Dynamics of waves in 1D electronic systems: from free fermions to hydrodynamics

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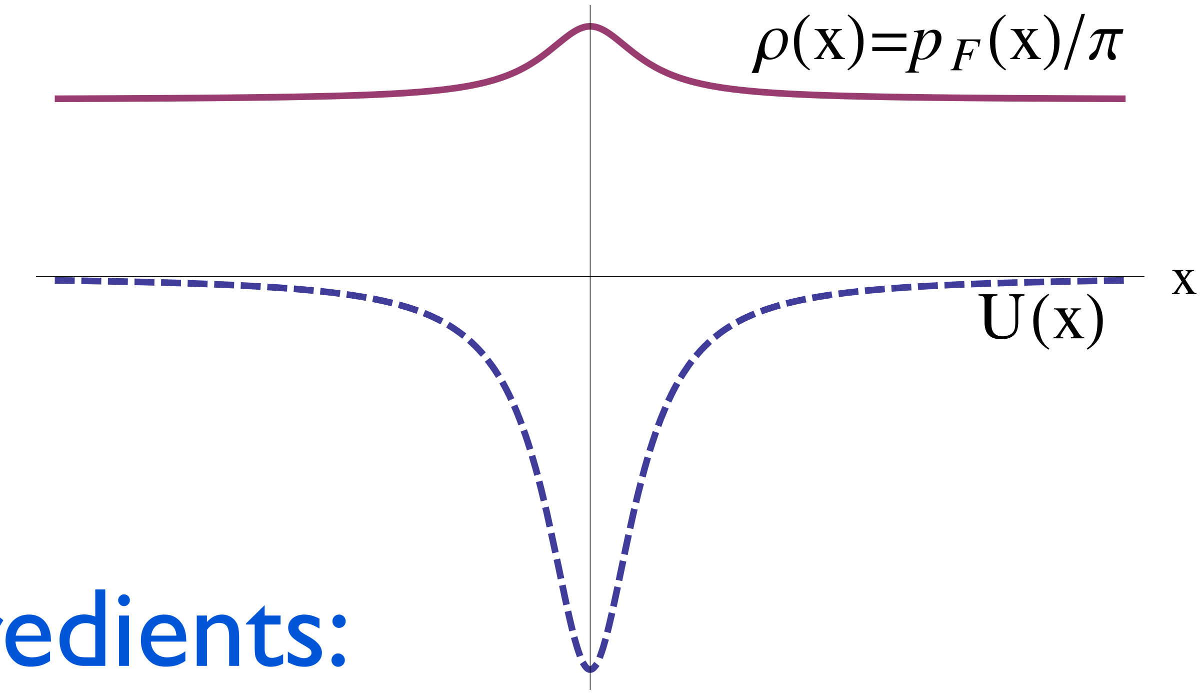
Outline

- Introductory remarks on dynamics of density perturbations in 1D.
- Free fermions and density oscillations driven by population inversion.
- Long-range interactions and hydrodynamics.
- Unifying picture: Kinetic equation for 1D fermions.
- Conclusion.

Coherent density perturbation of Fermi sea

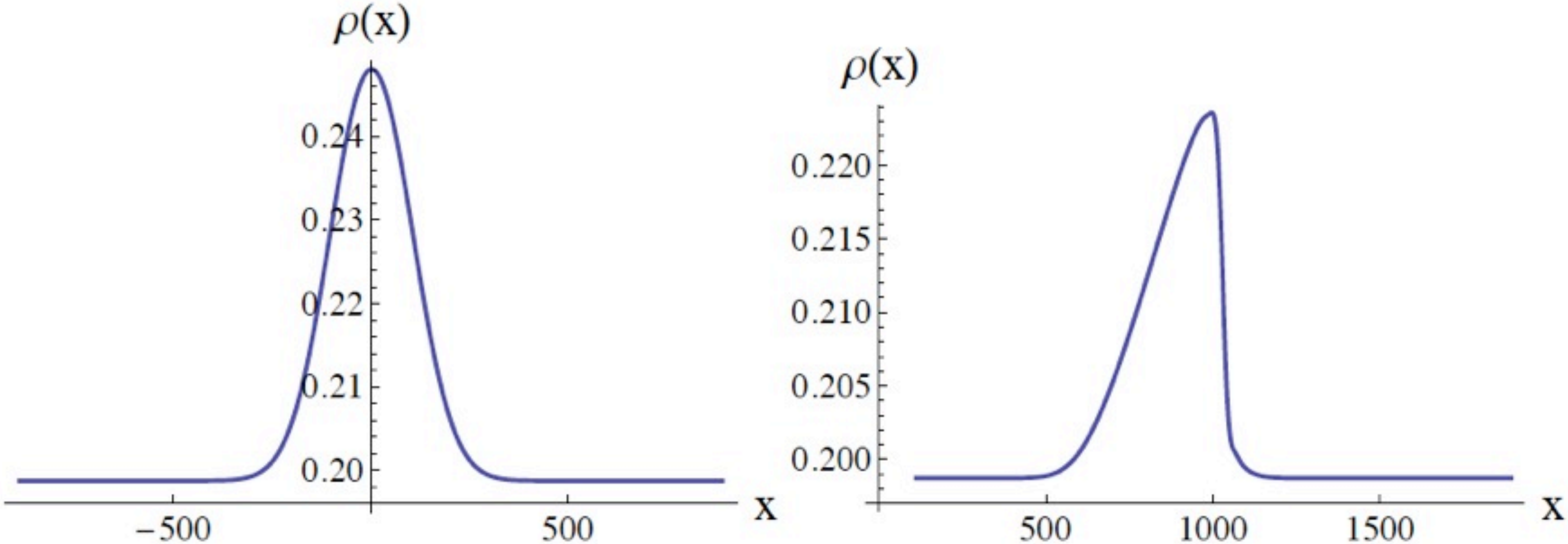


Important ingredients:

- Interaction (if any); also the interaction radius is important;
- Curvature of fermionic spectrum.

(E. Bettelheim, A. G. Abanov, and P. Wiegmann, PRL 2006, 2011)

Fermionic curvature and quantum shock

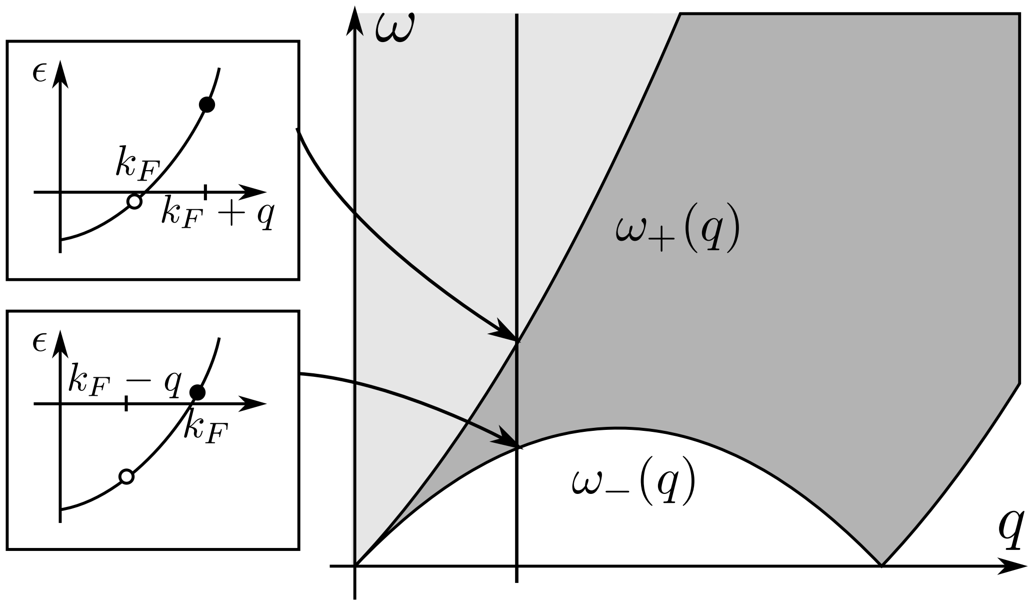


As time flows the fermionic curvature drives the system away from LL fixed point.

The shock time:

$$t_c \sim m\Delta x/\Delta\rho$$

Curvature effects at linear response level



$$S(q,\omega) = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dx e^{i(\omega t - qx)} \langle \rho(x,t)\rho(0,0)\rangle$$

$$S_{\rm LL} \propto |q|\delta(\omega - v|q|) \quad \Rightarrow \quad S_0(q,\omega) = (m/q)\theta(q^2/2m - |\omega - v_F q|)$$

A. Imambekov, T. L. Schmidt, and L. I. Glazman, RMP 2012

Free Fermions

Wigner function

$$f(X,p;t) = \int dy e^{-ipy} \left\langle \psi^{+}\left(X - \frac{y}{2};t\right) \psi\left(X + \frac{y}{2};t\right)\right\rangle$$

Exact Kinetic Equation

$$\partial_t f(X,p;t) + p\partial_X f(X,p;t) = 0$$

Exact Solution

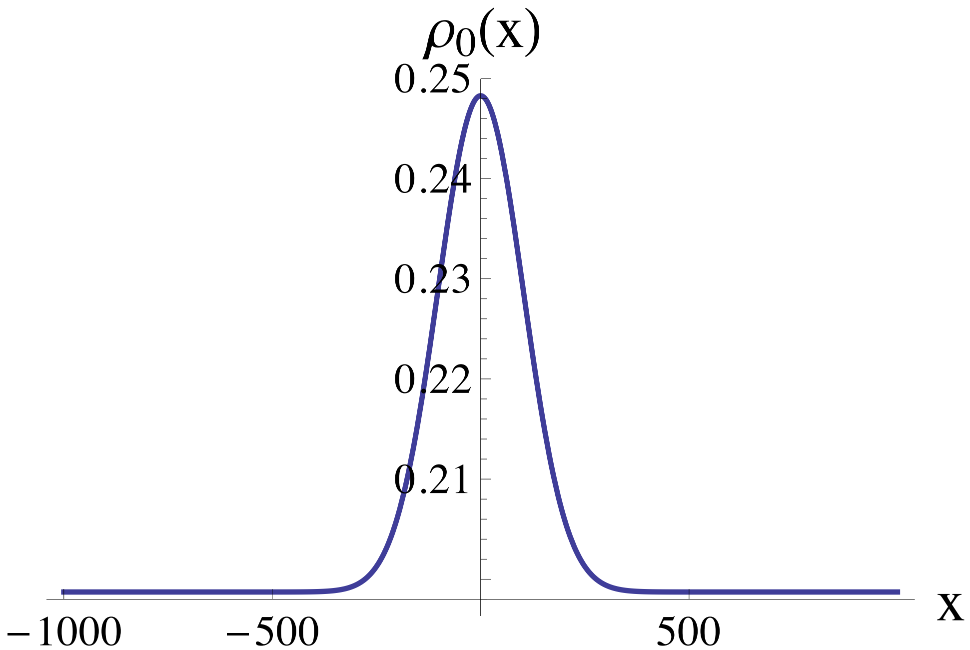
$$f(X,p;t) = f(X - pt, p; t = 0) \equiv f_0(X - pt, p)$$

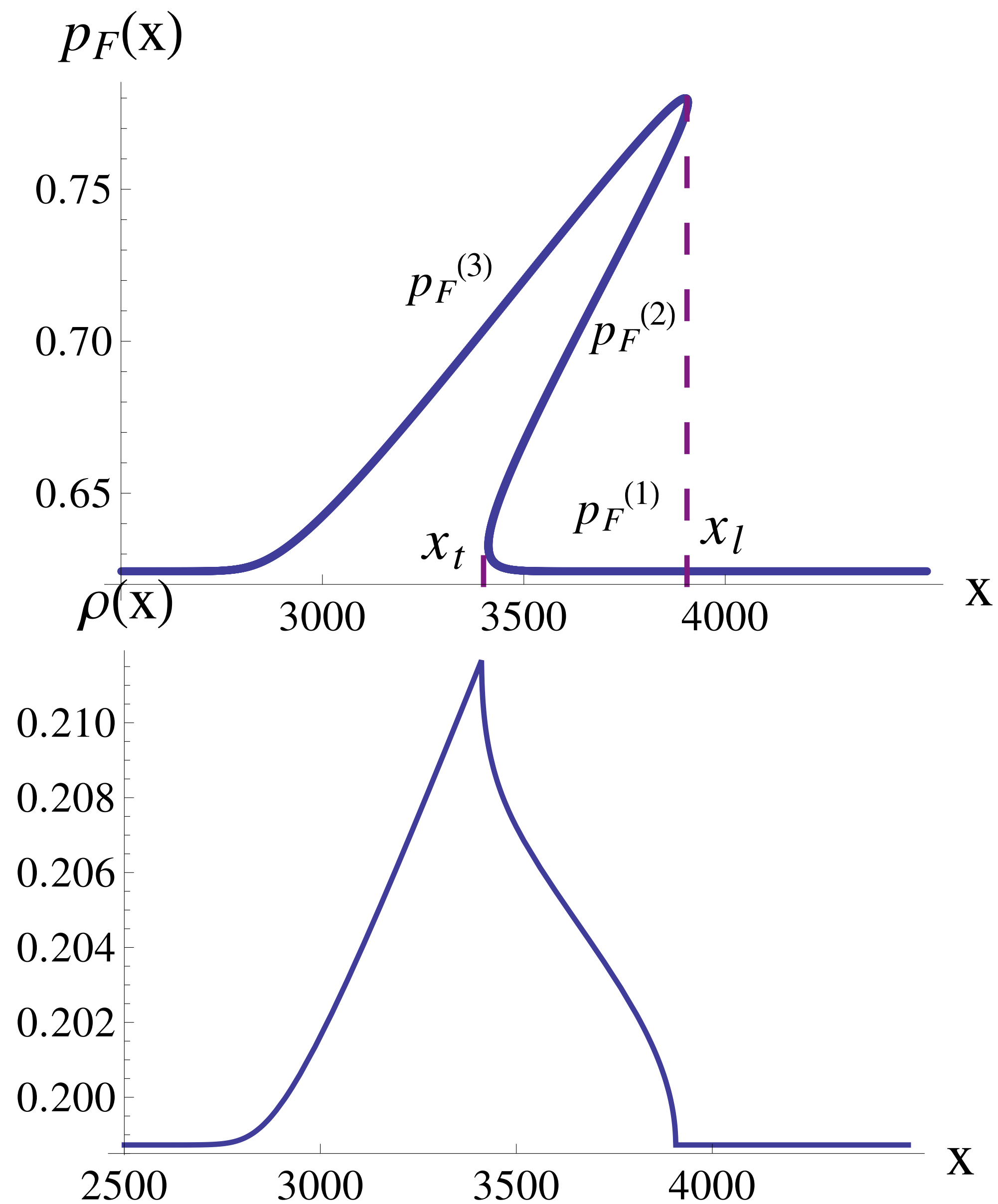
Semiclassical Approximation

$$f_0(X,p) = \Theta\left(p_F^2(X) - p^2\right)$$

Fermi Surface

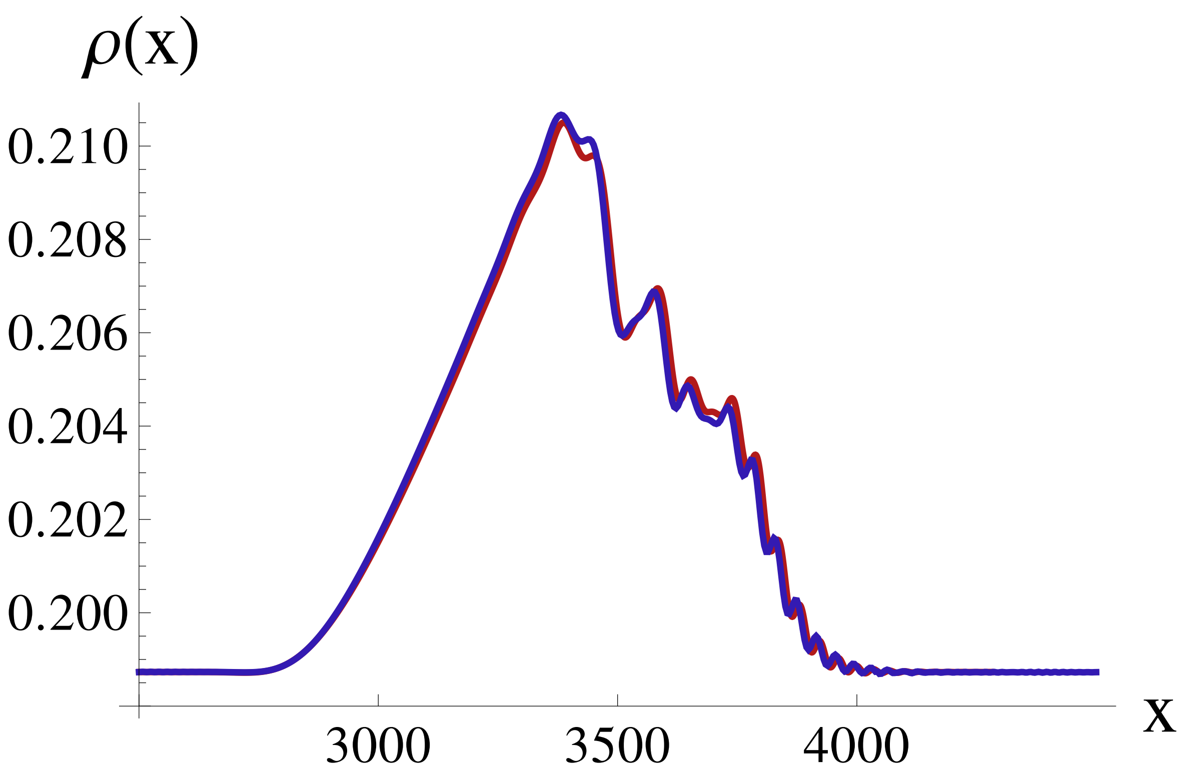
$$\partial_t p_F(x,t) + p_F(x,t)\partial_x p_F(x,t) = 0$$

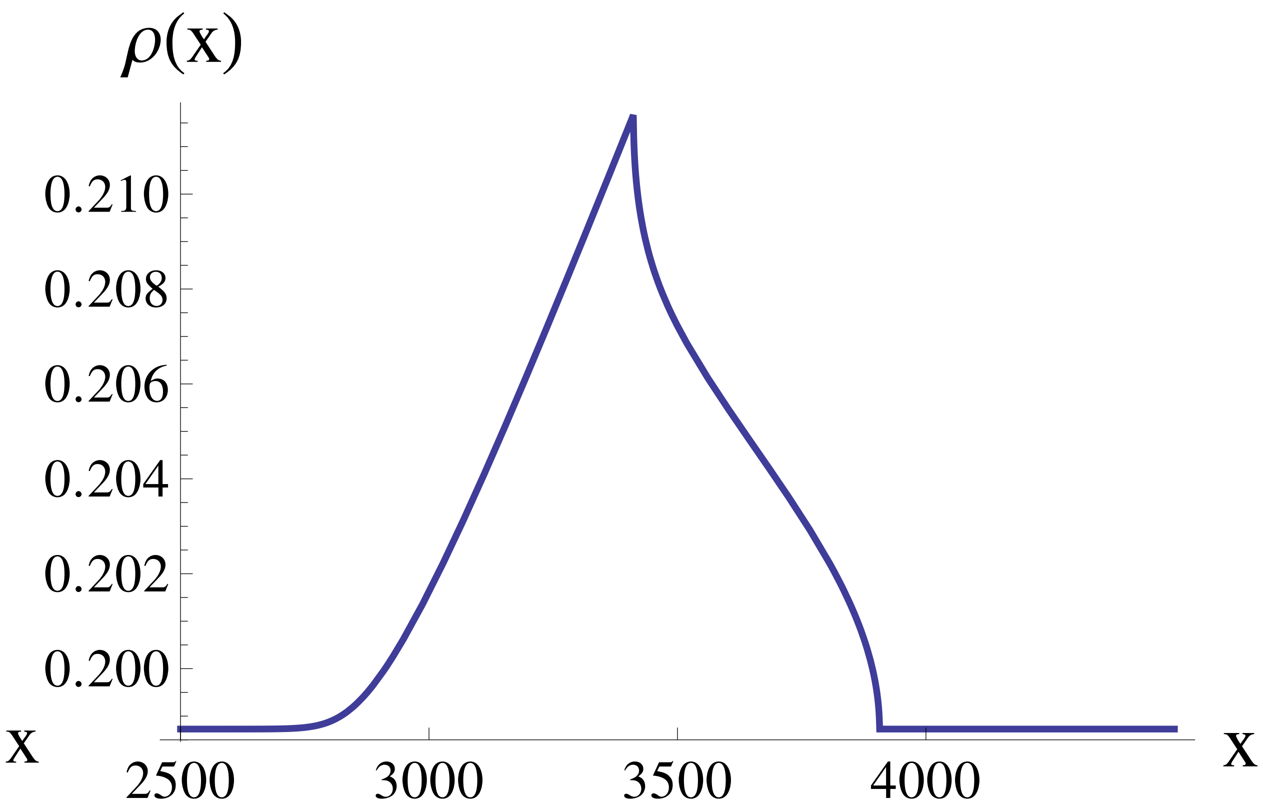




$$\rho_0(x) = \rho_\infty + \frac{N}{\sqrt{2\pi\sigma^2}} e^{-x^2/2\sigma^2}$$

FILM


ρ(x)
0.210
0.208
0.206
0.204
0.202
0.200
3000
3500
4000
x


ρ(x)
0.210
0.208
0.206
0.204
0.202
0.200
2500
3000
3500
4000
x

Wigner function, at t=0

(E. Bettelheim and P. Wiegmann, PRB 2011)

$$\partial_X f_0(X,y) - i\left[p_F\left(X+\frac{y}{2}\right) - p_F\left(X-\frac{y}{2}\right)\right] f_0(X,y) = 0$$

Boundary conditions

$$f_0(X=-\infty, y) = \int \frac{dp}{2\pi} e^{ipy}\Theta\left(p_\infty - p\right)$$

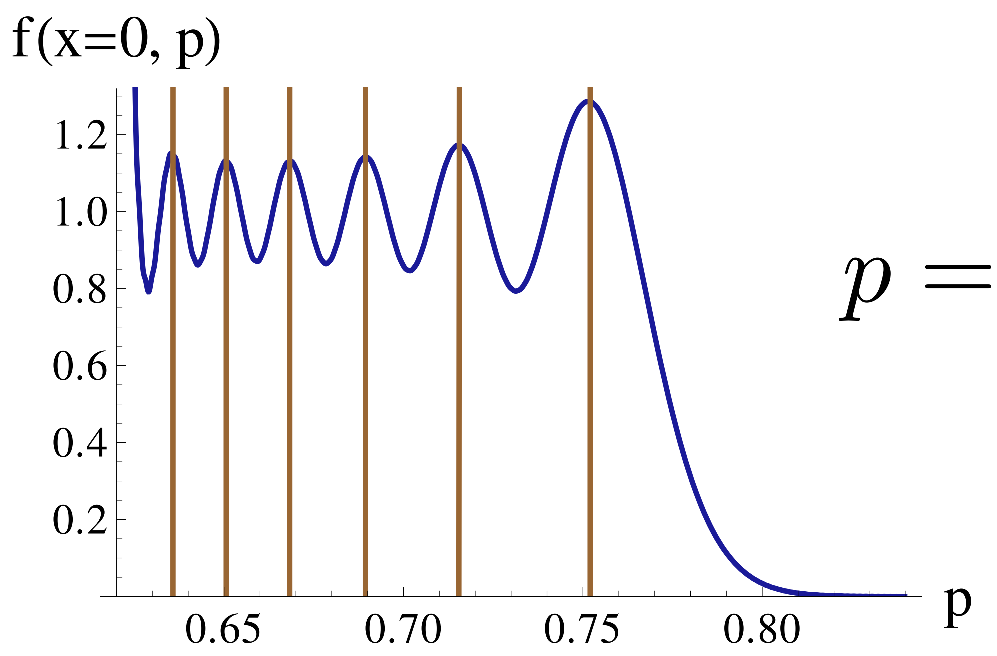
Approximate solution

$$f_0(X,p) = \int \frac{dy}{2\pi i(y-i0)} e^{-iS[y;X,p]}$$

$$S[y;X,p] = py - \int_{X-\frac{y}{2}}^{X+\frac{y}{2}} dX' p_F(X')$$

Oscillations in the phase space

$$S[y; X, p] = py - \int_{X-\frac{y}{2}}^{X+\frac{y}{2}} dX' p_F(X')$$



$$p = \frac{p_F(X + y/2) + p_F(X - y/2)}{2}$$

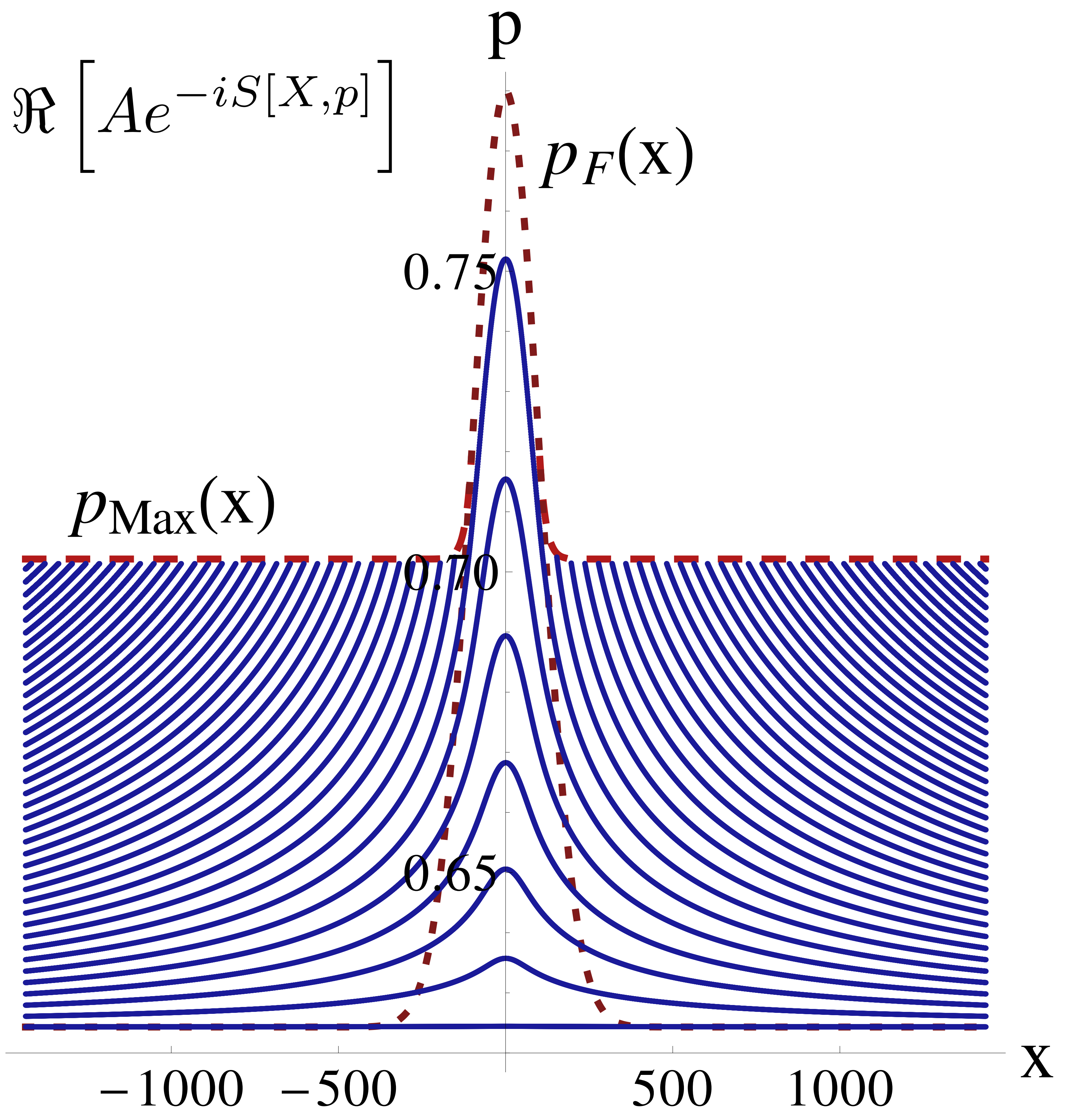
$$\delta f_0(X, p) \propto \Re\left[Ae^{-iS[X,p]}\right]$$

$$p_\infty < p < p_{\mathrm{Max}}(X)$$

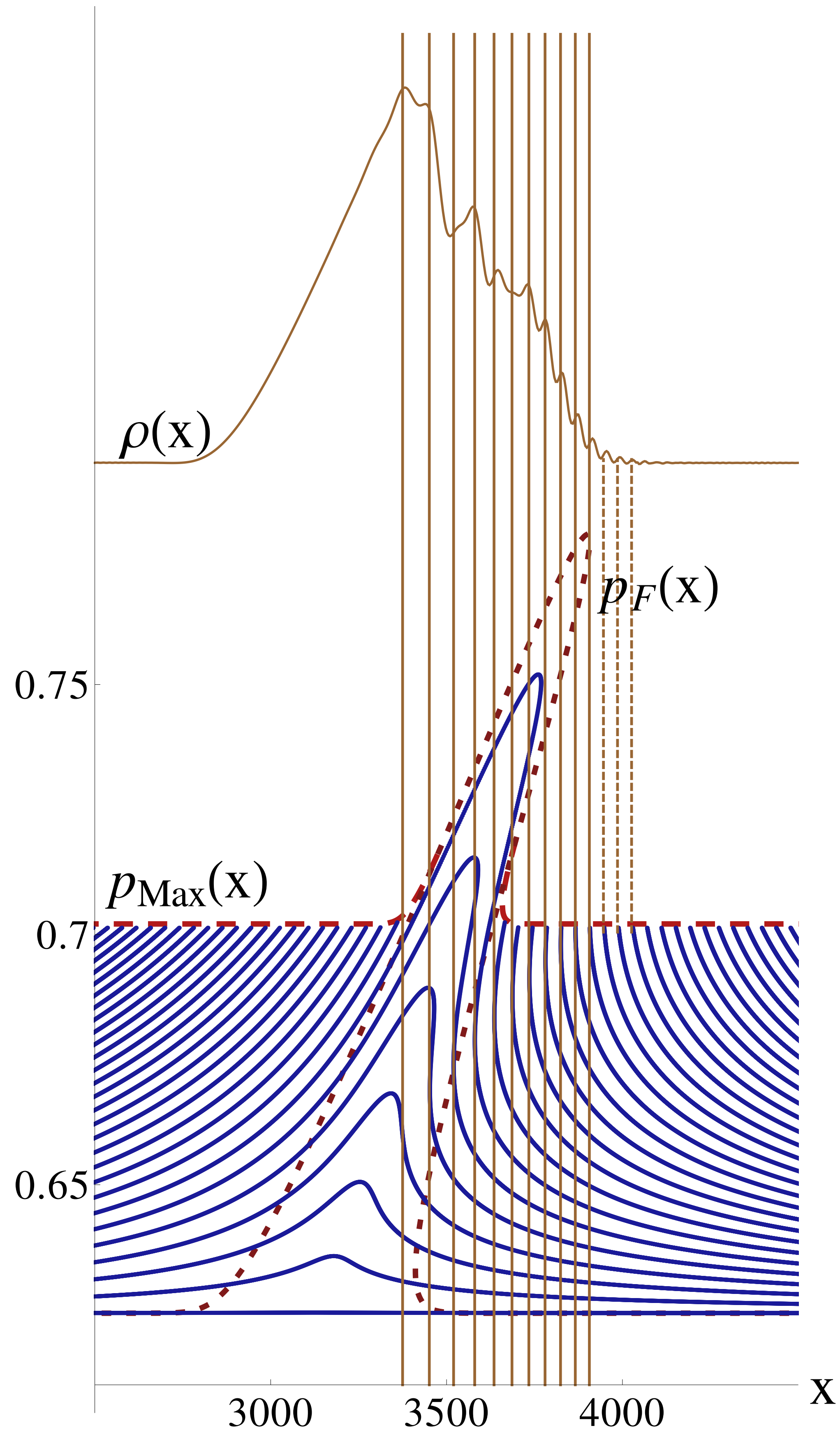
$$N = \frac{1}{2\pi}\int dx(p_F(x) - p_\infty)$$

Equipotentials of the action

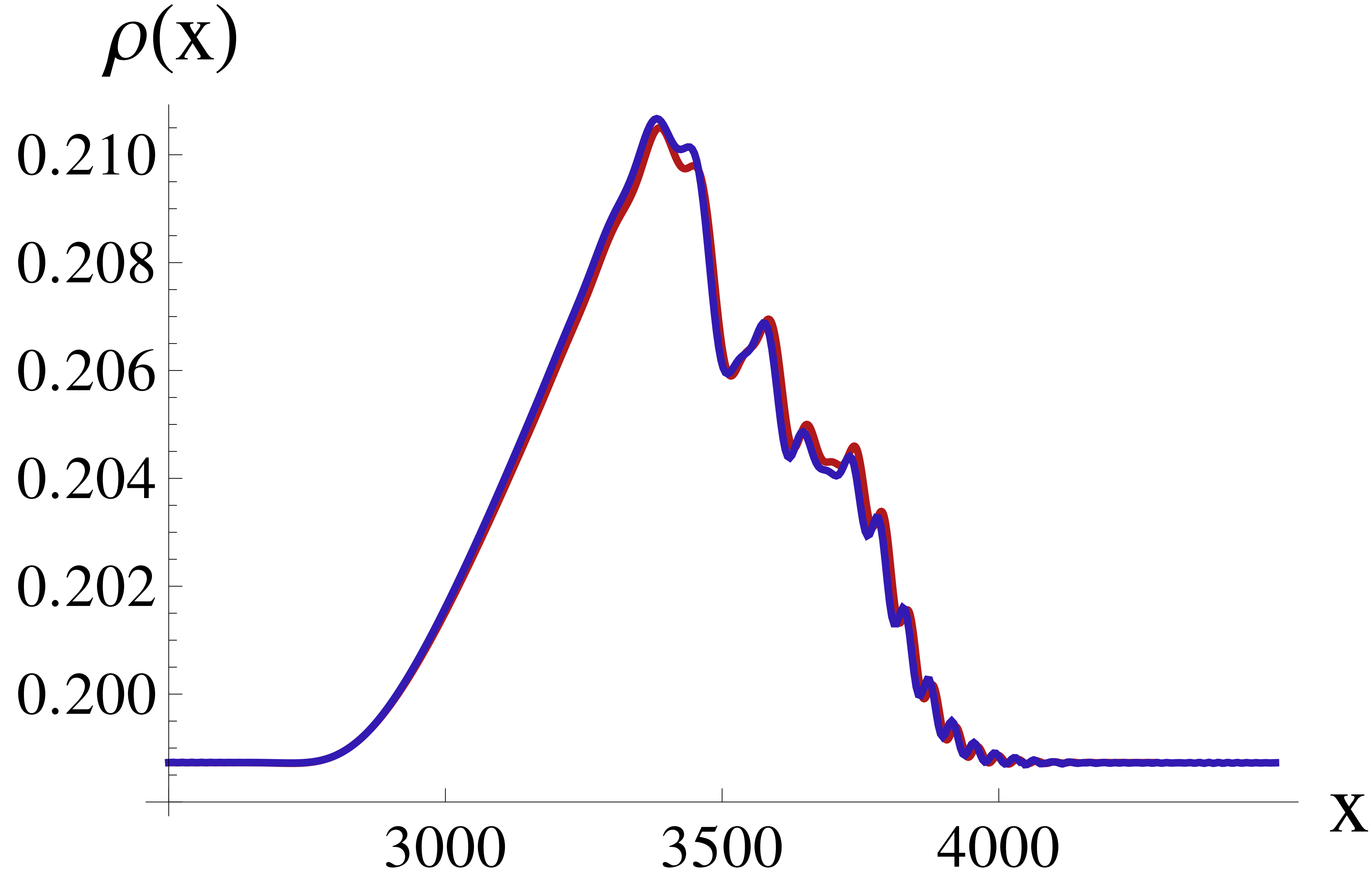
$$\delta f_0(X,p) \propto \Re\left[Ae^{-iS[X,p]}\right]$$



Density profile after the shock



Kinetic equation compared to numerics



(see also E. Bettelheim and L.I. Glazman, arXiv:1209.1881)

Bosonization and Hydrodynamic description

Quantum Hamiltonian
(Schick, Phys. Rev. 1968)

$$H = \int dx \left(\frac{\widehat{\rho}\widehat{v}^2}{2} + \frac{\pi^2 \widehat{\rho}^3}{6} \right)$$

Commutation relation

$$[\widehat{\rho}(x), \widehat{v}(y)] = -i\delta'(x-y)\,.$$

Equations of motion

$$\partial_t \widehat{\rho} + \partial_x (\widehat{\rho}\widehat{v}) = 0\,, \qquad \partial_t \widehat{v} + \partial_x \left(\frac{\widehat{v}^2}{2} + \frac{\pi^2 \widehat{\rho}^2}{2} \right) = 0$$

Can we remove hats?
Loop corrections should be taken into account.

Interaction and hydrodynamics

$$H_{\mathrm{int}} = (1/2) \int dx_1 dx_2 g(x_1 - x_2) \rho(x_1) \rho(x_2)$$

Inelastic time processes are strongly suppressed:

(M. Khodas et al. , PRB 2007; ... ;
A. Imambekov, T. L. Schmidt, and L. I. Glazman, RMP 2012)

Power-law interaction

$$g_\alpha(x) = \frac{1}{m l_0^{2-\alpha}} \frac{1}{x^\alpha}, \qquad 1 \le \alpha < 2, \qquad r_s = \lambda_F / l_0$$

$$H_{int,\alpha} \simeq \frac{1}{l_0^{2-\alpha}} \int dx \left(\rho^{\alpha+1} - \frac{1}{2} \rho \widehat{A}_\alpha \rho \right),$$

$$A_\alpha(q) = -2\Gamma[1-\alpha] \sin \frac{\pi\alpha}{2} |q|^{\alpha-1}, \qquad \alpha > 1$$

$$A_1(q) = \ln qd, \qquad qd \gg 1$$

$$H = \int dx \left[\frac{\rho v^2}{2} + \frac{\pi^2 \lambda^2 \rho^3}{6} - \frac{1}{2} \rho \widehat{A}_\alpha \rho \right].$$

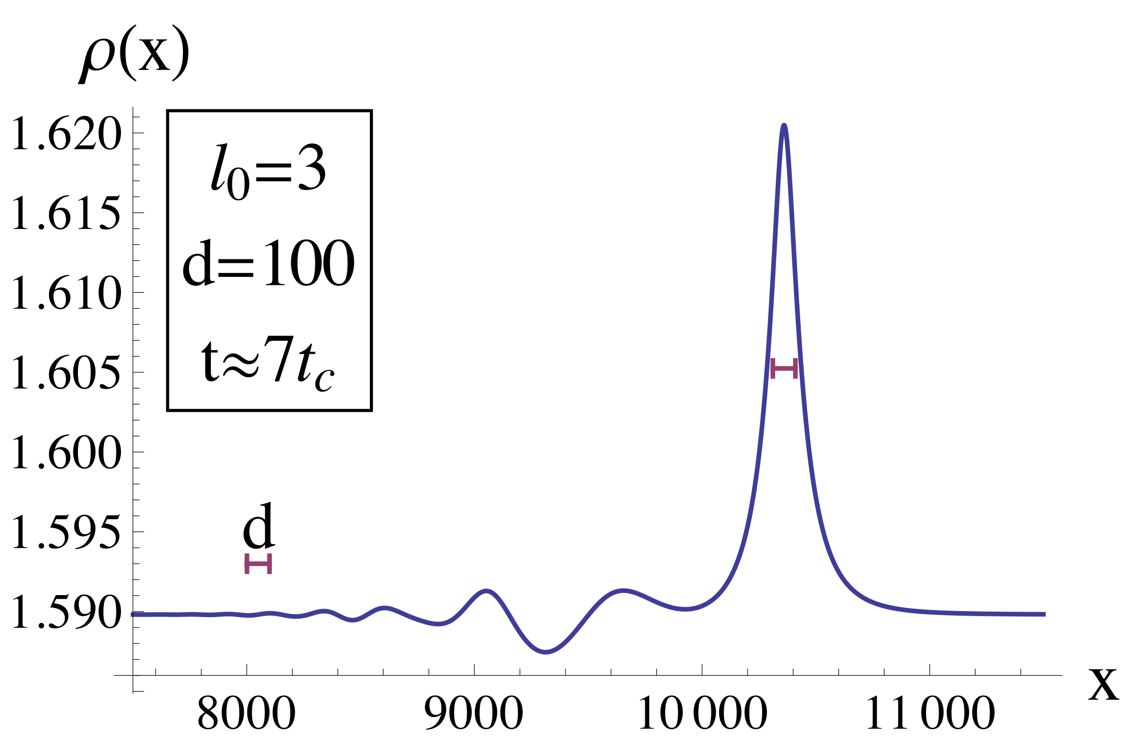
Power-law interaction

Period of oscillation $\delta x \sim l_0 \frac{1}{(l_0 \Delta\rho)^\beta}\,, \qquad \beta = \frac{1}{\alpha - 1}$

Hydrodynamics works if: $l_0\, \Delta\rho \ll 1$

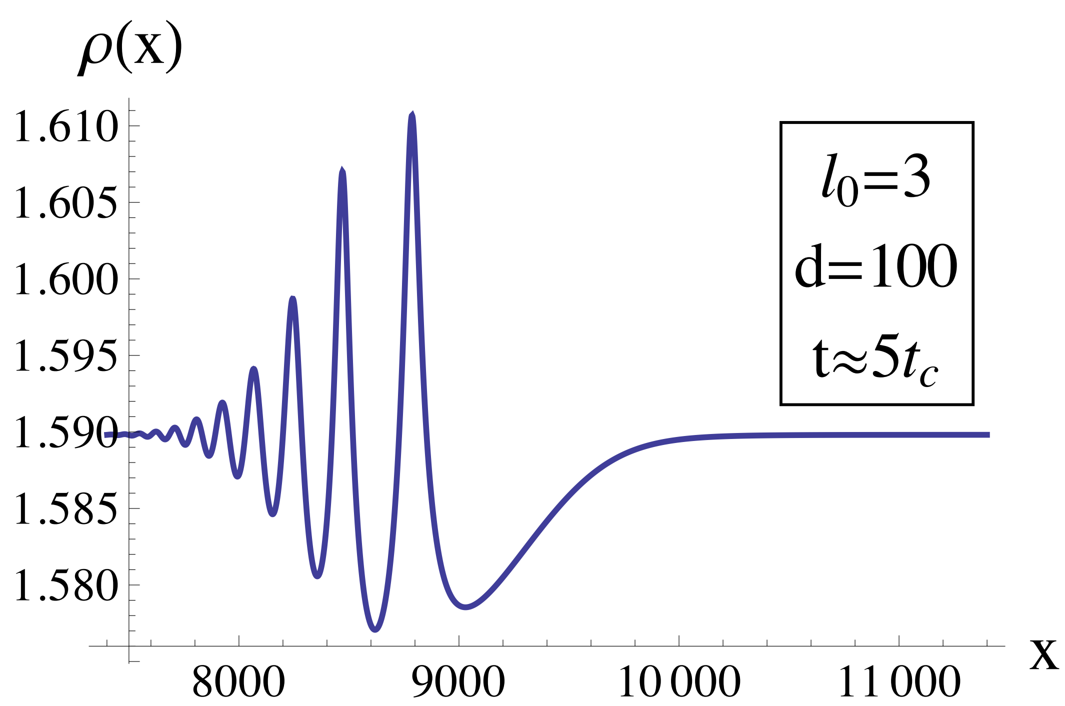
Coulomb: $\delta x \sim d \gg l_0$

Density Hump



solitons, charge much larger then 1

Density Dip



Ripples

Finite range interaction

$$g(q) = \frac{1}{ml_0} e^{-q^2 l_{int}^2} \simeq \frac{1}{ml_0} - \frac{q^2 l_{int}^2}{ml_0} = \frac{1}{ml_0^2} - \frac{l_1^2 q^2}{m}$$

$$H_{int} = \frac{1}{2} \int dx \left[\frac{\rho^2}{l_0} - l_1 \left(\partial_x \rho \right)^2 \right]$$

Hydrodynamics works if

$$l_0 \Delta\rho \ll 1 \,, \qquad l_{\mathrm{int}}^2 \Delta\rho / l_0 \gg 1$$

Kinetic equation for LL: from free fermions to hydrodynamics

We bosonize:

$$H_0 = \pi v_F \int dx\, (\rho_R^2 + \rho_L^2) + (4\pi^2/6m) \int (\rho_R^3 + \rho_L^3)$$

$$H_{\rm int} = (1/2) \int dx_1 dx_2 g(x_1 - x_2)\rho(x_1)\rho(x_2)$$

We make Bogolubov transform:

$$\rho_{R,q} = \cosh\kappa_q R_q - \sinh\kappa_q L_q\,,$$
$$\rho_{L,q} = -\sinh\kappa_q R_q + \cosh\kappa_q L_q$$

$$H^{(2)} = (\pi/L) \sum_q u_q\,(R_q R_{-q} + L_q L_{-q})$$

$$u_q = v_F (1 + g_q/\pi v_F)^{1/2} = v_F/K_q$$

$$H^{(3)} = (2\pi^2/3mL^2) \sum_{\mathbf{q}} \Gamma_{\mathbf{q}} [(R_1 R_2 R_3 + L_1 L_2 L_3) + 3\Gamma'_{\mathbf{q}} (R_1 R_2 L_3 + L_1 L_2 R_3)]$$

Kinetic equation for LL: from free fermions to hydrodynamics

We we make non-linear rotation: $\tilde{\rho}_L = U_3 L U_3^\dagger$

$$U_3 = \exp \sum_{\mathbf{q}} [f_{\mathbf{q}} R_1 R_2 L_3 - (L \leftrightarrow R)] \qquad f_{\mathbf{q}} = \frac{2\pi^2}{mL^2} \frac{\Gamma'_{\mathbf{q}}}{u_{q_1} q_1 + u_{q_2} q_2 - u_{q_3} q_3}$$

$$H = (\pi/L) \sum_{\eta,q} u_q \tilde{\rho}_{\eta,q} \tilde{\rho}_{\eta,-q} + (2\pi^2/3mL^2) \sum_{\eta,\mathbf{q}} \Gamma_{\mathbf{q}} \tilde{\rho}_{\eta,1} \tilde{\rho}_{\eta,2} \tilde{\rho}_{\eta,3} + O(\tilde{\rho}^4)$$

We perform refermionization now:

(A. Rozhkov, PRB 2006;
A. Imambekov and L. I. Glazman, Science 2009)

$$H = \sum_{\eta,k} \tilde{\Psi}^\dagger_{\eta,k} \left(\eta u_0 k - \frac{k^2}{2m^*} \right) \tilde{\Psi}_{\eta,k} + \frac{1}{2L} \sum_{\eta,q} V_q \tilde{\rho}_{\eta,q} \tilde{\rho}_{\eta,-q} + \frac{2\pi^2}{3mL^2} \sum_{\eta,\mathbf{q}} \gamma_{\mathbf{q}} \tilde{\rho}_{\eta,q_1} \tilde{\rho}_{\eta,q_2} \tilde{\rho}_{\eta,q_3}$$

Kinetic equation for LL: from free fermions to hydrodynamics

Parameters of Fermionic Hamiltonian:

$$1/m^* \simeq \Gamma_{\mathbf{q}=0}/m \qquad V_q = 2\pi(u_q - u_0) \qquad \gamma_{\mathbf{q}} = \Gamma_{\mathbf{q}} - \Gamma_{\mathbf{q}=0}$$

$$H = \sum_{\eta,k} \tilde{\Psi}^{\dagger}_{\eta,k}\left(\eta u_0 k - \frac{k^2}{2m^*}\right)\tilde{\Psi}_{\eta,k} + \frac{1}{2L}\sum_{\eta,q} V_q \tilde{\rho}_{\eta,q}\tilde{\rho}_{\eta,-q} + \frac{2\pi^2}{3mL^2}\sum_{\eta,\mathbf{q}} \gamma_{\mathbf{q}} \tilde{\rho}_{\eta,q_1}\tilde{\rho}_{\eta,q_2}\tilde{\rho}_{\eta,q_3}$$

Interaction of fermions is now RG irrelevant and can be treated in perturbation theory!

Kinetic equation for LL: from free fermions to hydrodynamics

Quantum Kinetic equation:

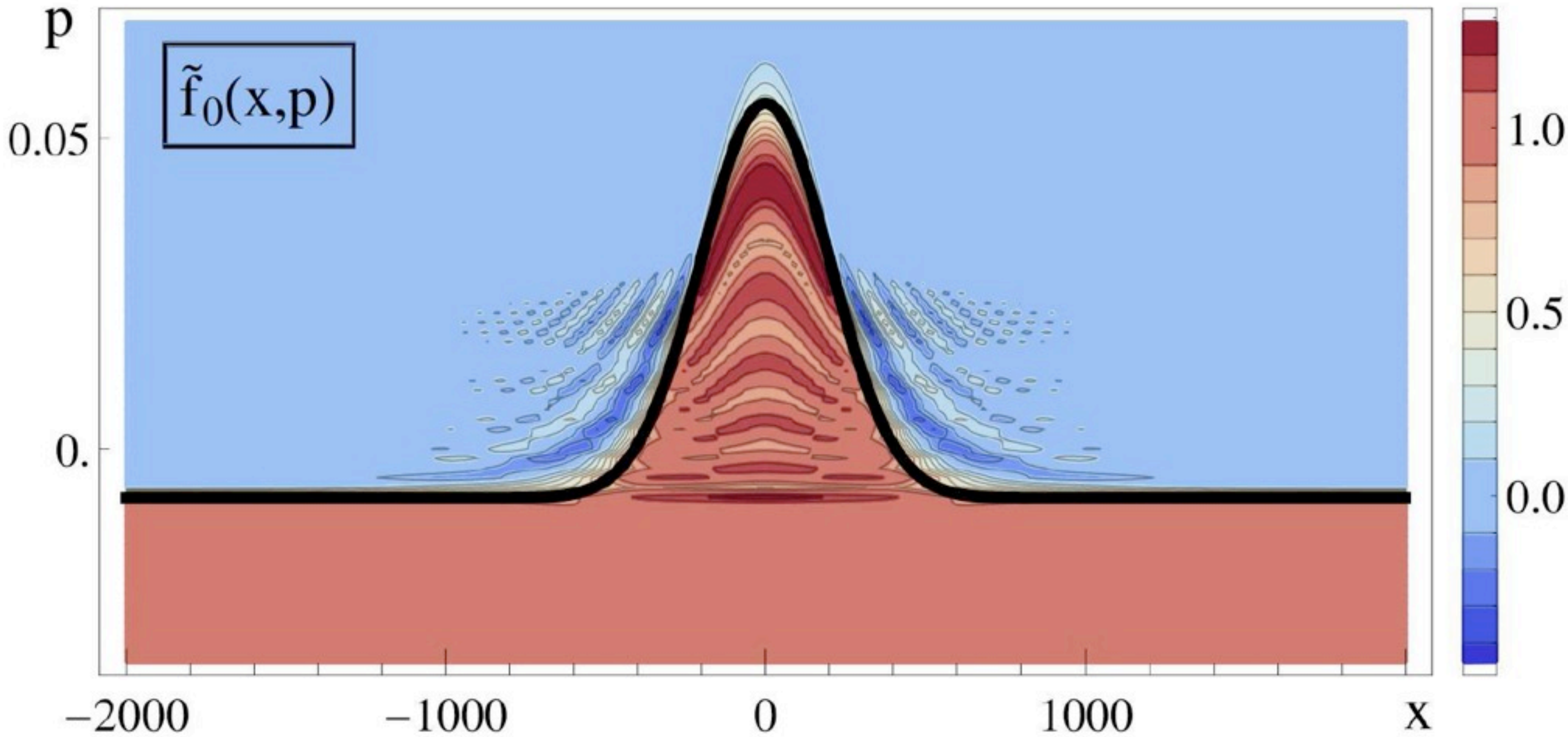
$$\partial_t \tilde{f}_\eta(p,x,t) + (p/m^*)\partial_x \tilde{f}_\eta(p,x,t) + \int (dp/2\pi) e^{-ipy} \times \tilde{f}_\eta(x,y,t)[\tilde{\phi}_\eta(x+y/2) - \tilde{\phi}_\eta(x-y/2)] = 0$$

$$\tilde{\phi}_\eta(x,t) = \int dx' V(x-x')\tilde{\rho}_\eta(x',t)$$

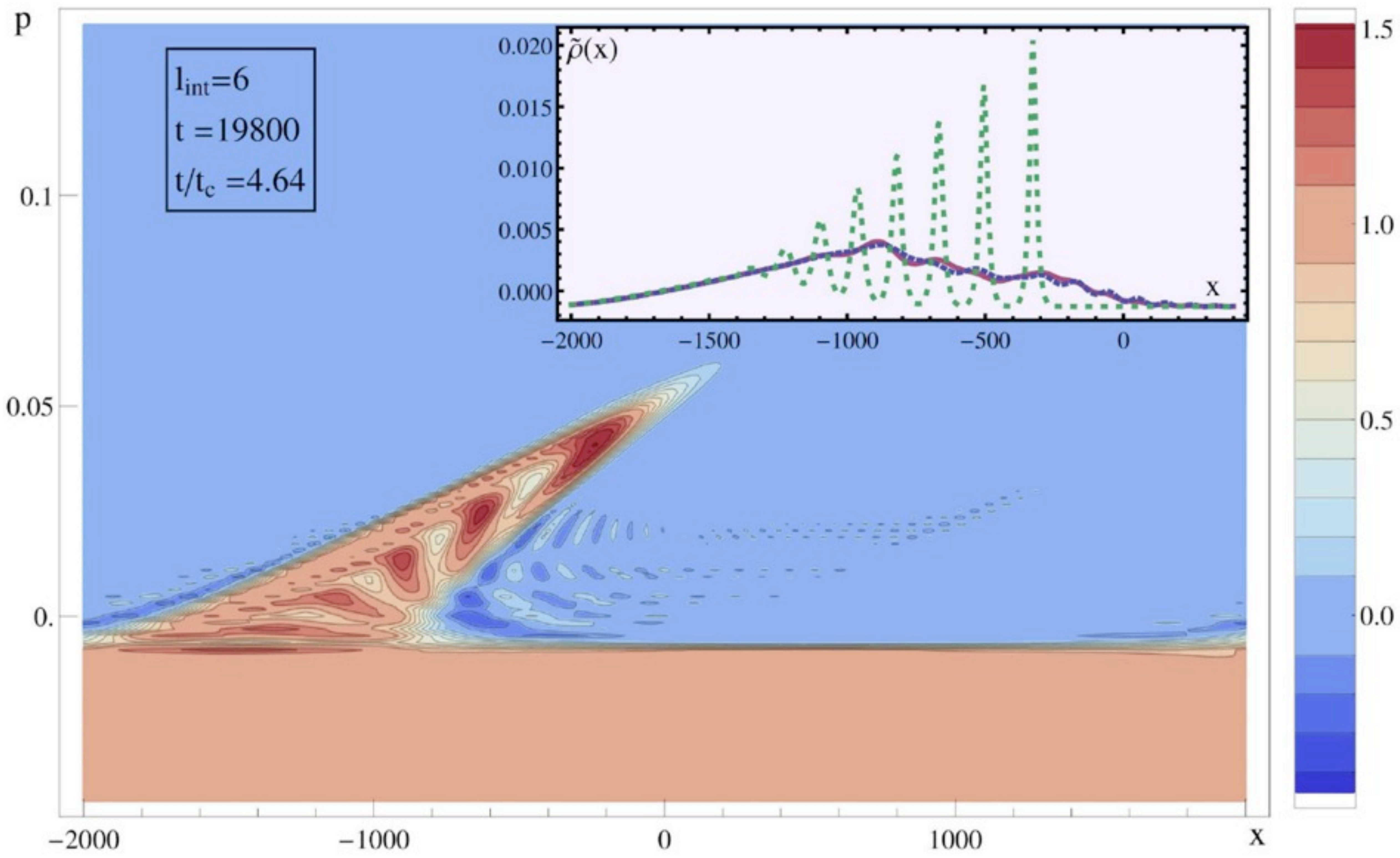
We concentrate on finite range interaction:

$$g(q) = \frac{1}{ml_0} e^{-q^2 l_{int}^2}$$

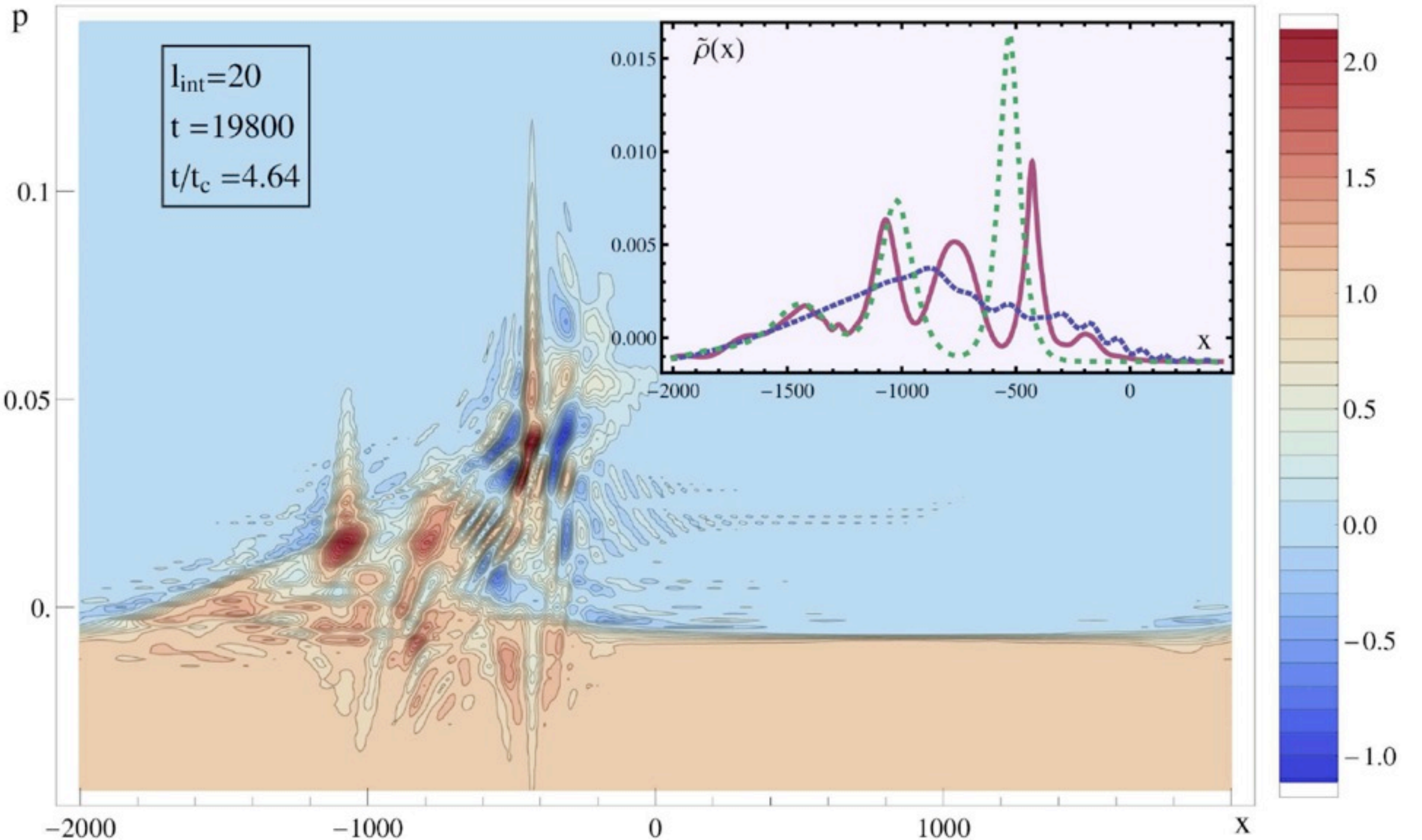
Kinetic equation for LL: from free fermions to hydrodynamics



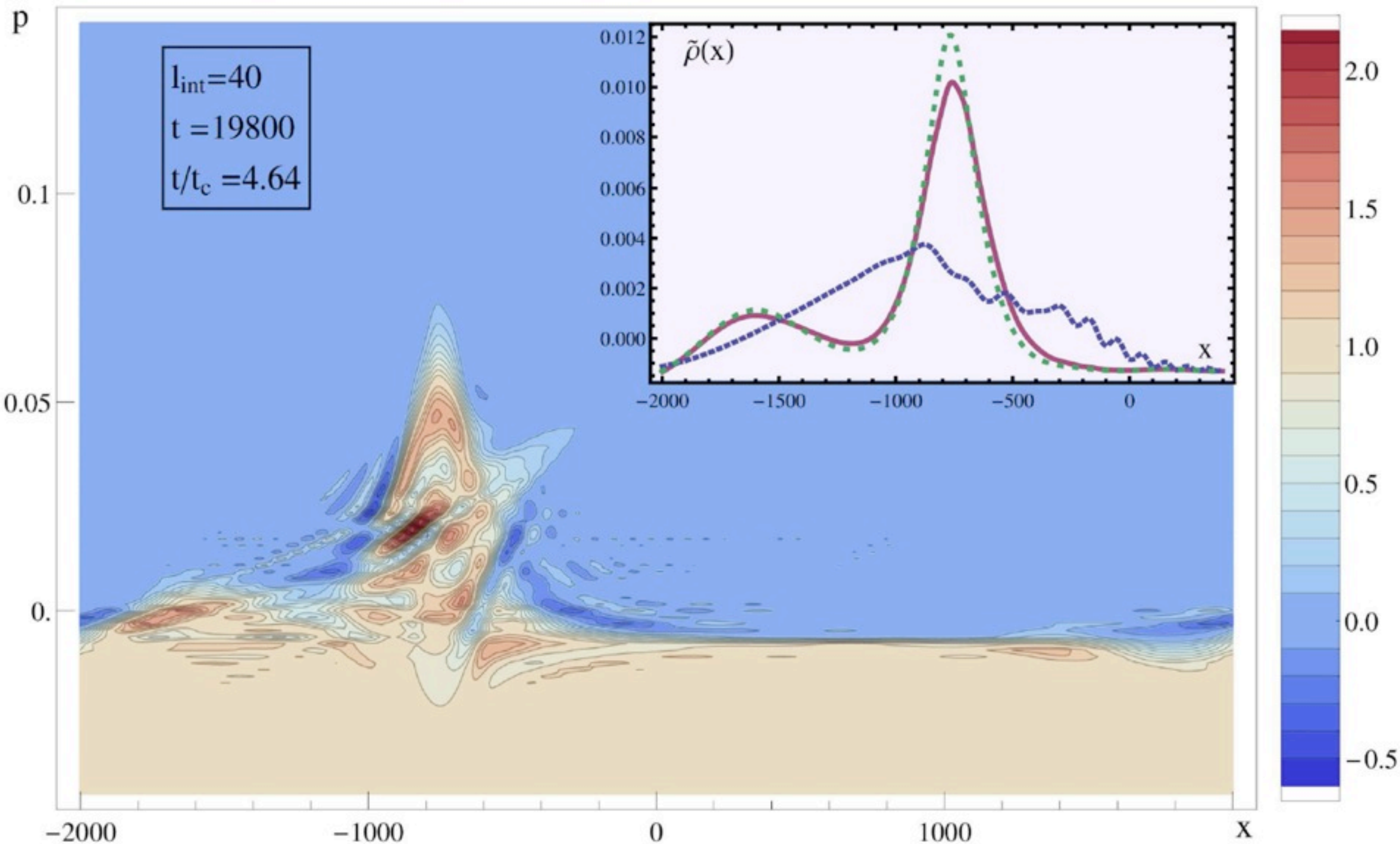
Kinetic equation for LL: from free fermions to hydrodynamics



Kinetic equation for LL: from free fermions to hydrodynamics



Kinetic equation for LL: from free fermions to hydrodynamics



Conclusions and Outlook

Free fermions or short range interaction

Oscillations in phase space, overturn and density ripples

Interacting Fermions (long range)

Solitons or Ripples
(depending on the sign of interaction and perturbation)

Intermediate regime

Kinetic equation in terms of proper fermions

Applications: cold atoms, QHE edges, top. insulators, ...

THANK YOU!