Density of states in a 2D chiral model with sublattice imbalance

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in collaboration with

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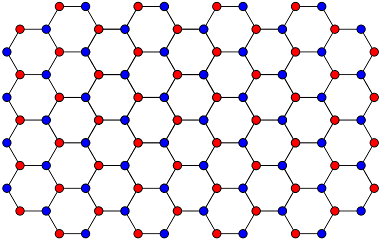
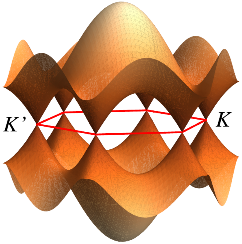
Outline

1. Introduction: chiral 2D metal
2. Quasiclassical approach: gap in DOS
3. Quantum corrections to DOS
4. Derivation of the sigma model
5. Perturbative renormalization
6. Non-perturbative effects: Lifshitz tails
7. Summary

Motivation: Graphene

MAX-PLANCK-GESELLSCHAFT



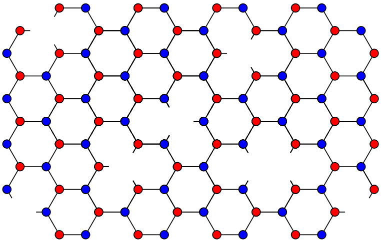


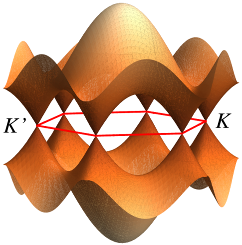
- Chiral structure: two sublattices: A, B
- Two valleys of the spectrum: K, K′
- Massless Dirac Hamiltonian in each valley:

$$H = v_0 \boldsymbol{\sigma} \mathbf{p}, \qquad \boldsymbol{\sigma} = \{\sigma_x, \sigma_y\}$$

Motivation: Graphene with vacancies

MAX-PLANCK-GESELLSCHAFT





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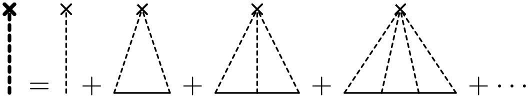
- **Vacancies (preserve chiral symmetry)**
- **Sublattice imbalance:** $n_A \neq n_B$

Self-consistent T-matrix



- T-matrix of an individual impurity:

$$T_{A/B} = \frac{V}{1 - Vg_{A/B}(0)} \xrightarrow{V\to\infty} -\frac{1}{g_{A/B}(0)}.$$

- Green function ($i\epsilon_{A/B} = i\epsilon - \Sigma_{A/B}$):

$$g(0) = \int \frac{d^2p}{(2\pi)^2} \begin{pmatrix} i\epsilon_A & -p_- \\ -p_+ & i\epsilon_B \end{pmatrix}^{-1} = -\frac{i}{4\pi} \begin{pmatrix} \epsilon_B & 0 \\ 0 & \epsilon_A \end{pmatrix} \ln \frac{B^2}{\epsilon_A \epsilon_B}.$$

- Self-consistency condition (non-crossing): $\Sigma_{A/B} = n_{A/B} T_{A/B}$.
- Average DOS: $\rho = -\frac{1}{\pi} \,\mathrm{Im}\,\mathrm{Tr}\, g(0)$.

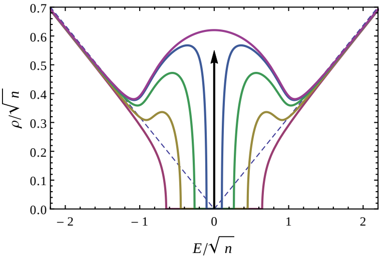
Self-consistent T-matrix

- Zero modes: $\rho(E \to 0) = |n_A - n_B|\delta(E)$
- A gap Δ around the peak
- Complete imbalance $n_B = 0$: $\Delta \approx \sqrt{\dfrac{2\pi n_A}{\ln(B^2/n_A)}}$
- Weak imbalance $|n_A - n_B| \ll n_A + n_B \equiv n$:

$$\frac{1}{\tau} = \sqrt{\frac{4\pi n}{\ln(B^2/n)}},$$

$$\Delta = \frac{|n_A - n_B|}{2\tau n},$$

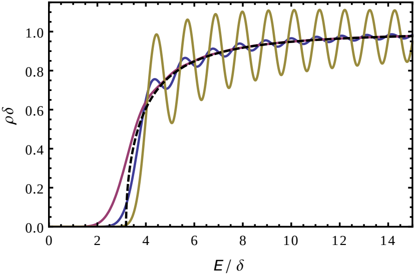
$$\rho(E \ll 1/\tau) = \frac{2\tau n}{\pi}\sqrt{1 - \frac{\Delta^2}{E^2}}.$$



Chiral random matrices

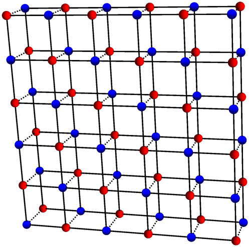
Ivanov '02

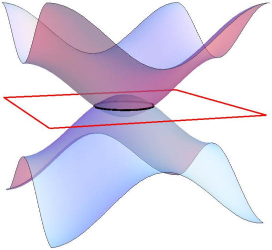
- Chiral matrix $H = \begin{pmatrix} 0 & h \\ h^\dagger & 0 \end{pmatrix}$
- h is an $M \times N$ random matrix: $\begin{cases} \text{real}, & \text{ChO (BDI)}, \\ \text{complex}, & \text{ChU (AIII)}, \\ \text{quaternion}, & \text{ChSp (CII)}. \end{cases}$
- Exactly $|M - N| \equiv m$ zero modes
- Universal limit: $M, N \to \infty$, $m = \text{const}$, $\delta = \text{const}$)
- Exact result (in terms of Bessel functions):
- Quasigap $\Delta = m\delta/\pi$
- Exponential tail



2D chiral metal

Gade, Wegner '91, Gade '93

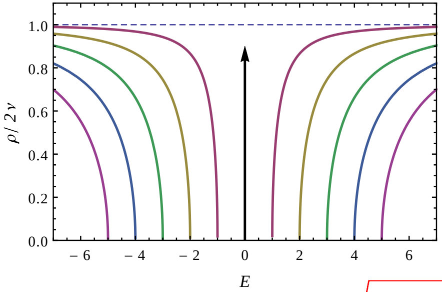




- Chiral structure: two sublattices: A, B
- Two hopping parameters: in-plane t and inter-plane $t' \lesssim 2t$
- Effective quadratic Hamiltonian: $H = \begin{pmatrix} 0 & \xi \\ \xi & 0 \end{pmatrix}, \quad \xi = \frac{p^2}{2m} - \mu$
- Vacancies with sublattice imbalance: $n_A \neq n_B$
- **Metallic limit:** $\mu \gg n/\nu, \quad \nu = m/2\pi$

Self-consistent T-matrix

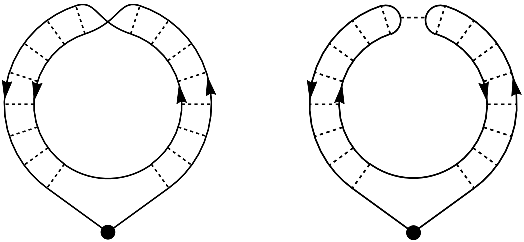
MAX-PLANCK-GESELLSCHAFT



- Density of states: $\rho = |n_A - n_B|\delta(E) + 2\nu\sqrt{1 - \frac{\Delta^2}{E^2}}$
- Gap: $\Delta = |n_A - n_B|/2\pi\nu$
- Scattering rate: $1/\tau = n/\pi\nu$
- Drude conductivity:

$\sigma =$  $+ \cdots = e^2\nu v^2\tau\left(1 - \frac{\Delta^2}{E^2}\right)$

Interference correction



- Cooperon loop (orthogonal class, BDI):

$$\frac{\delta\rho}{\rho} = \int \frac{(2\pi\nu)^{-1}(d^2q)}{D_0 q^2 + 2\sqrt{\epsilon^2+\Delta^2}} = \frac{\ln(L_E/l)}{(2\pi)^2\sigma}, \qquad L_E = \sqrt{\frac{D_0}{E,\Delta}}$$

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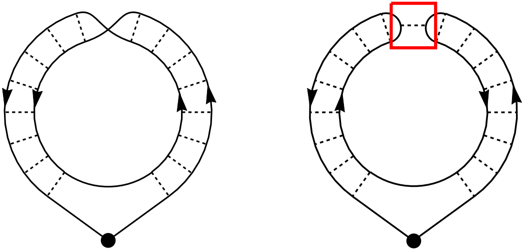

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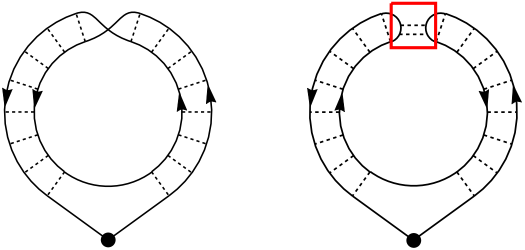

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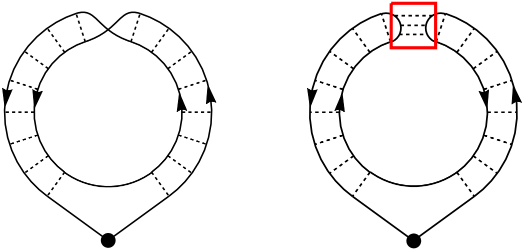

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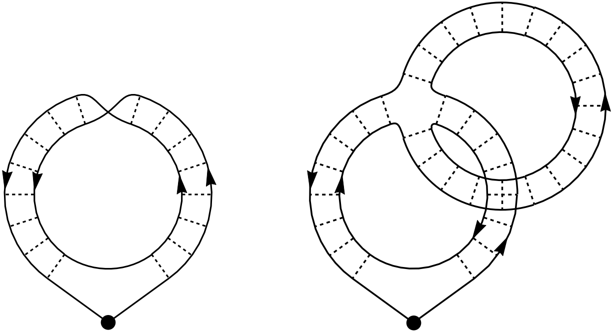

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- Diffuson: two loops $\Longrightarrow$ $\delta\rho/\rho \propto [\ln^2(L_E/l)]/\sigma^2$
- Systematic RG within sigma model [Gade, Wegner '91, Gade '93]

Sigma model for Poisson disorder

- Assume bosomic replica version of class AIII (unitary)
- Average Lagrangian

$$S = -i\Psi^\dagger \begin{pmatrix} E & -\xi \\ -\xi & E \end{pmatrix} \Psi + n_A \left(1 - e^{-iV\psi_A^\dagger \psi_A}\right) + n_B \left(1 - e^{-iV\psi_B^\dagger \psi_B}\right)$$

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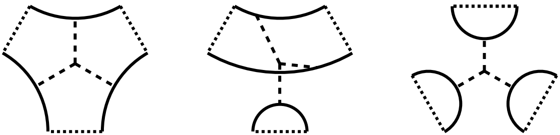
- Single out soft modes $\langle \Psi\Psi^\dagger \rangle_q \equiv \sum_{p \gg q} \Psi_{p+q}\Psi^\dagger_{-p}$



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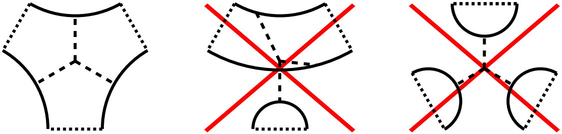

- Neglect reducible vertices

$$S_{\text{dis}} = n_A \,\mathrm{Tr}\ln\left(1 + iV\langle \psi_A \psi_A^\dagger \rangle\right) + n_B \,\mathrm{Tr}\ln\left(1 + iV\langle \psi_B \psi_B^\dagger \rangle\right)$$

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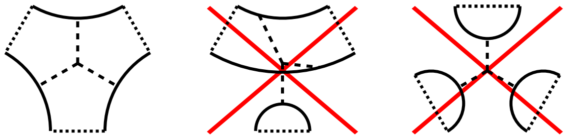

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- Decouple with the help of Fourier transform

Sigma model for Poisson disorder

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- Matrix-valued Fourier tranform

$$e^{-S_{\text{dis}}[\langle\psi\psi^\dagger\rangle]} = \int dR\, e^{-i\psi^\dagger R\psi}\, F(R) \quad \Rightarrow \quad F(R) = \int dM\, e^{i\,\mathrm{Tr}\, RM - S_{\text{dis}}[M]}$$

- Both $S_{\text{dis}}[M]$ and $F[R]$ depend on the eigenvalues of the argument
- Assume R is diagonal. Represent $M = U^{-1} m U$
- Integrate over U [Itzykson, Zuber '80]

$$F(R) = \frac{\Delta[\partial/\partial r]}{\Delta[r]} \int \left(\prod_a dm_a \right) \exp\left(i \sum_a r_a m_a - S_{\text{dis}}\{m\} \right)$$

- In our case $S_{\text{dis}}\{m\} = n \sum_a \ln(1 + iVm_a) \quad \Rightarrow$ **integrals decouple**

$$F(R) = \exp\left\{ -\mathrm{Tr}\left[\frac{R}{2V} + \left(N - \frac{n}{2}\right) \ln\left(\frac{R}{2V}\right) \right] \right\}$$

Sigma model for Poisson disorder

MAX-PLANCK-GESELLSCHAFT

- Integrate out Ψ. Take the limit $N \to 0$, $V \to \infty$

$$S = \frac{1}{2}\,\mathrm{Tr}\left[\ln \begin{pmatrix} E - R_A & -\xi \\ -\xi & E - R_B \end{pmatrix} - n_A \ln R_A - n_B \ln R_B\right]$$

- **At $n_A = n_B$, $E = 0$, the saddle point reproduces SCTMA!**
- $R_{A/B}$ play the role of $\Sigma_{A/B}$
- Gauge rotations of Ψ generate the saddle manifold

$$R_A = -iQ/2\tau \qquad \text{and} \qquad R_B = -iQ^{-1}/2\tau$$

- Expansion in small ∇Q, $n_A - n_B$, and E yields the sigma model

$$S = -\,\mathrm{Tr}\left[\frac{\sigma}{8\pi}\,\mathrm{Tr}(\nabla Q^{-1}\nabla Q) + \pi\nu\,\mathrm{Tr}\left(iE\left(Q + Q^{-1}\right) + 2\Delta \ln Q\right)\right]$$

Sigma model for Poisson disorder

- Sigma-model action

$$S = -\frac{1}{s}\int d^2r \left\{ \frac{\sigma}{8\pi}\,\mathrm{Tr}(\nabla Q^{-1}\nabla Q) + \overbrace{\frac{c}{8\pi}\left(\mathrm{Tr}\, Q^{-1}\nabla Q\right)^2}^{\text{Gade term}} + \pi\nu\,\mathrm{Tr}\left(iE\,(Q + Q^{-1}) + \underbrace{2\Delta \ln Q}_{\text{imbalance}} \right) \right\}$$

- Parameter s is 1 for AIII and 2 for BDI and CII
- Gade term is generated by the RG
- In 0D limit the model derived in [Ivanov '02] is reproduced
- Average density of states

$$\rho = \lim_{N\to 0}\frac{\nu}{2sN}\,\mathrm{Re}\int DQ\;\mathrm{Tr}(Q + Q^{-1})\,e^{-S[Q]}$$

- Neglecting gradients, the result of SCTMA is reproduced

Perturbative renormalization

Gade and Wegner '91, Gade '93

- Rewrite $Q = e^{\phi} U$ ($\det U = 1$)
- At $E = 0$

$$S[Q] = -\int d^2 r \left\{ \frac{\sigma}{8\pi s} \operatorname{Tr}\left[\nabla U^{-1} \nabla U\right] + N \left[\frac{\sigma - Nc}{8\pi s} (\nabla \phi)^2 - \frac{2\pi\nu\Delta}{s} \phi \right] \right\}$$

- Decoupled Gaussian theory in ϕ:

$$\frac{d}{d \ln L}(\sigma + Nc) = 0, \qquad \frac{d\Delta}{d \ln L} = 0$$

- Replica limit

$$\frac{d\sigma}{d \ln L} = -N \frac{dc}{d \ln L} \xrightarrow{N \to 0} 0$$

- **Absence of localization to all orders in perturbation theory!**

Perturbative renormalization

Gade and Wegner '91, Gade '93

- One-loop RG equations

$$\frac{d\sigma}{d\ln L} \equiv 0, \quad \frac{d\Delta}{d\ln L} \equiv 0, \quad \frac{dc}{d\ln L} = 1, \qquad \frac{d\ln E}{d\ln L} = \begin{cases} \frac{1}{\sigma} + \frac{2c}{\sigma^2}, & \text{BDI}, \\ \frac{c}{\sigma^2}, & \text{AIII}, \\ -\frac{1}{\sigma} + \frac{2c}{\sigma^2}, & \text{CII} \end{cases}$$

- RG stops at the scale L_c when $\frac{\sigma}{\nu L_c^2} \sim \max\{E, \Delta\}$
- Renormalized DOS (at $\Delta = 0$, class BDI)

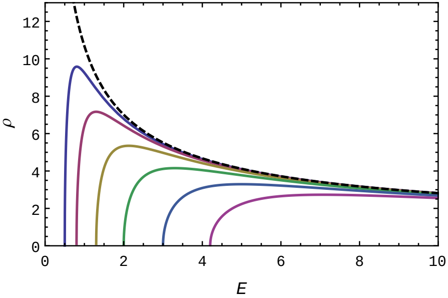
$$\rho_{\text{Gade}} \sim \frac{\sigma}{E_0 L_c^2} \propto \begin{cases} (E_0\tau)^{-1/2\sigma}, & |\ln(E_0\tau)| \ll \sigma^2, \\ \frac{1}{E_0} \exp\left[-2\sigma\sqrt{|\ln(E_0\tau)|}\right], & |\ln(E_0\tau)| \gg \sigma^2 \end{cases}$$

Perturbative renormalization

- Renormalized DOS with imbalance

$$\rho(E_0) = \rho_{\text{Gade}}(E_0)\sqrt{1 - \frac{\Delta^2}{E^2}} = \rho_{\text{Gade}}(E_0)\sqrt{1 - \frac{E_g^2}{E_0^2}}$$

- Effective gap $\quad E_g = \Delta \exp\left(-\frac{\ln^2 \Delta\tau}{4\sigma^2}\right)$



Subgap DOS: non-perturbative effect

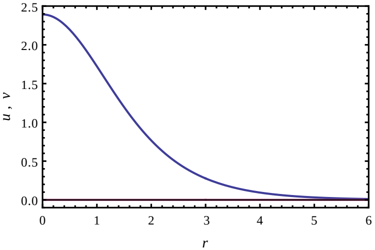
MAX-PLANCK-GESELLSCHAFT

- Assume $\epsilon = 1 - E/\Delta = 1 - E_0/E_g \ll 1$
- Two very close saddle points of the action
- Non-classical saddle point breaks replica symmetry
- Characterictic size of the instanton $\sim L_c \epsilon^{-1/4} \gg L_c$
- Equations of motion in rescaled parameters

$$-\nabla^2 u + u - u^2 = -\frac{c}{\sigma}\nabla^2(u - v),$$
$$-\nabla^2 v + v - v^2 = -\frac{c}{\sigma}\nabla^2(u - v)$$

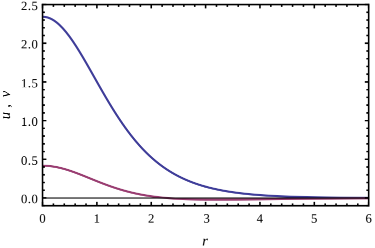
- **Gade term mixes variables!**
- Instanton action $S_{\text{inst}} = \frac{\sigma\epsilon}{6\pi}\int d^2x(u^3 - v^3) = \sigma\epsilon Y(c/\sigma)$

Subgap DOS: non-perturbative effect



Instanton shape

Subgap DOS: non-perturbative effect



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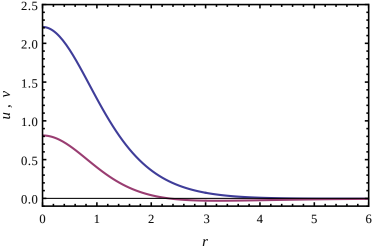

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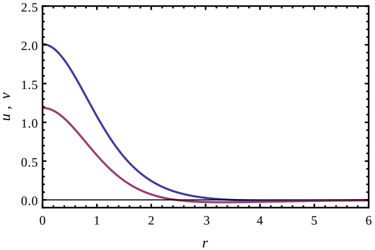

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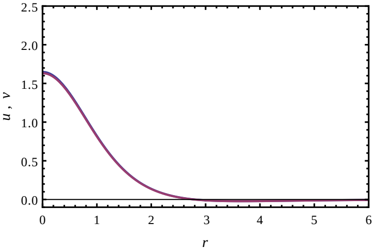

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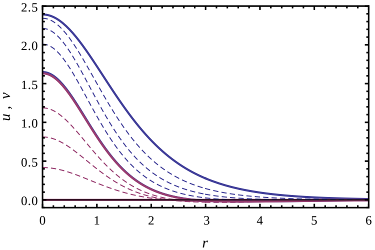

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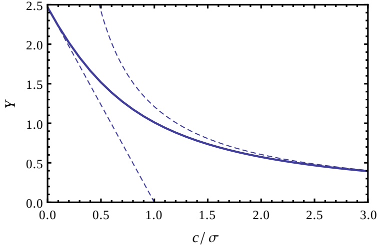

Instanton action

$$\rho \propto \begin{cases} \exp\left[-2.47\sigma\epsilon\left(1-|\ln E_g\tau|/2\right)\right], & |\ln E_g\tau| \ll \sigma, \\ \exp\left[-2.42\dfrac{\sigma^2\epsilon}{|\ln E_g\tau|}\right], & \sigma \ll |\ln E_g\tau| \ll \sigma^2, \\ \exp\left[-1.21\dfrac{\sigma\epsilon}{\sqrt{|\ln E_g\tau|}}\right], & |\ln E_g\tau| \gg \sigma^2 \end{cases}$$

Summary

MAX-PLANCK-GESELLSCHAFT

Results

1. Imbalance opens a gap in the spectrum of a 2D chiral metal
2. Sigma model for arbitrary local disorder: matrix Fourier transform
3. Action acquires an additional term due to imbalance
4. Localization effects renormalize DOS above the gap
5. Instanton in the sigma model provides exponential subgap tail

Outlook

1. Deep subgap tail
2. Numerical study of DOS in imbalanced graphene