How to build a holographic liquid crystal?

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Motivation

- Hydrodynamics is an effective theory that still requires better understanding
- Recent holographic models provided a lot of insight into relativistic hydrodynamics
- Low viscosity over entropy density in the hydrodynamic description of heavy-ion collision is understood
- Progress in the studies of QFT anomalies in hydrodynamics
- Superfluidity is investigated via AdS/CFT
- What else we can possibly shed some light on using holography?

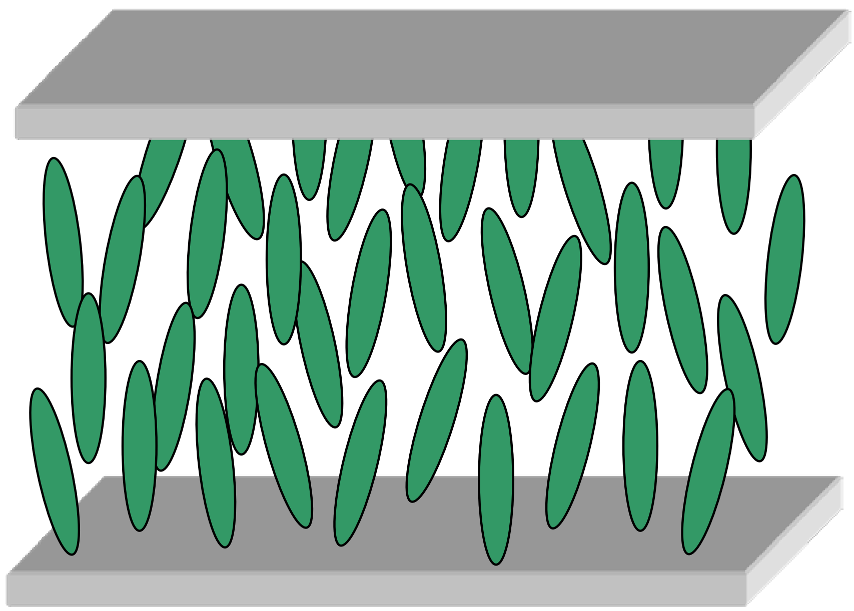
Solid → Liquid Crystal → Fluid

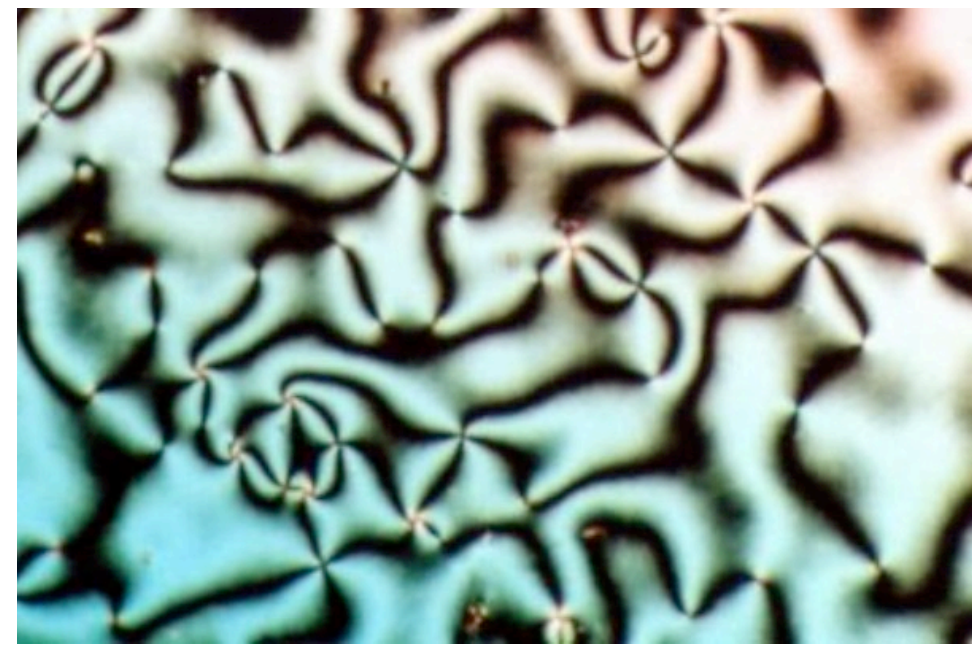
- Solid is characterized by a structural rigidity and resistance to changes of shape or volume. Described by classical mechanics.
- Fluid changes its shape under applied shear stress. Described by hydrodynamics.
- Liquid crystal is a state of matter ”in between”. It shares properties with fluids (eg. deforms under shear stress) and solids (eg. non-zero elasticity properties).
- Phase transition between isotropic un oriented phase and ordered liquid crystal phase. There is a number of phenomenological theories (eg. Maier Saupe mean field theory, Landau-De Gennes theory).

Liquid crystals

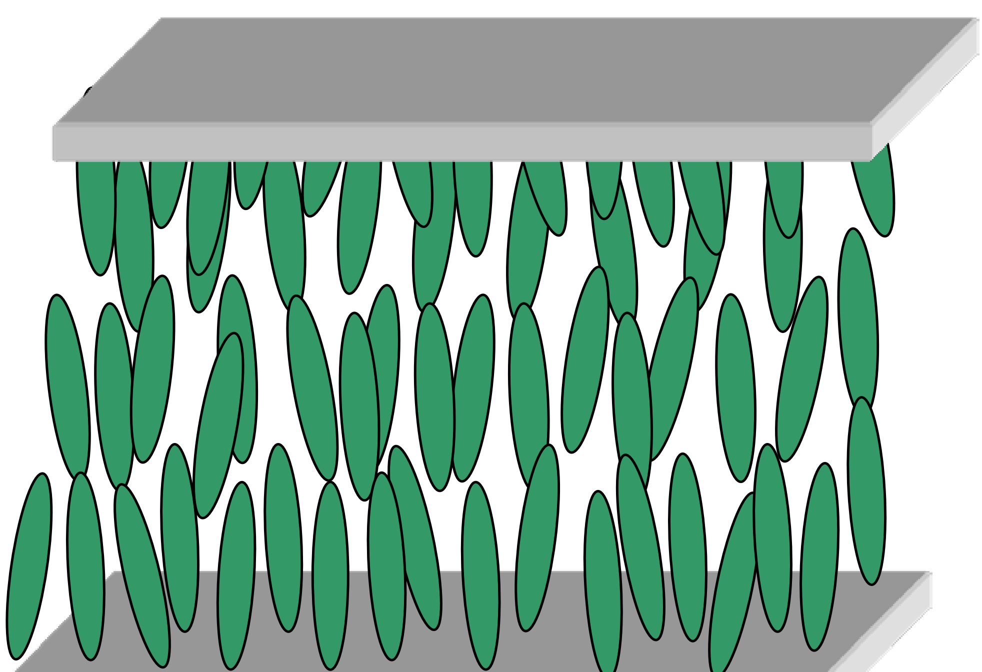
Texture - distribution of crystallographic orientations

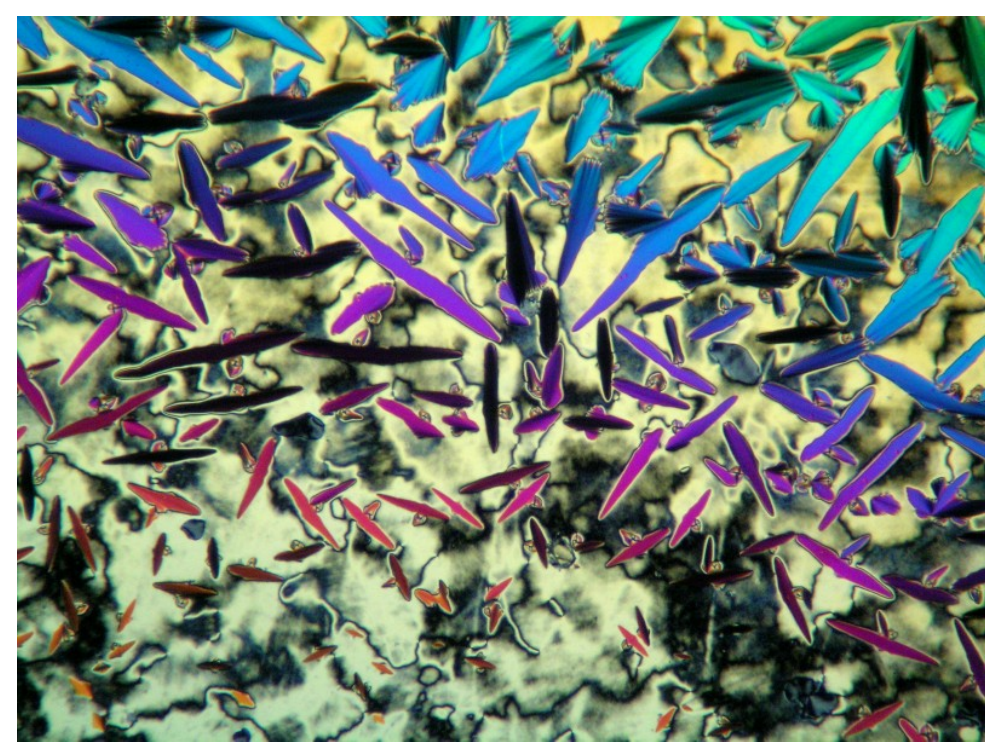
Nematic phase





Smectic A (layered) phase





Isotropic phase - hydrodynamics

Relativistic fluid with one conserved charge described by conservation laws:

$$\partial_\mu T^{\mu\nu} = 0$$
$$\partial_\mu j^\mu = 0$$

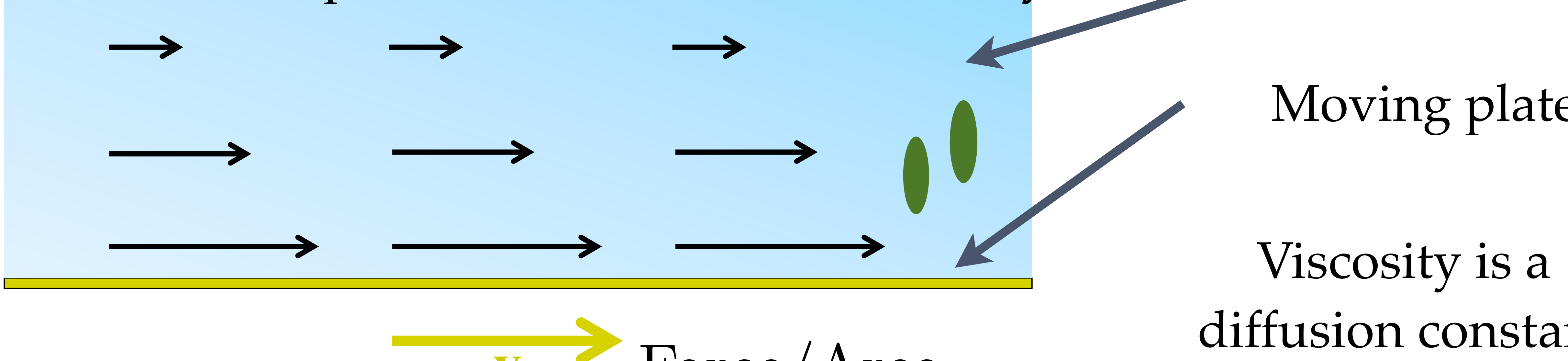
plus equations that express $T^{\mu\nu}$ and j^μ in terms of local temperature T, chemical potential μ, and fluid velocity u^μ:

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu}$$
$$j^\mu = n u^\mu + \nu^\mu$$

The definition of velocity is ambiguous beyond leading order. We fix it by imposing $u_\mu \tau^{\mu\nu} = u_\mu \nu^\mu = 0$.

Liquid crystal phase (nematic)

In the nematic phase we can measure viscosity

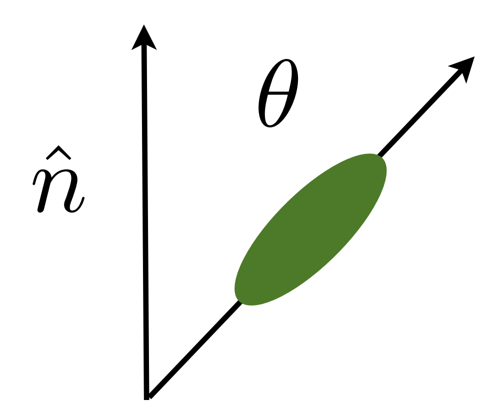


Viscosity is a diffusion constant for momentum

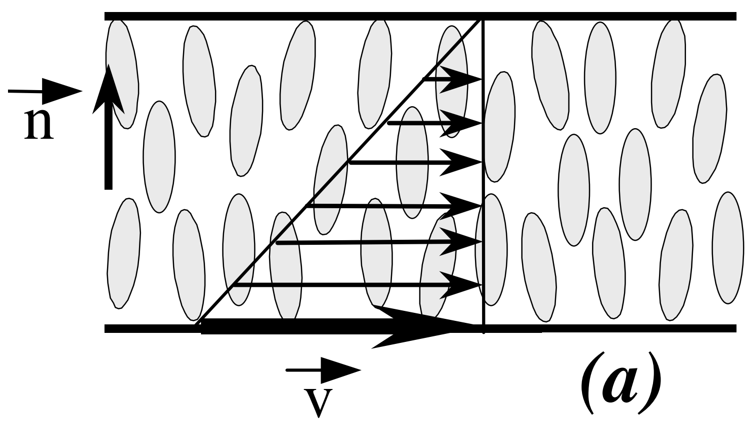
$$\text{Viscosity} = \frac{\text{Force/Area}}{\text{Unit} \quad \text{Velocity} \quad \text{Gradient}}$$

We can define a unit vector n called the director to be the average molecular orientation direction.

Scalar order parameter $\qquad S = \frac{1}{2}\left\langle 3\cos^2\theta - 1\right\rangle$



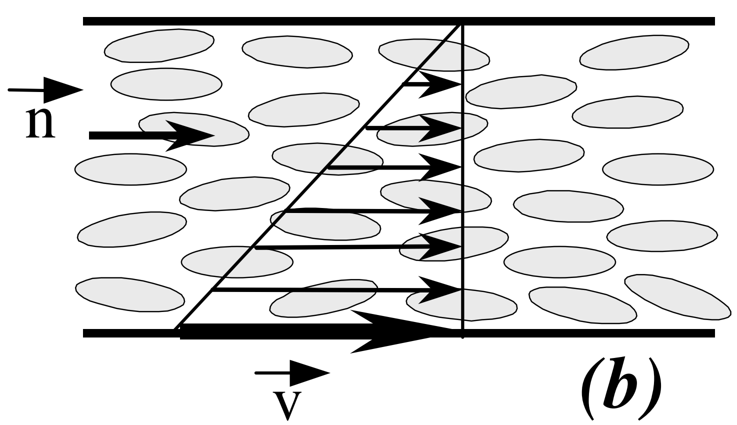
Viscosity in liquid crystals



η_1

$$n \perp v \text{ and } n \parallel \nabla v$$

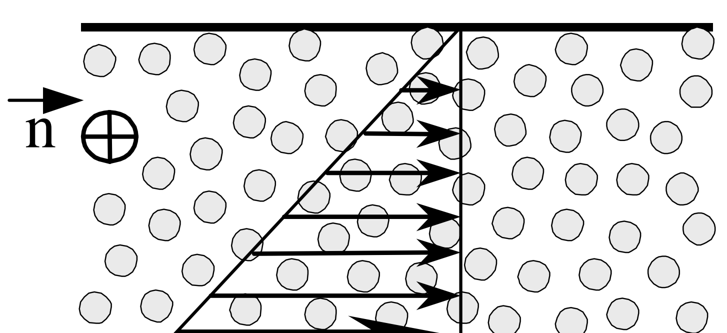
Universality?



η_2

$$n \parallel v \text{ and } n \perp \nabla v$$

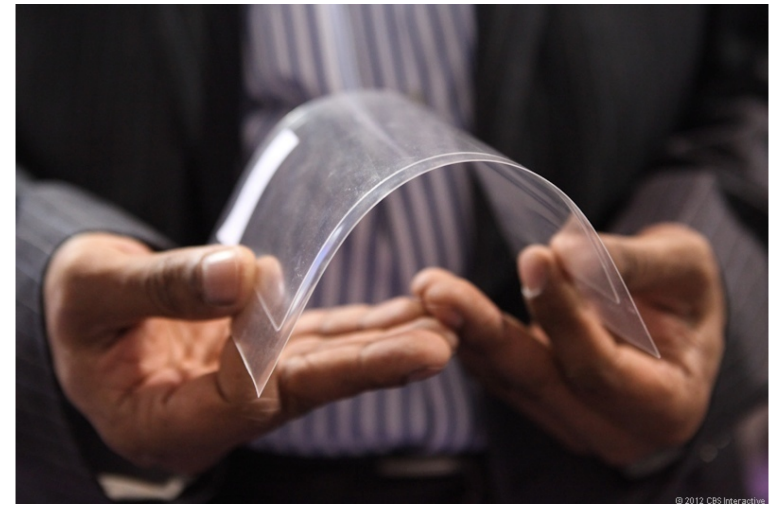
Magnitude?

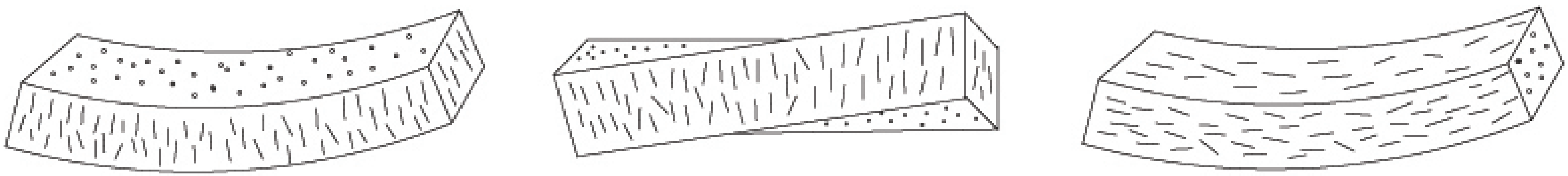


η_3

$$n \perp v \text{ and } n \perp \nabla v$$

Elasticity





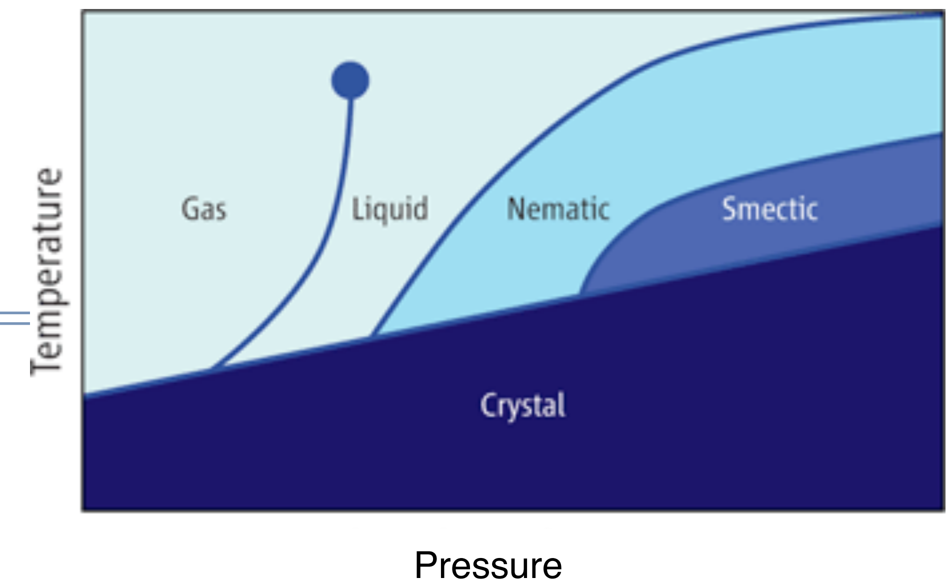
Splay Twist Bend

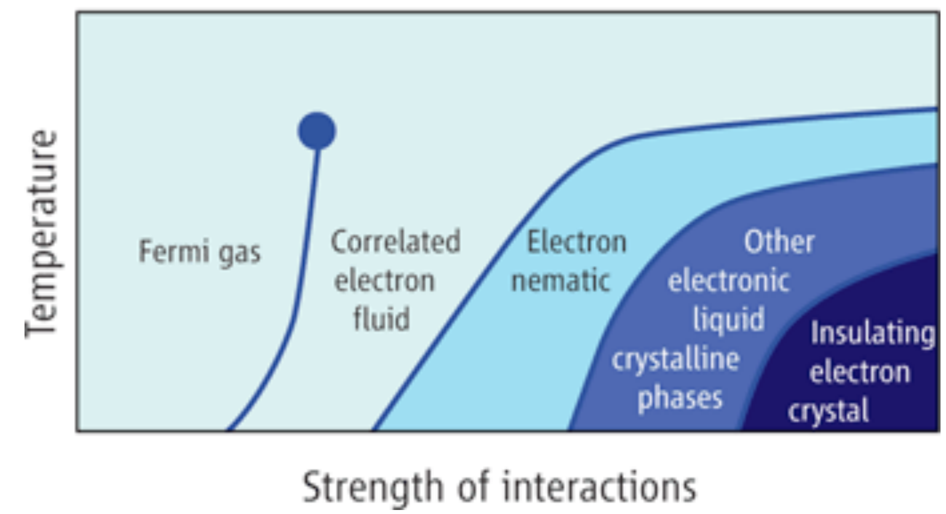
The question we would like to address here is: how much energy will it take to deform the director field?

The deformation of relative orientations away from equilibrium position will manifest itself as curvature strain. The restoring forces which arise to oppose these deformations will cause curvature stresses or torques. If these changes in molecular orientation vary slowly in space relative to the molecular distance scale, we may describe the response of the liquid crystal with a version of a continuum elastic theory.

Phase transitions

A phenomenological theory of phase transitions was established by Landau. He suggested that phase transitions were manifestation of a broken symmetry. In the simplest cases through the definition of an appropriate order parameter, Q, the macroscopic behaviour of a phase may be followed. Typically Q = 0 in the more symmetric (less ordered) phases and Q ≠ 0 in the less symmetric (more ordered) phases.





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The theory, though originally introduced to describe continuous phase transitions in solids, appeared (as we know today) to correctly account for symmetry change observed at majority of continuous and first order phase transitions.

The Landau theory generaly fails in the temperature adjacent to the transition, in which the behaviour of a system is dominated by fluctuations

Landau thery tailored to describe liquid crystals

The LT appears as a necessary intermediate step (and also as a tool) in constructing generalized theory that include fluctuations - it is known as Landau-Ginzburg-Wilson theory:

Majority of insight into physics of liquid crystals relies on the implementation of Landau's ideas (physics of defects, elasticity, hydrodynamics, relaxation, etc.), and the approach has been rather successful.

A major breakthrough in including liquid crystals in the Landau reasoning is due to de Gennes. He realized that instead of the scalar order parameter one should use a tensorial quantity. For the nematic phase it reads:

$$Q_{\alpha\beta} = \frac{S}{2}\left(3\hat{n}_\alpha \hat{n}_\beta - \delta_{\alpha\beta}\right)$$

Landau-de Gennes theory

Once the appropriate tensor order parameter of the system is identified we can assume, in a spirit of Landau theories, that the free energy density $\mathcal{F}$ is an analytic function of the order parameter. The Landau-de Gennes theory of the nematic-isotropic transition starts by assuming that a spatially invariant dimensionless, order parameter is small in the nematic phase close to the transition point. The difference in free energy density (per unit volume) of the two phases it thus expanded in powers of the order parameter. Since the free energy must be invariant under rigid rotations, all terms of the expansion must be scalar functions of the tensor.

$$\mathcal{F}_{LdG}(P,T,Q_{\alpha\beta}) = F_0 + \alpha_F Q_{\alpha\beta}Q_{\beta\alpha} + \beta_F Q_{\alpha\beta}Q_{\beta\gamma}Q_{\gamma\alpha} + \gamma_F Q_{\alpha\beta}Q_{\beta\alpha}Q_{\gamma\rho}Q_{\rho\gamma}$$

The most general form capturing the uniaxial phase is typically truncated at fourth order.

Holography=spherical cow approx.

- Strongly coupled QFTs are difficult to study
- Use $\mathcal{N}=4$ SYM. as a playground
- Apply holography, treat as a toy-model (spherical cow)
- Aim at universal properties, understand qualitative features
- It was particularly successful in the context of hydrodynamics. Quantitative understanding why viscosity over entropy density is small at RHIC. Useful in the context of anomalies due to their coupling independent nature. Natural language to investigate two-fluid models (superfluidity). Maybe the same tool can be used to better understand liquid crystals.

AdS/CFT correspondence

$\mathcal{N} = 4$ supersymmetric Yang-Mills theory in $\mathbb{R}^4$	$\equiv$	IIB superstrings in curved $AdS_5 \times S^5$ 10D spacetime

Strongly coupled gauge fields equivalent to weakly coupled strings

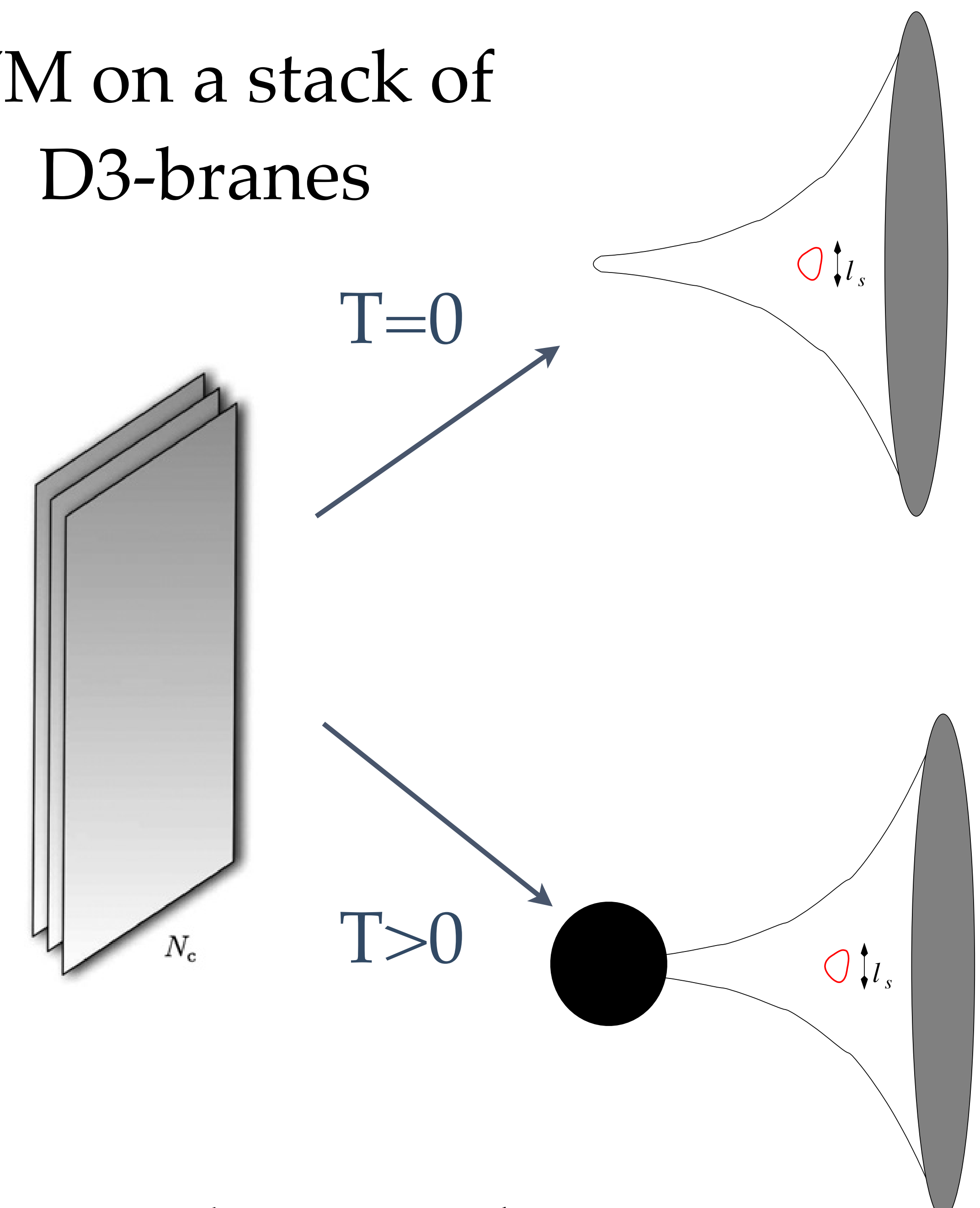
Potentially very useful tool but we need some practical implementation of the above equivalence:

$$e^{-S_{sugra}} \approx Z_{string} = Z_{CFT} \equiv \langle e^{\int d^4x \phi_0(x) \mathcal{O}(x)} \rangle$$

The dictionary	
Gauge side	String side
$\mathrm{Tr} F_{\mu\nu} F^{\mu\nu}$	dilaton
$T_{\mu\nu}$	graviton $g_{\mu\nu}$
dimension of operator	mass of the field
...	...

AdS/CFT at finite temperature

SYM on a stack of D3-branes



Anti-de Sitter geometry

$$ds^2 = \left(1 + \frac{L^4}{r^4}\right)^{-\frac{1}{2}} dx^\mu dx_\mu + \left(1 + \frac{L^4}{r^4}\right)^{\frac{1}{2}} (dr^2 + r^2 d\Omega_5^2)$$

AdS Schwarzschild geometry

$$ds^2 = \frac{r^2}{L^2}(-f(r)dt^2 + dx^i dx_i) + \frac{L^2}{f(r)r^2} dr^2 + L^2 d\Omega_5^2$$

where $f(r) = 1 - \frac{R^4}{r^4}$

Top-down vs. bottom-up

Spin 2 Lagrangian

Complicated plus various consistency issues

$$S = \int d^{d+1}x \sqrt{-g}\Big[-\frac{1}{2}(\nabla_\mu \varphi_{\alpha\beta})^2 + (\nabla_\mu \varphi^{\mu\alpha})^2 + \frac{1}{2}(\nabla_\mu \varphi)^2 - \nabla_\mu \varphi^{\mu\nu}\nabla_\nu \varphi - \frac{1}{2}m_\varphi^2(\varphi_{\mu\nu}\varphi^{\mu\nu} - \varphi^2) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_A^2 A_\mu^2 + R_{\mu\nu\alpha\beta}\varphi^{\mu\alpha}\varphi^{\nu\beta} - \frac{1}{2(d+1)}R\varphi^2 - \frac{\alpha}{2}A^2(\varphi_{\mu\nu}\varphi^{\mu\nu} - \varphi^2)\Big],$$

We need a quadratic coupling between the fields. A cubic coupling of the form $a_1 A_\mu A_\nu \varphi^{\mu\nu} + a_2 A^2 \varphi$ would not allow for spontaneous hairy black holes. The spin 1 field would just act as a source for the massive spin 2 field and all charged black holes would have a secondary hair. Instead we want to have a hair appearing below a critical temperature. The situation would be similar to trying to build a superconductor using a dilatonic coupling $e^\phi F^2$. It would not work since the electric field would source the dilaton.

Equations of motion

Our ansatz:

$$\varphi_{\mu\nu} = \mathrm{diag}(0, 0, \varphi_{x_1x_1}(z), \varphi_{x_2x_2}(z), \varphi_{x_3x_3}(z))$$
$$C_\mu dx^\mu = \phi(z)dt$$

where

$$\varphi_{ij} = \frac{l^2}{z^2}\psi(z)\sqrt{\frac{d-1}{d-2}}\left(\delta_i^1\delta_j^1 - \frac{1}{d-1}\right)$$

We get the following EOMs

$$\psi'' + \left(\frac{f'}{f} - \frac{d-1}{z}\right)\psi' + \left(+\frac{\alpha\phi^2}{f^2} - \frac{m_\varphi^2 l^2}{z^2 f}\right)\psi = 0,$$

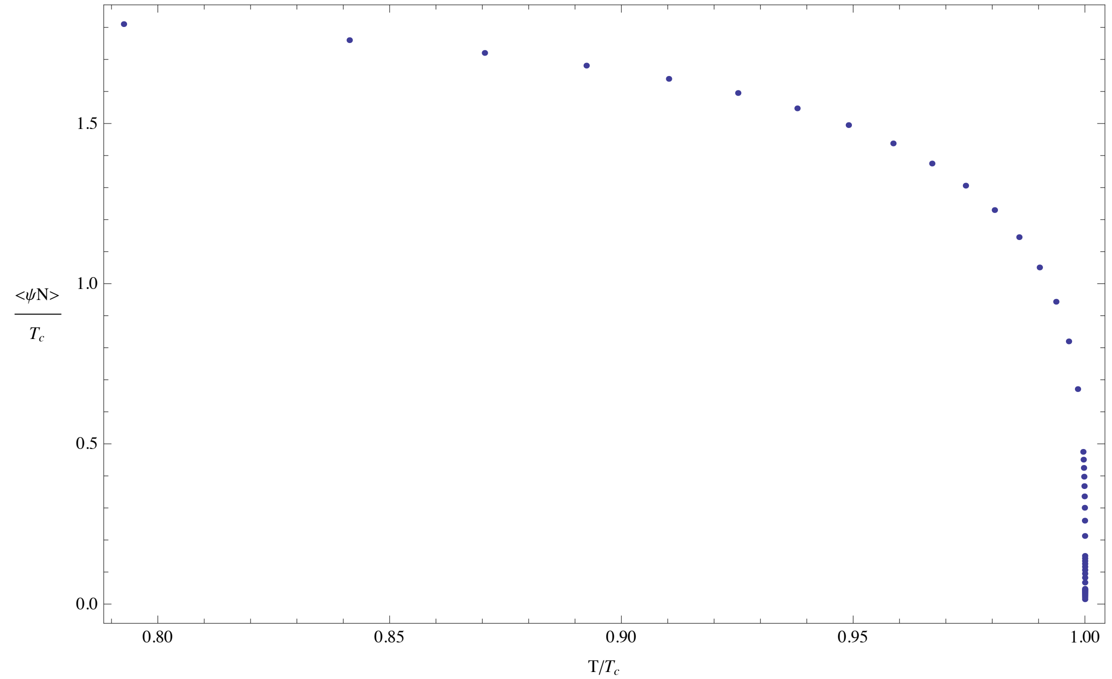
$$\phi'' - \frac{d-3}{z}\phi' + \left(-\frac{\alpha l^2}{z^2 f}\psi^2 - \frac{m_A^2 l^2}{z^2 f}\right)\phi = 0.$$

Looking for an instability

The dynamical fields ψ, A_1 obey hypergeometric equations when there is no coupling $\alpha = 0$. Their fall-off is

$$\psi(z) = \psi_D z^{\frac{d}{2} - \sqrt{\frac{d^2}{4} + m_\varphi^2 l^2}} + \psi_N z^{\frac{d}{2} + \sqrt{\frac{d^2}{4} + m_\varphi^2 l^2}} + \dots$$
$$\phi(z) = \mu z^{\frac{d-2}{2} - \sqrt{\frac{(d-2)^2}{4} + m_A^2 l^2}} + \phi_N z^{\frac{d-2}{2} + \sqrt{\frac{(d-2)^2}{4} + m_A^2 l^2}} + \dots$$

We impose regularity at the horizon and Dirichlet condition at the boundary



Instability!

We have spin-2 condensate

Partition function

The Euclidean black hole solution is interpreted as a saddle-point in the path integral corresponding to the thermal partition function. The supergravity action evaluated for this solution is interpreted as the leading contribution to the free energy. Free energy for a stack of D3-branes

$$F = TS_{sugra} = -\frac{\pi^2}{8} N_c^2 T^4$$

To prove the instability for the spin-2 system we need to calculate the partition functions for the isotropic phase and for the phase with the condensate and show that the system lowers the free energy by developing the condensate.

Free energy analysis

For simplicity, we will look at d = 4 but we will keep the masses general.

$$S_{tot} = S + c\int d^d x\,\sqrt{-\gamma}\,A_\mu A^\mu$$

The variation of the total action reduces for our ansatz to

$$\delta S_{tot} = \int d^5x\,\sqrt{-g}\,\left(E_{\mu\nu}\delta\varphi^{\mu\nu} + E_\mu \delta A^\mu\right) - \int dt d^3x\left[\frac{l}{z}\,\phi'\delta\phi + \frac{2c\,l^2}{z^2\sqrt{f}}\,\phi\,\delta\phi - \frac{l^3 f}{z^3}\psi'\delta\psi\right]_{z=0}$$

We fix c in order to have a well define Dirichlet problem

$$\delta S_{tot} = \int d^5x\,\sqrt{-g}\,\left(E_{\mu\nu}\delta\varphi^{\mu\nu} + E_\mu \delta A^\mu\right) + \int dt d^3x\,(J + \langle O\rangle\delta\mu)$$

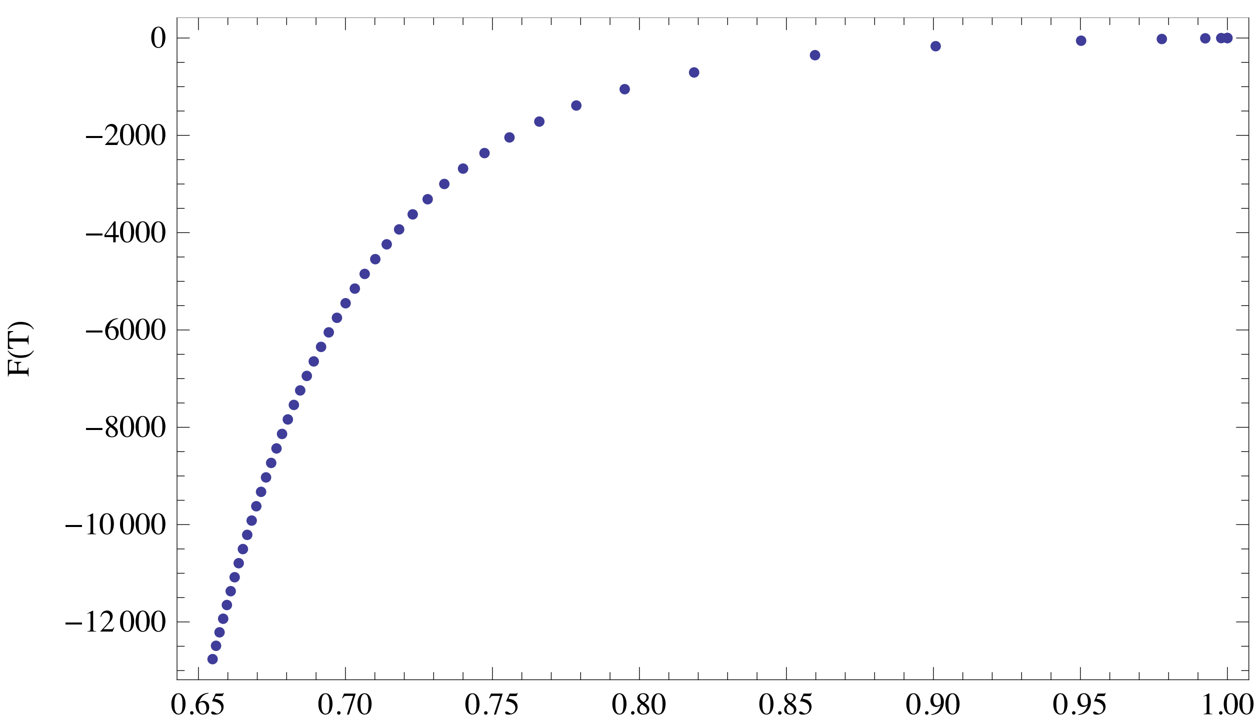
We pass to Euclidean signature and evaluate the action

$$F(\psi\neq 0) = S_E/\beta = V\int_0^{z_h} dz\,\frac{\alpha l^3\phi^2\psi^2}{2f(z)z^3} + V\left[\frac{l\phi\phi'}{2z} + \frac{c\,l^2\phi^2}{z^2\sqrt{f}} + \frac{l^3 f\psi\psi'}{2z^3}\right]_{z=0}$$

An instability confirmed

Finally we can write down an expression for the difference of the free energies in the normal and condensed phase

$$F(\psi \neq 0) - F(\psi = 0) = V\left[\int_0^{\beta/\pi} dz\, \frac{\alpha l^3 \phi^2 \psi^2}{2f(z)z^3} - \frac{1}{2}(\langle O\rangle - \langle O\rangle_n)\mu\right].$$



Difference of free energy between the uniaxial nematic phase and the normal phase as a function of the temperature T = 1/β. The uniaxial nematic phase is favored.

Future directions

- Check if the model gives the equations of nematodynamics
- Use an anisotropic ansatz for the metric and study backreaction
- Calculate viscosity coefficients
- Investigate the non-relativistic limit