Towards conformal cosmology

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Outline

- Introduction
- Two toy models for generating flat spectrum of field perturbations
    - Why do they have the same outcome
- Reprocessing field perturbations into adiabatic perturbations

  Nothing new
- Properties at non-linear level
- Two sub-scenarios and their predictions:
    - Statistical anisotropy
    - Non-Gaussianity
- Summary

Introduction

Primordial scalar perturbations (density perturbations and associated gravitational potentials)

- Existed before the hot cosmological epoch
  - Must have been generated at some earlier stage, preceeding the hot epoch
- Gaussian (or nearly Gaussian) random field $\zeta(\vec{x})$, obeys Wick theorem
  - This suggests the origin: enhanced vacuum fluctuations of some (almost) free quantum field

- Flat or nearly flat power spectrum

$$\langle \zeta(\vec{k})\zeta(\vec{k}')\rangle = \frac{1}{4\pi k^3}\mathscr{P}(k)\delta(\vec{k}+\vec{k}')$$

$\mathscr{P}(k) =$ power spectrum, gives fluctuation in logarithmic interval of momenta,

$$\langle(\zeta(\vec{x}))^2\rangle = \int_0^\infty \frac{dk}{k}\,\mathscr{P}(k)$$

$$\mathscr{P} \propto k^{n_s-1}$$

Flat spectrum, $n_s = 1$

Harrison' 70; Zeldovich' 72

Almost true: $n_s - 1 \approx -0.04$.

There must be some symmetry behind flatness of spectrum

- Inflation: symmetry of de Sitter space-time, $SO(4,1)$

$$ds^2 = dt^2 - \mathrm{e}^{2Ht} d\vec{x}^{\,2}$$

Symmetry: spatial dilatations supplemented by time translations

$$\vec{x} \to \lambda \vec{x}\,, \quad t \to t - \frac{1}{2H} \log \lambda$$

Inflation automatically generates nearly flat spectrum.

- Alternative: conformal symmetry $SO(4,2)$

Conformal group includes dilatations, $x^\mu \to \lambda x^\mu$.

$\Longrightarrow$ No scale, good chance for flatness of spectrum

First mentioned by Antoniadis, Mazur, Mottola' 97

Concrete models: V.R.' 09;

Creminelli, Nicolis, Trincherini' 10

What if our Universe started off from or passed through

an unstable conformal state

and then evolved to much less symmetric state we see today?

Further motivation: Alternatives to inflation:

- Contraction — Bounce — Expansion
- Start up from static state

Creminelli, Niclois, Trincherini '10

Difficult, but not impossible.

How to generate density perturbations with nearly flat spectrum?

Two ways of getting flat scalar spectrum

Way # 1: conformal rolling V.R.

Conformal plus global symmetry instead of de Sitter symmetry

- Main requirement: long evolution before the hot stage. But otherwise insensitive to regime of cosmological evolution. Can work at inflation and its alternatives.

Model:

$$S = S_{G+M} + S_\phi$$

S_{G+M}: gravity plus dominating matter

S_ϕ: conformal complex scalar field ϕ with negative quartic potential. Spectator until late epoch.

$$S = \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi^* \partial_\nu \phi + \frac{R}{6} |\phi|^2 - (-h^2 |\phi|^4) \right]$$

Conformal symmetry. Global symmetry $U(1)$. $\phi = 0$: unstable state with unbroken conformal symmetry. [Conformal symmetry explicitly broken at large fields. To be discussed later.]

Homogeneous and isotropic Universe,

$$ds^2 = a^2(\eta)[d\eta^2 - d\vec{x}^2]$$

In terms of the field $\chi(\eta,\vec{x}) = a(\eta)\phi(\eta,\vec{x}) = \chi_1 + i\chi_2$, evolution is Minkowskian,

$$\eta^{\mu\nu}\partial_\mu\partial_\nu\chi - 2h^2|\chi|^2\chi = 0$$

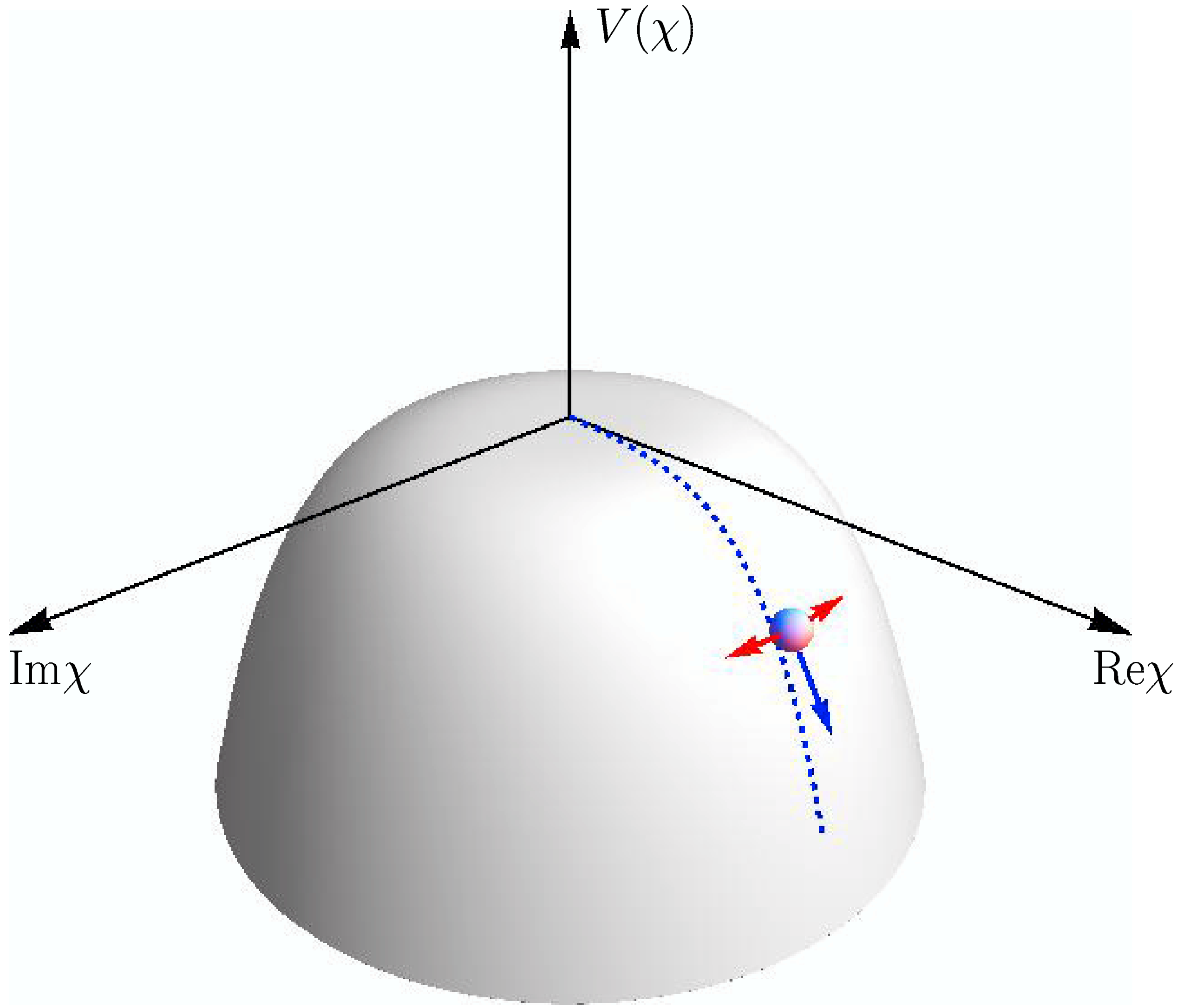
Homogeneous background solution, breaks $SO(4,2) \to SO(4,1)$

Attractor (real without loss of generality)

$$\chi_c(\eta) = \frac{1}{h(\eta_* - \eta)}$$

$\eta_* =$ constant of integration, end time of roll.

NB: Particular behavior $\chi_c \propto (\eta_* - \eta)^{-1}$
dictated by conformal symmetry.



$V(\chi)$
$\mathrm{Im}\chi$
$\mathrm{Re}\chi$

Fluctuations of Im χ

automatically have flat power spectrum

Linearized equation for fluctuation $\delta\chi_2 \equiv \mathrm{Im}\chi$. Mode of 3-momentum k:

$$\frac{d^2}{d\eta^2}\delta\chi_2 + k^2\delta\chi_2 - 2h^2\chi_c^2\delta\chi_2 = 0$$

[recall $h\chi_c = 1/(\eta_* - \eta)$]

Regimes of evolution:

- Early times, $k \gg 1/(\eta_* - \eta)$, short wavelength regime, χ_c negligible, free Minkowskian field

$$\delta\chi_2 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \mathrm{e}^{-ik\eta} A_{\vec{k}} \ + \ \mathrm{h.c.}$$

- Late times, $k \ll 1/(\eta_* - \eta)$, long wavelength regime,
  term with χ_c dominates,

$$\delta\chi_2 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \cdot \frac{1}{k(\eta_* - \eta)} \cdot A_{\vec{k}} \quad + \quad \text{h.c.}$$

  - Phase of the field ϕ freezes out:

$$\delta\theta = \frac{\delta\chi_2}{\chi_c} = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \cdot \frac{h}{k} \cdot A_{\vec{k}} \quad + \quad \text{h.c.}$$

  - Power specrum of phase is flat:

$$\langle\delta\theta^2\rangle = \frac{h^2}{2(2\pi)^3} \int \frac{d^3k}{k^3} \implies \mathscr{P}_{\delta\theta} = \frac{h^2}{(2\pi)^2}$$

  - This is automatic consequence of global $U(1)$
    and conformal symmetry

To see this, consider long wavelength regime:

$\vec{k}$ negligible,
equation for $\delta\chi_2$ is equation for spatially homogeneous perturbation.
χ_c is solution to full field equation, $\mathrm{e}^{i\alpha}\chi_c$ also $\Longrightarrow$
$\delta\chi = i\alpha\chi_c$ is solution to perturbation equation $\Longrightarrow$

$$\delta\chi_2 : \quad \mathrm{e}^{-ik\eta} \;\Longrightarrow\; C(k)\chi_c(\eta) = \frac{1}{k(\eta_* - \eta)}$$

NB: $1/k$ on dimensional grounds.

NB: In fact, equation for $\delta\chi_2$ is precisely the same as equation for minimally coupled massless scalar field in inflating Universe

Comments:

- Mechanism requires long cosmological evolution: need

$$(\eta_* - \eta) \gg 1/k$$

early times, short wavelength regime,
well defined vacuum of the field $\delta\chi_2$.

For $k \sim H_0$ this is precisely the requirement that the horizon problem is solved, at least formally.

This is a pre-requisite for most mechanisms that generate density perturbations

- Small explicit breaking of conformal invariance $\Longrightarrow$ tilt of the spectrum

Osipov, V.R.

- Depends both on the way conformal invariance is broken and on the evolution of scale factor

Way # 2 of getting flat spectrum

Creminelli, Nicolis, Trincherini '10

Galilean Genesis

Begin with galileon field π, Lagrangian

$$L_\pi = -f^2 \mathrm{e}^{2\pi} \partial_\mu \pi \partial^\mu \pi + \frac{f^3}{\Lambda^3} \partial_\mu \pi \partial^\mu \pi \cdot \Box \pi + \frac{f^3}{2\Lambda^3} (\partial_\mu \pi \partial^\mu \pi)^2$$

Conformally invariant. Under dilatations

$$\mathrm{e}^{\pi(x)} \to \lambda \mathrm{e}^{\pi(\lambda x)}$$

Universe begins from Minkowski space-time. Galileon rolls as

$$\mathrm{e}^{\pi_c} = \frac{1}{H_G(t_* - t)} , \qquad t < t_* ,$$

where $H_G^2 = \frac{2\Lambda^3}{3f}$. Again dictated by conformal invariance.

Initial energy density is zero, then it slowly builds up,

$$H(t) = \frac{1}{3}\frac{f^2}{M_{Pl}^2}\frac{1}{H_G^2(t_* - t)^3}$$

until $(t_* - t_e) \sim H_G^{-1} \cdot f/M_{PL}$. NB: Hubble parameter grows in time. Strong violation of all energy conditions. Yet fully consistent theory, no ghosts, tachyons, other pathologies.

At some point galileon is assumed to transmit its energy to conventional matter, hot epoch begins.

Galileon perturbations are not suitable for generating scalar perturbations.
Introduce another field θ of conformal weight 0,

$$L_\theta = \mathrm{e}^{2\pi}(\partial_\mu \theta)^2 \implies L_\theta = \frac{\text{const}}{(t_* - t)^2} \cdot (\partial_\mu \theta)^2$$

Dynamics of perturbations $\delta\theta$ in background π_c is exactly the same as in conformal rolling model.

Similarity is not an accident

Hinterbichler, Khouri

Hinterbichler, Joyce, Khouri

General setting:

- Effectively Minkowski space-time
- Conformally invariant theory
- Field ρ of conformal weight $\Delta \neq 0$
  - $\rho = \text{const} \cdot |\phi|$ in conformal rolling model
  - $\rho = \text{const} \cdot \mathrm{e}^{\pi}$ in Galilean Genesis; $\Delta = 1$ in both models.

Homogeneous classical solution

$$\rho_c(t) = \frac{1}{(t_* - t)^{\Delta}}$$

by conformal invariance.

NB: t is conformal time in conformal rolling scenario

- Another scalar field θ of conformal weight 0.
- Kinetic term dictated by conformal invariance (modulo field rescaling)

$$L_\theta = \rho^{2/\Delta}(\partial_\mu \theta)^2$$

- Assume potential terms negligible $\Longrightarrow$ Lagrangian in rolling background

$$L_\theta = \frac{1}{(t_* - t)^2} \cdot (\partial_\mu \theta)^2$$

Exactly like scalar field minimally coupled to gravity in de Sitter space, with $t =$ conformal time, $a(t) = \text{const}/(t_* - t)$.

θ develops perturbations with flat power spectrum.

Use conformal rolling model in what follows for definiteness.

Reprocessing field perturbations into adiabatic perturbations

- Assume that conformal evolution ends up at some late time. Scalar potential actually has a minimum at large field.

Modulus of the field ϕ freezes out at the minimum of the scalar potential. Assume that energy density of ϕ is negligible at that time (probably, unimportant).

There are at least two known mechanisms of converting phase perturbations into adiabatic perturbations.

- Modulated decay

Dvali, Gruzinov, Zaldarriaga' 03
Kofman' 03

- Phase = pseudo-Goldstone curvaton

Linde, Mukhanov' 97;
Enqvist, Sloth' 01; Lyth, Wands' 01; Moroi, Takahashi' 01;
K. Dimopoulos et. al.' 03

In either case

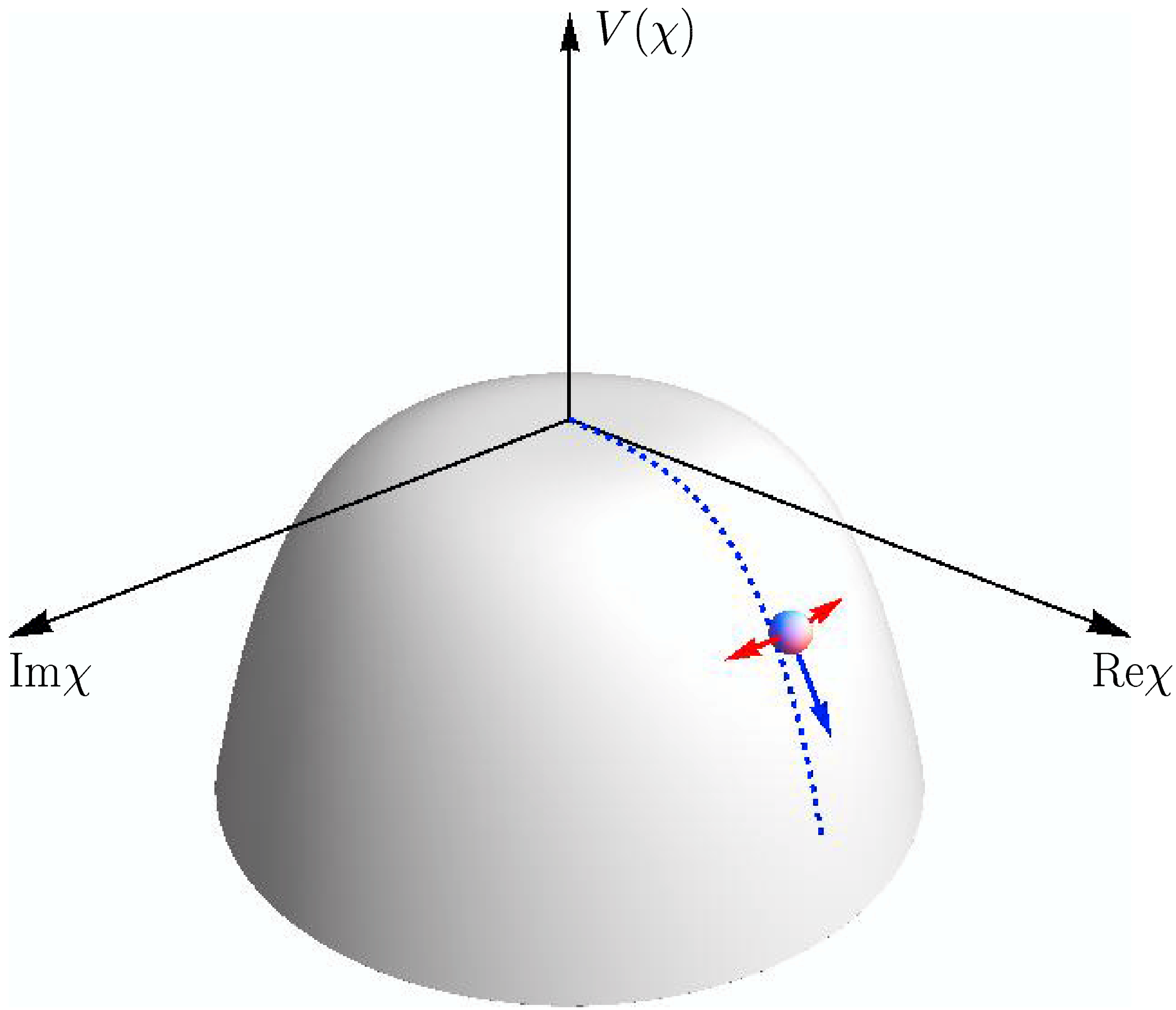
$$\zeta = \text{const} \cdot \delta\theta \ + \ \text{possible non-linear terms}$$

Adiabatic perturbations inherit shape of power spectrum and correlation properties from $\delta\theta$, plus possible additional non-Gaussianity.

const < 1 and may be $\ll 1 \Longrightarrow \delta\theta \gg \zeta$ quite possible $\Longrightarrow$ $h < 1$, but not necessarily $h \sim 10^{-4}$.

In any case, no relationship with tensor perturbations

Order h effects: back to conformal evolution



Peculiarity: raidial perturbations.

- Linear analysis of perturbations of $\chi_1 = \mathrm{Re}\chi$ about the homogeneous real solution χ_c:

$$\frac{d^2}{d\eta^2}\delta\chi_1 + k^2\delta\chi_1 - 6h^2\chi_c^2\delta\chi_1 = 0$$

[recall $h\chi_c = 1/(\eta_* - \eta)$].
Again initial condition

$$\delta\chi_1 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \mathrm{e}^{i\vec{k}\vec{x} - ik\eta} B_{\vec{k}} \ + \ \text{h.c.}$$

But now the solution is

$$\delta\chi_1 = \frac{1}{4\pi}\sqrt{\frac{\eta_* - \eta}{2}} H_{5/2}{}^{(1)}\left[k(\eta_* - \eta)\right] \cdot B_{\vec{k}} \ + \ \text{h.c.}$$

- In long wavelength regime, $k \ll 1/(\eta_* - \eta)$,

$$\delta\chi_1 = \frac{3}{4\pi^{3/2}} \frac{1}{k^2\sqrt{k}(\eta_* - \eta)^2} B_{\vec{k}} \;+\; \text{h.c.}$$

  - Red spectrum:

$$\langle \delta\chi_1^2 \rangle \propto \int \frac{d^3k}{k^5}$$

  - Large $\delta\chi_1$ at small $(\eta_* - \eta)$

    [Recall $\chi_c = 1/[h(\eta_* - \eta)]$]

- Again by symmetry: now translations of conformal time: $\chi_c \propto 1/(\eta_* - \eta) \Longrightarrow$ spatially homogeneous solution to perturbation equation $\delta\chi = \partial_\eta \chi_c$.

Modulo field redefinition and notations, properties of galileon perturbations are exactly the same as properties of radial perturbations in conformal rolling scenario.

Libanov, Mironov, V.R.

Furthermore, these properties are unambiguously determined by conformal invariance

Libanov, Mironov, V.R.

Hinterbichler, Khouri

Hence, we are dealing with the whole class of models

- Interpretation: time shift $\eta_* \longrightarrow \eta_* + \delta\eta_*(\vec{x})$

$$\mathrm{Re}\chi = \chi_c(\eta) + \delta\chi_1(\eta, \vec{x})$$
$$= \frac{1}{\eta_* - \eta} + \frac{F(\vec{x})}{(\eta_* - \eta)^2} = \frac{1}{\eta_* + \delta\eta_*(\vec{x}) - \eta}$$

  - Background for perturbations $\delta\chi_2 = \mathrm{Im}\chi$ (in other words, for phase θ) is no longer spatially homogeneous.
  - Red spectrum of $\delta\eta_*(\vec{x})$: $\sqrt{\mathscr{P}_{\delta\eta_*}} = \frac{3h}{2\pi k}$
- η_* itself is irrelevant: overall time shift. Relevant are derivatives

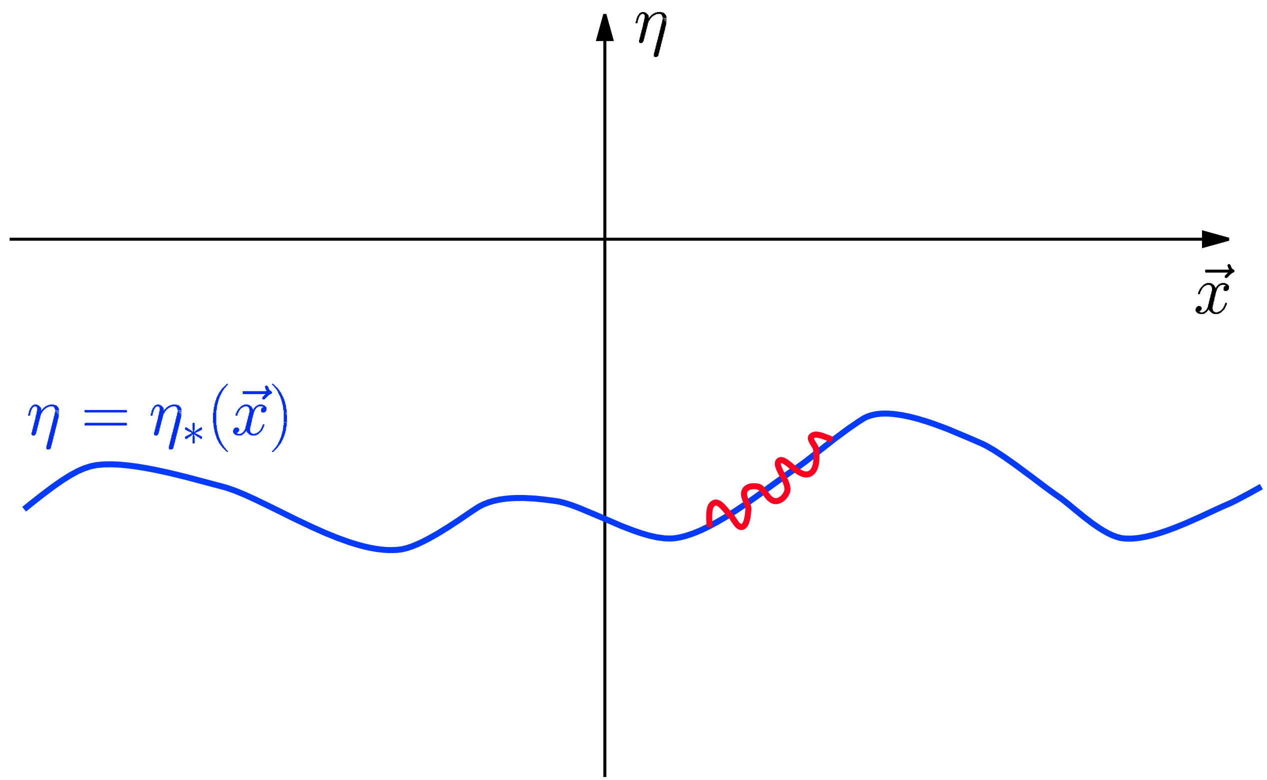
$$\vec{\partial}\eta_*(\vec{x}) \equiv -\vec{v}\,, \qquad \text{etc.}$$

  - $\vec{v}$ has flat power spectrum

$$\sqrt{\mathscr{P}_{\vec{v}}} = \frac{3h}{2\pi}$$

- Potentially dangerous effects of infrared radial modes: have to study perturbations of $\mathrm{Im}\chi$ in *spatially inhomogeneous background*, slowly varying in space,

$$\chi_c = \frac{1}{h(\eta_*(\vec{x}) - \eta)}$$



- Back to equation for perturbations of $\delta\chi_2 = \mathrm{Im}\chi$

$$\frac{d^2}{d\eta^2}\delta\chi_2 - \frac{\partial^2}{\partial\vec{x}^2}\delta\chi_2 - \frac{2}{(\eta_*(\vec{x}) - \eta)}\delta\chi_2 = 0$$

  - Initial condition as $\eta \to -\infty$:

$$\delta\chi_2 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \mathrm{e}^{i\vec{k}\vec{x} - ik\eta} A_{\vec{k}} \quad + \quad \text{h.c.}$$

  - $\eta_*(\vec{x})$: long ranged field, derivative expansion appropriate
    - Zeroth order in $\partial_i\eta_*$: local shift of conformal time.
    - First order in $\partial_i\eta_*$:

      $\eta_*(\vec{x}) - \eta = \eta_*(0) - (\eta - \vec{\partial}\eta_* \cdot \vec{x})$

      $\Longrightarrow$ local Lorentz boost with $\vec{v} = -\vec{\partial}\eta_*$;
      background is locally homogeneous and isotropic in a reference frame other than cosmic frame.

Solution to the first order in derivative expansion: time shift and Lorentz boost of the original solution $f(k,\eta) \propto H_{3/2}[k(\eta_* - \eta)]$:

$\vec{q} = \vec{k} + k\vec{v} =$ boosted momentum; $v_i(\vec{x}) = -\partial_i \eta_*(\vec{x})$.

Phase field freezes out at

$$\delta\theta = \frac{\delta\chi_2}{\chi_c} = \frac{h}{(2\pi)^{3/2}\sqrt{2k}(k + \vec{k}\vec{v})} \mathrm{e}^{i\vec{k}\vec{x} - ik\eta_*(\vec{x})} \cdot A_{\vec{k}} \quad + \quad \text{h.c.}$$

Potentially observable effects depend on what happens to phase perturbations after the end of conformal rolling stage.

NB: Once radial field $|\phi|$ has settled down to minimum of its potential, phase θ is massless scalar field minimally coupled to gravity

Two sub-scenarios

- Sub-scenario # 1. Phase perturbations superhorizon in conventional sense after end of conformal rolling stage

Libanov, V.R.

- Sub-scenario # 2 (more natural in conformal rolling model, less natural in Galilean Genesis): Phase perturbations sub-horizon in conventional sense after end of conformal rolling stage

Libanov, Ramazanov, V.R.

- Sub-scenario # 1. Phase perturbations superhorizon in conventional sense after end of conformal rolling stage

Libanov, V.R.

$\delta\theta$ remains frozen until the time it gets reprocessed into adiabatic perturbations $\Longrightarrow$

$$\mathscr{P}_\zeta \propto \mathscr{P}_{\delta\theta}$$

No effect to the linear order in $\vec{v} = -\vec{\partial}\eta_*$ (!!). Two-point function

$$\langle \delta\theta(\vec{x})\delta\theta(\vec{x}')\rangle \propto \int \frac{d^3k}{k} \frac{1}{(k+\vec{k}\vec{v})^2} \mathrm{e}^{i\vec{k}(\vec{x}-\vec{x}') - ik(\eta_*(\vec{x})-\eta_*(\vec{x}'))}$$

$$= \int \frac{d^3k}{k} \frac{1}{(k+\vec{k}\vec{v})^2} \mathrm{e}^{i\vec{k}(\vec{x}-\vec{x}') + ik\vec{v}(\vec{x}-\vec{x}')}$$

$$= \int \frac{d^3q}{q} \frac{1}{q^2} \mathrm{e}^{i\vec{q}(\vec{x}-\vec{x}')}$$

Lorentz-invariance does the job.

- Derivative expansion to the second order: perturbative solution.

Long wavelength regime:

$$\delta\theta = \frac{h}{(2\pi)^{3/2}\sqrt{2k}q} e^{i\vec{k}x - ik\eta_*(\vec{x})} \left(1 - \frac{\pi}{2k}\frac{k_i k_j}{k^2}\partial_i\partial_j\eta_*\right)\cdot A_{\vec{k}} \ + \ \text{h.c.}$$

Scalar power spectrum ($n_{k\,i} = k_i/k =$ unit vector along $\vec{k}$)

$$\mathscr{P}(\vec{k}) = \mathscr{P}_0(k)\left(1 - \frac{\pi}{k} n_{k\,i} n_{k\,j} \partial_i\partial_j\eta_*\right)$$

Statistical anisotropy due to constant in space tensor $\partial_i\partial_j\eta_*|_{\text{long wavelengths}} \Longrightarrow$ CMB correlators $\langle a_{l,m} a^*_{l\pm 2,m}\rangle$, etc.

- Quadrupole of general form
- Momentum dependence $1/k$

NB: Power spectrum of $\partial^2\eta_*$ is blue $\Longrightarrow$

$$\langle(\pi\partial_i\partial_j\eta_*)^2\rangle_{\text{long wavelengths}} \simeq \frac{9h^2}{4}\int_0^{H_0} kdk \simeq h^2 H_0^2$$

Statistical anisotropy effect on perturbations of wave vector k:

$$\mathscr{P}(\vec{k}) = \mathscr{P}^{(0)}(k)\left(1 + \frac{hH_0}{k} w_{ij} n_{ki} n_{kj}\right)$$

(with $w_{ij}w_{ij} = 1$) $\Longrightarrow$ effect on CMB behaves as $1/l$

Difficult case for observations: only low CMB multipoles feel the effect, large cosmic variance.

WMAP data: essentially no limit on h

Ramazanov, Rubtsov ’12

- Non-Gaussianity to order h^2

Libanov, Mironov, V.R.

Over and beyond non-Gaussianity which can be generated when perturbations in θ are converted into adiabatic perturbations.

Invariance $\theta \to -\theta \Longrightarrow$ bispectrum vanishes.

Trispectrum fully calculated. Most striking property: singularity in folded limit:

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle = c \cdot \delta\left(\sum_{i=1}^{n} \vec{k}_i\right) \cdot \frac{1}{k_{12} k_1^4 k_3^4} \left[1 - 3\left(\frac{\vec{k}_{12} \cdot \vec{k}_1}{k_{12} k_1}\right)^2\right] \left[1 - 3\left(\frac{\vec{k}_{12} \cdot \vec{k}_3}{k_{12} k_3}\right)^2\right]$$

$$\vec{k}_{12} = \vec{k}_1 + \vec{k}_2 \to 0$$

This is in sharp contrast to single field inflationary models.

Origin: infrared enhancement of radial perturbations $\delta\chi_1$

- Sub-scenario # 2 (more natural in conformal rolling model, less natural in Galilean Genesis): Phase perturbations sub-horizon in conventional sense after end of conformal rolling stage

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$\delta\theta$ evolves non-trivially before it becomes super-horizon and freezes out again.

Motivation. Fairly generic feature of alternatives to inflation: long stage of almost Minkowskian evolution

Example: contracting Universe. Need stiff equation of state, otherwise strong inhomogeneity and anisotropy towards the end of contraction (Belinsky–Lifshits–Khalatnikov phenomenon)

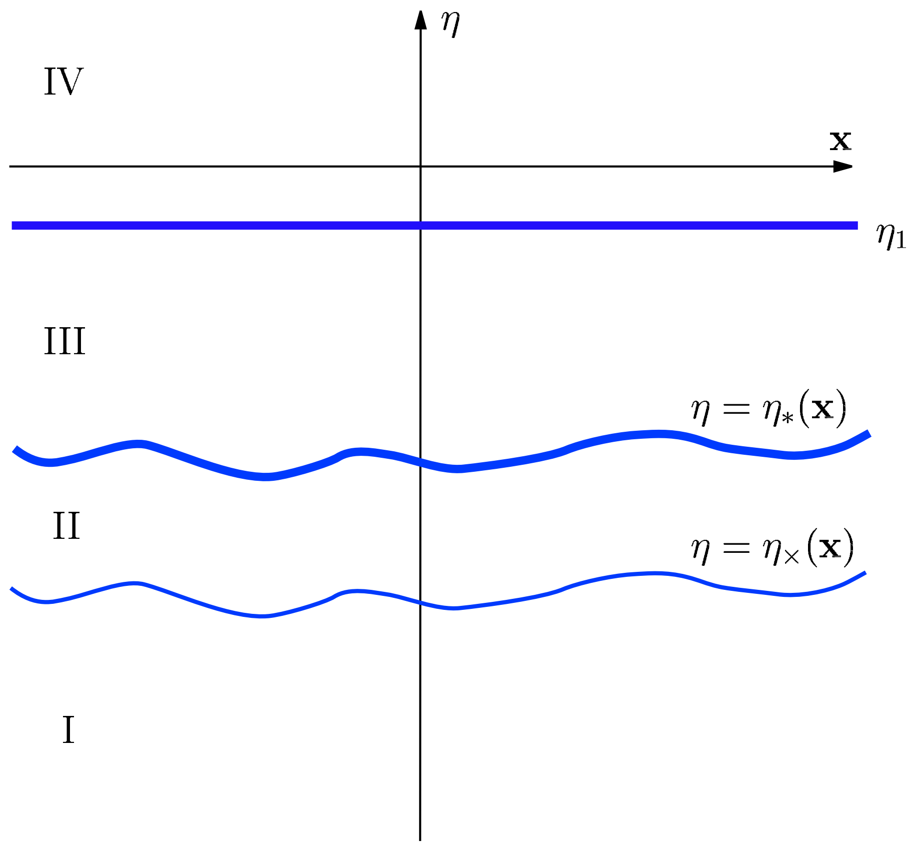
Erickson et. al. ’03;

Garfinkle et. al. ’08

$\Longrightarrow$ Ekpyrotic models (in broad sense): at contraction

$$a(t) = |t|^p , \qquad p \ll 1$$

Almost Minkowski.



- Nearly Minkowskian evolution at intermediate stage III

Otherwise power spectrum becomes tilted!

- Two-fold effect of radial perturbations
  - $\eta \approx \eta_*$: initial field $\delta\theta(\vec{x})$ non-trivial
  - Cauchy hypersurface $\eta = \eta_*(\vec{x})$ non-trivial

- For given $\vec{k}$, phase perturbation after second freeze-out is a linear combination of waves coming from direction of $\vec{k}$ and from opposite direction and traveling distance $r = \eta_1 - \eta_* \Longrightarrow$ Imprint on $\delta\theta(\vec{k})$ of random field $\vec{v}(\pm\vec{n}_k r)$, which depends on $\vec{n}_k$ only,

$$\delta\theta = F\left[\vec{v}(\pm\vec{n}_k r)\right]\cdot A_{\vec{k}} \quad + \quad \text{h.c.}$$

In particular,

$$\mathscr{P}_{\delta\theta}(\vec{k}) = \mathscr{P}_0\left\{1 + n_{k\,i}\left[v_i(+\vec{n}_k r) - v_i(-\vec{n}_k r)\right]\right\} \; , \qquad \vec{v} = -\vec{\partial}\eta_*(\vec{x})$$

Non-trivial dependence on $\vec{n}_k$. Statistical anisotropy with all even multipoles.

- Resulting statistical anisotropy

$$\mathscr{P}_\zeta(\vec{k}) = \mathscr{P}_\zeta^{(0)}(k)\left[1 + \mathscr{Q}\cdot w_{ij}\left(n_{ki}n_{ki} - \frac{1}{3}\delta_{ij}\right) + \text{higher multipoles}\right]$$

with $w_{ij}w_{ij} = 1$ and

$$\langle \mathscr{Q}^2 \rangle = \frac{675}{32\pi^2}h^2\ .$$

NB: multipoles $\mathscr{Q}$, etc., are independent of $k \Longrightarrow$ no suppression of effect on CMB at large l, unlike in sub-scenario # 1.

Put in another way:

$$\mathscr{P}_\zeta(\vec{k}) = \mathscr{P}_\zeta^{(0)}(k)\,[1+Q(\vec{n}_k)]$$

Multipoles in statistical anisotropy

$$Q(\vec{n}_k) = \sum_{lm} q_{lm} Y_{lm}(\vec{n}_k)$$

Variances:

$$\langle q_{lm} q^*_{l'm'} \rangle = \frac{3h^2}{\pi} \frac{1}{(l-1)(l+2)} \delta_{ll'} \delta_{mm'} \;, \qquad \text{even} \quad l \neq 0$$

Easier to search in data.

Analysis of WMAP data: $h^2 \leq 0.045$
(dominated by systematics)

Ramazanov, Rubtsov '12

Planck: much better sensitivity, down to $h^2 \sim 10^{-3}$

non-Gaussianity

- Non-trivial part of tri-spectrum: dependence on directions of momenta

$$\langle \zeta(\vec{k})\zeta(\vec{k}')\zeta(\vec{q})\zeta(\vec{q}')\rangle = \frac{\mathscr{P}_\zeta^{(0)}(k)}{4\pi k^3}\frac{\mathscr{P}_\zeta^{(0)}(q)}{4\pi q^3}\delta(\vec{k}+\vec{k}')\delta(\vec{q}+\vec{q}')\cdot[1+F_{NG}(\vec{n}_k,\vec{n}_q)]$$

$$+ \text{ permutations}$$

with

$$F_{NG} = \frac{3h^2}{\pi^2}\log\frac{\text{const}}{|\vec{n}_k - \vec{n}_q|}$$

NB: recall that power spectrum of $\vec{v}$ is flat $\Longrightarrow$ log behavior of F_{NG}.

To summarize:

- Flat (or nearly flat) spectrum of scalar perturbations may be a consequence of conformal symmetry (+ possibly global symmetry), rather than de Sitter symmetry

  - Models of this sort: (i) conformally coupled complex scalar field with negative quartic potential
    (ii) Galilean Genesis

  - Properties of perturbations dictated by conformal invariance

  - Predictions are model-independent, at least to the leading non-linear level (modulo effects due to conversion of field perturbations into adiabatic perturbations)

- Peculiar property which has potentially observable consequences: fluctuations along rolling direction
    - Interpretation in terms of local time shift
- Interplay between phase perturbations, responsible for density perturbations in the end, and local time shift $\delta\eta_*(\vec{x}) \Longrightarrow$ non-trivial correlation properties of density perturbations

Sub-scenario # 1:

- Statistical anisotropy of quadrupole form

$$\mathscr{P}(\vec{k}) = A_s(k)\left(1 + \frac{hH_0}{k} w_{ij} n_{ki} n_{kj}\right)$$

- Trispectrum singular in folded limit

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle \propto \frac{1}{|\vec{k}_1 + \vec{k}_2|}$$

Sub-scenario # 2:

- Statistical anisotropy of a general form
- Non-Gaussianity of a peculiar kind

What if the world started out

conformal indeed?

Observability in general

With our normalization

$$L_{\delta\theta} = \frac{1}{2}\rho^2(\partial_\mu \delta\theta)^2\,, \quad \rho_c = -\frac{1}{ht}$$

Then

$$\delta\theta \sim h\,, \qquad \delta\rho/\rho \sim h$$

General relationship with density perturbation

$$\zeta = r\frac{\delta\theta}{\bar{\theta}}$$

$r \lesssim 1$: dilution factor; $\bar{\theta}$: homogeneous part of the field of conformal weight 0

Small ζ: (i) small h, weak non-linearity effects;
(ii) small r, large non-Gaussianity due to reprocessing
(iii) large $\bar{\theta} \Longrightarrow$ no bounds on h

In the case (iii) both statistical anisotropy and non-Gaussianity may be large.

NB: $r \sim 1 \Longrightarrow$ non-Gaussianity due to reprocessing small.

NB: the case (iii) impossible in conformal rolling scenario:
there $\theta \lesssim \pi$
$\Longrightarrow$ statistical anisotropy at most of order 10^{-2};
intrinsic trispectrum of order of trispectrum due to reprocessing
(except for modulated decay reprocessing with $\Gamma_X = a + b\theta$)

How strong non-Gaussianity?

Standard definition of magnitude of trispectrum

$$\langle \zeta_{\mathbf{k_1}} \dots \zeta_{\mathbf{k_4}} \rangle_{k_i = k_{12} = k_{14} \equiv k} = (2\pi)^9 \mathscr{P}_\zeta^3 \delta\left(\sum_{i=1}^4 \mathbf{k}_i\right) \frac{1}{k^9} t_{NL} \ .$$

We can calculate

$$\langle \theta_{\mathbf{k_1}} \dots \theta_{\mathbf{k_4}} \rangle_{k_i = k_{12} = k_{14} \equiv k} = (2\pi)^9 \mathscr{P}_\theta^3 \delta\left(\sum_{i=1}^4 \mathbf{k}_i\right) \frac{1}{k^9} t_{NL}^{(\theta)} \ ,$$

where $\mathscr{P}_\theta = h^2/(4\pi^2)$, and $t_{NL}^{(\theta)} = 2.87$ is the size of the non-Gaussianity in θ.
Hence

$$t_{NL} = \frac{\mathscr{P}_\theta}{\mathscr{P}_\zeta} \cdot t_{NL}^{(\theta)} = 2.87 \frac{h^2}{4\pi^2 \mathscr{P}_\zeta}$$

Generically, can be as large as 10^7 $\Longrightarrow$ observable.

Scalar tilt

- Tilt in $\mathscr{P}_{\delta\theta}$: emerges at order $v^2 \propto h^2$ (in a tricky way)

- Physics: interaction between phase perturbation $\delta\theta(\vec{k})$ and modes of $\vec{v}$ with momenta $r^{-1} < p < k \Longrightarrow$ more modes of $\vec{v}$ contribute for larger k

  - Logarithmic effect because of flat spectrum of $\vec{v}$:

$$\mathscr{P}_{\delta\theta} \propto 1 - \frac{3h^2}{4\pi^2} \ln(kr)$$

Tilt

$$n_{\delta\theta} - 1 = -\frac{3h^2}{4\pi^2} \; .$$

NB: Other possible sources of scalar tilt: violation of conformal symmetry, non-Minkowskian evolution from η_* to η_1.

- To speculate:

if $n_{\delta\theta} - 1 = n_s - 1 \simeq -0.04$, then statistical anisotropy of order 1. Probably inconsistent with data.

On the other side: if smallness of $\mathscr{P}_\zeta \simeq 2.5 \cdot 10^{-9}$ is entirely due to small $\mathscr{P}_{\delta\theta}$, then

$$\mathscr{P}_\zeta = \mathscr{P}_{\delta\theta} = \frac{h^2}{8\pi^2} ;$$

statistical anisotropy at the level 10^{-3}. In particular, $\mathscr{Q} \sim 7 \cdot 10^{-4}$. Probably too small.

Dynamical version of conformal rolling

Hinterbichler, Khouri ’11

Instead of assuming that ϕ is spectator, consider ϕ as the only relevant field.

- Universe begins from nearly Minkowski state

$$\phi_c = \frac{1}{h(t_* - t)} \Longrightarrow p = \frac{h^2}{(t_* - t)^4}\,, \quad \rho \ll p$$

- Super-stiff equation of state, $p \gg \rho$
- Universe contracts and speeds up:

$$H = -\frac{1}{6M_{Pl}^2 h^2 (t_* - t)^3}$$

A version of ekpyrosis

Beware: Very low strong coupling scales claimed by Hinterbichler and Khoury: spurious

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