1$

$$\frac{dt}{dy} = t^2, \qquad \frac{d}{dy}\begin{pmatrix}\gamma_s\\ \gamma_t\\ \gamma_c\end{pmatrix} = -\frac{t}{2}\begin{pmatrix}1 & 3 & 2\\ 1 & -1 & -2\\ 1 & -3 & 0\end{pmatrix}\begin{pmatrix}\gamma_s\\ \gamma_t\\ \gamma_c\end{pmatrix}$$

Eigenvalues and eigenvectors

$$\lambda = 2t \;:\; \begin{pmatrix}-1\\ 1\\ 1\end{pmatrix}; \qquad \lambda' = -t \;:\; \begin{pmatrix}1\\ 1\\ -1\end{pmatrix} \text{ and } \begin{pmatrix}1\\ -1\\ 2\end{pmatrix}.$$

- attraction to the (BCS) line $-\gamma_s = \gamma_t = \gamma_c \equiv \gamma$
- Anomalous dimensions of operators with two Q's in NLσM: $\Delta_2 = -\lambda + \ldots$ and $\mu_2 = -\lambda' + \ldots$
- $\Delta_2 < 0$ is due to multifractality of electron wave functions without interactions

Projected RG equations:

$$\frac{dt}{dy} = t^2, \qquad \frac{d\gamma}{dy} = 2t\gamma - 2\gamma^2/3$$

$$t(0) = t_0, \qquad \gamma(0) = \gamma_0 = (-\gamma_{s0} + 3\gamma_{t0} + 2\gamma_{c0})/6, \qquad |\gamma_0| \ll t_0 \ll 1$$

Two-step renormalization

1. $t > \gamma$: neglect Cooper instability (γ^2);
   enhancement of interaction matrix element due to weak multifractality
2. $\gamma > t$: neglect disorder-induced term ($t\gamma$) $\Longrightarrow$ conventional BCS,
   but with the new “bare” coupling constant t determined by disorder
   $\Longrightarrow$ exponential enhancement of the mean-field T_c

- If $t_0 \ll \sqrt{|\gamma_0|} \ll 1$, superconductor wins:

$$|\gamma(y_*)| \sim 1, \qquad t(y_*) \sim t_0^2/|\gamma_0| \ll 1, \qquad y_* \sim \frac{1}{t_0}$$

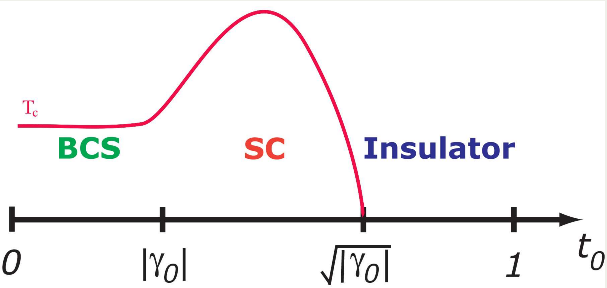
superconductor transition temperature $T_c \propto e^{-2/t_0} \gg T_c^{BCS}$

Enhancement of T_c due to weak multifractality!

- If $\sqrt{|\gamma_0|} \ll t_0 \ll 1$, insulator wins:

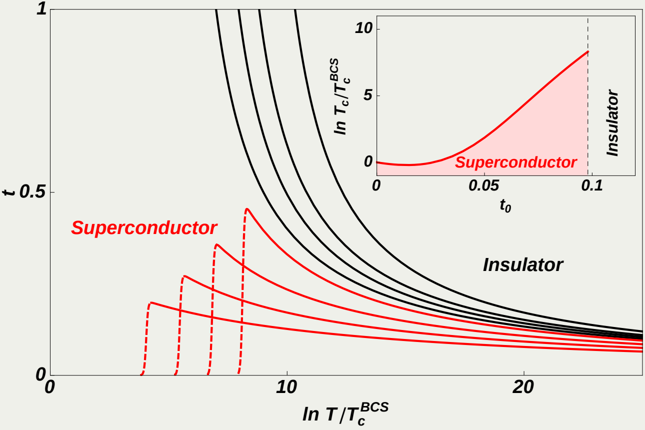
$$t(y_*) \sim 1, \qquad |\gamma(y_*)| \sim |\gamma_0|/t_0^2 \ll 1, \qquad y_* \sim \frac{1}{t_0}$$

Sketch of phase diagram



Superconductor-Insulator Transition (SIT)

- Resistivity t near SIT and dependence of T_c on t_0



$\gamma_{s0} = -0.005$, $\gamma_{t0} = 0.005$, $\gamma_{c0} = -0.04$,

$t_0 = 0.065, 0.075, 0.085, 0.095, 0.10, 0.105, 0.11, 0.12$ (from bottom to top)

- Non-monotonous dependence of T_c on disorder (bare conductance g_0)
- Exponentially strong enhancement of superconductivity by multifractality in the intermediate disorder range, $|\gamma_0|^{-1/2} \lesssim g_0 \lesssim |\gamma_0|^{-1}$

$$\begin{aligned} T_c \sim \exp\{-1/|\gamma_{c,0}|\} \quad \text{(BCS)}\,, & \qquad g_0 \gtrsim |\gamma_0|^{-1} \\ T_c \sim \exp\{-2g_0\}\,, & \qquad |\gamma_0|^{-1/2} \lesssim g_0 \lesssim |\gamma_0|^{-1} \\ \text{insulator}, & \qquad g_0 \lesssim |\gamma_0|^{-1/2} \end{aligned}$$

2D electrons: BKT transition

- SC transition in 2D is of Berezinskii-Kosterlitz-Thouless type
- Mean-field T_c differs only slightly from T_c^{BKT} for weak disorder

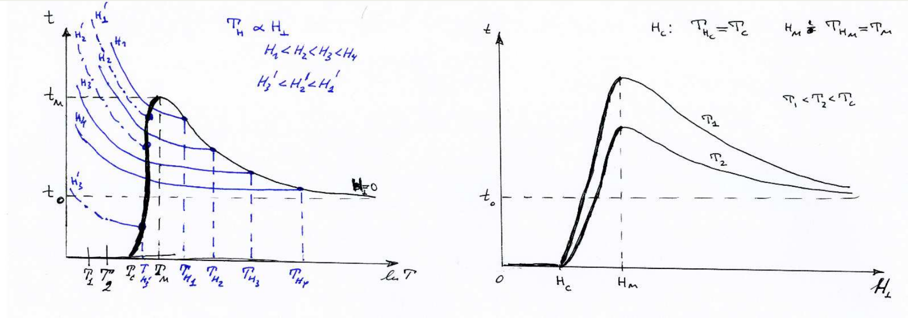
  Beasley, Mooij, Orlando '79, Halperin, Nelson '79

- Future work: effect of multifractality on BKT transition

- Coulomb (rather than short-range) interaction in typical experiments on SIT in 2D
- $LaAlO_3/SrTiO_3$ interface: dielectric constant $\epsilon \approx 10^4 \quad \Longrightarrow \quad \varkappa^{-1} \gg l.$
- Our results for T_c in 2D are valid for Coulomb interaction (with $\varkappa l \ll 1$), provided
$$\ln 1/(\varkappa l) \gg \min\{|\gamma_{c0}|^{-1}, t_0^{-1}\}$$
- Screening of Coulomb interaction $\longrightarrow$ high-T_c superconductivity?

Role of perpendicular magnetic field $H_\perp$ in 2D for $|\gamma_{c0}| \ll t_0 \ll \sqrt{|\gamma_{c0}|}$

Sketch of magnetoresistance



RG equations near free electron fixed point $t = t_c$, $\gamma = 0$

$$\frac{dt}{dy} = \frac{1}{\nu}(t - t_c) + \eta\gamma, \qquad \frac{d\gamma}{dy} = -\Delta_2\gamma - A\gamma^2\,, \quad A \sim 1$$

$t(0) = t_0$ and $\gamma(0) = \gamma_0 < 0$ at the UV energy scale $E_0 \sim 1/(\nu_0 l^d)$

- Correlation length:

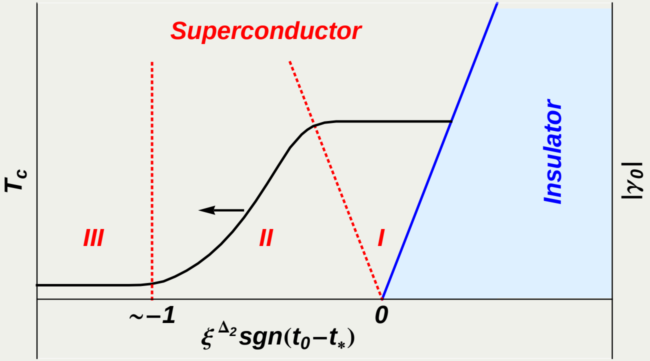
$$\xi = \left|\tilde{t}_0 - t_c\right|^{-\nu}$$

  $\tilde{t} = t - \eta\nu\gamma/(|\Delta_2|\nu - 1)$ and $\tilde{t}_0 = \tilde{t}(0)$

- 3D Anderson transition (orth. symmetry class):

$$\nu = 1.57 \pm 0.02 \text{ and } \Delta_2 = -1.7 \pm 0.05$$

Schematic phase diagram in the interaction–disorder plane and T_c



I: $T_c = E_0|\gamma_0|^{d/|\Delta_2|}$ II: $T_c = \xi^{-d} E_0 \exp\left(-\frac{d}{a|\gamma_0|\xi^{|\Delta_2|}}\right)$ III: $T_c = T_c^{BCS}$

T_c for region I agrees with Feigelman, Ioffe, Kravtsov, Yuzbashyan (2007)

Conclusions

- Strong enhancement of T_c for 2D electrons (short-range interactions)
- Strong enhancement of T_c near (free electron) 3D Anderson transition (short-range interactions)
- In both cases enhancement of superconductivity is due to multifractality
- No Coulomb interaction: Anderson localization facilitates superconductivity $\Rightarrow$ high-T_c superconductivity?
- Anderson '59 vs Anderson '58: Anderson wins!