# Looking beyond Majorana zero modes: Engineering non-Abelian anyons

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Joint work with: Jason Alicea, Caltech
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Support:
DARPA QuEST

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# Ising model (of all things)

1D quantum Ising chain:

$$H = -J \sum_{j=1}^{L-1} \sigma_j^z \sigma_{j+1}^z - h \sum_{j=1}^{L} \sigma_j^x$$

Jordan—Wigner transformation:

$$\gamma_{2j-1} = \sigma_j^z \prod_{i<j} \sigma_i^x, \qquad \gamma_{2j} = \sigma_j^y \prod_{i<j} \sigma_i^x$$

$\gamma$'s are *Majorana* operators:

$$\gamma_j^2 = 1, \quad \gamma_j^\dagger = \gamma_j, \qquad \gamma_j \gamma_k = -\gamma_k \gamma_j$$

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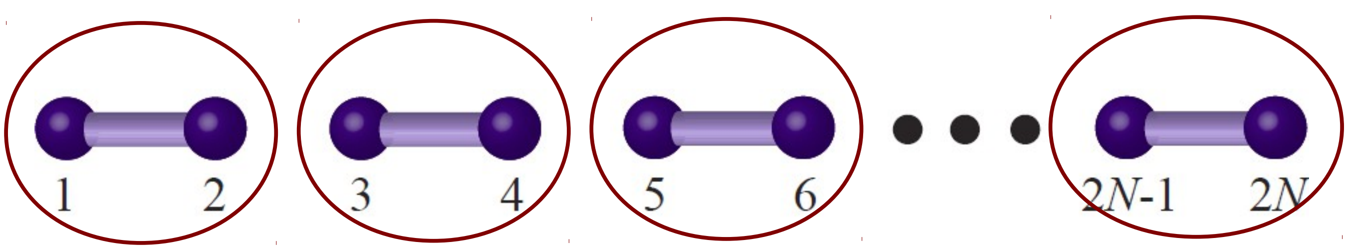
$J = 0:$



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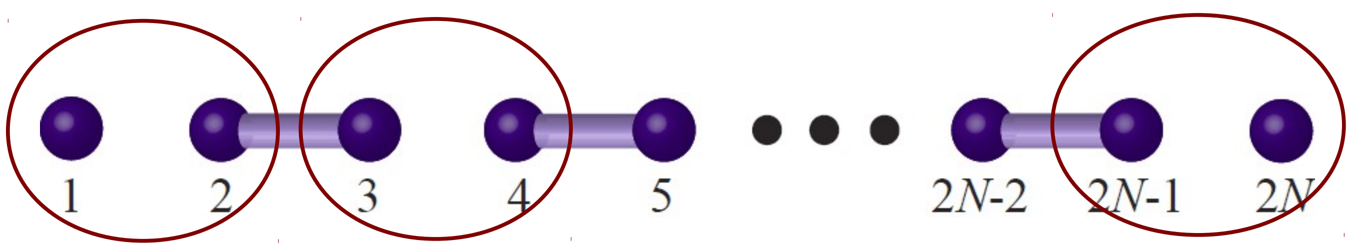

# From Ising to Majorana wires

1D spinless p-wave superconductor(Kitaev 2001):

$$H = \mu \sum_{x=1}^{N} c_x^\dagger c_x - \sum_{x=1}^{N-1} \left( t c_x^\dagger c_{x+1} + |\Delta| e^{i\phi} c_x c_{x+1} + h.c. \right)$$

$$\mu = 0$$

$$t = |\Delta|$$

$$c_x = \frac{1}{2} e^{-i\frac{\phi}{2}} \left( \gamma_{B,x} + i \gamma_{A,x} \right)$$

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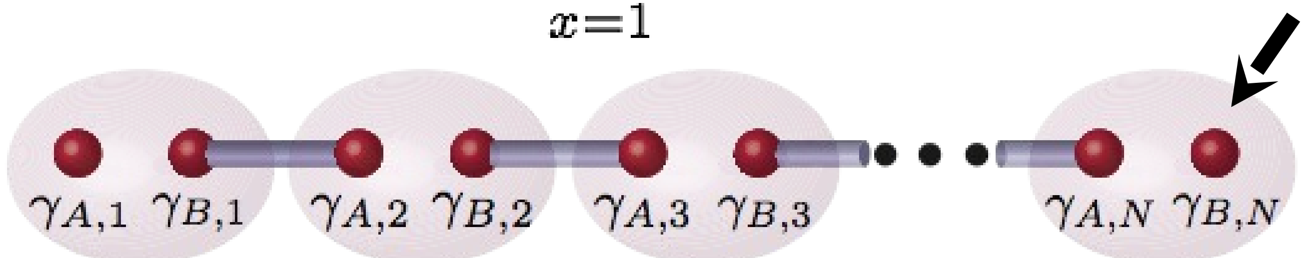
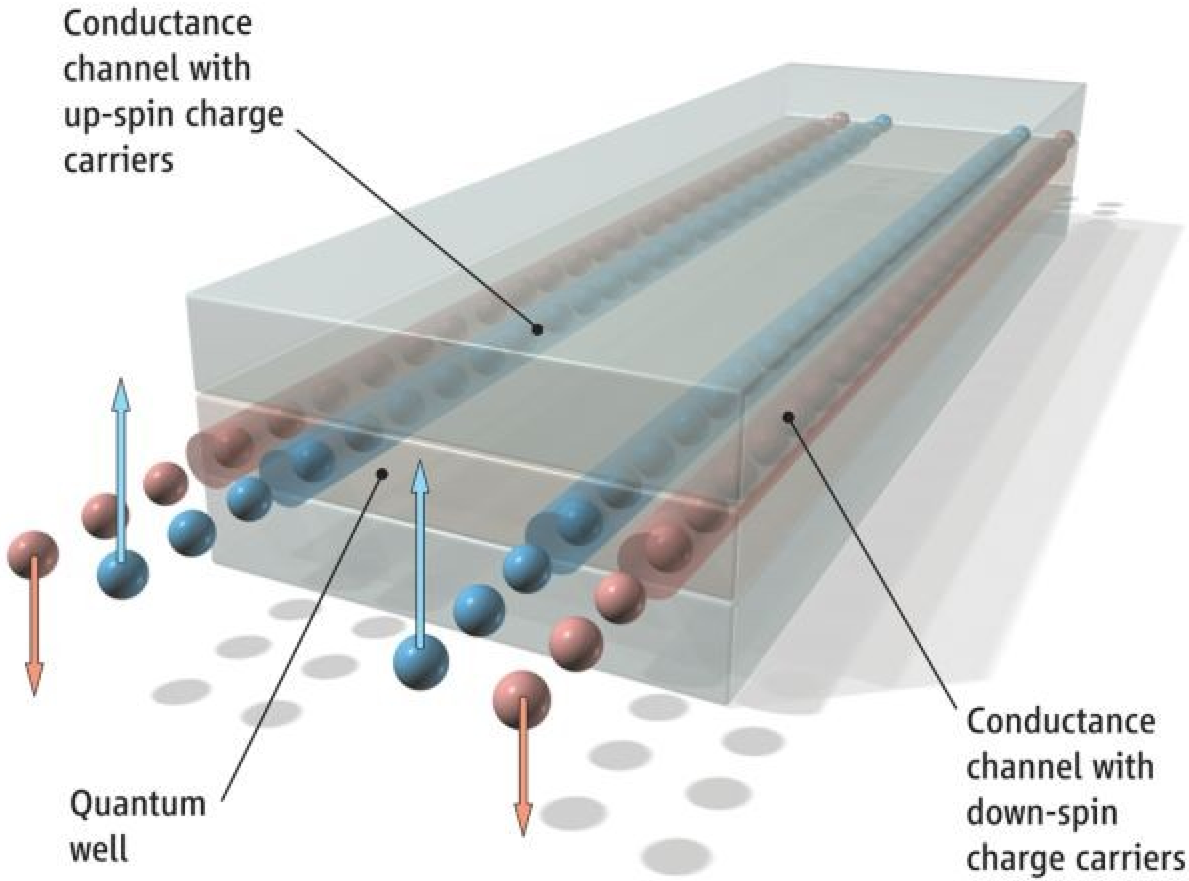
$$c_x = \frac{1}{2} e^{-i\frac{\phi}{2}} (\gamma_{B,x} + i\gamma_{A,x})$$

$$H = -it \sum_{x=1}^{N-1} \gamma_{B,x}\gamma_{A,x+1}$$

Unpaired Majorana fermions at the ends!



# Realization in topological insulator edges




Kane & Mele, 2005; Bernevig, Hughes, Zhang, 2006; Fu & Kane, 2008

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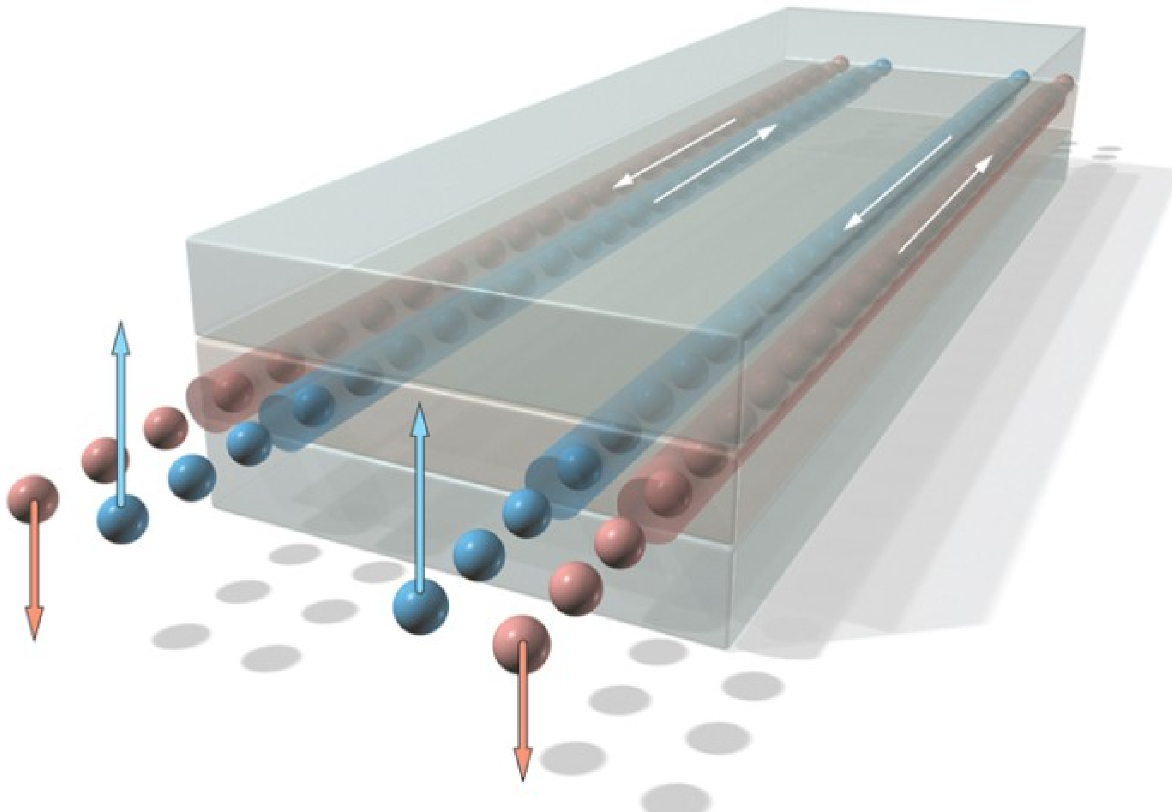

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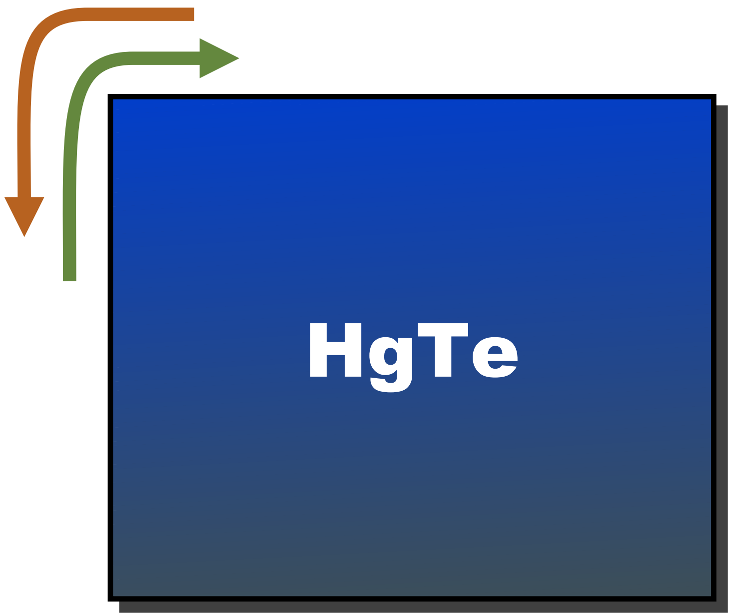
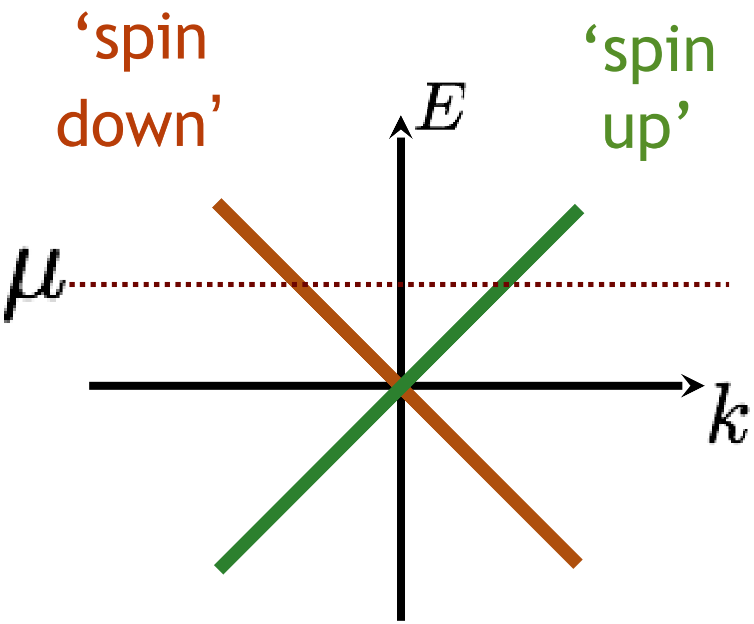


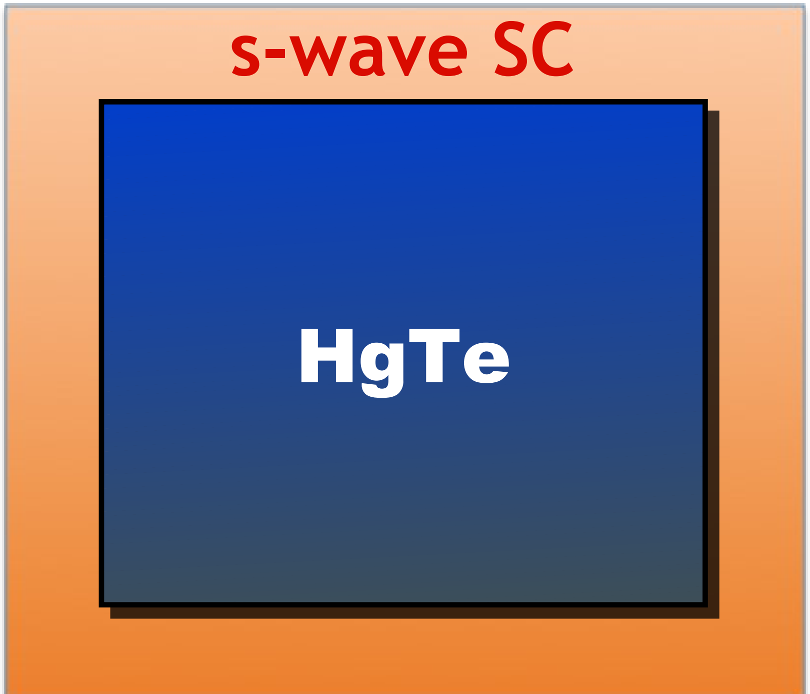



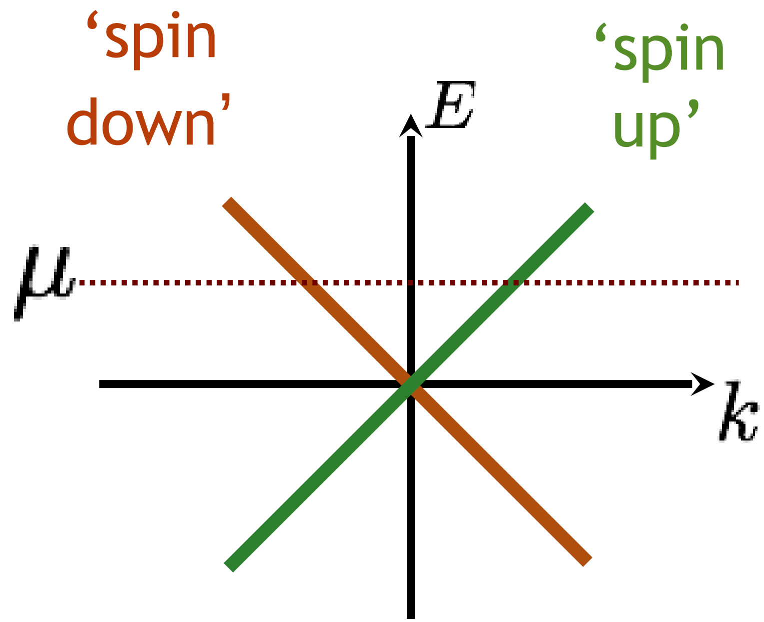

$$H_{\rm edge} = \int dx[-\mu(\psi_R^\dagger\psi_R + \psi_L^\dagger\psi_L) - i\hbar v(\psi_R^\dagger\partial_x\psi_R - \psi_L^\dagger\partial_x\psi_L)]$$

1D and effectively 'spinless'! Just need superconductivity...

Fu & Kane, 2008

# Realization in topological insulator edges

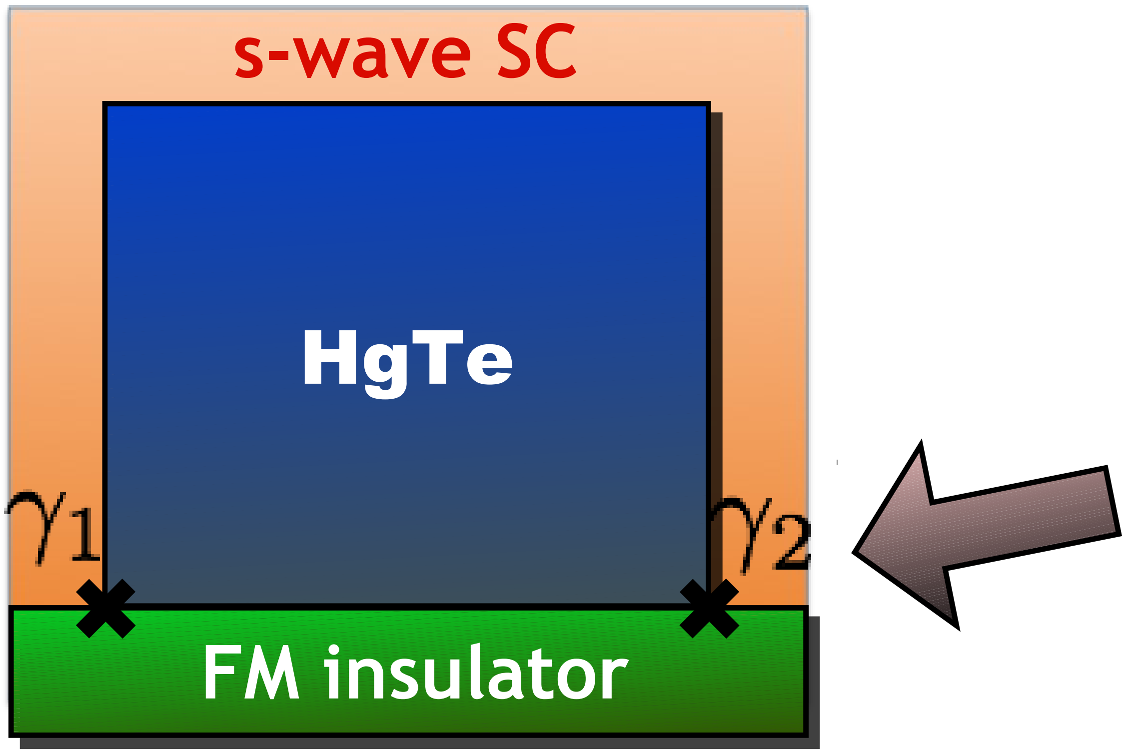



$$H_{\rm edge} = \int dx[-\mu(\psi_R^\dagger \psi_R + \psi_L^\dagger \psi_L) - i\hbar v(\psi_R^\dagger \partial_x \psi_R - \psi_L^\dagger \partial_x \psi_L)]$$
$$+[\Delta \psi_R \psi_L + h.c.]$$

Fu & Kane, 2008

# Realization in topological insulator edges

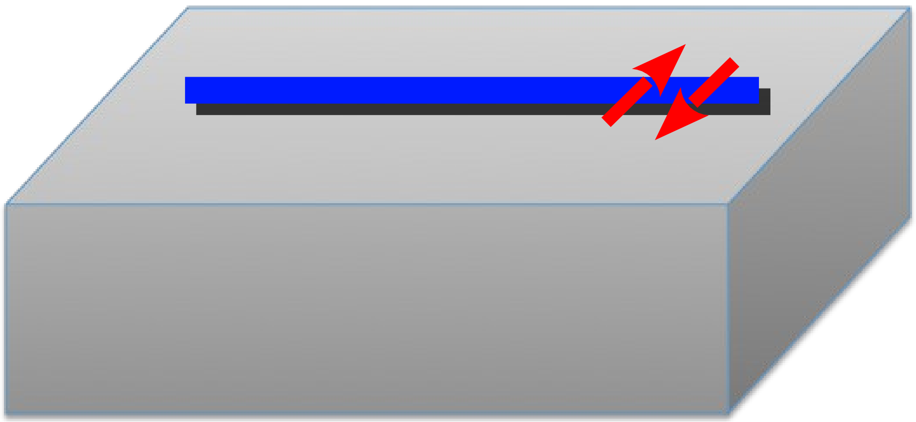


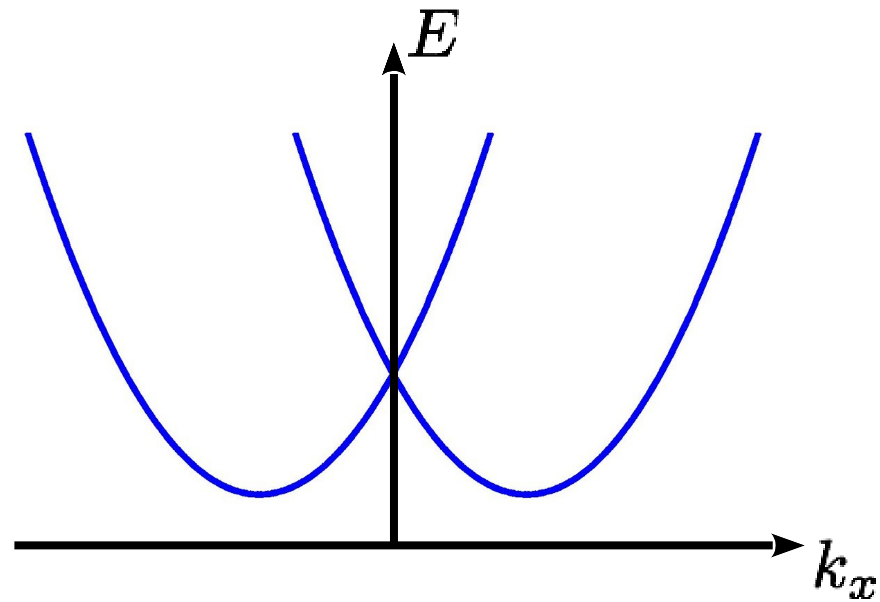

"Terminating" the SC wire by a magnetic gap: Majorana zero modes localised at the ends

Fu & Kane, 2008

# Realization in 1D wires

1D spin-orbit-coupled
wire (e.g. InAs)






$$H = \int dx \psi^\dagger \left[ -\frac{\partial_x^2}{2m} - \mu - i\hbar v \partial_x \sigma^y \right] \psi$$

(Lutchyn, Sau, Das Sarma 2010; Oreg, Refael, von Oppen 2010)

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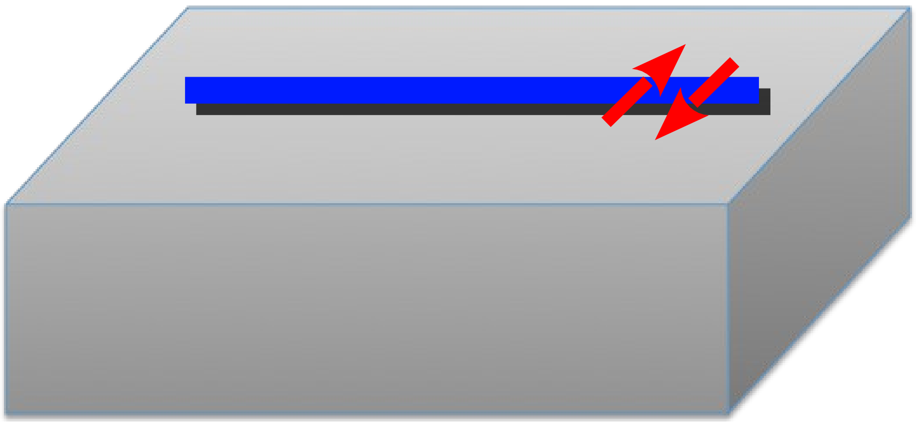

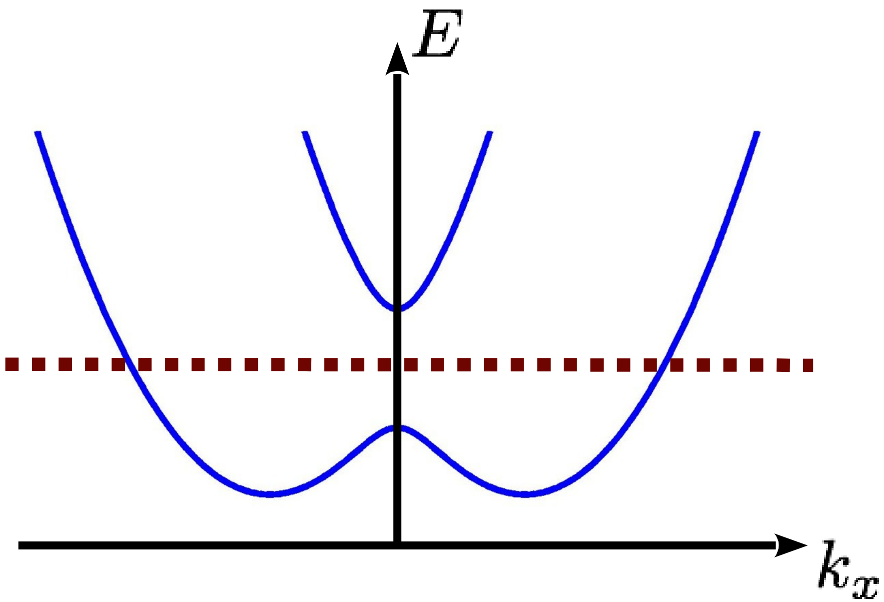


$$H = \int dx \psi^\dagger \left[ -\frac{\partial_x^2}{2m} - \mu - i\hbar v \partial_x \sigma^y - \frac{g\mu_B B}{2}\sigma^z \right] \psi$$

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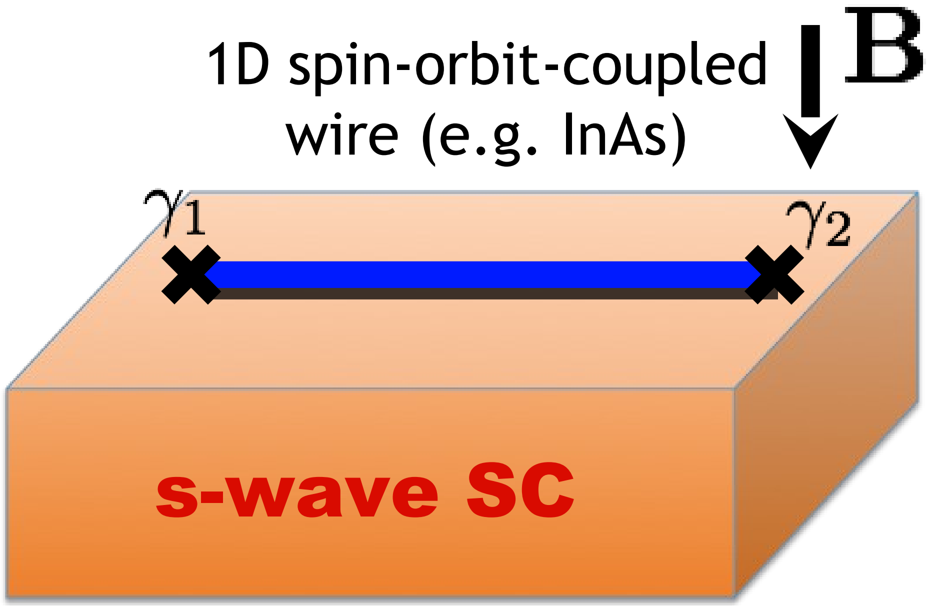


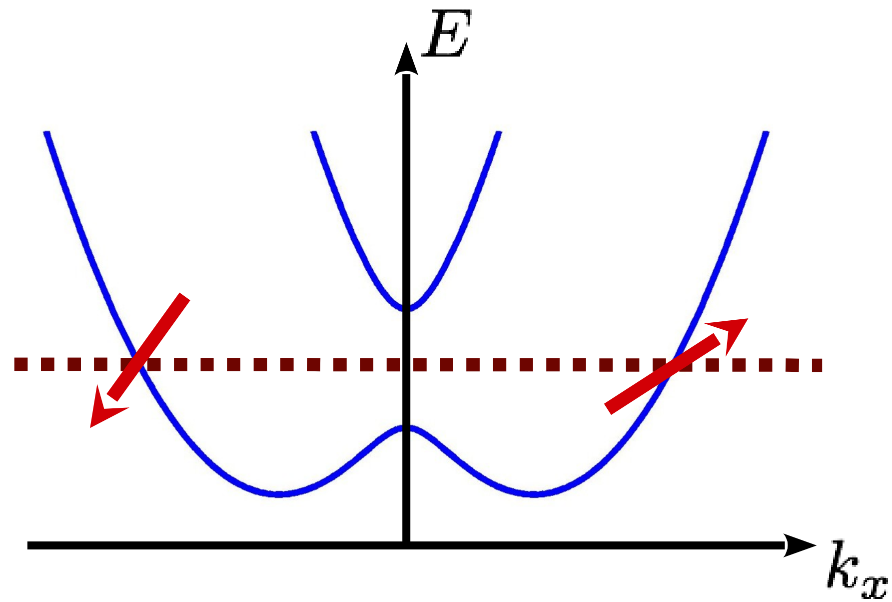


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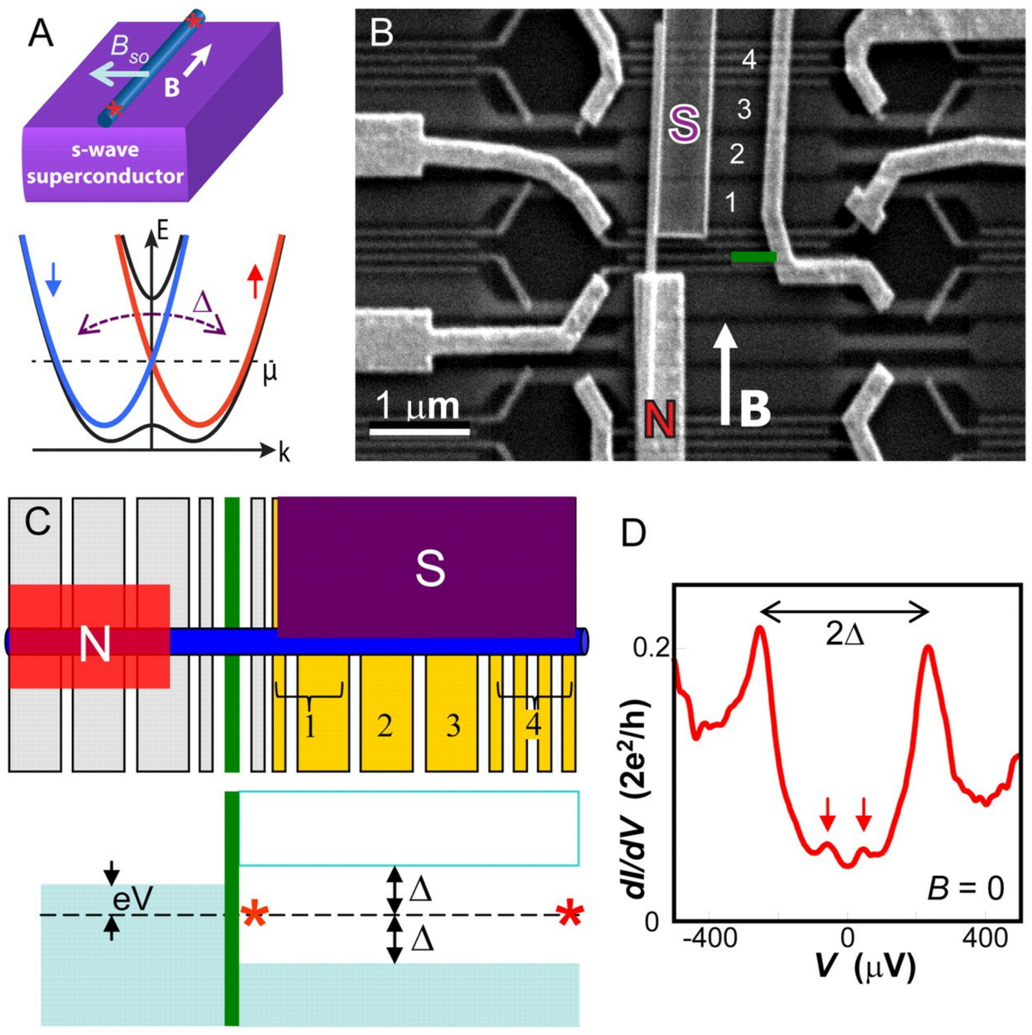
$$+ (\Delta \psi_\uparrow \psi_\downarrow + h.c.)$$

**Generates a1D 'spinless' SC state with Majorana fermions!**

(Lutchyn, Sau, Das Sarma 2010; Oreg, Refael, von Oppen 2010)

# First possible experimental realization




Mourik et al., Science 2012 (Kouwenhoven's group, Delft) following proposals by Lutchyn, Sau & Das Sarma, 2010; Oreg, Refael & von Oppen, 2010.

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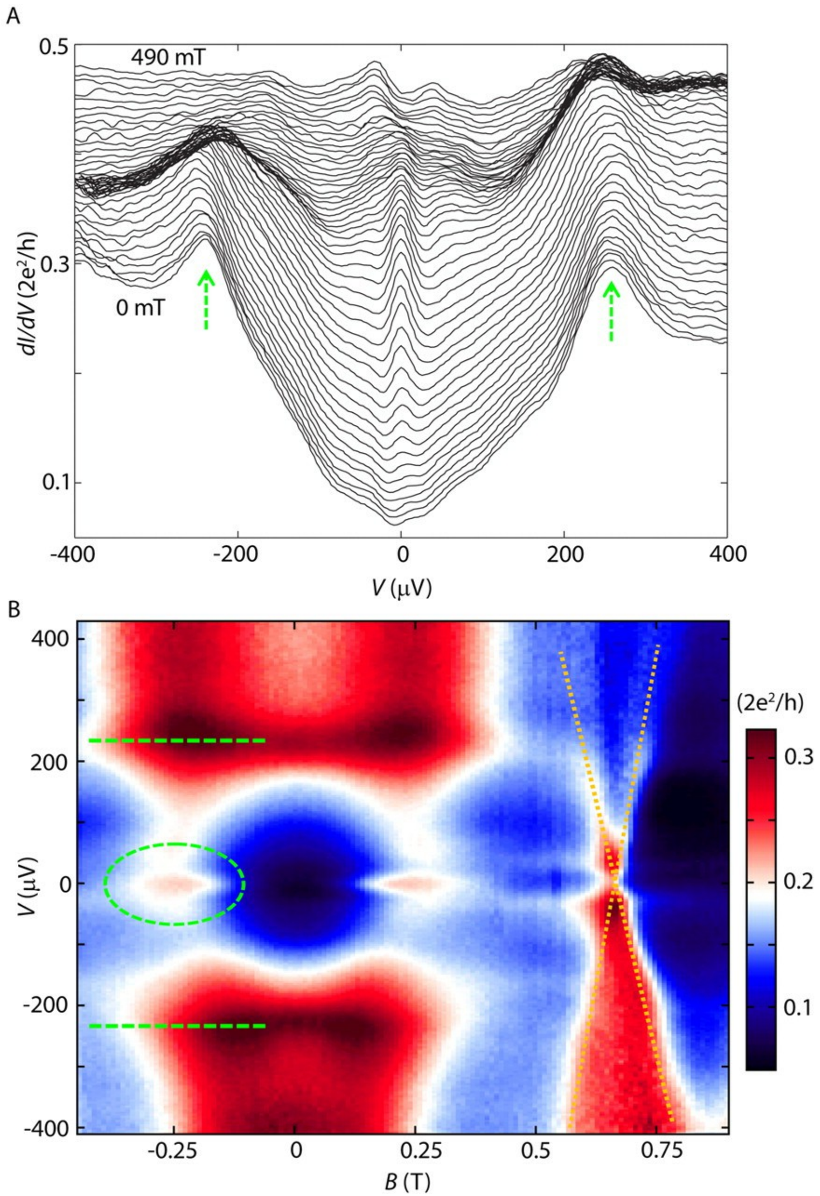


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# Great! Anything else?

Can this idea be generalised to other types of exotic (non-Abelian) zero modes?

At first glance, this is very doubtful:

- $\mathbb{Z}_8$ classification of phases of interacting fermions in 1D (Fidkowski & Kitaev, 2009; Turner, Pollmann & Berg, 2010)

Nevertheless, it is possible!

# Taking a cue from Stat Mech

1D quantum clock model (Fendley, arXiv:1209.0472):

$$H = -J \sum_{j=1}^{L-1} (\sigma_j^\dagger \sigma_{j+1} + H.c.) - h \sum_{j=1}^{L} (\tau_j^\dagger + \tau_j)$$

$$\sigma_j^N = 1 \qquad \sigma_j^\dagger = \sigma_j^{N-1}$$

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$$\sigma_j \tau_j = \tau_j \sigma_j e^{2\pi i/N}$$

$N$=2:
quantum Ising chain

$$\sigma \equiv \sigma^z$$

$$\tau \equiv \sigma^x$$

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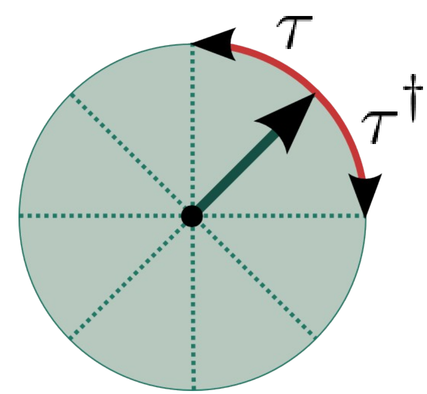
$N \neq 2$:
quantum clock

$$\sigma |q\rangle = e^{2\pi i q/N} |q\rangle$$

$$\tau^\dagger |q\rangle = |q+1\rangle$$



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$\alpha$'s are *parafermionic* operators:

$$\alpha_j^N = 1, \quad \alpha_j^\dagger = \alpha_j^{N-1}, \quad \alpha_j \alpha_k = \alpha_k \alpha_j e^{i\frac{2\pi}{N}\mathrm{sgn}(k-j)}$$

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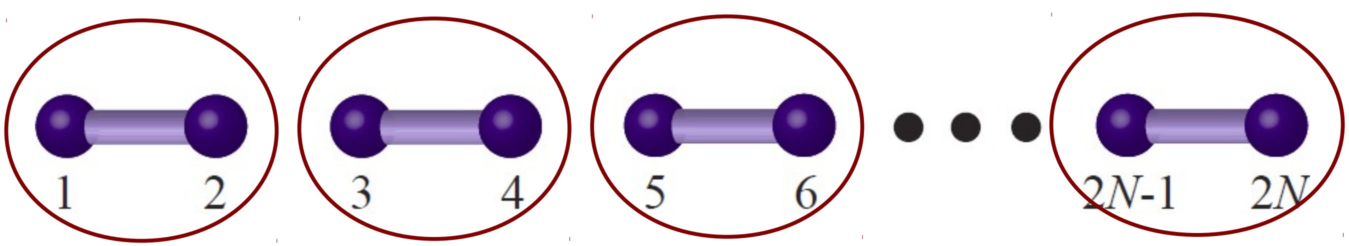
$J = 0:$



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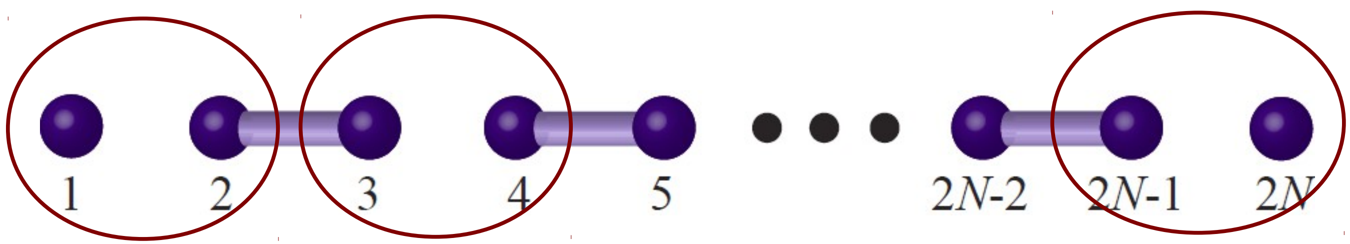
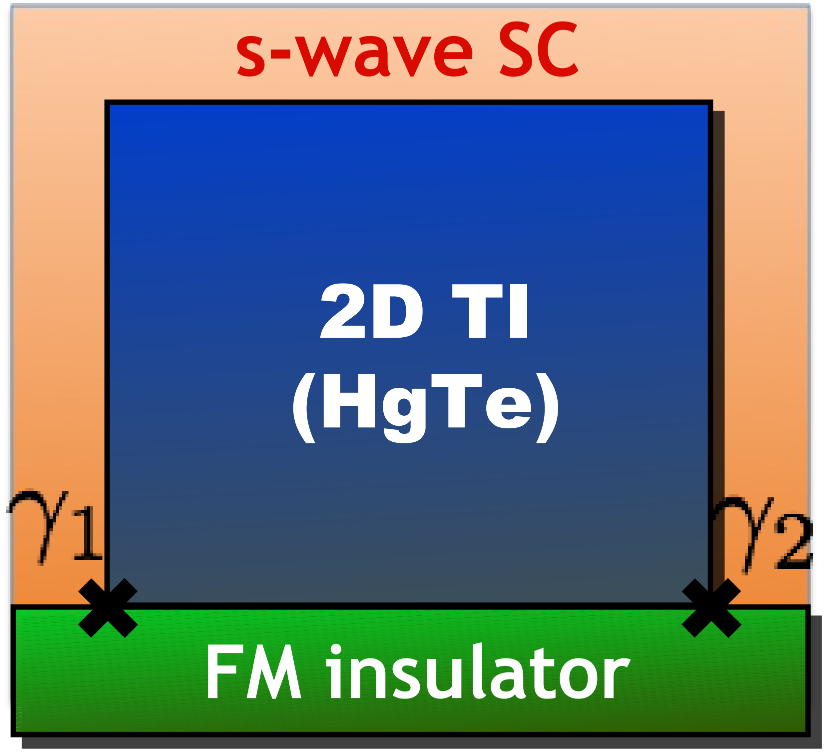
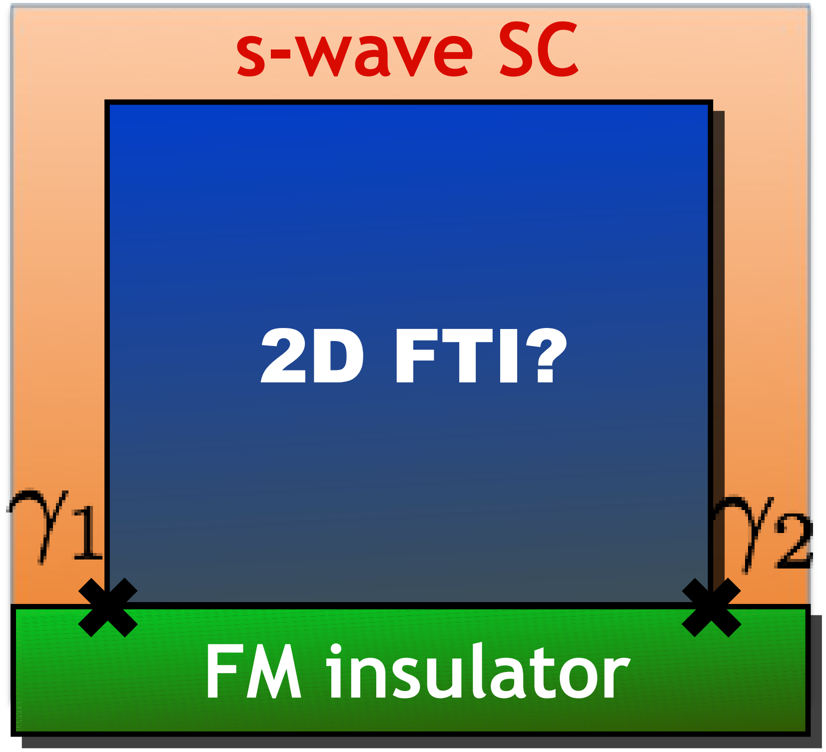


# Recall topological insulator edges




Fu & Kane, 2008

# What about fractional TI edges?




We could envision playing the same game with 2D fractional topological insulators (*à la* Levin & Stern, 2009), but...

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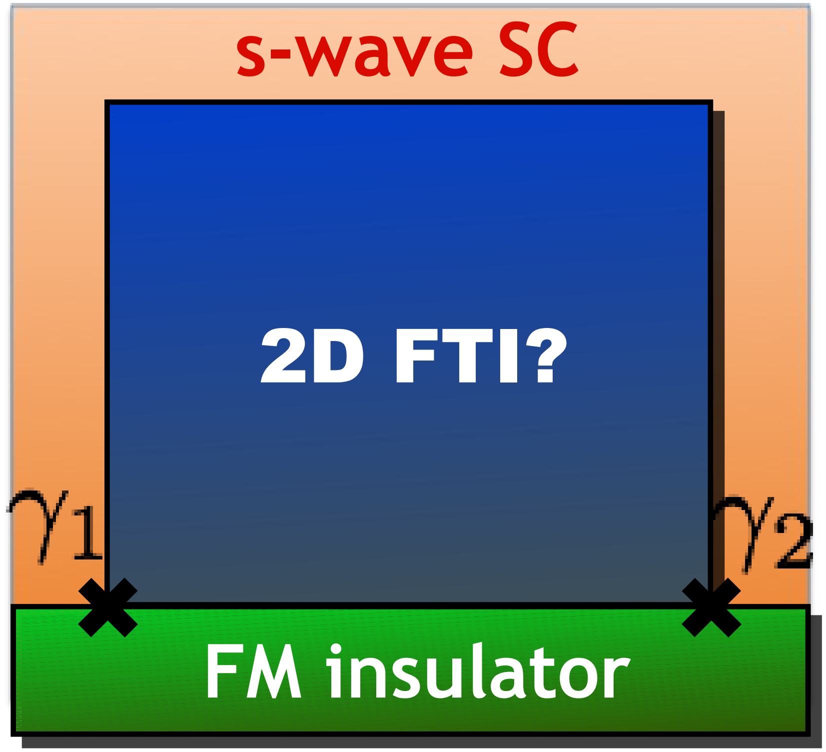
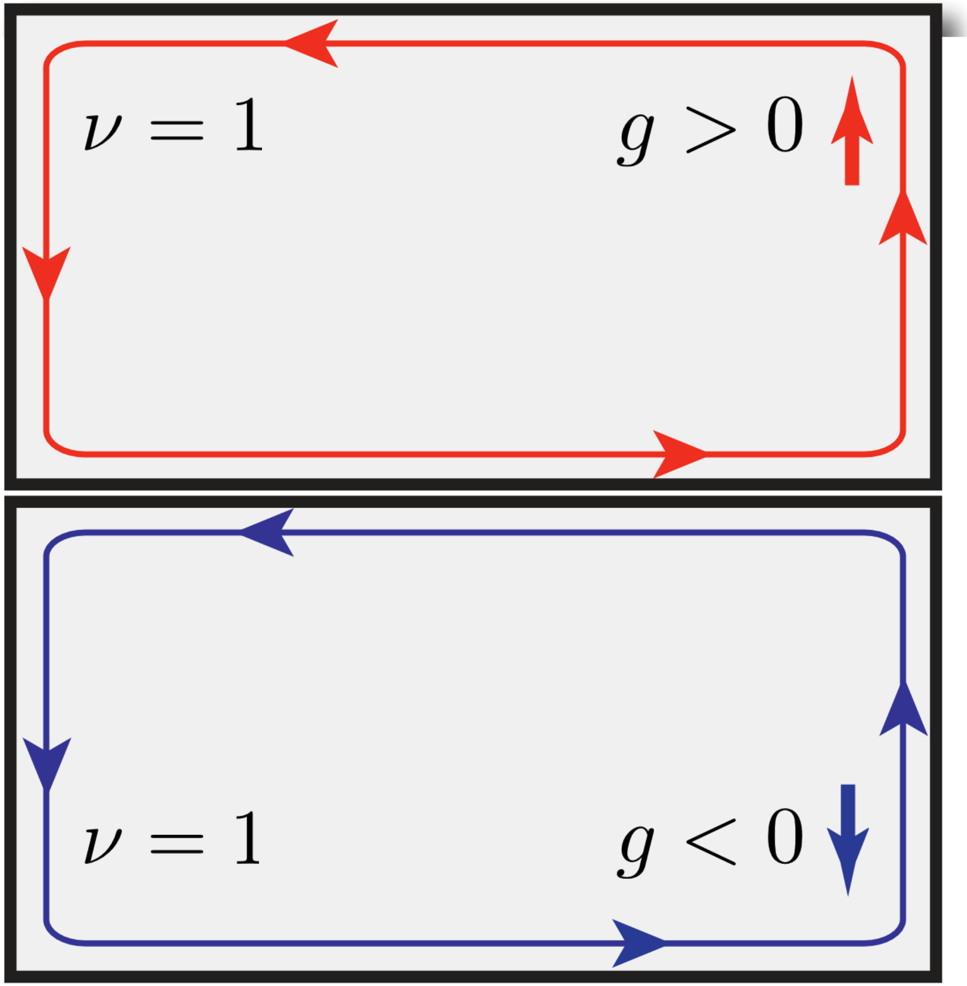


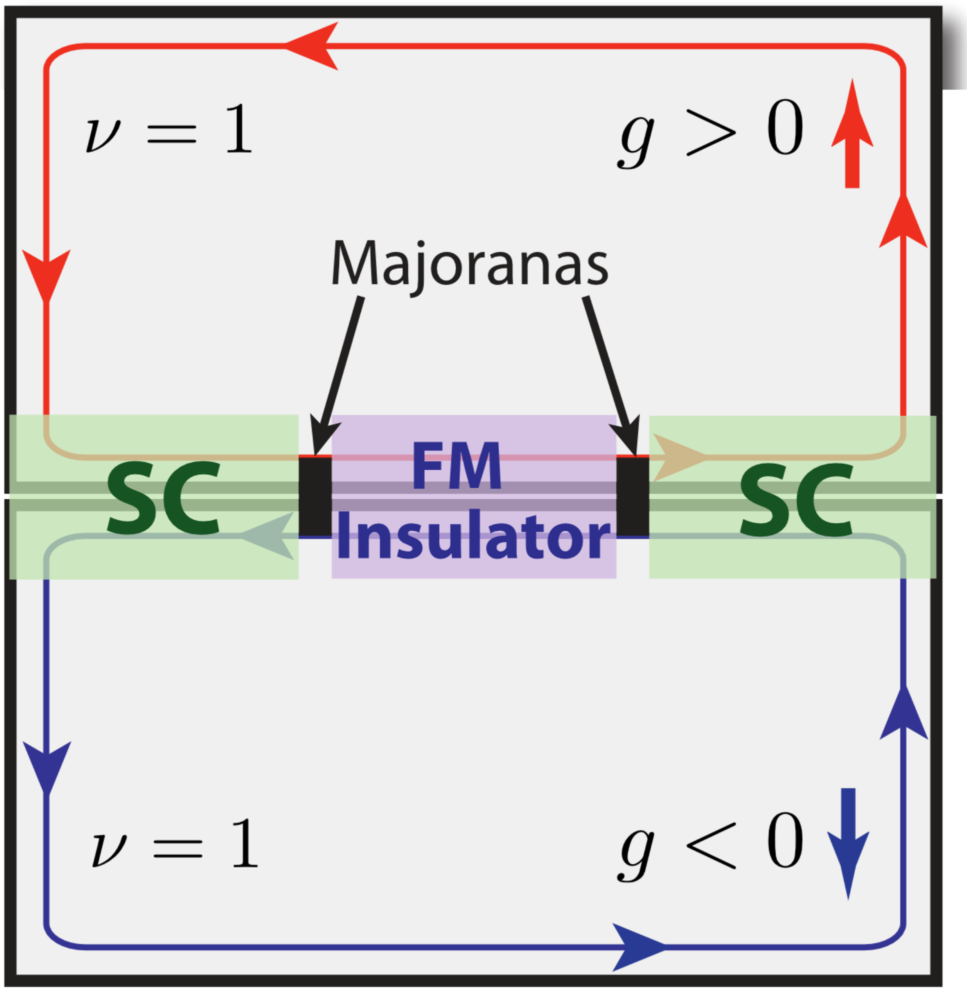

There are no known fractional topological insulators (yet).
But could we 'fake' the same physics elsewhere?

# Realization in quantum Hall edges




Counter-propagating edge modes at the boundary between $g > 0$ and $g < 0$.
The sign of $g$ can be changed by stress.

# Realization in quantum Hall edges

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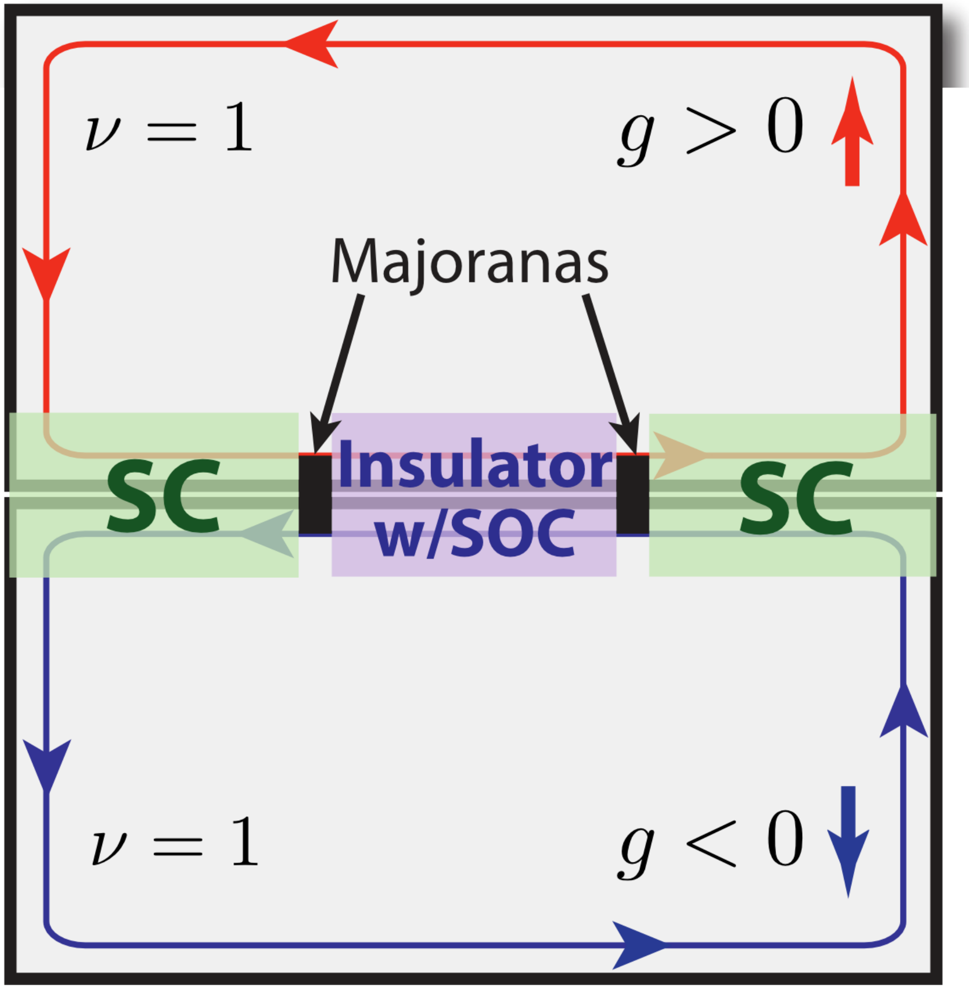


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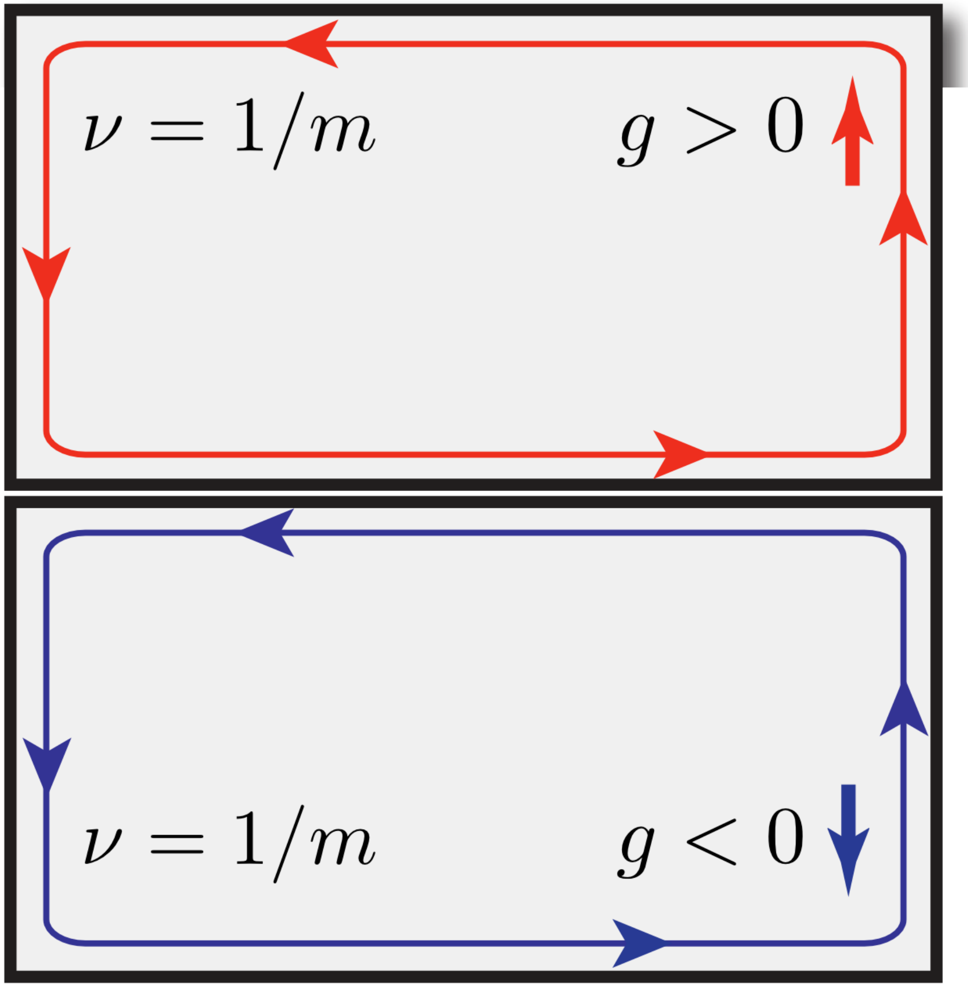


Counter-propagating ***fractionalized*** edge modes at the boundary between $g > 0$ and $g < 0$. The sign of $g$ can be changed by stress.

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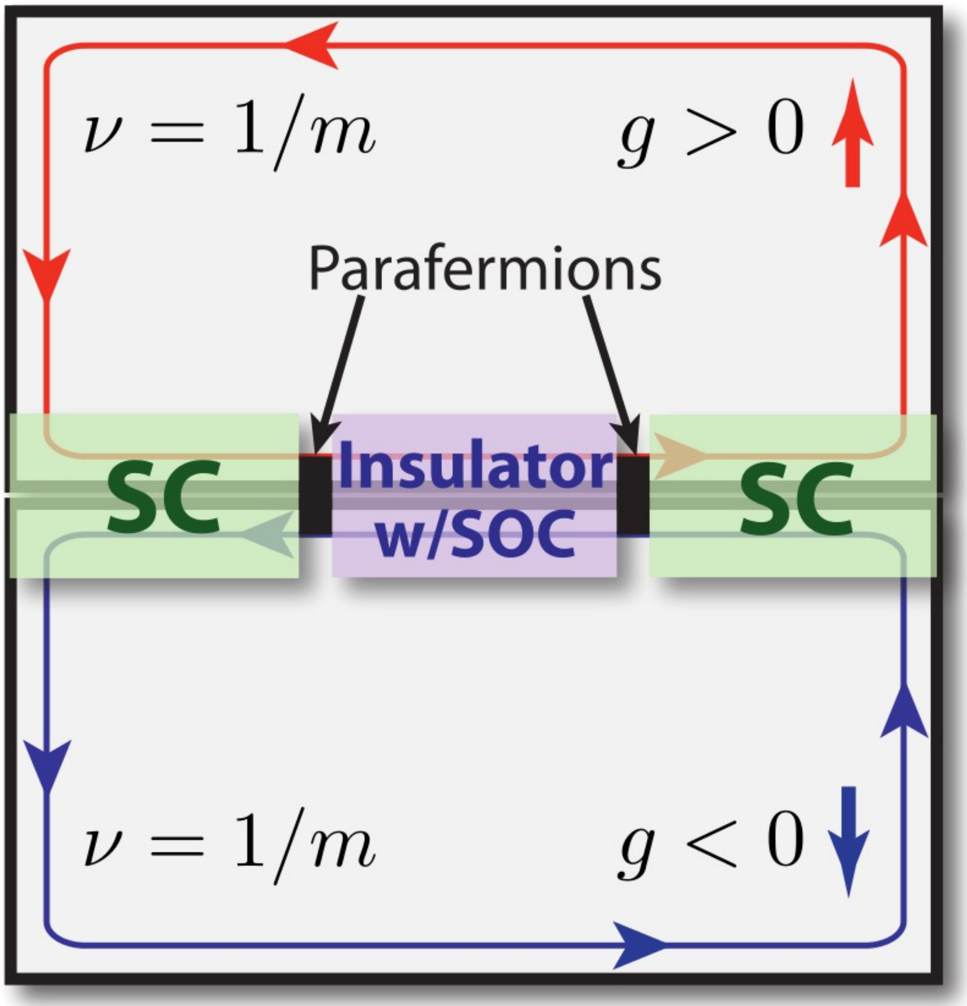



Counter-propagating ***fractionalized*** edge modes at the boundary between $g > 0$ and $g < 0$. The sign of $g$ can be changed by stress.

# Parafermions vs Majoranas

*Upshot:*

Majorana Fermions:

$$\gamma^2 = 1$$

$$\gamma_y \gamma_x = -\gamma_x \gamma_y$$

Parafermions:

$$\alpha^N = 1$$

$$\alpha_y \alpha_x = \alpha_x \alpha_y e^{\frac{2\pi i}{N} \mathrm{sgn}(x-y)}$$

Majoranas ←→ 1D quantum Ising model

Parafermions ←→ 1D quantum Clock/Potts model

Paul Fendley, arXiv:1209.0472

# Parafermions from quantum Hall edges

A Laughlin edge state at $\nu = 1/m$ is a natural starting point since

$$[\phi(x), \phi(y)] = i\frac{\pi}{m}\mathrm{sgn}(x-y)$$

and hence

$$e^{i\phi(x)}e^{i\phi(y)} = e^{i\phi(y)}e^{i\phi(x)}e^{i\frac{\pi}{m}\mathrm{sgn}(y-x)}$$

for chiral edge excitations of charge $e/m$.
Now, we have two counter-propagating modes, $\phi_{R/L}$ , which obey

$$[\phi_{R/L}(x), \phi_{R/L}(y)] = \pm i\frac{\pi}{m}\mathrm{sgn}(x-y)$$

The electron fields are $\psi_{R/L} \sim e^{im\phi_{R/L}}$

# Parafermions from quantum Hall edges

Change of variables: $\phi_{R/L} = \varphi \pm \theta$

Free Hamiltonian: $$\mathcal{H}_0 = \frac{mv}{2\pi}\int dx\left[(\partial_x\varphi)^2 + (\partial_x\theta)^2\right]$$

Just need to show that a zero mode is bound at a domain wall between

$$\mathcal{H}'_{\rm s}(x) = \Delta(x)\psi_R\psi_L + H.c. \sim -\Delta(x)\cos(2m\varphi)$$

and

$$\mathcal{H}'_{\rm m}(x) = \mathcal{M}(x)\psi_R^\dagger\psi_L + H.c. \sim -\mathcal{M}(x)\cos(2m\theta)$$

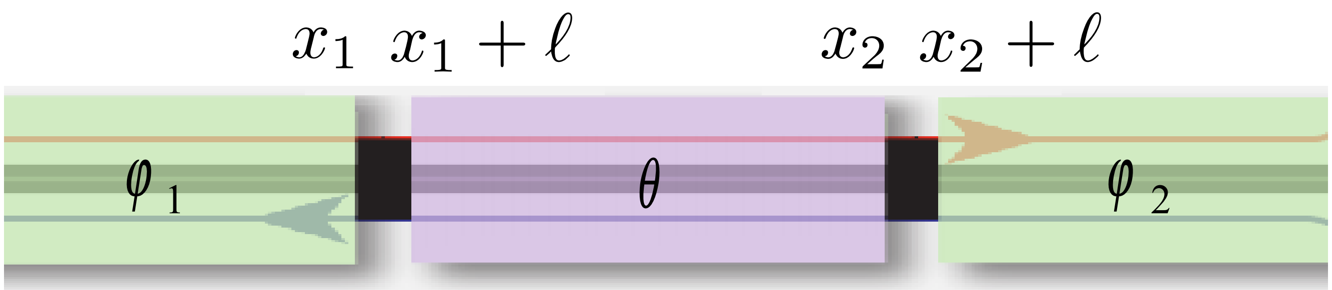
where $\psi_{R/L} \sim e^{im\phi_{R/L}}$

# Parafermionic zero mode

Assuming strong tunnelling and pairing,

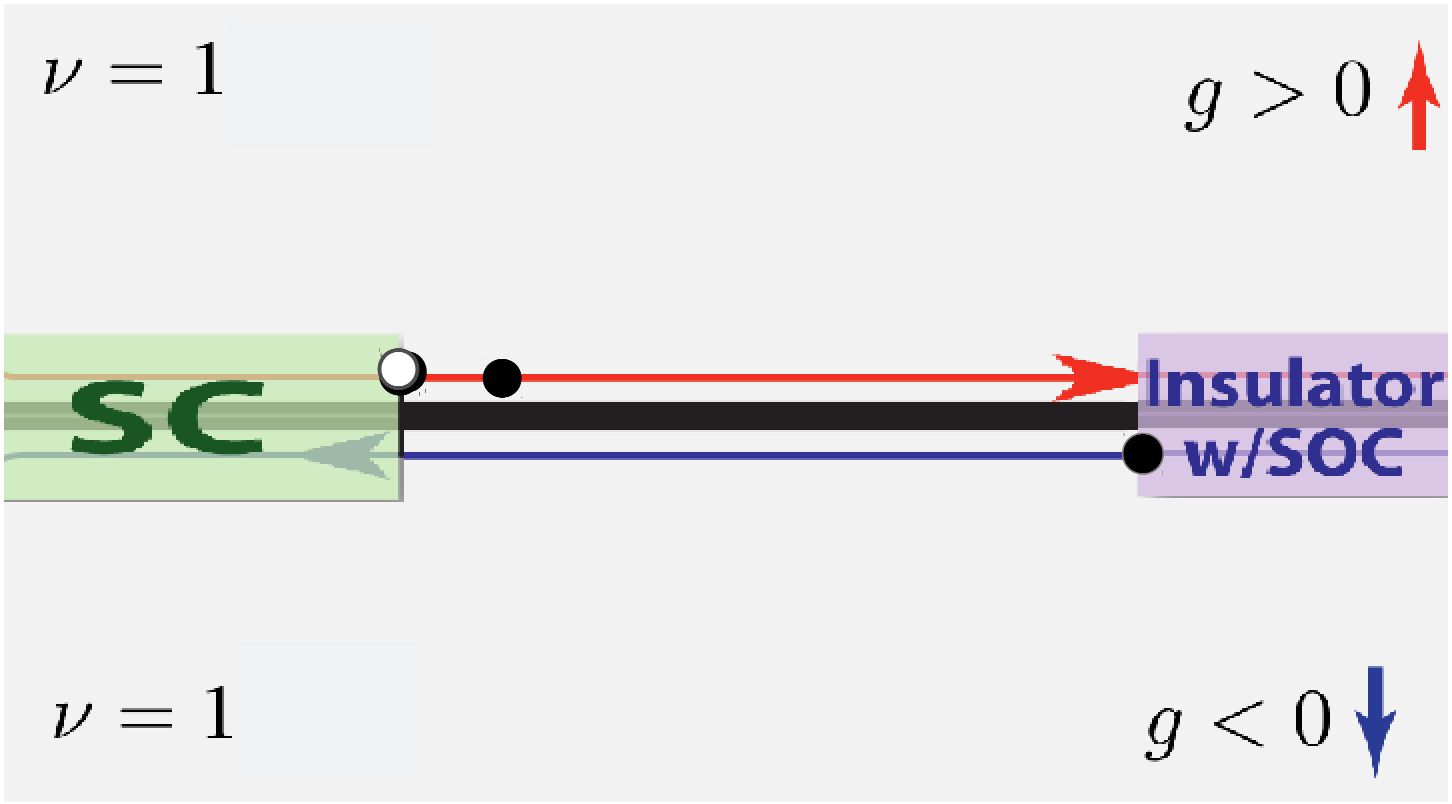
$\varphi = \frac{\pi n_\varphi}{m}$ under the superconductors

$\theta = \frac{\pi n_\theta}{m}$ under the SO coupled insulators




$$\alpha_j = e^{i\frac{\pi}{m}(\hat{n}_\varphi^{(j)} + \hat{n}_\theta)} \int_{x_j}^{x_j+\ell} dx \Big[ e^{-i\frac{\pi}{m}(\hat{n}_\varphi^{(j)} + \hat{n}_\theta)} e^{i(\varphi+\theta)} + e^{-i\frac{\pi}{m}(\hat{n}_\varphi^{(j)} - \hat{n}_\theta)} e^{i(\varphi-\theta)} + H.c. \Big]$$

# Majorana zero mode



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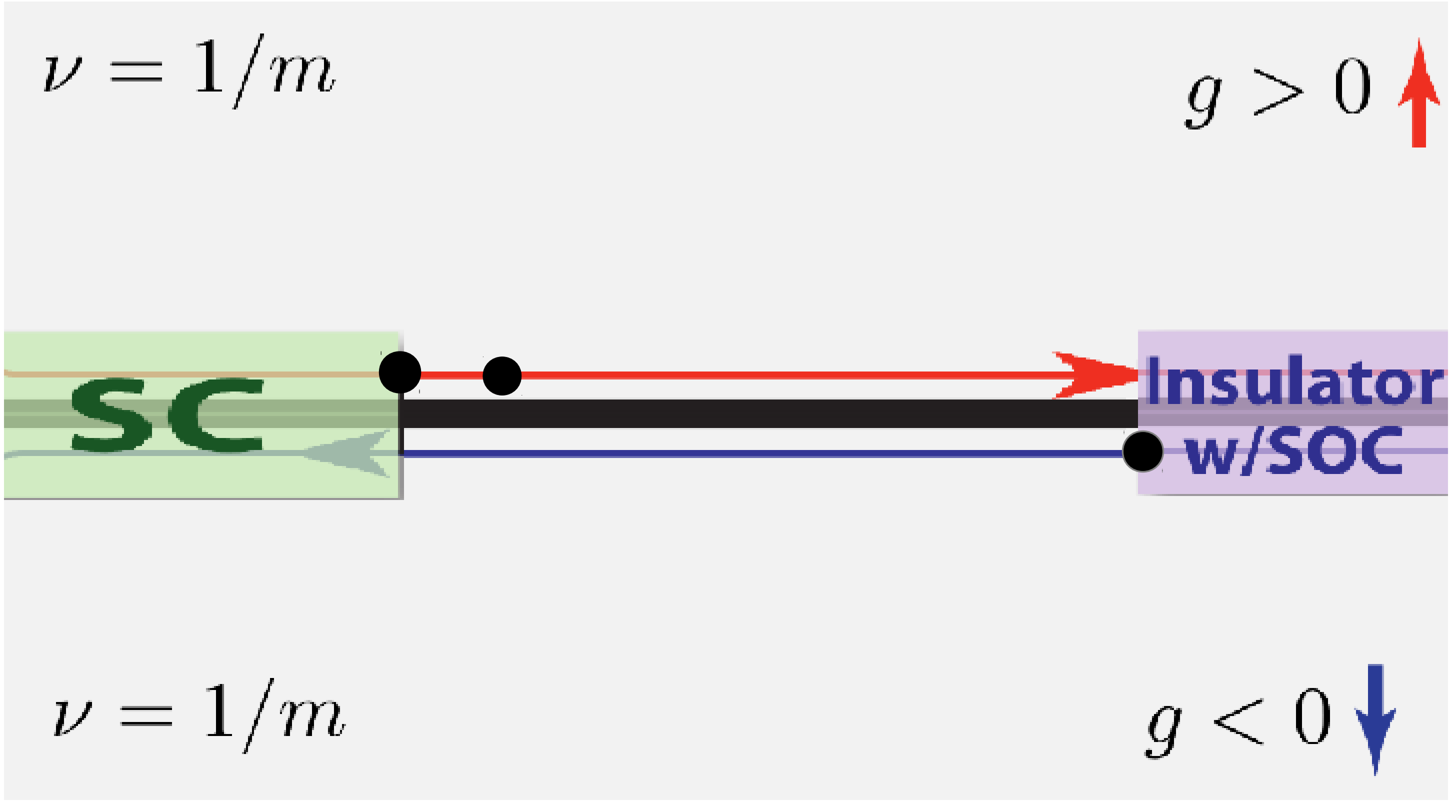
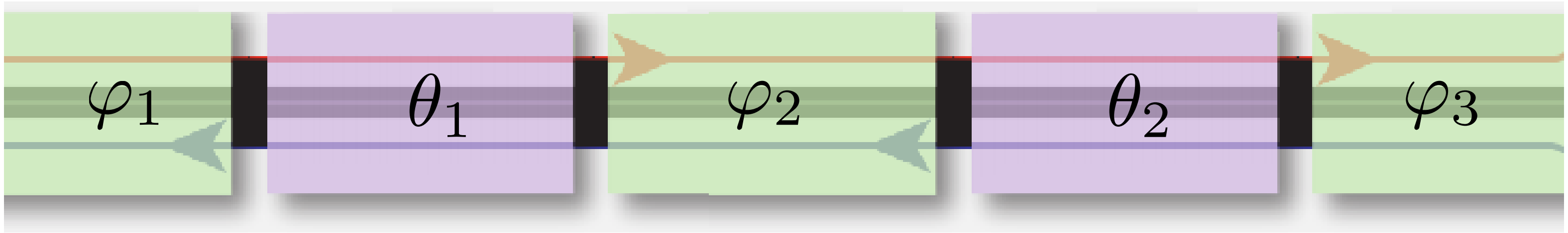


# Quantum Dimension

Reminder: in the strong backscattering and pairing limit,

$$\varphi = \frac{\pi n_\varphi}{m}$$ in the SC regions,

$$\theta = \frac{\pi n_\theta}{m}$$ in the 'magnetic' regions



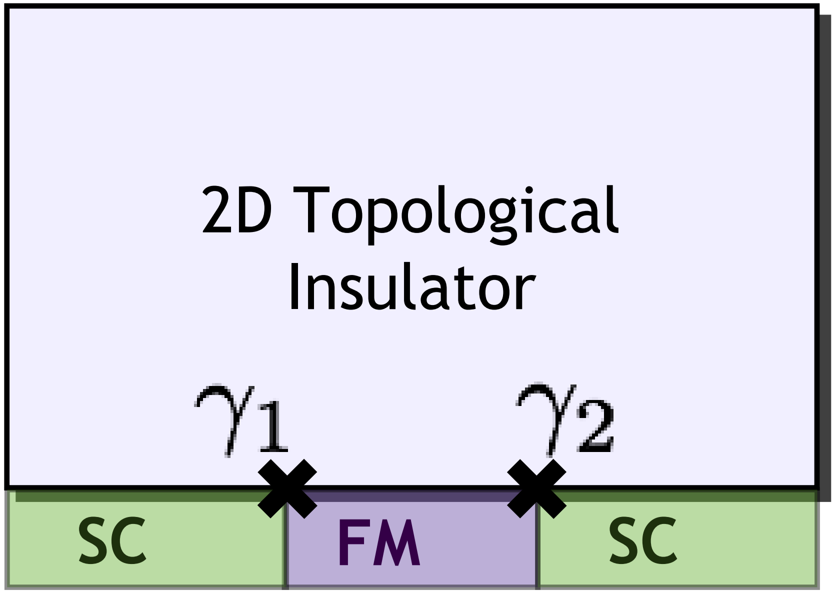

Each $n$ has eignenvalues $0, \ldots 2m-1$, but they don't commute!

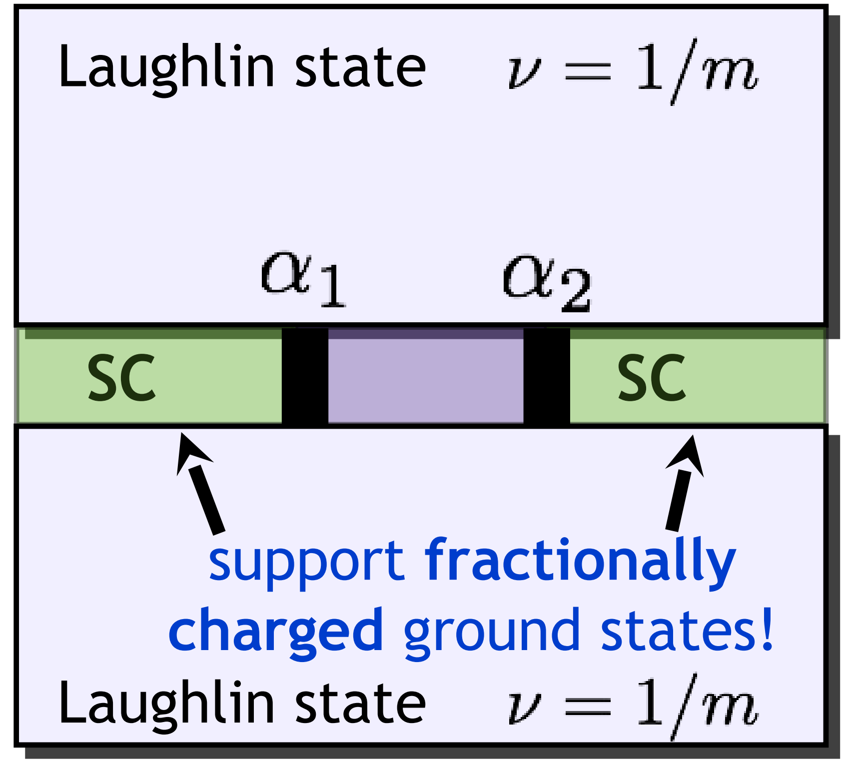
$$[\varphi(x), \theta(y)] = i\frac{\pi}{m}\Theta(x-y)$$ $\Rightarrow$ One can fix $n_\varphi$'s or $n_\theta$'s

$(2m)^N$ degenerate states for $2N$ modes $\Rightarrow$ $d = \sqrt{2m}$

# Majorana vs. parafermion zero-modes

Fu & Kane (2009)




$\gamma_i^2 = 1 \qquad \gamma_1\gamma_2 = -\gamma_2\gamma_1$

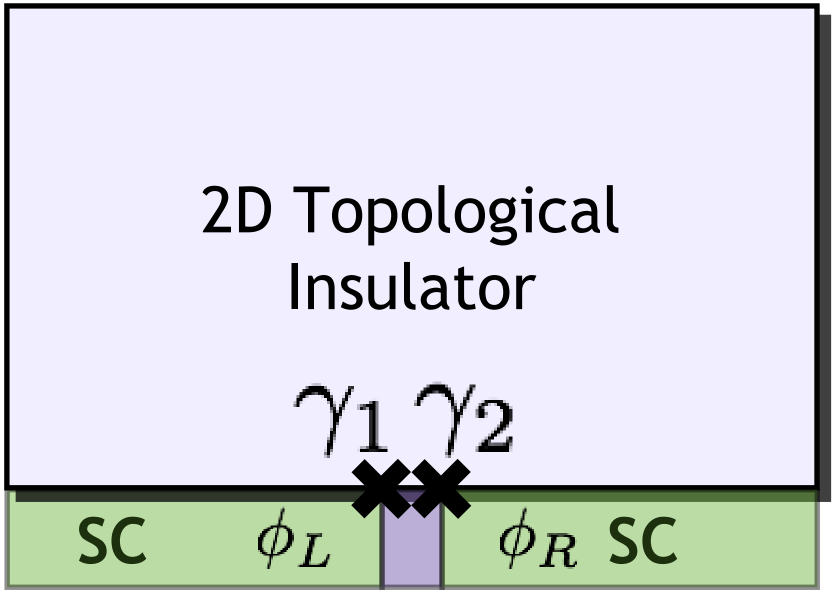
$\alpha_i^{2m} = 1 \qquad \alpha_1\alpha_2 = e^{i\frac{\pi}{m}}\alpha_2\alpha_1$

$|\mathrm{even}\rangle \equiv |0\rangle \overset{\gamma_i}{\longleftrightarrow} |\mathrm{odd}\rangle \equiv |1\rangle$

$\alpha_i^\dagger$: $|0\rangle \to |1/m\rangle \to |2/m\rangle \cdots \to |2 - 1/m\rangle$

# Majorana vs. parafermion zero-modes

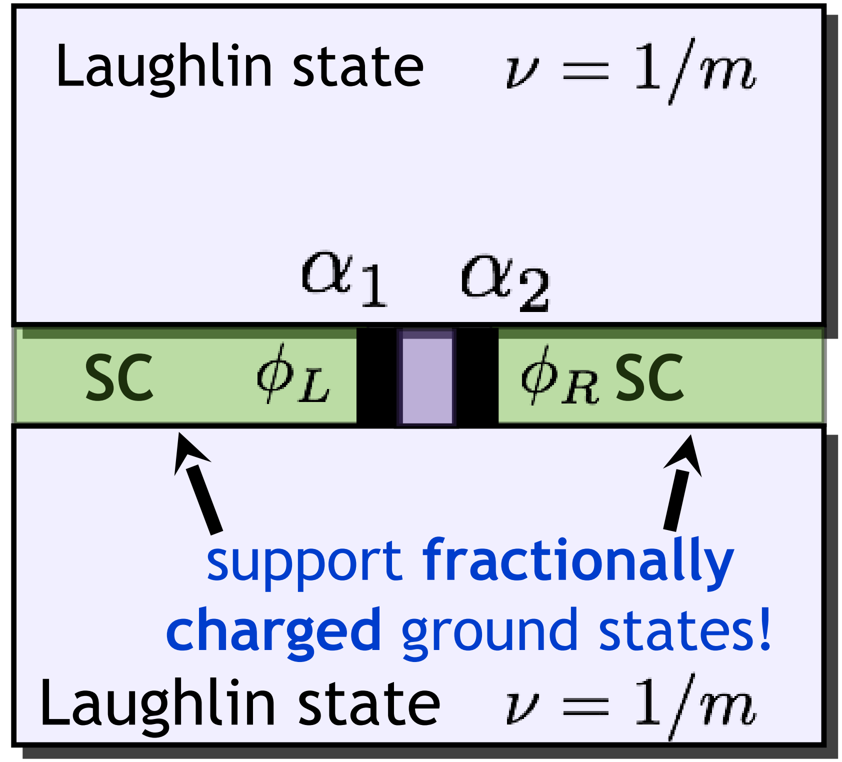
Fu & Kane (2009)




Fractional Josephson effect

$$I \propto \sin\left(\frac{\Delta\phi}{2}\right)$$

Kitaev (2001)




$$I \propto \sin\left(\frac{\Delta\phi}{2m}\right)$$

Effect persists even with **electronic** relaxation processes

# Is this zero-mode a (non-Abelian) anyon?

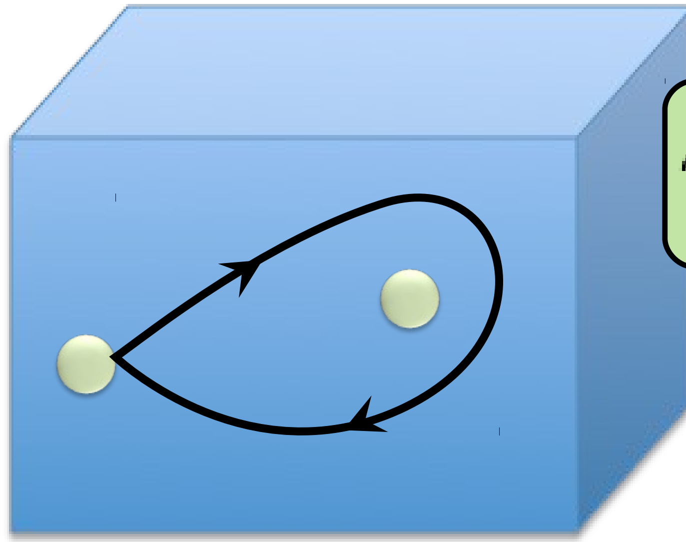
"It depends upon what the meaning of the word 'is' is"
Bill Clinton

- First and foremost, it is ***not*** a low-energy quasiparticle!
  It is a zero mode bound to a very high-energy topological defect – c.f. Majorana mode bound to a SC vortex or 'genons'
  (M. Barkeshli, X-L. Qi, arXiv:1112.3311, arXiv:1208.4834)
- Instead of the usual braiding statistics, one should think of *projective braiding statistics* (statistics up to a phase)
- "Metaplectic anyons"
  – M. Hastings, C. Nayak, Z. Wang, arXiv:1210.5477
- What are braiding statistics in 1D anyway?

# Braiding statistics in 1D?

## d = 3

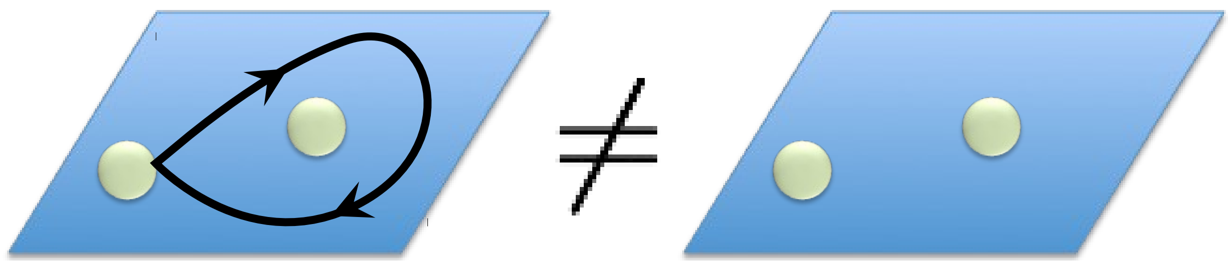
Only bosons & fermions (no knots in 3+1 dimensions)



$$\psi \longrightarrow \pm\psi$$

## d = 2

**Anyons** are now possible!

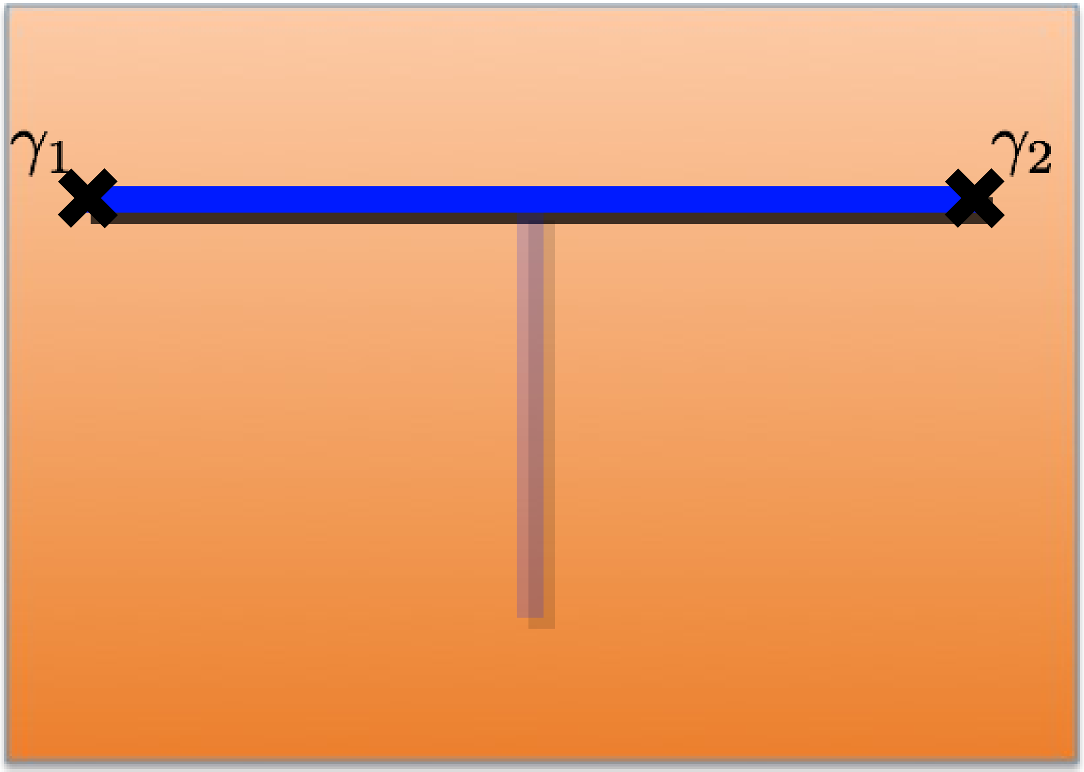


## d = 1

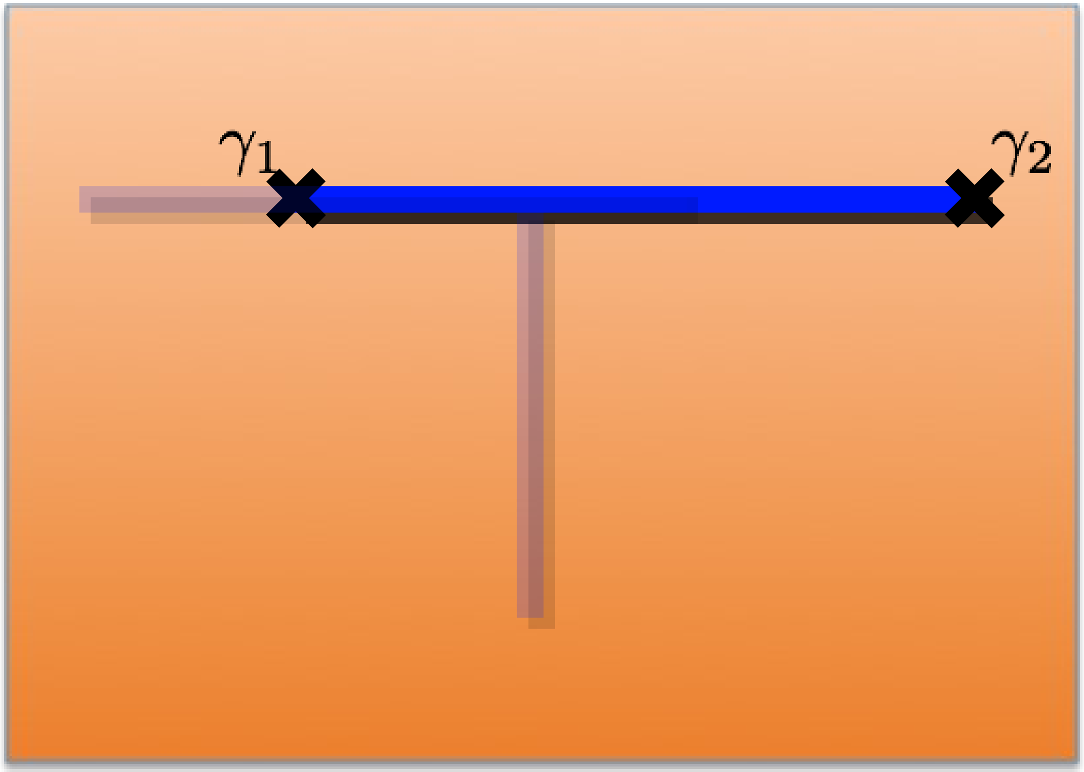
Exchange not well defined...

...because particles inevitably “collide”

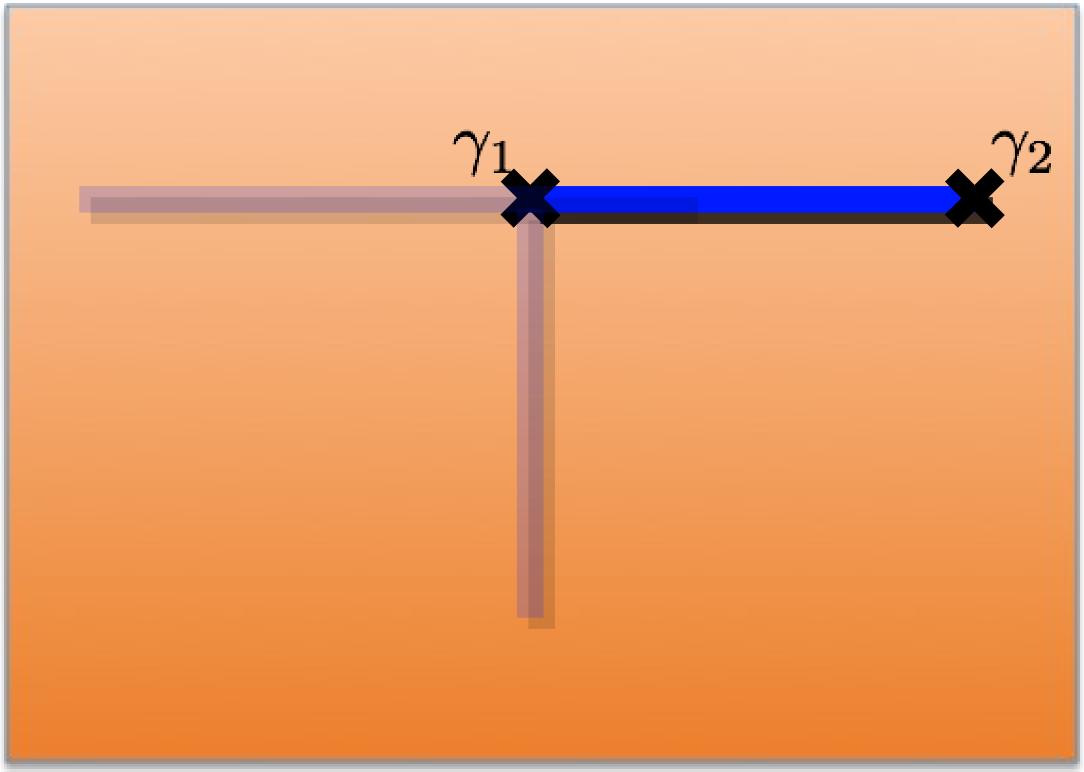
# Exchanging end modes in 1D wires



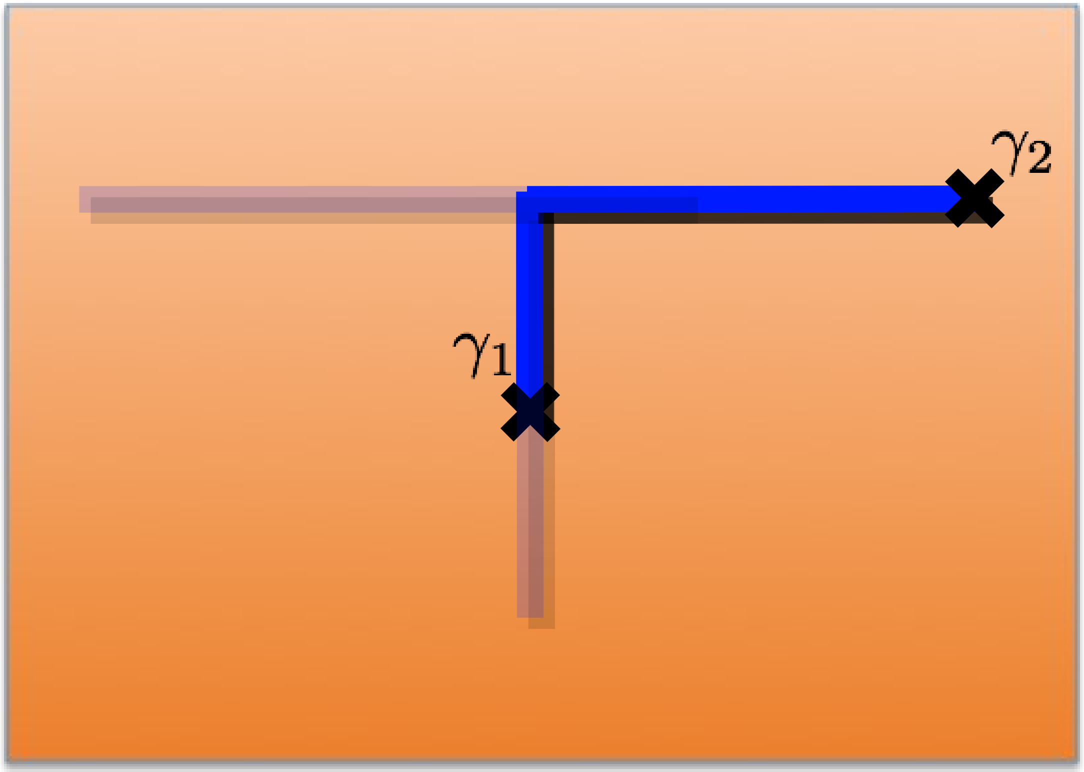
# Exchanging end modes in 1D wires



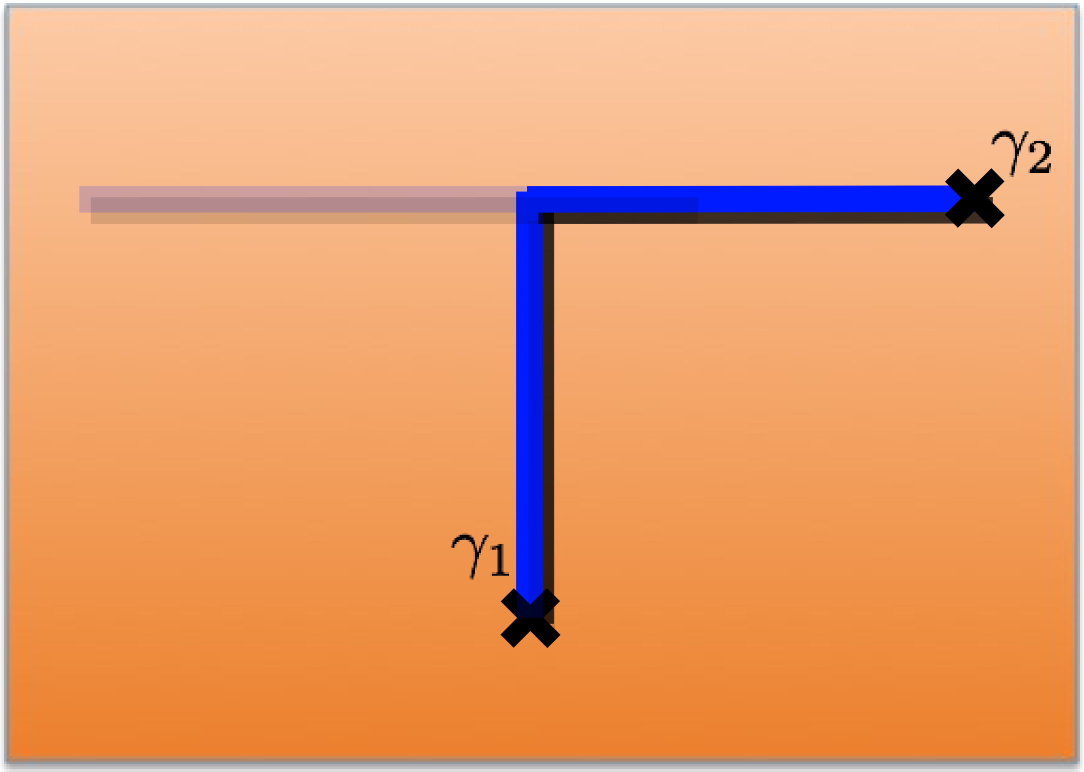
# Exchanging end modes in 1D wires



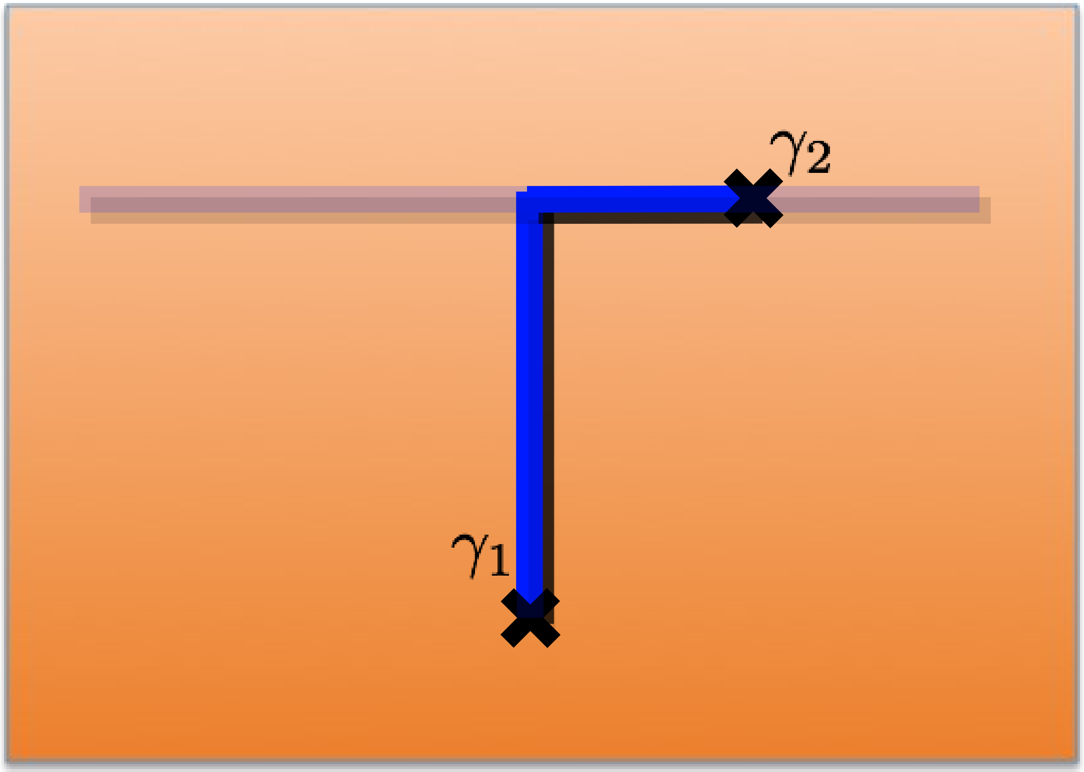
# Exchanging end modes in 1D wires



# Exchanging end modes in 1D wires



# Exchanging end modes in 1D wires



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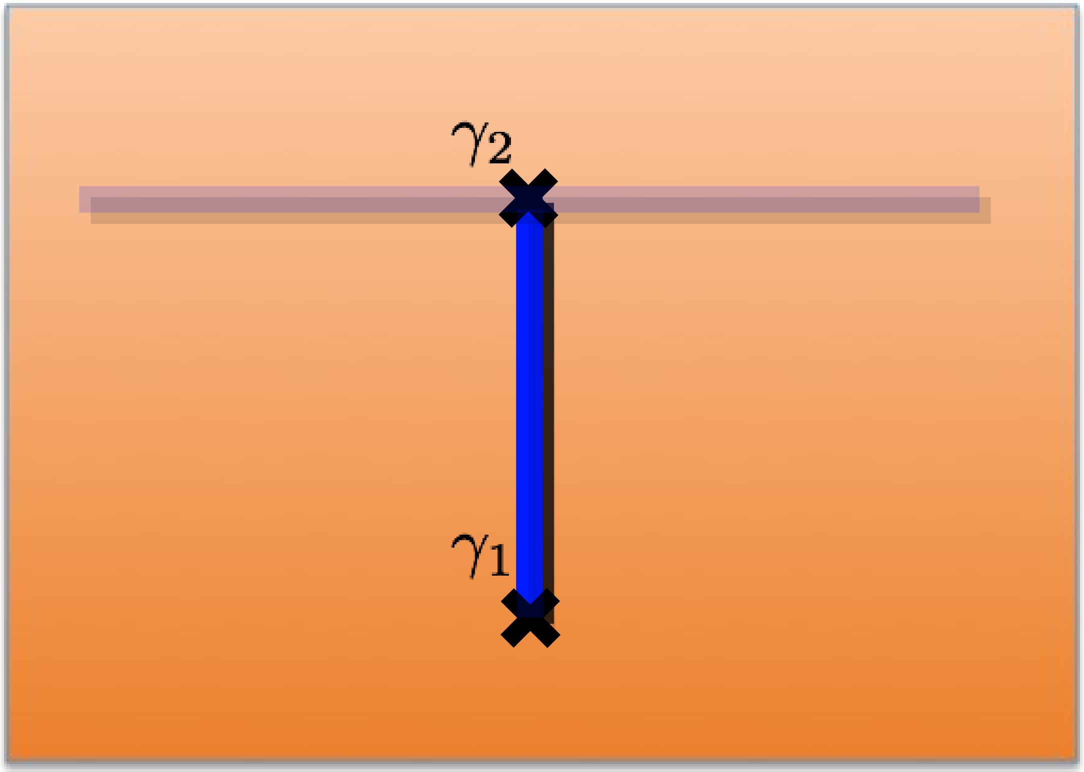

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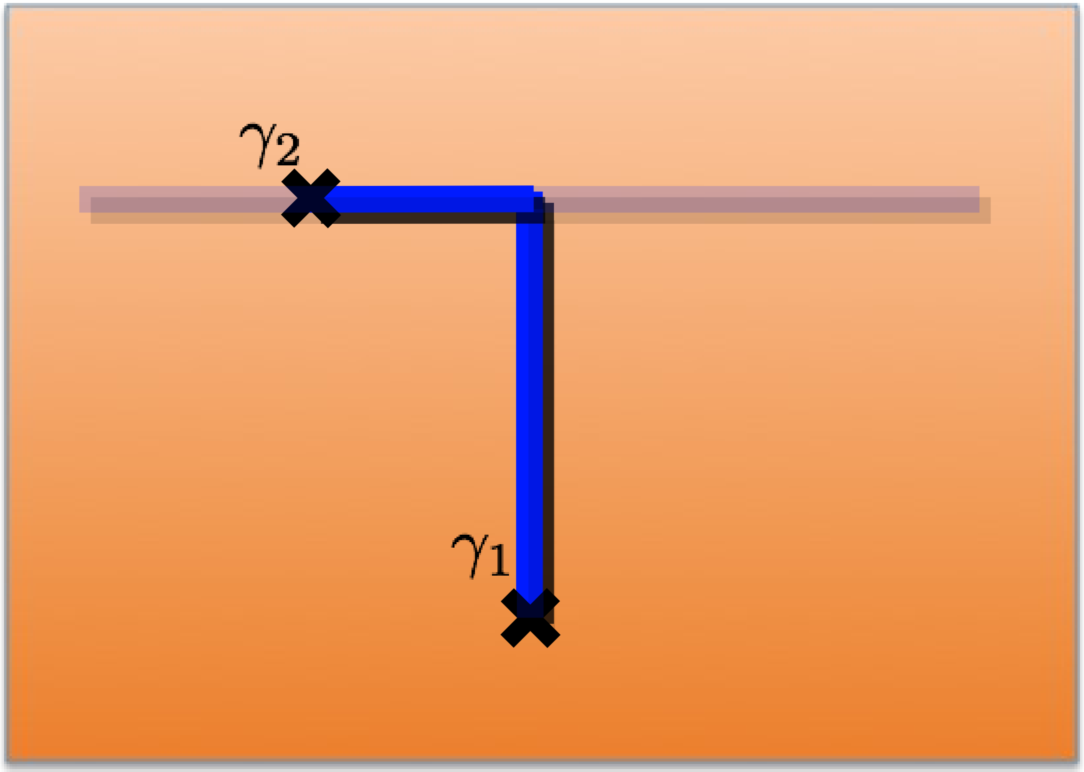
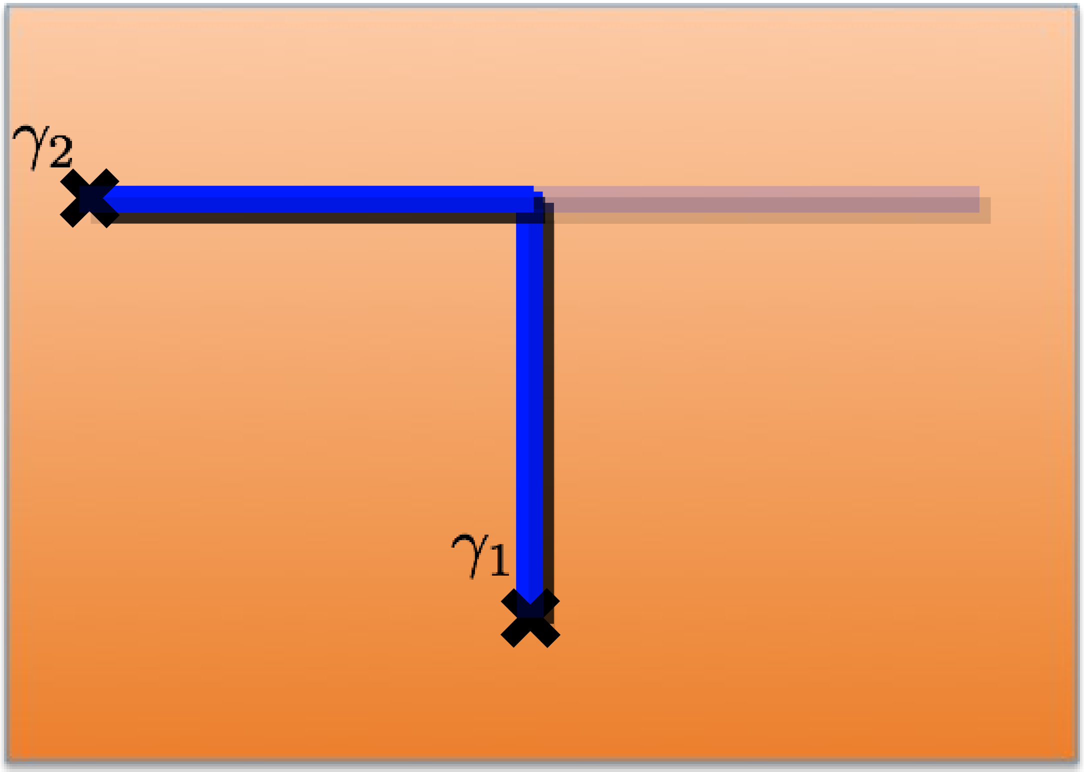


# Exchanging end modes in 1D wires



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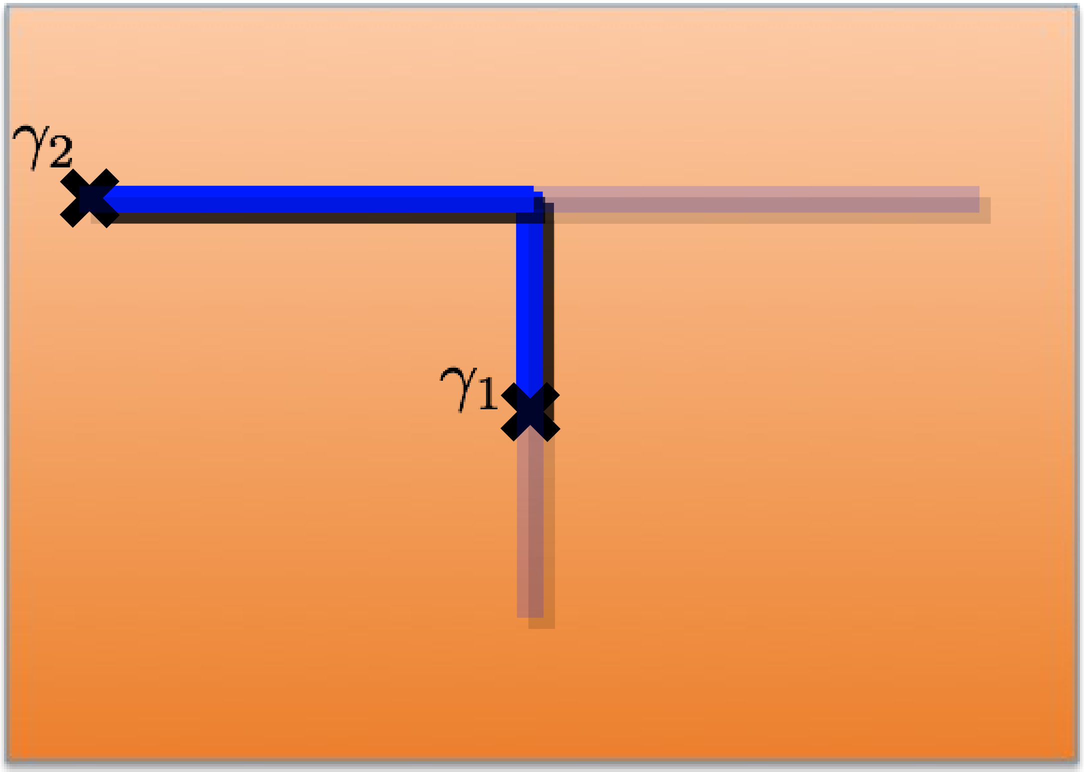
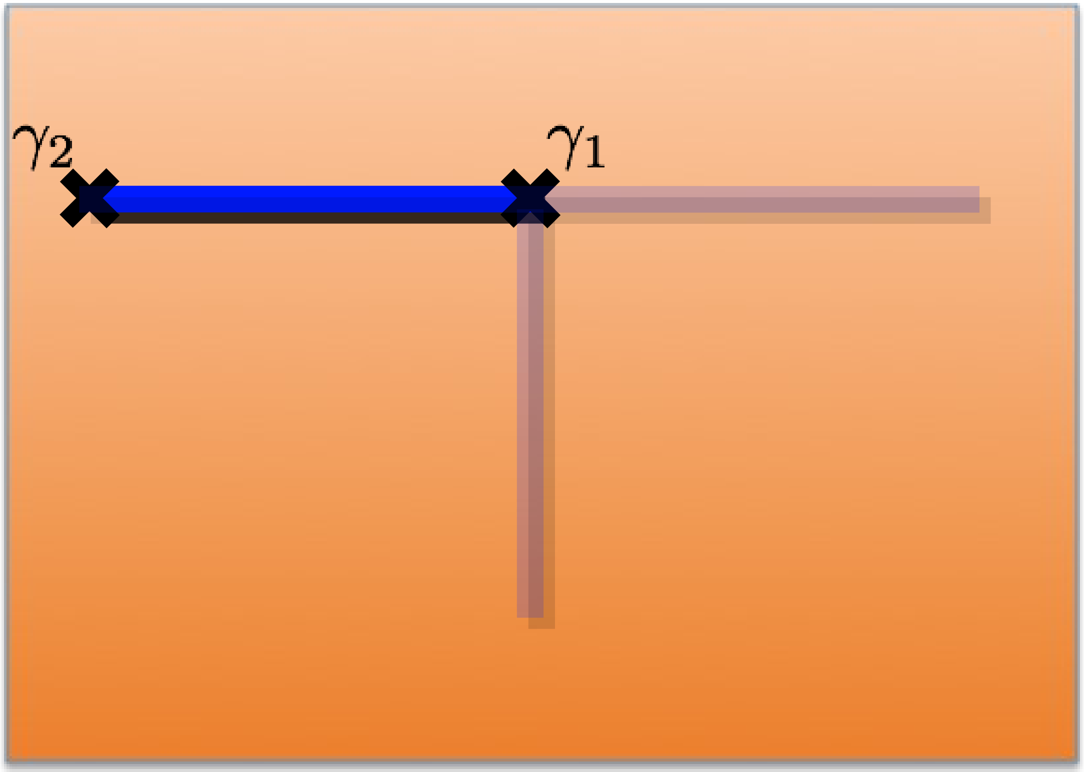
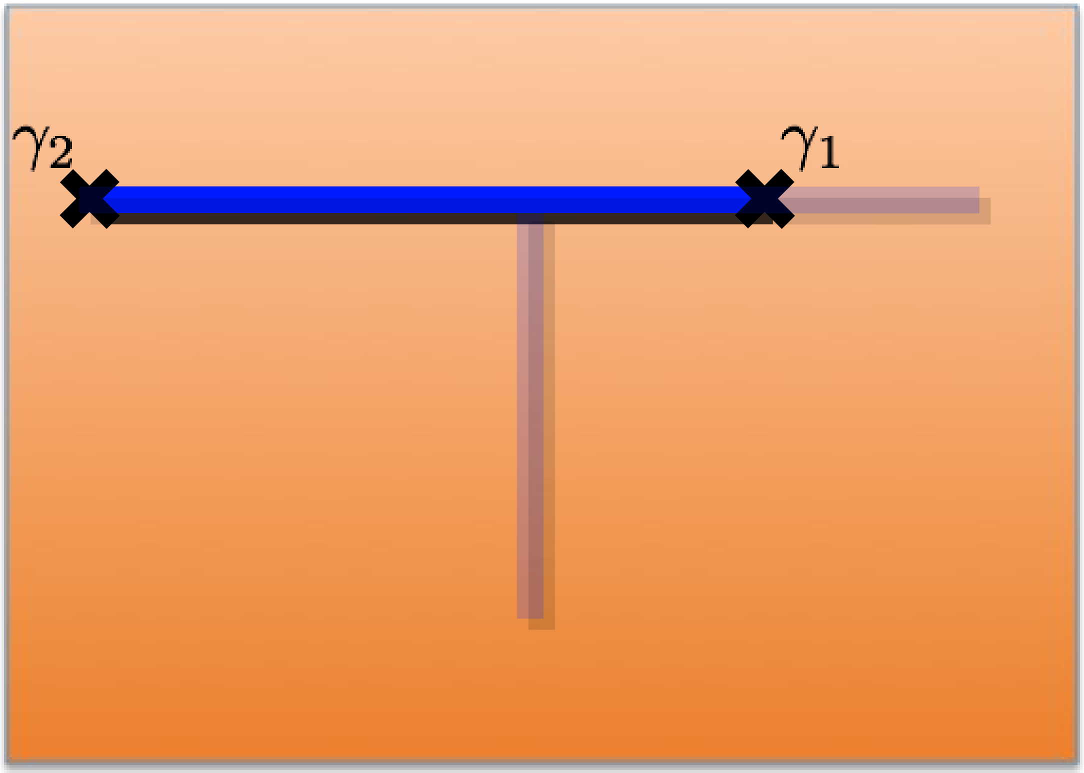


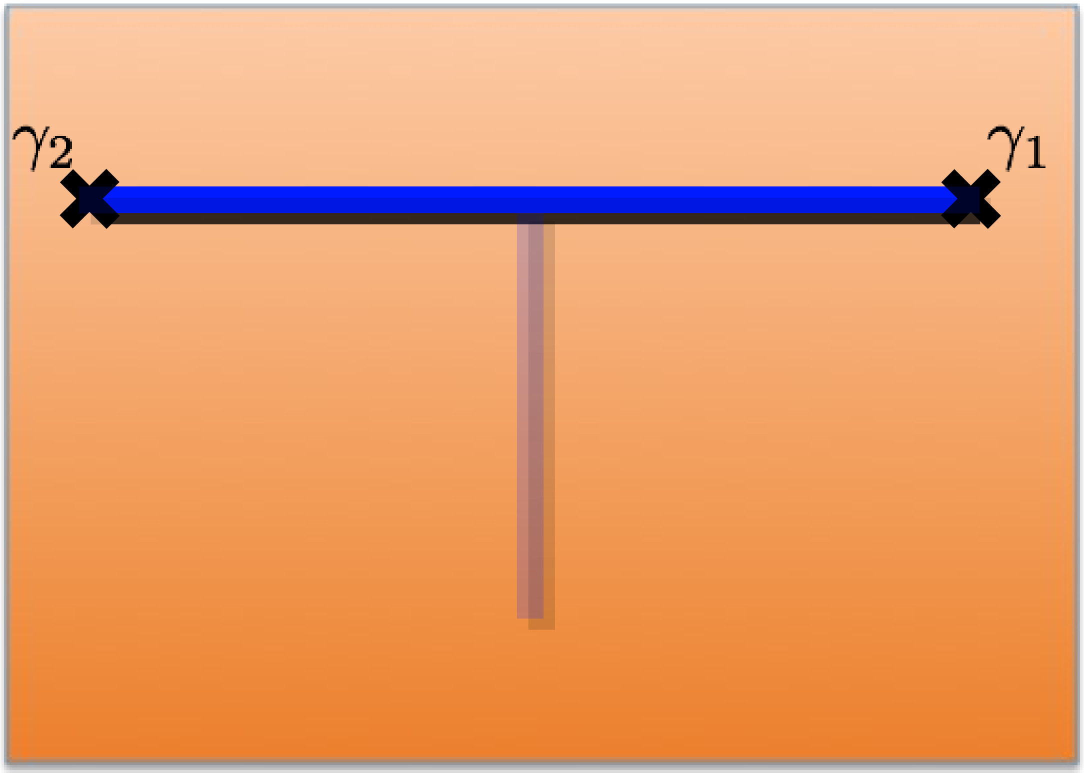
# Exchanging end modes in 1D wires



# Exchanging end modes in 1D wires



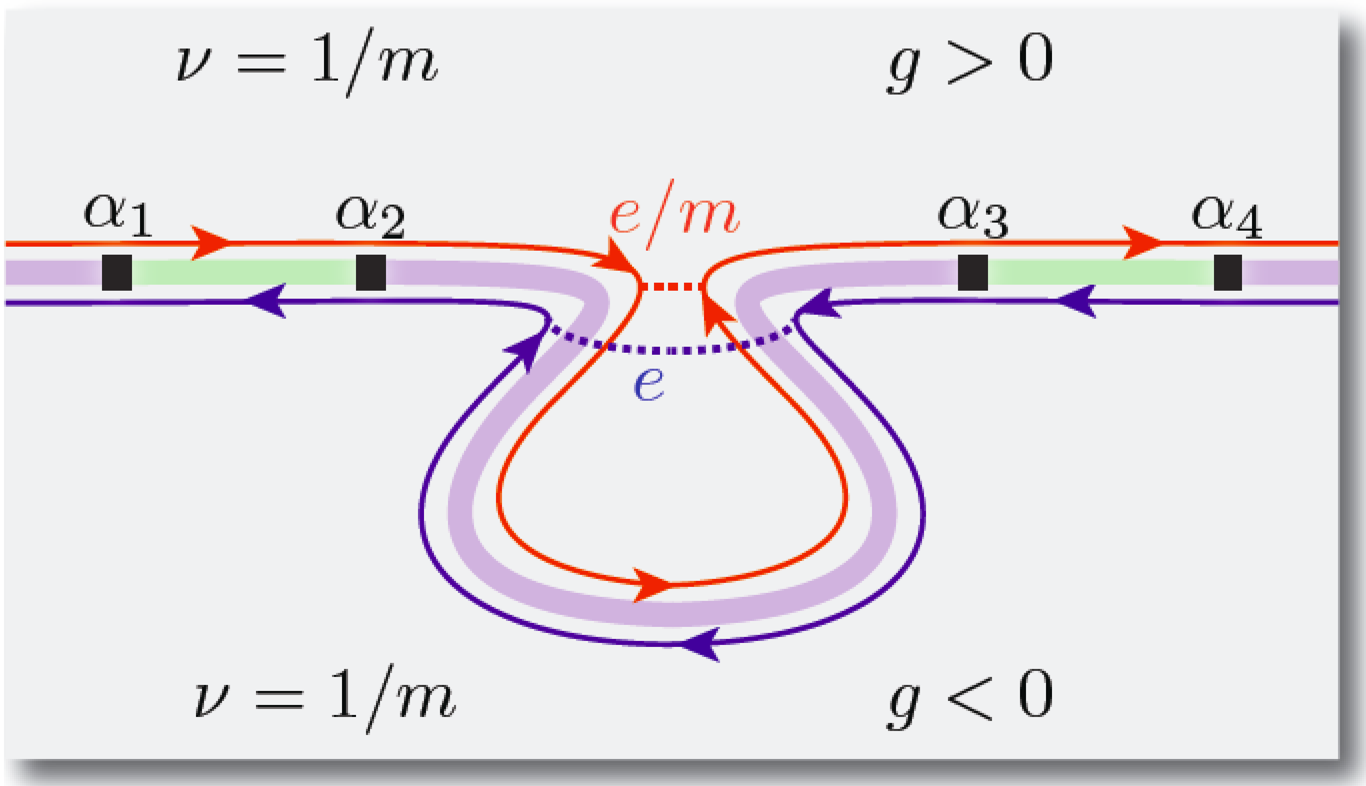
# Exchanging end modes in 1D wires



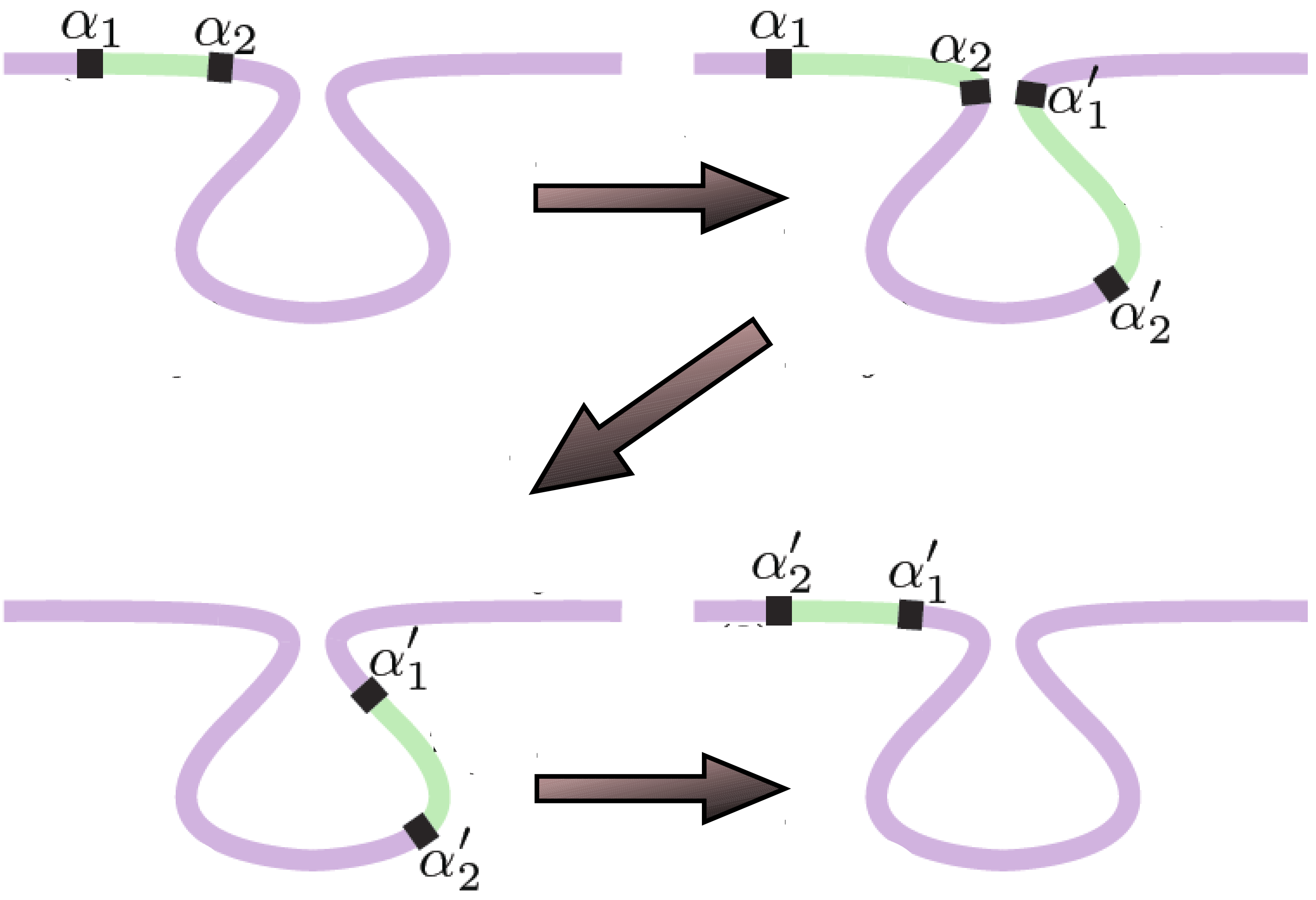
# Exchanging end modes in our case

**Apparent problem:**

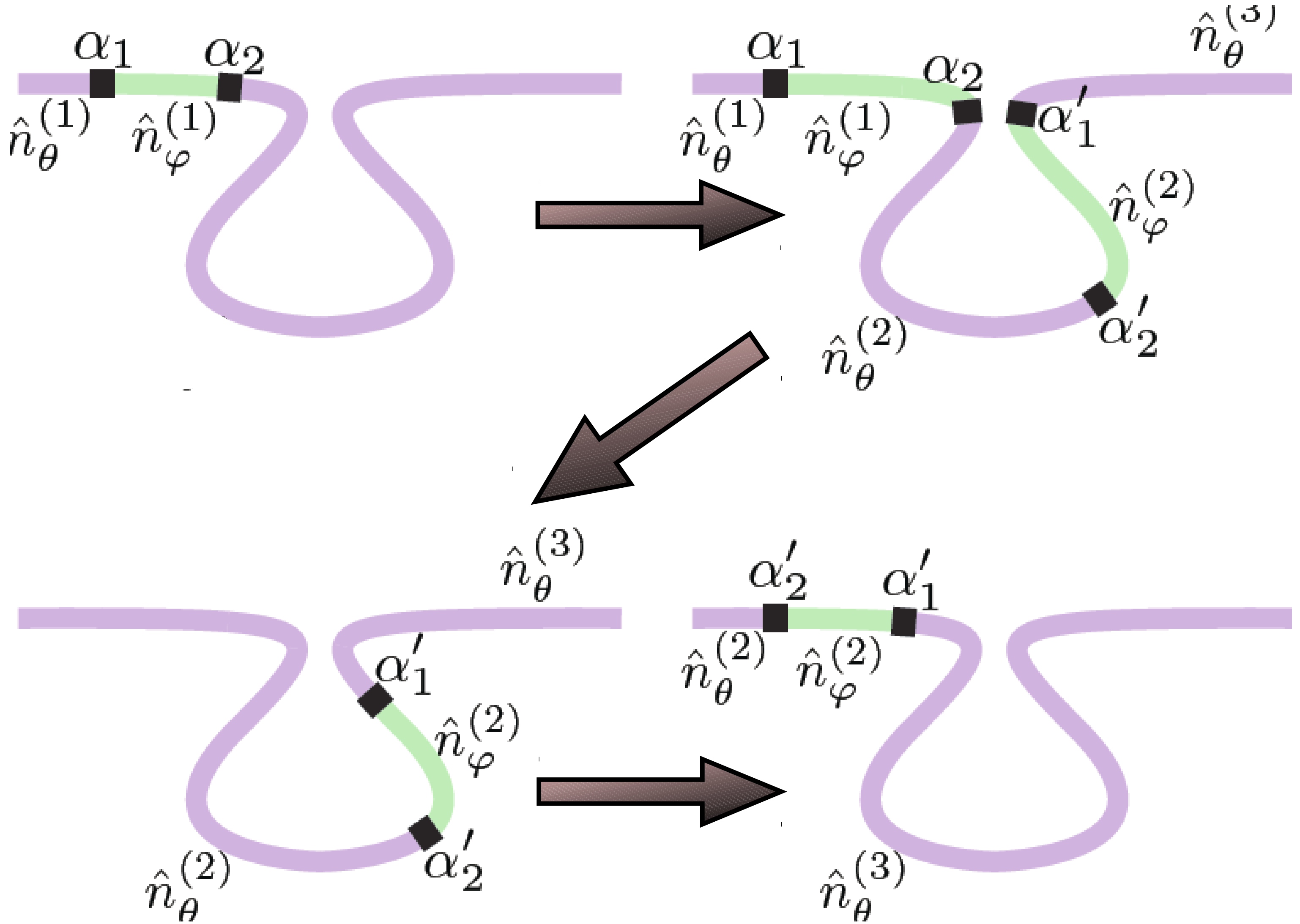
- We cannot have Y-junctions: our modes live on the domain walls..
- We can still exchange them:



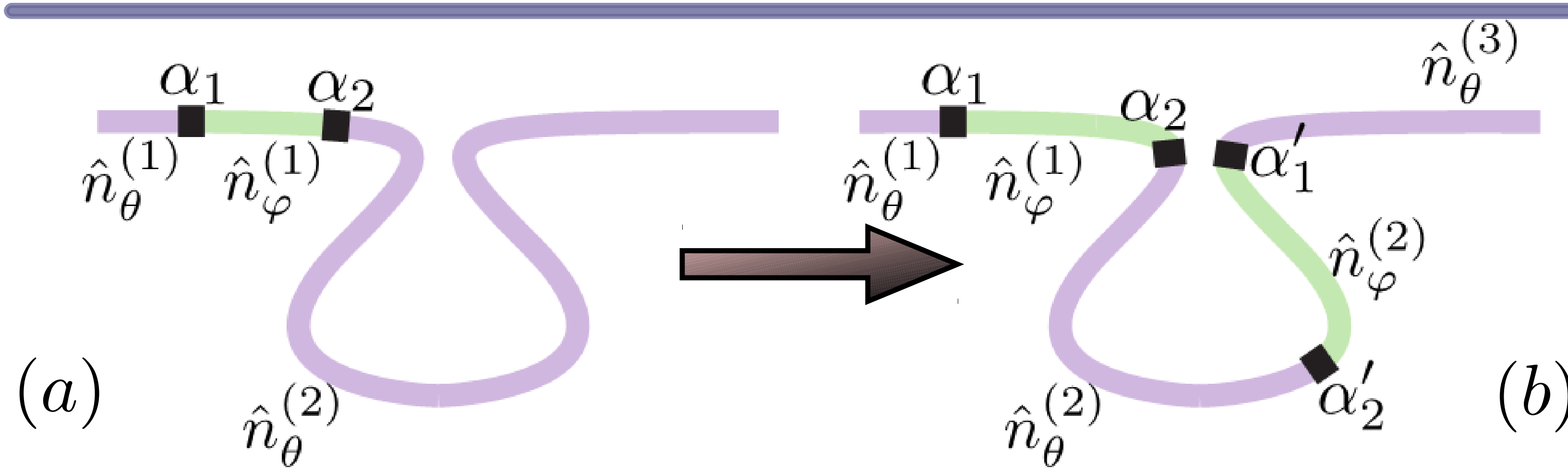
# Exchanging end modes in our case



# Exchanging end modes in our case

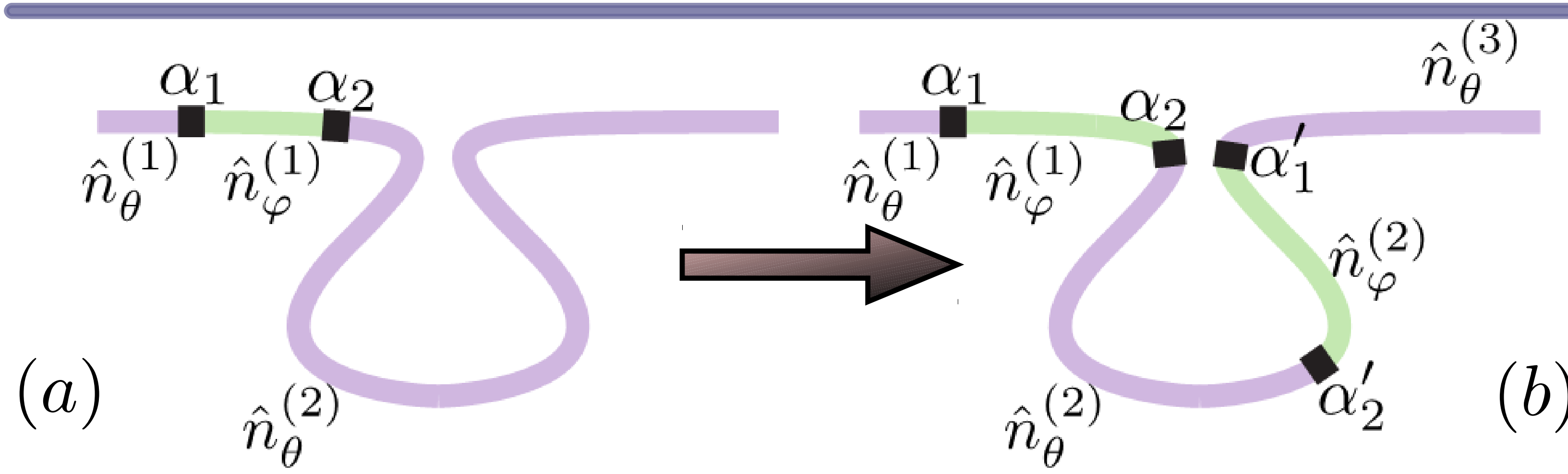


# Exchanging end modes in our case




$$
\begin{aligned}
H_{a\to b} &= (t_J \alpha_2^\dagger \alpha_1' + H.c.) + (t \alpha_1'^\dagger \alpha_2' + H.c.) \\
&= -|t_J| \cos\left[\frac{\pi}{m}\left(\hat{n}_\varphi^{(2)} + \hat{n}_\theta^{(3)} - \hat{n}_\varphi^{(1)} - \hat{n}_\theta^{(2)}\right) + \beta\right] \\
&\quad -|t| \cos\left[\frac{\pi}{m}\left(\hat{n}_\theta^{(2)} - \hat{n}_\theta^{(3)}\right)\right]
\end{aligned}
$$

# Exchanging end modes in our case




Integral of motion:

$$\chi \equiv e^{i\frac{\pi}{2m}}\alpha_2 \alpha_2'^{\dagger}\alpha_1' = e^{i\frac{\pi}{m}(\hat{n}_\varphi^{(1)}+\hat{n}_\theta^{(3)})}$$

Energy-minimizing condition:

$$\hat{n}_\varphi^{(2)} + \hat{n}_\theta^{(3)} - \hat{n}_\varphi^{(1)} - \hat{n}_\theta^{(2)} = k(\beta) \in \mathbb{Z}$$

# Parafermion ZM Braiding

<u>*Upshot:*</u>

$$\alpha_1 \rightarrow e^{-i\frac{\pi}{m}k}\alpha_2$$

$$\alpha_2 \rightarrow e^{i\frac{\pi}{m}(1-k)}\alpha_1^\dagger \alpha_2^2$$

$m = 1$ (Majorana zero modes):

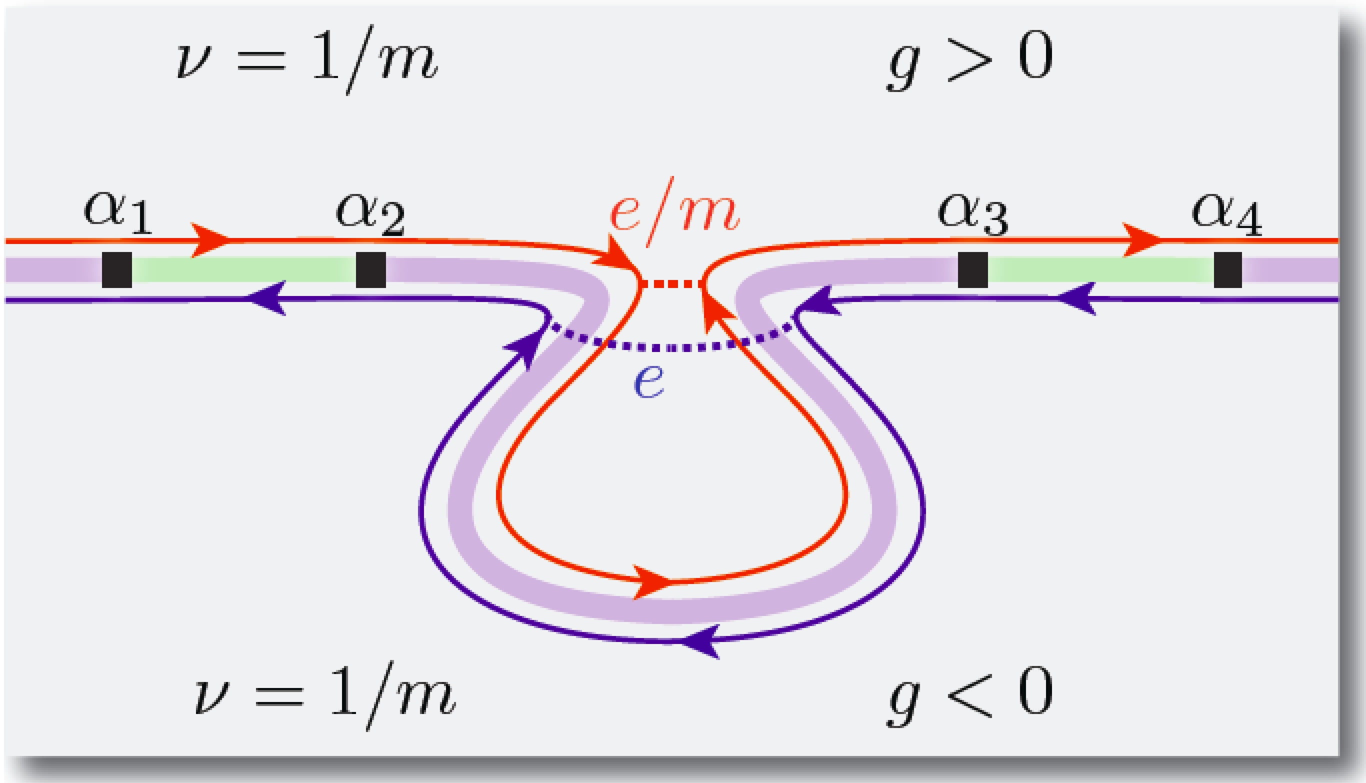
$$\gamma_1 \rightarrow \gamma_2$$

$$\gamma_2 \rightarrow -\gamma_1$$

# Parafermion ZM Braiding

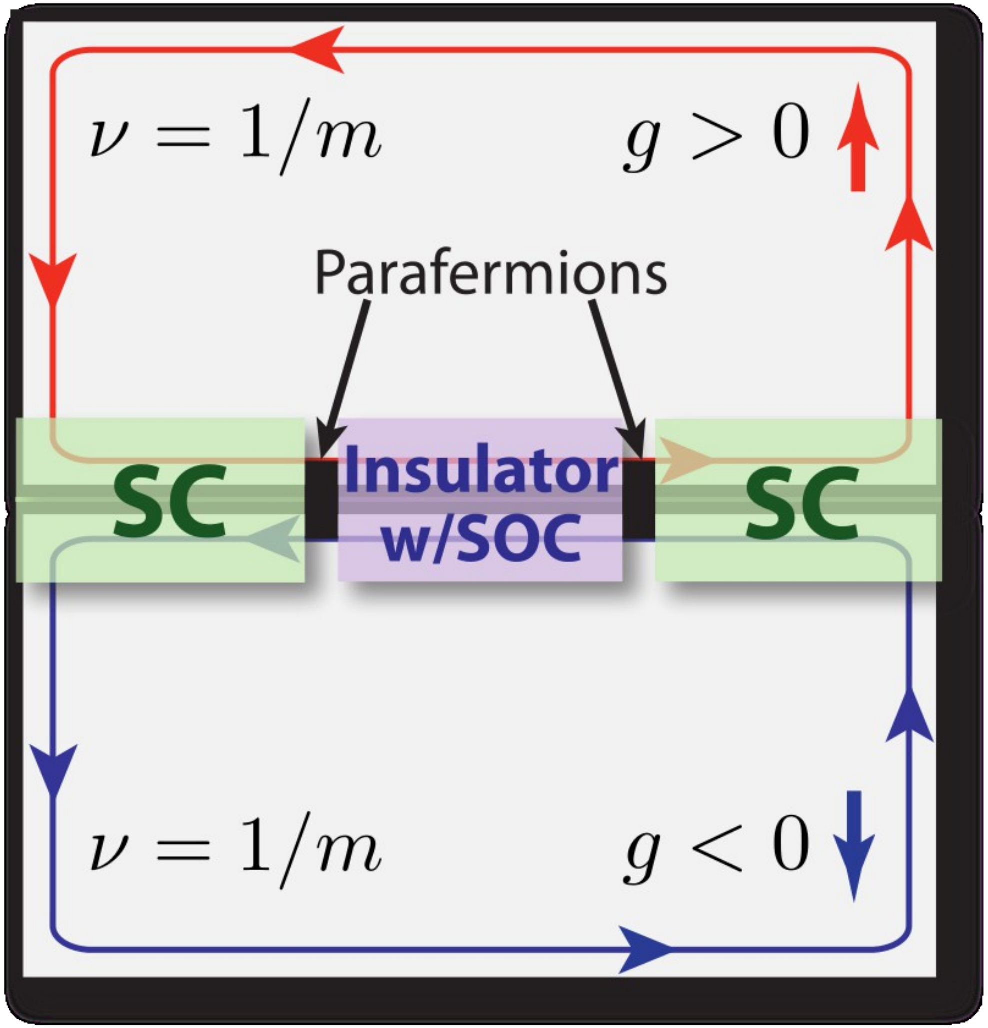
**Important observation:**

- If quasiparticles of both chiralities are allowed to tunnel, the braiding is not universal $\Rightarrow$ Potential problem for fractional TI!



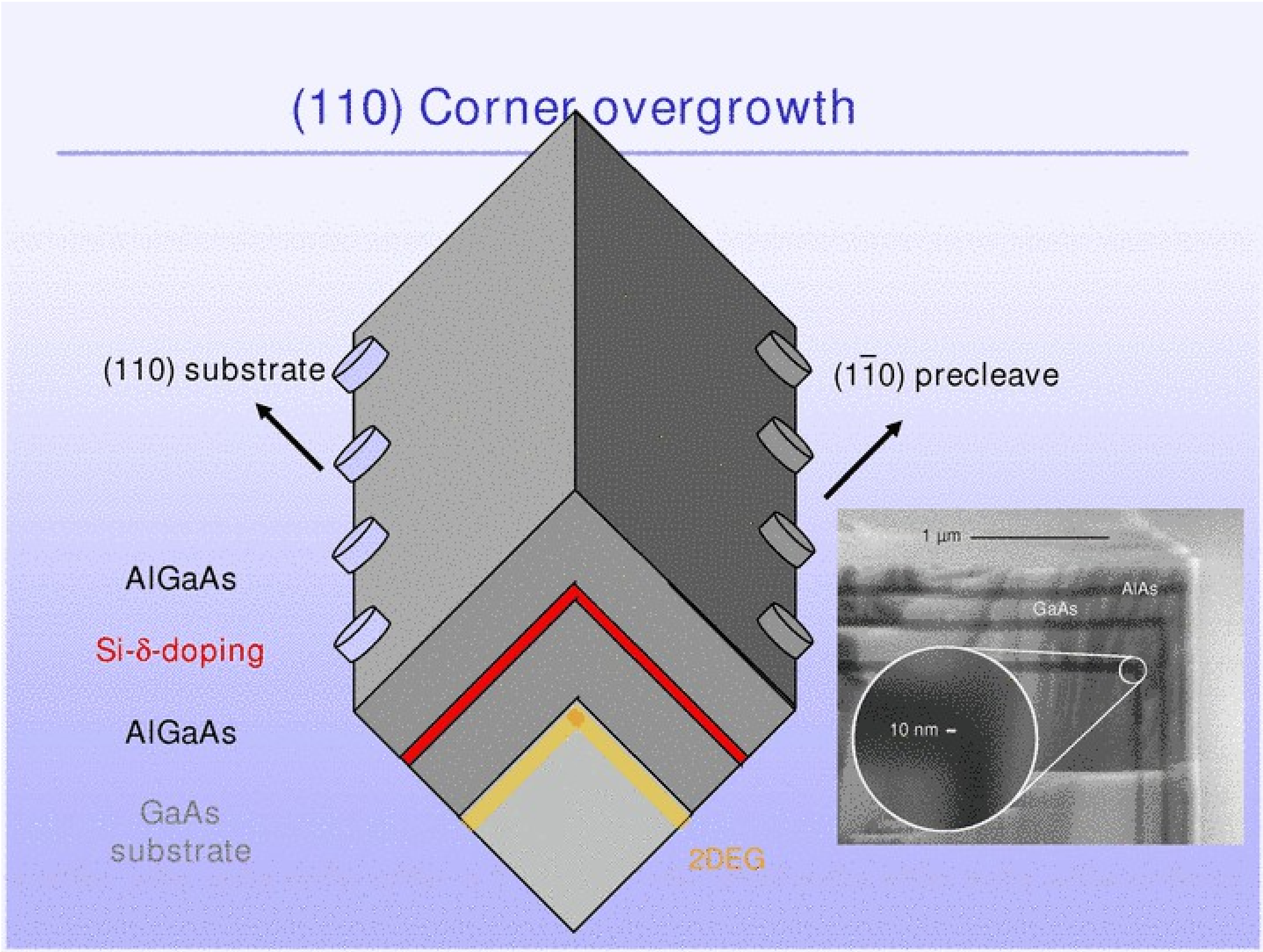
# Experimental realizations?

Is this feasible?



# Experimental realizations?

Bending the Quantum Hall Effect (Matt Grayson's group,TU München)

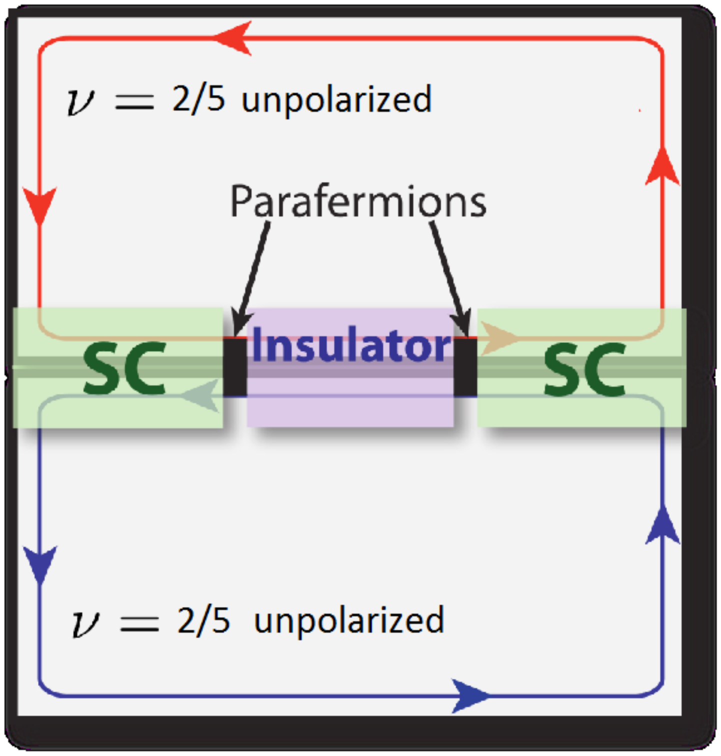


# Which Parts are Really Necessary?

*Do we need opposite g-factors (or exotic materials)?*

No! Use spin-unpolarized states, like 2/3 or 2/5

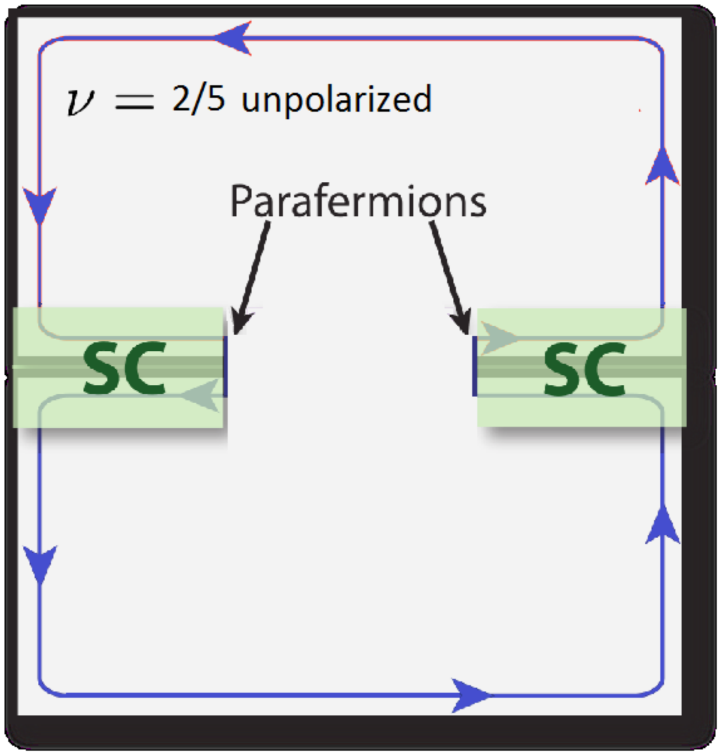
D. Clarke, J. Alicea, KS, in preparation



# Which Parts are Really Necessary?

***So do we really need the insulator at all?***

Not in this case!

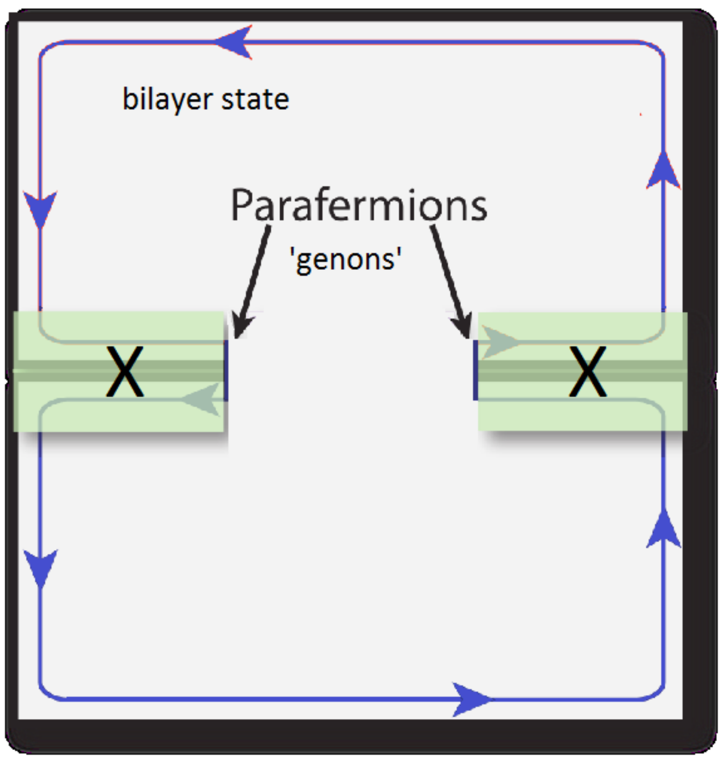


# Which Parts are Really Necessary?

***Ok, but we need the superconductor, right?***

Not if we use a bilayer system, then particle<-> hole is replaced by layer<->layer

Barkeshli, Jian, Qi, arxiv:1208.4834



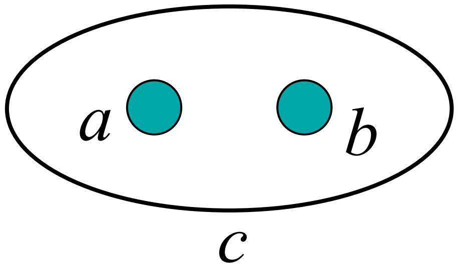
# Topological Quantum Computation

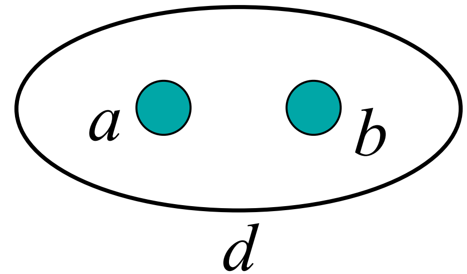
(Kitaev, Preskill, Freedman, Larsen, Wang)

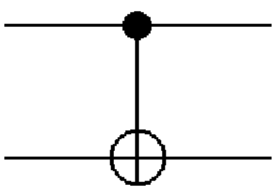
Things we need:

- Multidimensional Hilbert space where we can encode information → Qubits
- Ability to initialise and read-out a qubit
- Unitary operations → Quantum gates

# Topological Quantum Computation

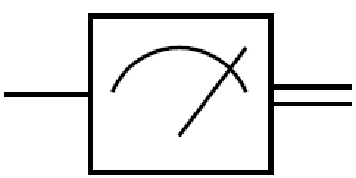
$|0\rangle \leftrightarrow$ 


$|1\rangle \leftrightarrow$ 


 $\leftrightarrow$ 

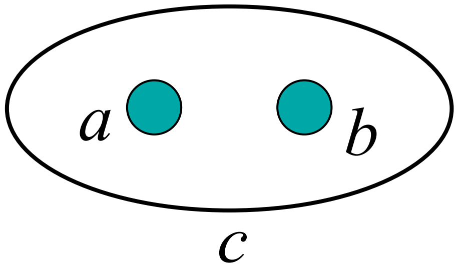
(Hormozi, Bonesteel, et. al.)

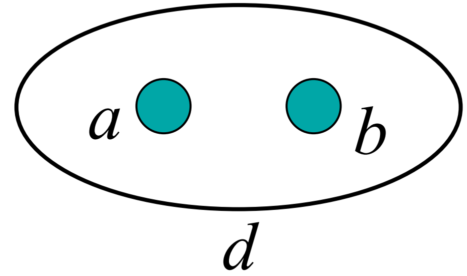
Or, perhaps use measurements to generate brading!
(Bonderson, Freedman, Nayak, 2009)

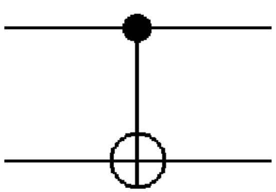
 $\leftrightarrow$ Interferometer

Bonderson, KS & Slingerland, PRL 2006, PRL 2007, Ann. Phys. 2008

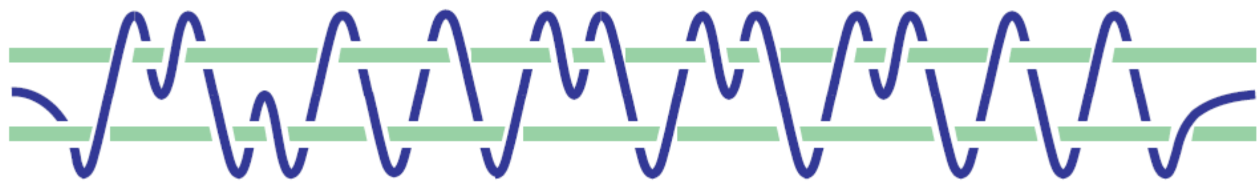
# Topological Quantum Computation

$|0\rangle \leftrightarrow$ 


$|1\rangle \leftrightarrow$ 

$\leftrightarrow$

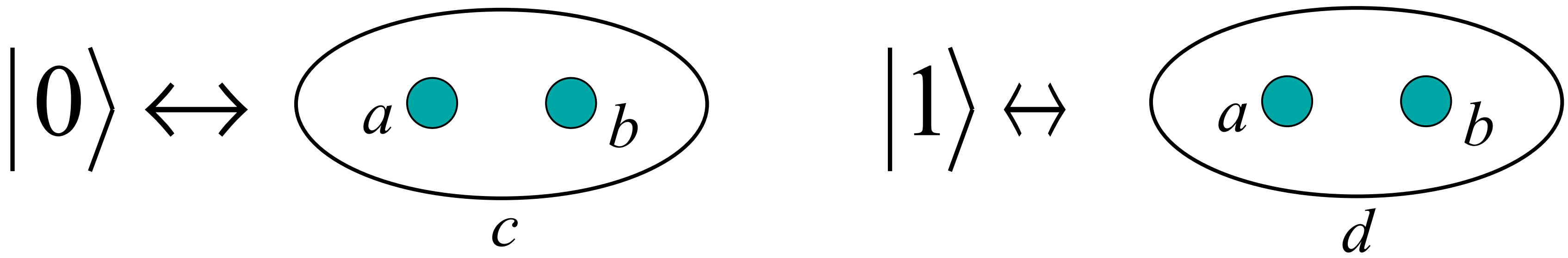


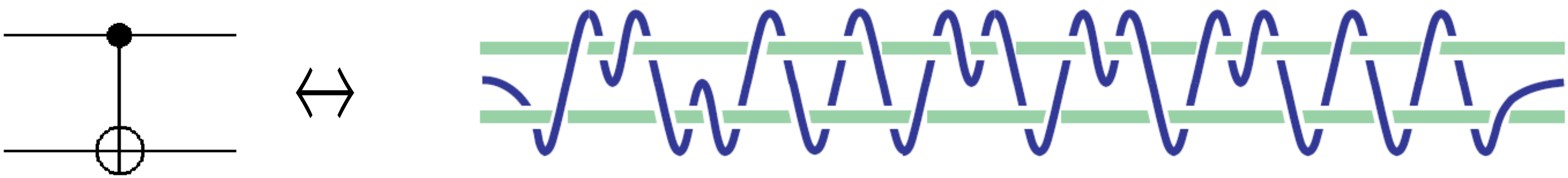
(Hormozi, Bonesteel, et. al.)

Or, perhaps use measurements to generate brading!
(Bonderson, Freedman, Nayak, 2009)

- Majorana zero modes are not universal!
  - No entangling gates with braiding alone
  - No phase gate

# Topological Quantum Computation



(Hormozi, Bonesteel, et. al.)

Or, perhaps use measurements to generate brading!
(Bonderson, Freedman, Nayak, 2009)

- Parafermionic zero modes are still not universal...
    - Can do entangling gates!
    - No phase gate?

# Conclusions

- Parafermionic zero modes can be localised in systems with counter-propagating fractionalised edge modes (FQHE, or fractional topological insulators)
  - Fractional Josephson effect with periodicity $4m\pi$
  - Zero-bias anomaly – similar to the Majorana case, but with fractionalised charge tunnelling
- Potential utility for quantum computing?

- D. Clarke, J. Alicea & KS, arXiv:1204.5479
- Parallel work:
  - N. Lindner, E. Berg, G. Refael & A. Stern, arXiv:1204.5733
  - M. Cheng, arXiv:1204.6084