Looking beyond Majorana zero modes: Engineering non-Abelian anyons

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**Ising model (of all things)**

1D quantum Ising chain:

\[
H = -J \sum_{j=1}^{L-1} \sigma_j^z \sigma_{j+1}^z - h \sum_{j=1}^{L} \sigma_j^x
\]

Jordan–Wigner transformation:

\[
\gamma_{2j-1} = \sigma_j^z \prod_{i<j} \sigma_i^x, \quad \gamma_{2j} = \sigma_j^y \prod_{i<j} \sigma_i^x
\]

\(\gamma's\) are **Majorana** operators:

\[
\gamma_j^2 = 1, \quad \gamma_j^\dagger = \gamma_j, \quad \gamma_j \gamma_k = -\gamma_k \gamma_j
\]
1D quantum Ising chain:

\[ H = -J \sum_{j=1}^{L-1} \sigma_j^z \sigma_{j+1}^z - h \sum_{j=1}^{L} \sigma_j^x \]

Hamiltonian after Jordan–Wigner transformation:

\[ H = -J \sum_{j=1}^{L-1} i\gamma_{2j} \gamma_{2j+1} - h \sum_{j=1}^{L} i\gamma_{2j-1} \gamma_{2j} \]

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\]

\[J = 0:\]

1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad \cdots \quad 2N-1 \quad 2N
Ising model (of all things)

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Hamiltonian after Jordan–Wigner transformation:

\[ H = -J \sum_{j=1}^{N-1} i \gamma_{2j} \gamma_{2j+1} - h \sum_{j=1}^{N} i \gamma_{2j-1} \gamma_{2j} \]

\( h = 0 \):
From Ising to Majorana wires

1D spinless p-wave superconductor (Kitaev 2001):

\[
H = \mu \sum_{x=1}^{N} c_x^\dagger c_x - \sum_{x=1}^{N-1} (tc_x^\dagger c_{x+1} + |\Delta|e^{i\phi}c_x c_{x+1} + \text{h.c.})
\]

\[
\begin{align*}
\mu &= 0 \\
t &= |\Delta| \\
c_x &= \frac{1}{2} e^{-i\frac{\phi}{2}} (\gamma_{B,x} + i\gamma_{A,x})
\end{align*}
\]
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- $\mu = 0$
- $t = |\Delta|$
- $c_x = \frac{1}{2} e^{-i \frac{\phi}{2}} (\gamma_{B,x} + i \gamma_{A,x})$

Unpaired Majorana fermions at the ends!
Realization in topological insulator edges

Kane & Mele, 2005; Bernevig, Hughes, Zhang, 2006; Fu & Kane, 2008
Realization in topological insulator edges

Kane & Mele, 2005; Bernevig, Hughes, Zhang, 2006; Fu & Kane, 2008
HgTe

Realization in topological insulator edges

\[ H_{\text{edge}} = \int dx \left[ -\mu (\psi_R^\dagger \psi_R + \psi_L^\dagger \psi_L) - i\hbar v (\psi_R^\dagger \partial_x \psi_R - \psi_L^\dagger \partial_x \psi_L) \right] \]

1D and effectively ‘spinless’! Just need superconductivity...

Fu & Kane, 2008
Realization in topological insulator edges

\[ H_{\text{edge}} = \int dx \left[ -\mu (\psi_R^\dagger \psi_R + \psi_L^\dagger \psi_L) - i\hbar v (\psi_R^\dagger \partial_x \psi_R - \psi_L^\dagger \partial_x \psi_L) \right] + [\Delta \psi_R \psi_L + \text{h.c.}] \]

Fu & Kane, 2008
Realization in topological insulator edges

“Terminating” the SC wire by a magnetic gap: Majorana zero modes localised at the ends

Fu & Kane, 2008
Realization in 1D wires

1D spin-orbit-coupled wire (e.g. InAs)

\[ H = \int dx \psi^\dagger \left[ -\frac{\partial^2_x}{2m} - \mu - i\hbar v \partial_x \sigma^y \right] \psi \]

(Lutchyn, Sau, Das Sarma 2010; Oreg, Refael, von Oppen 2010)
1D spin-orbit-coupled wire (e.g. InAs)

\[ H = \int dx \psi^\dagger \left[ -\frac{\partial_x^2}{2m} - \mu - i\hbar v \partial_x \sigma^y - \frac{g \mu_B B}{2} \sigma^z \right] \psi \]

(Lutchyn, Sau, Das Sarma 2010; Oreg, Refael, von Oppen 2010)
1D spin-orbit-coupled wire (e.g. InAs)

Realization in 1D wires

s-wave SC

\[ H = \int dx \psi^\dagger \left[ -\frac{\partial^2}{2m} - \mu - i\hbar v \partial_x \sigma^y - \frac{g\mu_B B}{2} \sigma^z \right] \psi \]

\[ + (\Delta \psi^\uparrow \psi^\downarrow + h.c.) \]

Generates a 1D ‘spinless’ SC state with Majorana fermions!

(Lutchyn, Sau, Das Sarma 2010; Oreg, Refael, von Oppen 2010)
First possible experimental realization

First possible experimental realization

Great! Anything else?

Can this idea be generalised to other types of exotic (non-Abelian) zero modes?

At first glance, this is very doubtful:
- $\mathbb{Z}_8$ classification of phases of interacting fermions in 1D (Fidkowski & Kitaev, 2009; Turner, Pollmann & Berg, 2010)

Nevertheless, it is possible!
1D quantum clock model (Fendley, arXiv:1209.0472):

\[ H = -J \sum_{j=1}^{L-1} (\sigma_j^\dagger \sigma_{j+1} + H.c.) - \hbar \sum_{j=1}^{L} (\tau_j^\dagger + \tau_j) \]

\[ \sigma_j^N = 1 \quad \sigma_j^\dagger = \sigma_j^{N-1} \]
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\[ \sigma_j \tau_j = \tau_j \sigma_j e^{2\pi i/N} \]

\( N=2: \)
quantum Ising chain
\[ \sigma \equiv \sigma^\sim \]
\[ \tau \equiv \sigma^x \]
Taking a cue from Stat Mech

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\( N \neq 2: \) quantum clock

\[ \sigma |q\rangle = e^{2\pi i q/N} |q\rangle \]

\[ \tau^\dagger |q\rangle = |q + 1\rangle \]
Taking a cue from Stat Mech

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Jordan–Wigner transformation:

\[ \alpha_{2j-1} = \sigma_j \prod_{i<j} \tau_i, \quad \alpha_{2j} = -e^{i\pi/N} \tau_j \sigma_j \prod_{i<j} \tau_i \]

\( \alpha \)'s are parafermionic operators:

\[ \alpha_j^N = 1, \quad \alpha_j^\dagger = \alpha_j^{N-1}, \quad \alpha_j \alpha_k = \alpha_k \alpha_j e^{i\frac{2\pi}{N} \text{sgn}(k-j)} \]

\( N=2 \): these are Majorana fermions \( (\alpha_j^2 = 1, \quad \alpha_j^\dagger = \alpha_j) \)
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Hamiltonian after Jordan–Wigner transformation:

\[
H = J \sum_{j=1}^{L-1} \left( e^{-i \frac{\pi}{N}} \alpha_{2j}^\dagger \alpha_{2j+1} + H.c. \right) + \hbar \sum_{j=1}^{L} \left( e^{i \frac{\pi}{N}} \alpha_{2j-1}^\dagger \alpha_{2j} + H.c. \right)
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\[ + \hbar \sum_{j=1}^{L} \left( e^{i \frac{\pi}{N}} \alpha_{2j-1}^\dagger \alpha_{2j} + H.c. \right) \]

\[ J = 0 : \]
Taking a cue from Stat Mech

1D quantum clock model (Fendley, arXiv:1209.0472):

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Hamiltonian after Jordan–Wigner transformation:

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H = J \sum_{j=1}^{L-1} \left( e^{-i \frac{\pi}{N}} \alpha_{2j}^+ \alpha_{2j+1} + H.c. \right)
\]

\[
+ \hbar \sum_{j=1}^{L} \left( e^{i \frac{\pi}{N}} \alpha_{2j-1}^+ \alpha_{2j} + H.c. \right)
\]

\(\hbar = 0: \)

\[
\begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & \cdots & 2N-2 & 2N-1 & 2N
\end{array}
\]
Recall topological insulator edges

2D TI (HgTe)

s-wave SC

FM insulator

Fu & Kane, 2008
What about fractional TI edges?

We could envision playing the same game with 2D fractional topological insulators (à la Levin & Stern, 2009), but...
There are no known fractional topological insulators (yet). But could we 'fake' the same physics elsewhere?
Realization in quantum Hall edges

Counter-propagating edge modes at the boundary between $g > 0$ and $g < 0$. The sign of $g$ can be changed by stress.
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Realization in quantum Hall edges
What about fractional quantum Hall edges?

Counter-propagating *fractionalized* edge modes at the boundary between $g > 0$ and $g < 0$. The sign of $g$ can be changed by stress.
Counter-propagating *fractionalized* edge modes at the boundary between \( g > 0 \) and \( g < 0 \).

The sign of \( g \) can be changed by stress.

What about fractional quantum Hall edges?
Parafermions vs Majoranas

Upshot:

Majorana Fermions: \[ \gamma^2 = 1 \]
\[ \gamma_y \gamma_x = -\gamma_x \gamma_y \]

Parafermions:
\[ \alpha^N = 1 \]
\[ \alpha_y \alpha_x = \alpha_x \alpha_y e^{\frac{2\pi i}{N} \text{sgn}(x-y)} \]

Majoranas \(\leftrightarrow\) 1D quantum Ising model
Parafermions \(\leftrightarrow\) 1D quantum Clock/Potts model

Paul Fendley, arXiv:1209.0472
Parafermions from quantum Hall edges

A Laughlin edge state at $\nu = 1/m$ is a natural starting point since

$$[\phi(x), \phi(y)] = i \frac{\pi}{m} \text{sgn}(x - y)$$

and hence

$$e^{i\phi(x)} e^{i\phi(y)} = e^{i\phi(y)} e^{i\phi(x)} e^{i \frac{\pi}{m} \text{sgn}(y-x)}$$

for chiral edge excitations of charge $e/m$.

Now, we have two counter-propagating modes, $\phi_{R/L}$, which obey

$$[\phi_{R/L}(x), \phi_{R/L}(y)] = \pm i \frac{\pi}{m} \text{sgn}(x - y)$$

The electron fields are $\psi_{R/L} \sim e^{i m \phi_{R/L}}$
Parafermions from quantum Hall edges

Change of variables: $\phi_{R/L} = \varphi \pm \theta$

Free Hamiltonian: $\mathcal{H}_0 = \frac{m v}{2\pi} \int dx \left[ (\partial_x \varphi)^2 + (\partial_x \theta)^2 \right]$

Just need to show that a zero mode is bound at a domain wall between

$\mathcal{H}'_s(x) = \Delta(x) \psi_R \psi_L + H.c. \sim -\Delta(x) \cos(2m\varphi)$

and

$\mathcal{H}'_m(x) = \mathcal{M}(x) \psi_R^\dagger \psi_L + H.c. \sim -\mathcal{M}(x) \cos(2m\theta)$

where $\psi_{R/L} \sim e^{im\phi_{R/L}}$
Parafermionic zero mode

Assuming strong tunnelling and pairing,

\[ \varphi = \frac{\pi n_\varphi}{m} \quad \text{under the superconductors} \]

\[ \theta = \frac{\pi n_\theta}{m} \quad \text{under the SO coupled insulators} \]

\[ \alpha_j = e^{i \frac{\pi}{m} (\hat{n}_\varphi^{(j)} + \hat{n}_\theta)} \int_{x_j}^{x_j + \ell} dx \left[ e^{-i \frac{\pi}{m} (\hat{n}_\varphi^{(j)} + \hat{n}_\theta)} e^{i(\varphi + \theta)} + e^{-i \frac{\pi}{m} (\hat{n}_\varphi^{(j)} - \hat{n}_\theta)} e^{i(\varphi - \theta)} + H.c. \right] \]
Majorana zero mode

\[ \nu = 1 \quad g > 0 \]

\[ \nu = 1 \quad g < 0 \]
Parafermionic zero mode

\[ \nu = \frac{1}{m} \]

\[ g > 0 \]

\[ g < 0 \]

[Diagram showing the transition between different states and parameters]
Reminder: in the strong backscattering and pairing limit,

\[ \varphi = \frac{\pi n \varphi}{m} \] in the SC regions,

\[ \theta = \frac{\pi n \theta}{m} \] in the 'magnetic' regions

Each \( n \) has eignenvalues 0,... \( 2m-1 \), but they don't commute!

\[ [\varphi(x), \theta(y)] = i \frac{\pi}{m} \Theta(x - y) \] \( \Rightarrow \) One can fix \( n_\varphi \)'s or \( n_\theta \)'s

\( (2m)^N \) degenerate states for \( 2N \) modes \( \Rightarrow \) \( d = \sqrt{2m} \)
Majorana vs. parafermion zero-modes

Fu & Kane (2009)

2D Topological Insulator

\[ \gamma_i^2 = 1 \quad \gamma_1 \gamma_2 = -\gamma_2 \gamma_1 \]

\[ |\text{even}\rangle \equiv |0\rangle \quad |\text{odd}\rangle \equiv |1\rangle \]

Laughlin state \( \nu = 1/m \)

\[ \alpha_i^{2m} = 1 \quad \alpha_1 \alpha_2 = e^{i \frac{\pi}{m}} \alpha_2 \alpha_1 \]

\[ \alpha_i^\dagger \]

\[ |0\rangle \quad |1/m\rangle \quad |2/m\rangle \cdots \quad |2 - 1/m\rangle \]

support fractionally charged ground states!
Majorana vs. parafermion zero-modes

Fu & Kane (2009)

2D Topological Insulator

\[ \gamma_1 \ \gamma_2 \]

Fractional Josephson effect

\[ I \propto \sin \left( \frac{\Delta \phi}{2} \right) \]

Kitaev (2001)

Laughlin state \( \nu = \frac{1}{m} \)

\[ \alpha_1 \ \alpha_2 \]

Support fractionally charged ground states!

Laughlin state \( \nu = \frac{1}{m} \)

\[ I \propto \sin \left( \frac{\Delta \phi}{2m} \right) \]

Effect persists even with electronic relaxation processes
Is this zero-mode a (non-Abelian) anyon?

“It depends upon what the meaning of the word 'is' is”
   Bill Clinton

First and foremost, it is not a low-energy quasiparticle!
It is a zero mode bound to a very high-energy topological
defect - c.f. Majorana mode bound to a SC vortex or 'genons'

Instead of the usual braiding statistics, one should think of
**projective braiding statistics** (statistics up to a phase)

“Metaplectic anyons”

What are braiding statistics in 1D anyway?
Braiding statistics in 1D?

**d = 3**

Only bosons & fermions (no knots in 3+1 dimensions)

\[ \psi \rightarrow \pm \psi \]

**d = 2**

Anyons are now possible!

**d = 1**

Exchange not well defined...

...because particles inevitably “collide”
Exchanging end modes in 1D wires
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Exchanging end modes in 1D wires
Exchanging end modes in our case

Apparent problem:

- We cannot have Y-junctions: our modes live on the domain walls..
- We can still exchange them:
Exchanging end modes in our case
Exchanging end modes in our case
Exchanging end modes in our case

\[ H_{a\rightarrow b} = (t_J \alpha_2^\dagger \alpha_1' + H.c.) + (t \alpha'_1 \alpha_2' + H.c.) \]

\[ = -|t_J| \cos \left[ \frac{\pi}{m} \left( \hat{n}_\varphi^{(2)} + \hat{n}_\theta^{(3)} - \hat{n}_\varphi^{(1)} - \hat{n}_\theta^{(2)} \right) + \beta \right] \]

\[ - |t| \cos \left[ \frac{\pi}{m} \left( \hat{n}_\varphi^{(2)} - \hat{n}_\varphi^{(3)} \right) \right] \]
Exchanging end modes in our case

Integral of motion:

\[ \chi \equiv e^{i \frac{\pi}{2m} \alpha_2 \alpha_2^{\dagger} \alpha_1'} = e^{i \frac{\pi}{m} (\hat{n}_\varphi^{(1)} + \hat{n}_\theta^{(3)})} \]

Energy-minimizing condition:

\[ \hat{n}_\varphi^{(2)} + \hat{n}_\theta^{(3)} - \hat{n}_\varphi^{(1)} - \hat{n}_\theta^{(2)} = k(\beta) \in \mathbb{Z} \]
**Parafermion ZM Braiding**

**Upshot:**

\[ \alpha_1 \rightarrow e^{-i \frac{\pi}{m} k} \alpha_2 \]

\[ \alpha_2 \rightarrow e^{i \frac{\pi}{m} (1-k)} \alpha_1^\dagger \alpha_2^2 \]

\[ m = 1 \quad \text{(Majorana zero modes):} \]

\[ \gamma_1 \rightarrow \gamma_2 \]

\[ \gamma_2 \rightarrow -\gamma_1 \]
Important observation:

- If quasiparticles of both chiralities are allowed to tunnel, the braiding is not universal ⇒ Potential problem for fractional TI!

Parafermion ZM Braiding
Experimental realizations?

Is this feasible?

\[ \nu = \frac{1}{m} \quad g > 0 \]

Parafermions

\[ \nu = \frac{1}{m} \quad g < 0 \]
Experimental realizations?

Bending the Quantum Hall Effect (Matt Grayson's group, TU München)
Which Parts are Really Necessary?

*Do we need opposite g-factors (or exotic materials)?*

No! Use spin-unpolarized states, like $2/3$ or $2/5$

D. Clarke, J. Alicea, KS, in preparation
Which Parts are Really Necessary?

So do we really need the insulator at all?

Not in this case!
Which Parts are Really Necessary?

Ok, but we need the superconductor, right?

Not if we use a bilayer system, then particle <-> hole is replaced by layer <-> layer

Barkeshli, Jian, Qi, arxiv:1208.4834
Topological Quantum Computation
(Kitaev, Preskill, Freedman, Larsen, Wang)

Things we need:

• Multidimensional Hilbert space where we can encode information → Qubits
• Ability to initialise and read-out a qubit
• Unitary operations → Quantum gates
Topological Quantum Computation

\[ |0\rangle \leftrightarrow a \quad b \quad \leftrightarrow \quad |1\rangle \leftrightarrow c \quad d \]

(Hormozi, Bonesteel, et. al.)

Or, perhaps use measurements to generate braiding!
(Bonderson, Freedman, Nayak, 2009)

\[ \text{Interferometer} \]

Majorana zero modes are not universal!

- No entangling gates with braiding alone
- No phase gate

(Or, perhaps use measurements to generate braiding! (Bonderson, Freedman, Nayak, 2009))
Topological Quantum Computation

- Parafermionic zero modes are still not universal…
  - Can do entangling gates!
  - No phase gate?

Or, perhaps use measurements to generate braiding!

(Hormozi, Bonesteel, et al., 2009)
Conclusions

- Parafermionic zero modes can be localised in systems with counter-propagating fractionalised edge modes (FQHE, or fractional topological insulators)
  - Fractional Josephson effect with periodicity $4m\pi$
  - Zero-bias anomaly - similar to the Majorana case, but with fractionalised charge tunnelling
- Potential utility for quantum computing?

- Parallel work:
  - M. Cheng, arXiv:1204.6084