DISORDERED WEAK TOPOLOGICAL INSULATORS (AND A Z_2 CHIRAL-ANOMALY)

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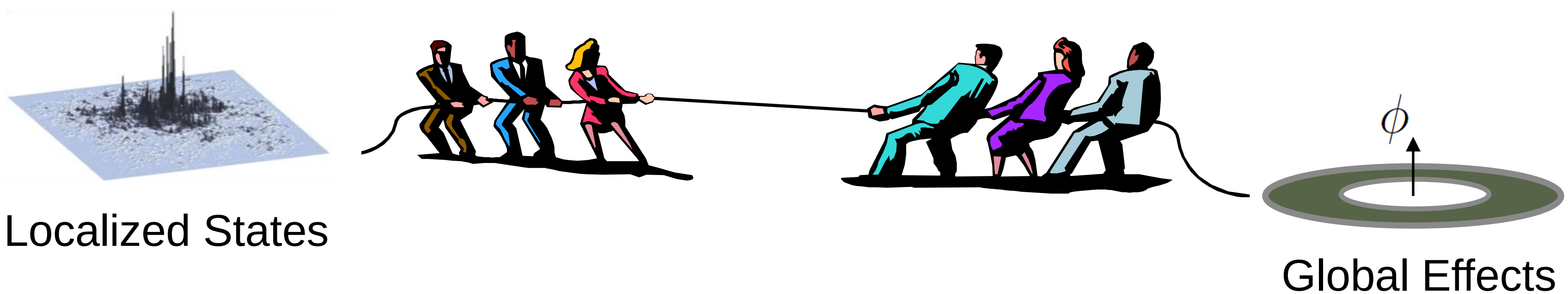
Outline

- Disordered Weak topological insulators (WTIs)
  - Motivation & Introduction to WTIs
  - WTI and Disorder – past conjecture.
  - WTI and Disorder – protection mechanism.
- A Z_2 anomaly on boundaries of topological insulators.

Motivation : Disorder Vs. Topology



The simplest Top. Phase – Integer Quantum Hall Effect

Z: Z = Hall Conductnace

Bulk effective action

$$S_{eff} = \int d^2x dt \varepsilon_{ijk} A_i \partial_j A_k$$

Gapped + Topological term

Correspondence

Edge effective action

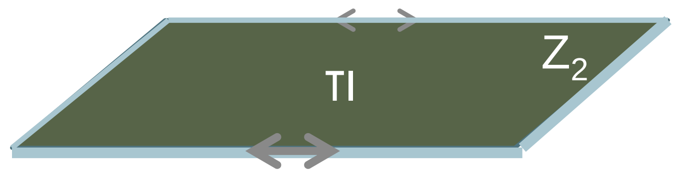
$$S_{eff} = \int dx dt \overline{\psi} (i\partial_t + i\partial_x) \psi$$

Critical + Charge anomaly

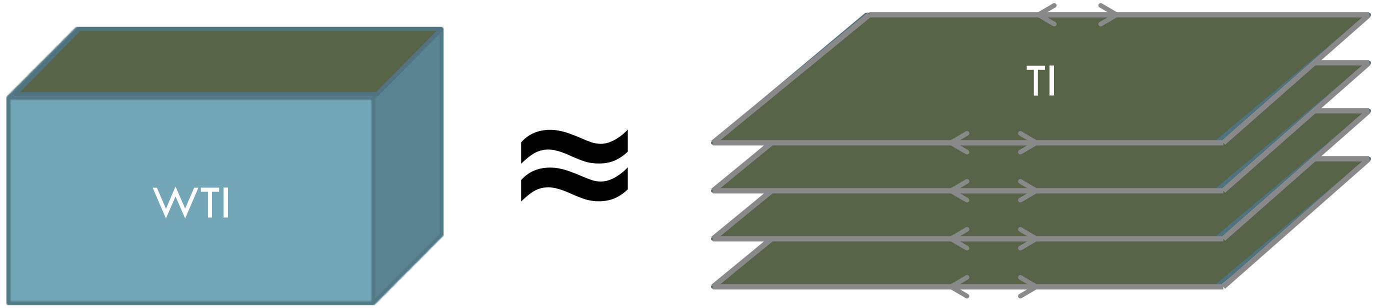
Newer Top. Phases: TIs and WTIs

- 2D Top. Insulator (TI)

Z_2: TI + TI = Trivial



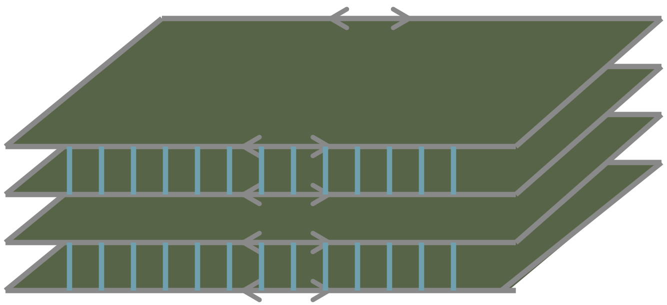
- 3D Weak Top. Insulator (WTI)



Kane & Mele (2005), Bernevig et. al (2006), Roy (2007), Kane & Fu (2007), Moore & Balents (2007), Fu & Kane (2007)

Why Weak

- A WTI with even # layer = topologically trivial



Z_2: TI + TI = Trivial

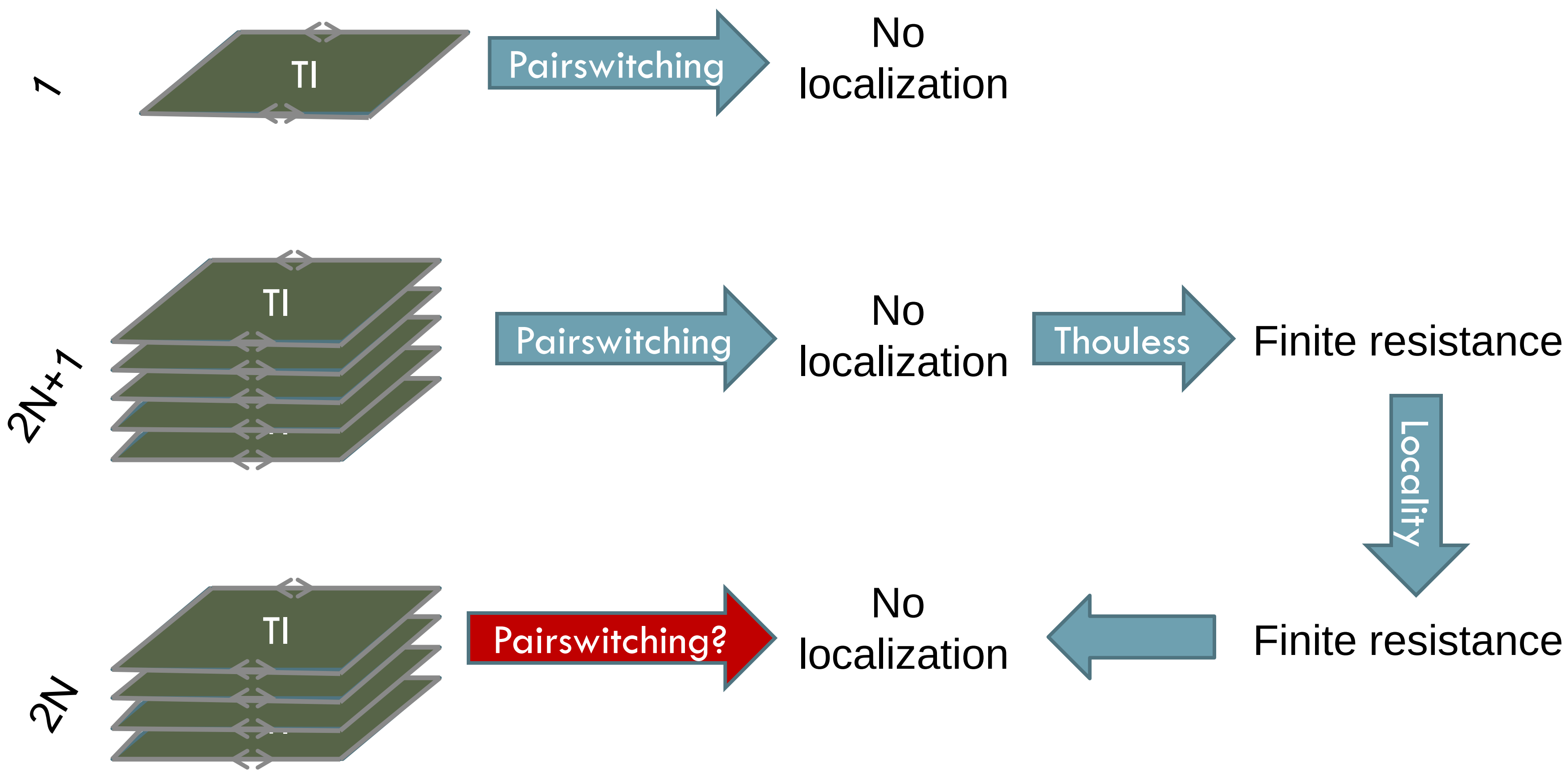
Topologically Protected if translation symmetry is preserved

Folklore about disorder

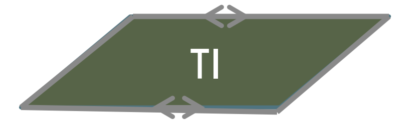
- If a metal can be made insulating – strong disorder will make it insulating.

- Weak Top. Insulators were thus discarded.

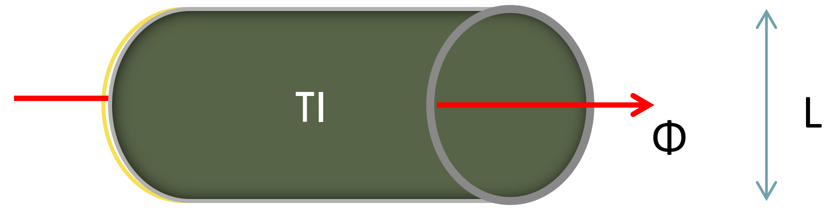
Protection argument - Outline



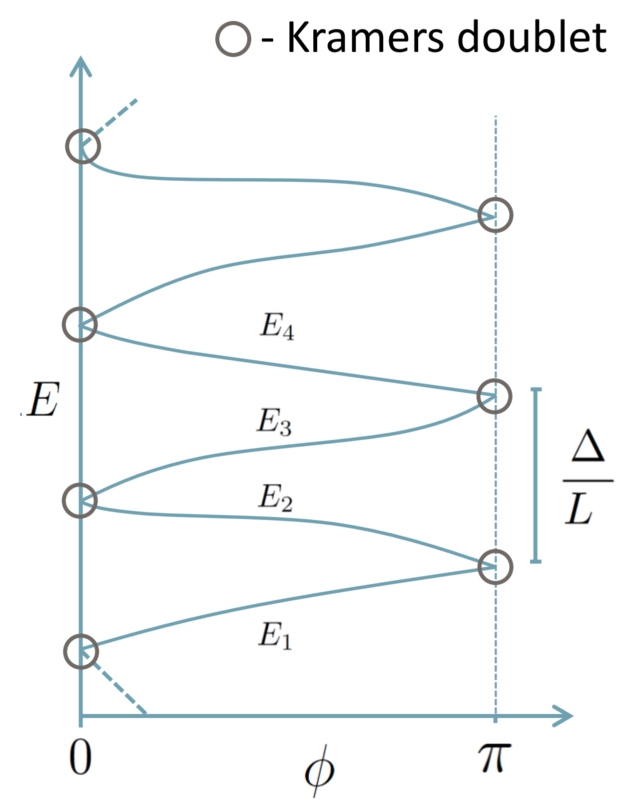
One Layer



- Put a TI on a cylinder

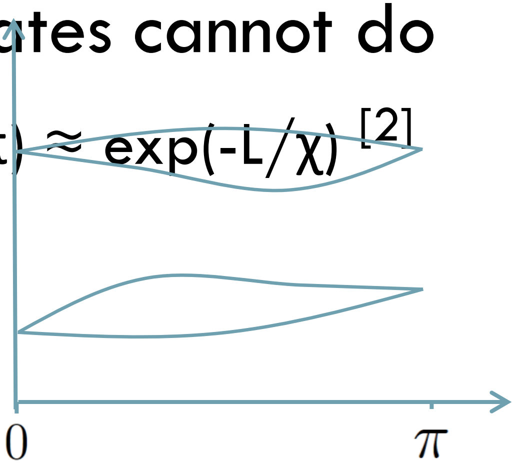


- Surface Spectrum does pairswitching[1]



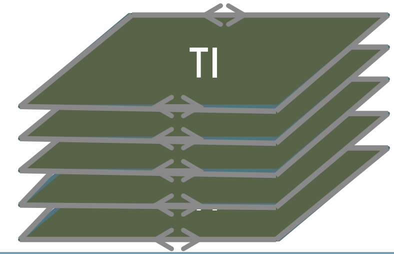
- Localized states cannot do this: $E(0) - E(\pi) \approx \exp(-L/\chi)$ [2]

No pairswitching

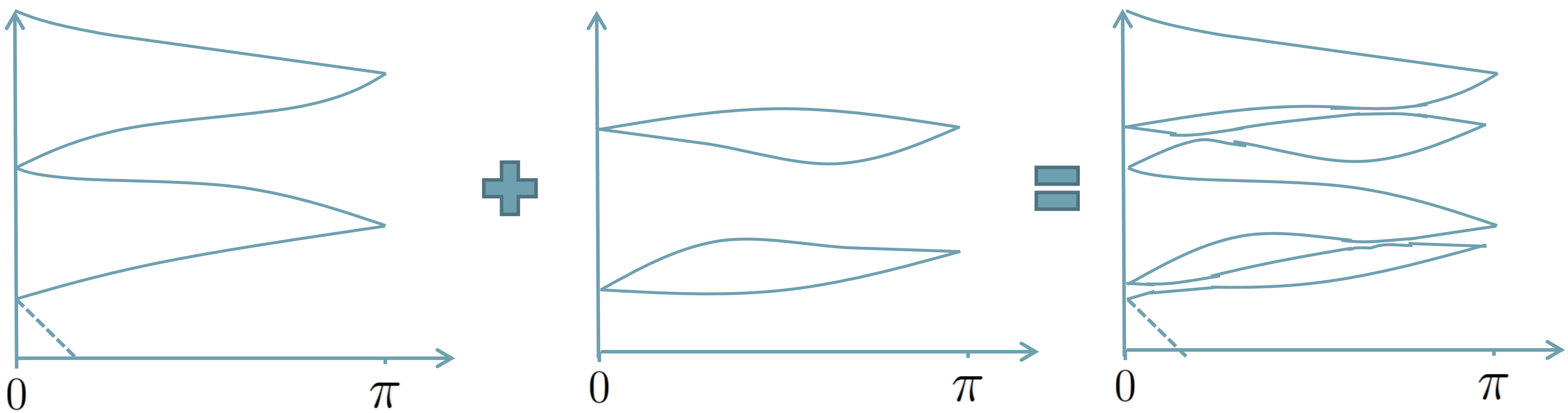


1. Fu & Kane (2006).
2. Nomura, Koshino & Ryu (2007)

Odd # of layers



- Pair-switching has a Z_2 algebra

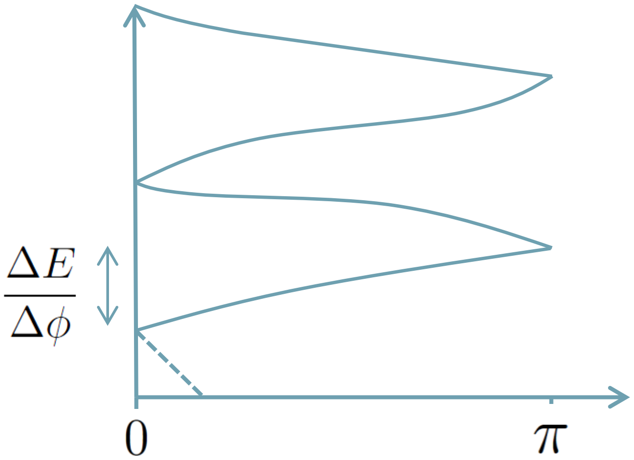


- Any odd number of layers cannot localize

Odd # of Layer - Conductance

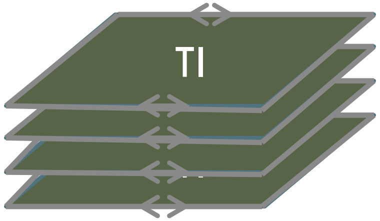
- The Thouless formula[1] relates flux sensitivity with conductance
- Pair-switching implies

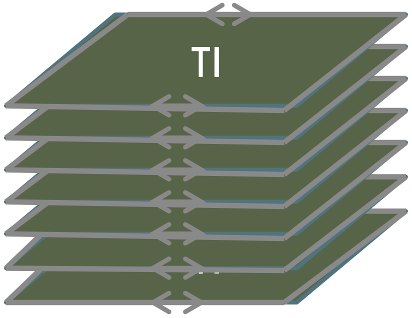
$$\sigma \approx \frac{e^2}{h} \left\langle \frac{\Delta E}{\Delta \phi} \right\rangle \frac{dN}{dE} \geq \frac{e^2}{h}$$



1. Thouless (1974)

Even # of Layers



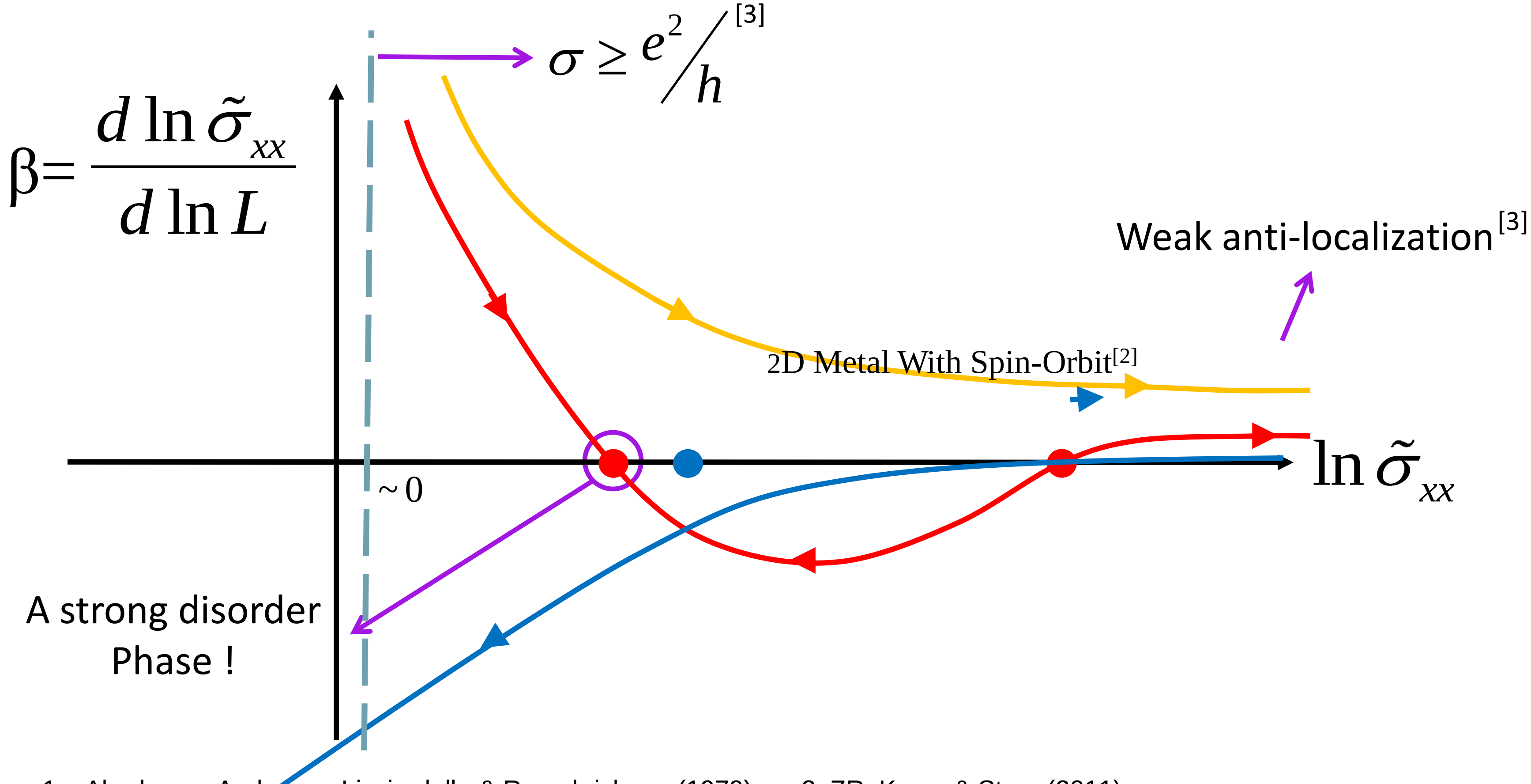


- Before cut, disorder is uniform so conductance is everywhere
- Removing a specific layer has **a weak effect on average**

Regardless of the layer parity - a WTI surface must conduct

RG analysis

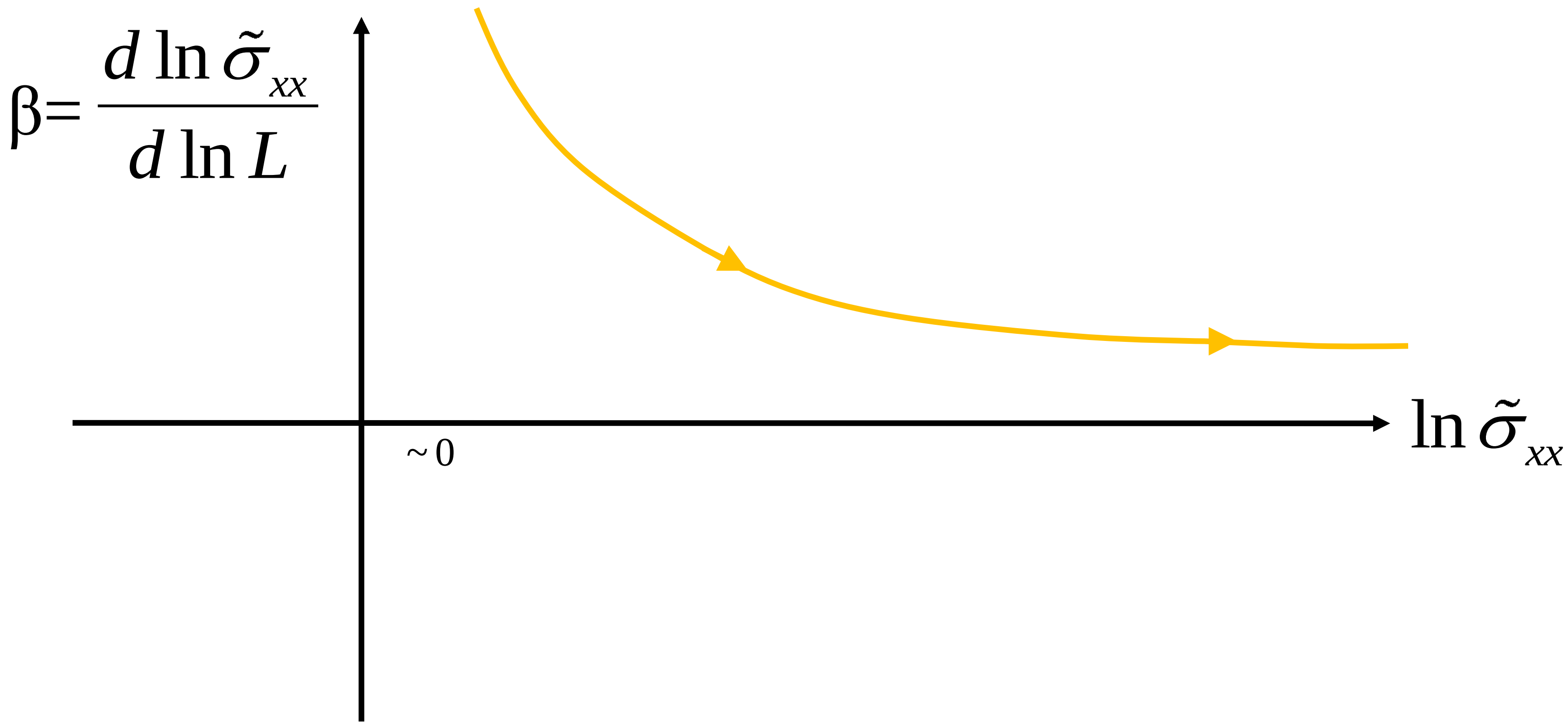
- Using Single Parameter Scaling[1]



1. Abrahams, Anderson, Licciardello & Ramakrishnan (1979) ; 3. ZR, Kraus & Stern (2011)
2. Hikame, Larkin & Nagaoka (1980).

RG analysis – Numerical results

- Transfer matrix methods show[1,2]

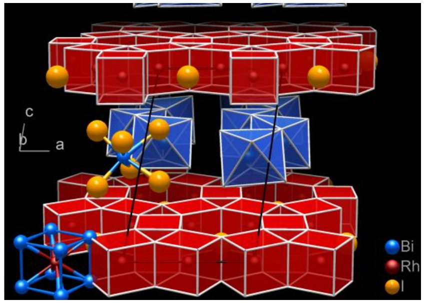


1. Mong, Bardarson & Moore (2011)
2. ZR (to be published).

Outlook

- Recently, a robust weak top. Insulator material was discovered[1]

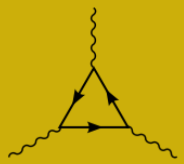
three-dimensional TI's.[4–8] We have synthesized the first bulk material belonging to an entirely different, *weak*, topological class, built from stacks of two-dimensional TI's: $Bi_{14}Rh_3I_9$. Its Bi-Rh sheets are graphene analogs, but with a honeycomb net composed of $RhBi_8$-cubes rather than carbon atoms. The strong bismuth-related spin-orbit interaction renders each graphene-like layer a TI with a 2400K band-gap.

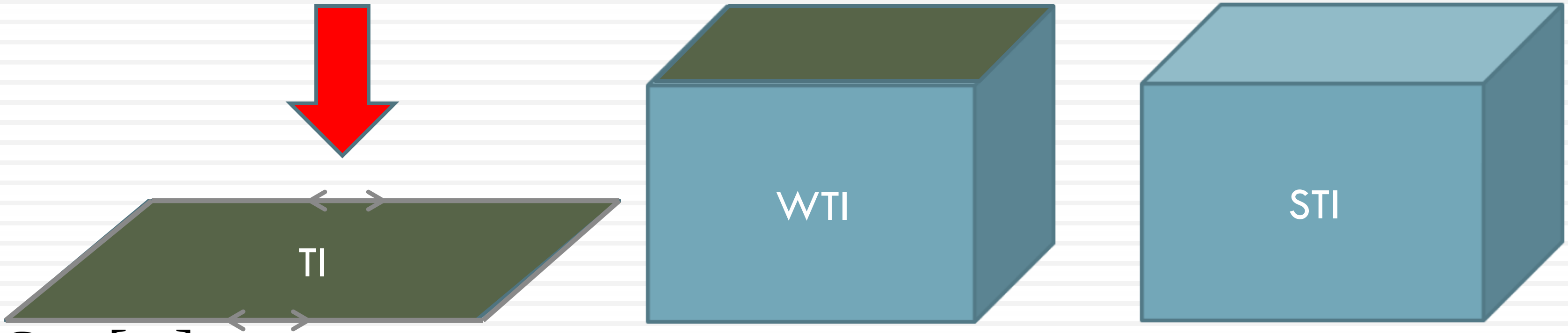


- Our results have been generalized to other supposedly weak topological phases.

[1] B. Rasche et. al., Nature Materials 12, 422–425 (2013).

A novel Z_2 Chiral Anomaly in the surface theory of TIs





$S_{edge}[\psi]$

Ringel & Stern (2012)

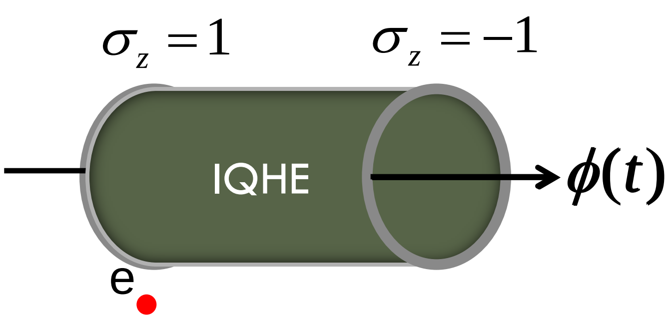
What can the anomaly do for you?

- Field theory formulation of TI boundaries
- No localization
- No go theorem
- Expands TIs to the interacting regime in an exact way[1]

1. ZR & Maciek Koch-Janusz (To be published)

IQHE: Pumping = Charge anomaly

- The Action



$$S_{2\otimes edge} = \int dt dx \overline{\psi} (\partial_t + \sigma_z H_{edge}) \psi$$

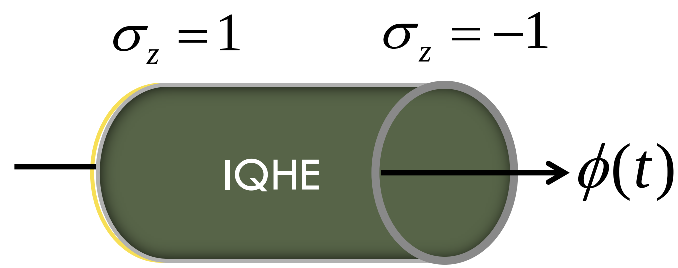
- Conserved Quantities and Symmetries
  - Q_{L+R} : $\overline{\psi} \to \overline{\psi} e^{-i\alpha}; \psi \to e^{i\alpha} \psi$
  - Q_{L-R} : $\overline{\psi} \to \overline{\psi} e^{-i\alpha\sigma_z}; \psi \to e^{i\alpha\sigma_z} \psi$
- Following flux insertions we expect[1]
  - $\Delta Q_{L-R} = \sigma_{xy} N_{flux} \neq 0$

1. Laughlin (1981)

IQHE: Edge theory = 1+1 QED

- The Action

$$S_{2\otimes edge} = \int dtdx\bar{\psi}(\partial_t + \sigma_z H_{edge})\psi$$



- Chiral Transformation

$$S_{2\otimes edge} \underset{\bar{\psi}\to\bar{\psi}i\sigma_x}{\Rightarrow} S_{ch} = \int dtdx\bar{\psi}(\sigma_x i\partial_t + \sigma_y H_{edge})\psi$$

For $H_{edge} = [i\partial_x - \phi(t)]$

This is 1+1 QED with a spinor.

A canonical example of the chiral anomaly[1]

1. Wen (1991), Froelich (1991) Kao & Lee (1996)

Pumping = Chiral Anomaly in QED

- Action

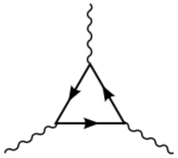
$$S_{ch} = \int_0^T dtdx\bar{\psi}(\overbrace{\sigma_x i\partial_t + \sigma_y[i\partial_x - \phi(t)]}^{\hat{S}})\psi$$

- Chiral symmetry

$$\{\hat{S}, \sigma_z\} = 0 \qquad \bar{\psi} \to \bar{\psi}e^{i\alpha\sigma_z}; \psi \to e^{i\alpha\sigma_z}\psi$$

- Chiral current is anomalous

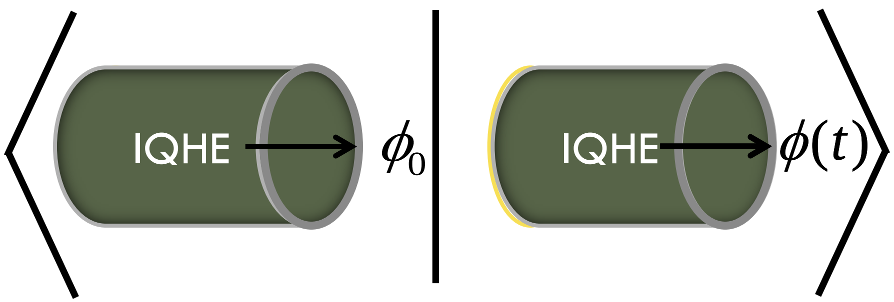
$$\int dtdx \nabla J_{ch} \equiv \boxed{\Delta Q_{L-R} =^{[1]} N_{flux}}$$



1. Adler (1969) ; Bell & Jackiw (1969)

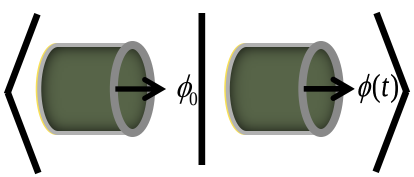
Orthogonality test for topology (IQHE)

- Pumping of Charge implies

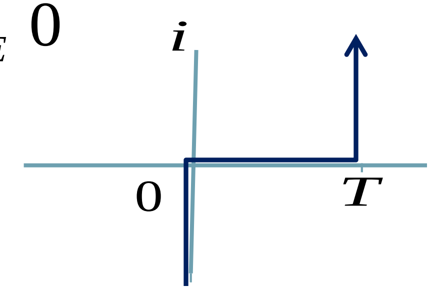


$$= \begin{cases} \mathbf{0}\ \text{(IQHE)} \\ \mathbf{1}\ \text{(Trivial)} \end{cases}$$

- Capturing this effect with the chiral anomaly[1]



$$= \int D[\psi,\overline{\psi}]e^{-S_{ch}} = Det[S_{ch}] =_{IQHE} 0$$

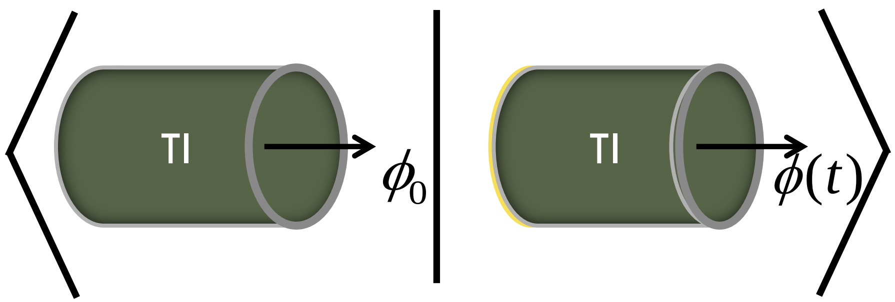


Zero Mode of S_{ch}
Implies Orthogonality

1. ZR & Stern (2012)

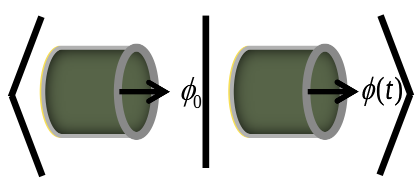
Generalization to TIs

- Pumping of some quantity



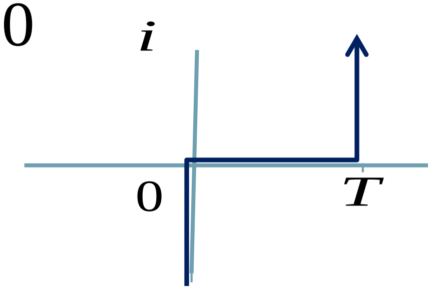
$$= \begin{cases} \mathbf{0}\ \text{(TI)} \\ \mathbf{1}\ \text{(Trivial)} \end{cases}$$

- Capturing this effect with the Z_2 Chiral Anomaly[1]



$$= \int D[\psi,\overline{\psi}]e^{-S_{ch}} = Det[S_{ch}] =_{TI} 0$$

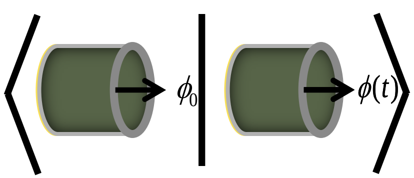
Kramer pair of
Action-zero-modes



1. ZR & Stern (2012)

Robustness of the Z_2 Anomaly

- The Z_2 Anomaly persists as long as TRS and charge conservation are not explicitly broken.
- This includes interactions and disorder[1] in an exact manner.
- Persists even as Time reversal symmetry is spontaneously broken on the edge.



$$= \int D[\psi, \overline{\psi}] e^{-S_{ch}} = Det[S_{ch}] =_{TI} 0$$

1. ZR & M. Koch Janusz (to be published)

Summary

- Weak phases do not localize unless excatly-dimerized.
- TI ground state goes to an orthogonal state following a full-flux insertion.
- The above holds with interactions+disorder and even when the flux insertion in not adiabatic.

Outlook

- A Z_2 Anomaly for Partons and Fractional topological insulators.

Thank You !