Duality and "Instead-of-Confinement" Mechanism in Supersymmetric QCD

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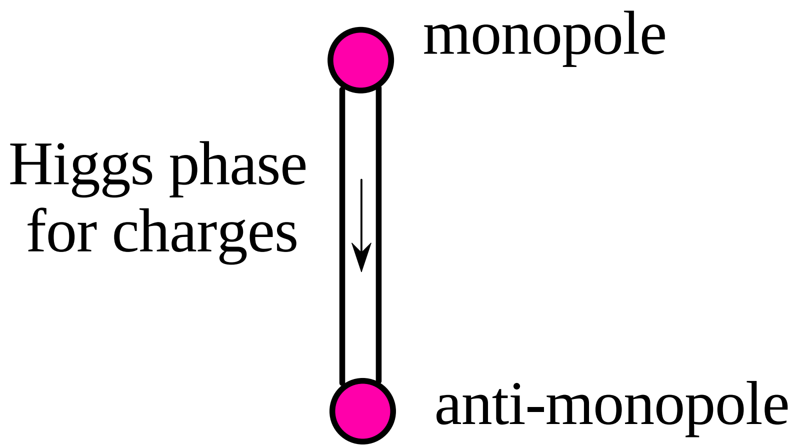
1 Introduction

Nambu, Mandelstam and 't Hooft 1970's:

Confinement is a dual Meissner effect upon condensation of monopoles.

Ordinary Meissner effect:

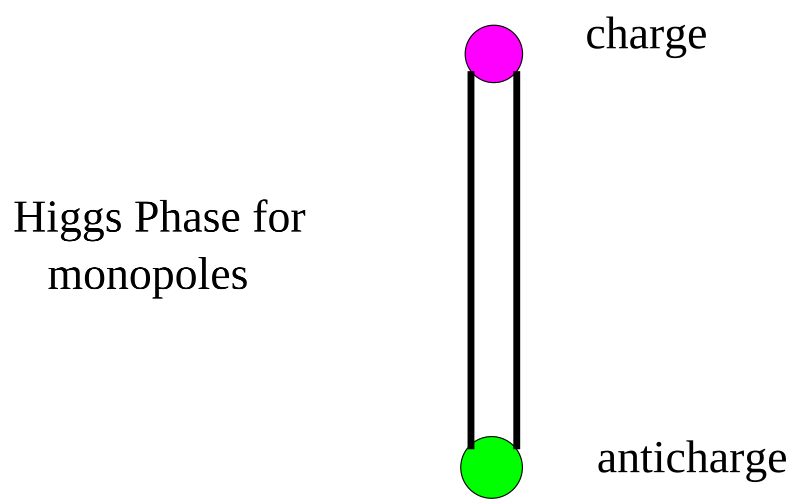
Electric charges condense $\rightarrow$ magnetic Abrikosov-Nielsen-Olesen flux tubes (strings) are formed $\rightarrow$ monopoles are confined



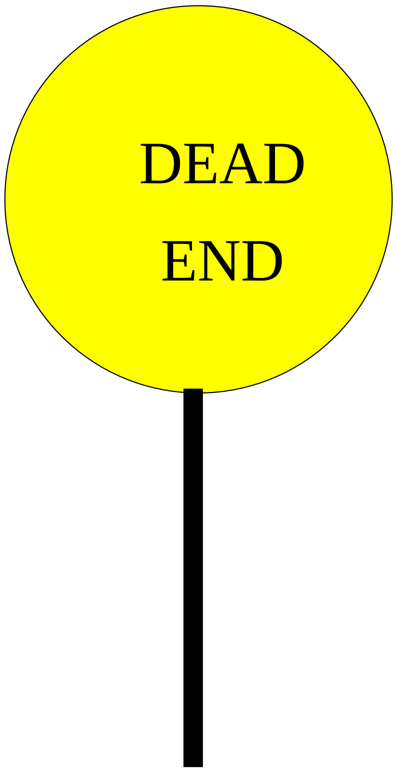
Nambu, Mandelstam and 't Hooft:

Dual Meissner effect:

Monopoles condense → electric Abrikosov-Nielsen-Olesen flux tubes are formed → electric charges are confined



No progress for many years...



QCD:

- No monopoles
- No confining strings
- Strong coupling

Seiberg and Witten 1994 : Confinement in $\mathcal{N}=2$ QCD

Cascade gauge symmetry breaking:

- SU(N)$\rightarrow$ U(1)$^{N-1}$ — VEV's of adjoint scalars
- U(1)$^{N-1}$ $\rightarrow 0$ (or discrete subgroup) — VEV's of monopoles

At the last stage Abelian Abrikosov-Nielsen-Olesen flux tubes are formed.

"Wrong" confinement: Abelian

In both QCD or $\mathcal{N}=1$ supersymmetric QCD there are
no adjoint fields $\rightarrow$ no Abelianization

Problem: Cannot decouple adjoint fields in monopole vacua

Masses in low energy U(1)$^{N-1}$ theory are $\sim \sqrt{\mu\Lambda}$

To have weak coupling we need $\mu \ll \Lambda$

Non-Abelian setup:

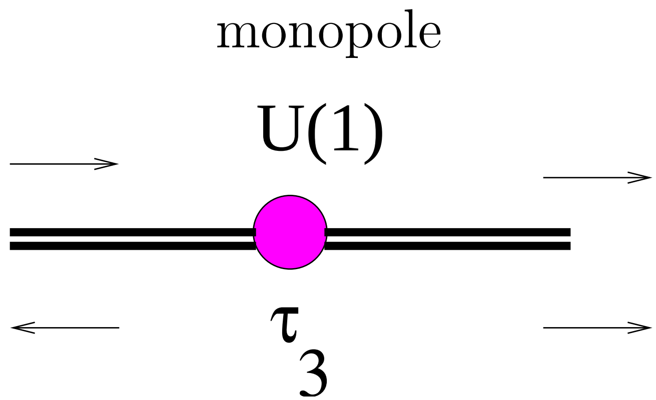
$\mathcal{N} = 2$ QCD with $U(N)$ gauge group and $N_f > N$ fundamental flavors (quarks), $N + 1 < N_f < \frac{3}{2}N$.
deformed by mass term for adjoint matter μ.

Quark vacuum

Scalar quarks condense with VEV's $\sim \sqrt{\xi}$, $\xi \sim \mu m$.
Large $\xi \to$ theory is at weak coupling

Non-Abelian strings confine monopoles

Example in $U(2)$



What happens if we reduce ξ and go to strong coupling?

Two steps:

- Reduce ξ at small μ (Near $\mathcal{N} = 2$ limit)
- Increase μ.

2 r Vacua at large ξ

$\mathcal{N}=2$ QCD with gauge group $U(N)=SU(N)\times U(1)$ and
N_f flavors of fundamental matter – quarks

The field content:
U(1) gauge field A_μ
SU(N) gauge field A^a_μ, $a=1,...,N^2-1$
complex scalar fields a, and a^a
+ fermions

Complex scalar fields q^{kA} and $\tilde{q}_{Ak}$ (squarks) + fermions
$k=1,...,N$ is the color index, A is the flavor index, $A=1,...,N_f$

Mass term for the adjoint chiral field

$$\mathcal{W}_{\rm br}=\mu\,{\rm Tr}\,\Phi^2,$$

where

$$\Phi=\frac{1}{2}\,\mathcal{A}+T^a\,\mathcal{A}^a.$$

r Vacuum at large $\xi \sim \mu m$

First r (s)quarks condense, $0 \leq r \leq N$

F-terms in the potential

$$\left| \tilde{q}_A q^A + \sqrt{2}\, \frac{\partial \mathcal{W}_{\rm br}}{\partial \Phi} \right|^2 , \qquad \left| (\sqrt{2}\Phi + m_A) q^A \right|^2$$

Adjoint fields:

$$\langle {\rm diag}\Phi \rangle \approx -\frac{1}{\sqrt{2}} \left[m_1, ..., m_r, 0, ..., 0 \right] ,$$

For $r = N$ $U(N)$ gauge group is Higgsed

For $r < N$ classically unbroken gauge group

$U(N-r) \quad \rightarrow \quad U(1)^{N-r} \quad \rightarrow \quad U(1)$

adjoints $\qquad (N-r-1)$ monopoles

$r = N$ Vacuum

Adjoint VEVs:

$$\langle \frac{1}{2}\, a + T^a\, a^a \rangle = -\frac{1}{\sqrt{2}} \begin{pmatrix} m_1 & \ldots & 0 \\ \ldots & \ldots & \ldots \\ 0 & \ldots & m_N \end{pmatrix},$$

Quark VEV's

$$\langle q^{kA} \rangle = \langle \bar{\tilde{q}}^{kA} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{\xi_1} & \ldots & 0 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & \sqrt{\xi_N} & 0 & \ldots & 0 \end{pmatrix},$$

$$k = 1, ..., N\,, \qquad A = 1, ..., N_f\,,$$

where

$$\xi_P \approx 2\, \mu m_P, \qquad P = 1, ..., N,$$

In the equal mass limit $\mathrm{U}(N)_{\text{gauge}} \times \mathrm{SU}(N_f)_{\text{flavor}}$
is broken down to

$$\mathrm{SU}(N)_{C+F} \times \mathrm{SU}(\tilde{N})_F \times \mathrm{U}(1)\,,$$

where $\tilde{N} = N_f - N$.

Quarks and gauge fields fill following representations of the global group:

$$(1,1) \quad (N^2-1,1) \quad (\bar{N},\tilde{N}) \quad (N,\bar{\tilde{N}})$$

3 r-Duality at small ξ

Small ξ

$$|\sqrt{\xi_P}| \ll \Lambda_{\mathcal{N}=2}\,, \qquad |m_A - m_B| \ll \Lambda_{\mathcal{N}=2}$$

Use Seiberg-Witten curve on the Coulomb branch at $\mu = 0$

- r-dual theory with gauge group

$$U(\nu) \times U(1)^{N-\nu}, \qquad \nu = \begin{cases} r, & r \leq \frac{N_f}{2} \\ N_f - r, & r > \frac{N_f}{2}, \end{cases}$$

  and N_f light dyons
  (with *weight*-like electric charges)

- non-Abelian strings which

  still confine **monopoles**

  (with *root*-like electric charges)

The non-Abelian gauge factor $U(\nu)$ is not broken by adjoint VEV's in the equal mass limit because this theory is infrared-free and stays at weak coupling.

Our case $r = N$ vacuum, so

$$\nu = N_f - N = \tilde{N}.$$

Argyres Plesser Seiberg:
$SU(\nu) \times U(1)^{(N-\nu)}$ was identified at roots of Higgs branches in SU(N) theory with massless quarks and $\mu = 0$.

$\nu = \tilde{N}$ Baryonic branch

$\nu < \tilde{N}$ Non-baryonic branches

Vacuum

Dyons

$$\langle D^{lA}\rangle = \langle \bar{\tilde{D}}^{lA}\rangle = \frac{1}{\sqrt{2}}\left(\begin{array}{cccccc} 0 & \ldots & 0 & \sqrt{\xi_1} & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & 0 & 0 & \ldots & \sqrt{\xi_{\tilde{N}}} \end{array}\right),$$

$$\langle D^J\rangle = \langle \bar{\tilde{D}}^J\rangle = \sqrt{\frac{\xi_J}{2}}, \qquad J = \tilde{N}+1, ..., N\,.$$

"Vacuum leap"

$$(1, ..., N)_{\sqrt{\xi}\gg\Lambda_{\mathcal{N}=2}} \to (N+1, ..., N_f,\ (\tilde{N}+1), ..., N)_{\sqrt{\xi}\ll\Lambda_{\mathcal{N}=2}}\,.$$

$$\xi_P = -2\sqrt{2}\,\mu\, e_P, \qquad P = 1, ..., N,$$

where e_P are the double roots of the Seiberg–Witten curve,

$$y^2 = \prod_{P=1}^{N} (x - \phi_P)^2 - 4\left(\frac{\Lambda}{\sqrt{2}}\right)^{N-\tilde{N}} \prod_{A=1}^{N_f}\left(x + \frac{m_A}{\sqrt{2}}\right) = \prod_{P=1}^{N}(x - e_P)^2$$

At small masses the double roots of the Seiberg–Witten curve are

$$\sqrt{2}e_I = -m_{I+N}, \qquad \sqrt{2}e_J = \Lambda_{\mathcal{N}=2}\, \exp\left(\frac{2\pi i}{N-\tilde{N}}J\right)$$

for $\tilde{N} < N - 1$, where

$$I = 1, ..., \tilde{N} \;\; \text{and} \;\; J = \tilde{N}+1, ..., N\,.$$

The $\tilde{N}$ first roots are determined by the masses of the last $\tilde{N}$ quarks — a reflection of the fact that the non-Abelian sector of the dual theory is infrared-free and is at weak coupling in the domain.

4 "Instead-of-confinement" mechanism

In the equal mass limit the global group is broken to

$$\mathrm{SU}(N)_F \times \mathrm{SU}(\tilde{N})_{C+F} \times \mathrm{U}(1)$$

Now dyons and dual gauge fields fill following representations of the global group:

$$\text{small } \ \xi : \qquad (1,1) \quad (1, \tilde{N}^2 - 1) \quad (\bar{N}, \tilde{N}) \quad (N, \bar{\tilde{N}})$$

Recall that quarks and gauge bosons of the original theory are in

$$\text{large } \ \xi : \qquad (1,1) \quad (N^2 - 1, 1) \quad (\bar{N}, \tilde{N}) \quad (N, \bar{\tilde{N}})$$

$(N^2 - 1)$ of $\mathrm{SU}(N)$ and $(\tilde{N}^2 - 1)$ of $\mathrm{SU}(\tilde{N})$

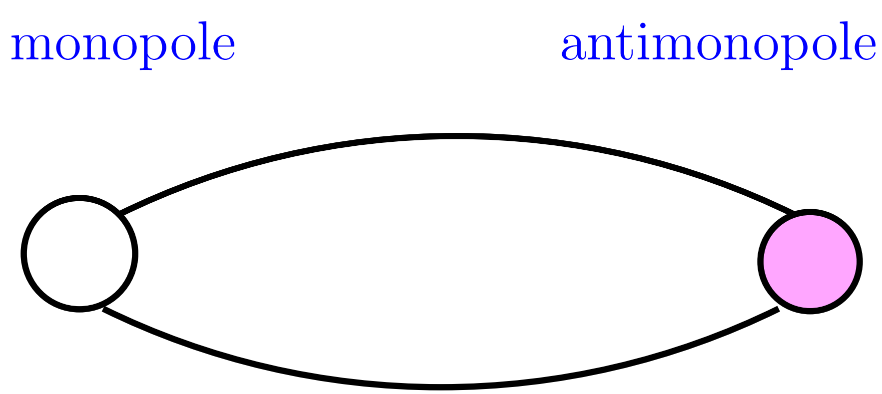
are different states

CROSSOVER

What is the physical nature of $(N^2 - 1)$ adjoints at small ξ?

- Higgs-screened quarks and gauge bosons decay into monopole-antimonopole pairs at CMS.

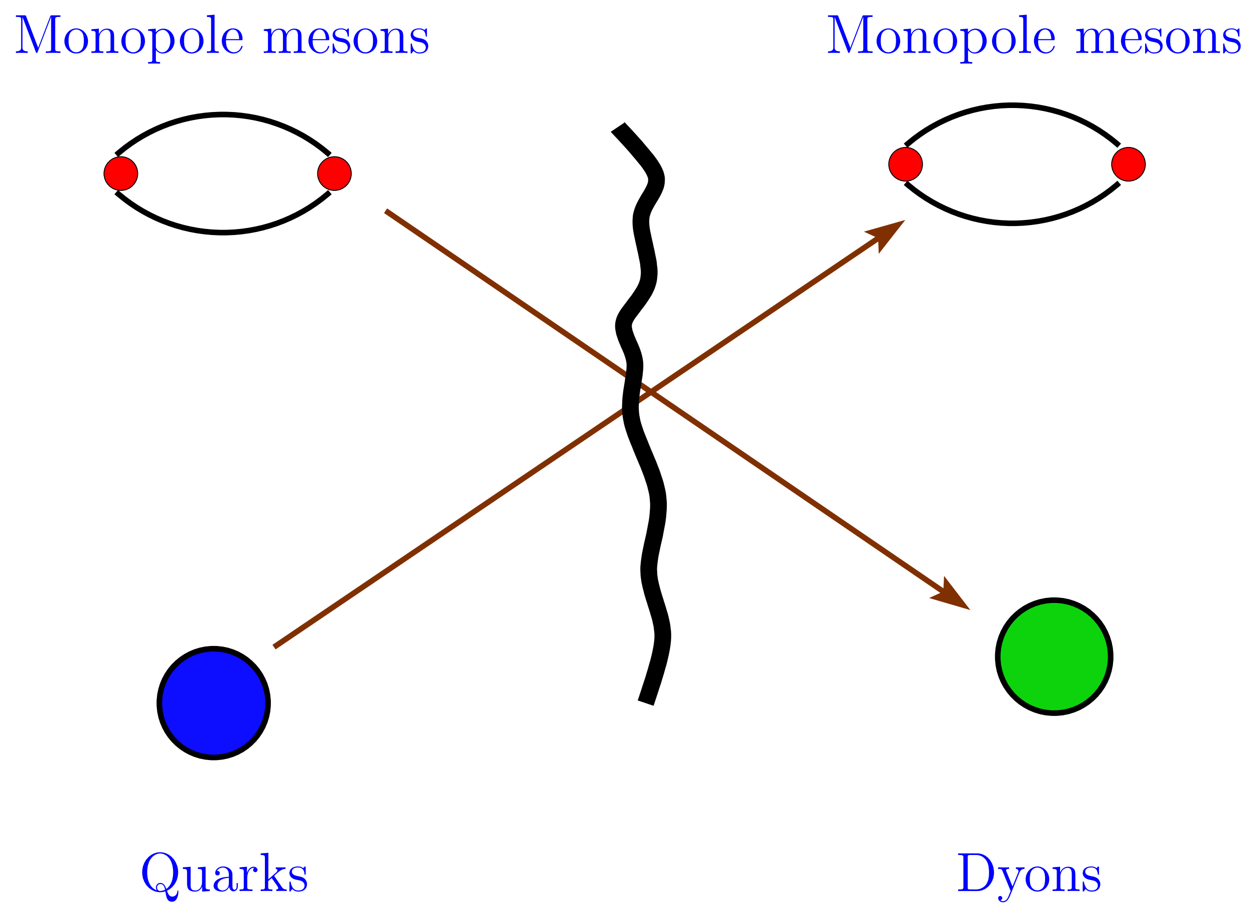
At $\xi \neq 0$ monopoles are confined and cannot move apart



In the region of small ξ $(N^2 - 1)$ of SU(N) are stringy mesons formed by pairs of monopoles and antimonopoles connected by two strings

Crossover

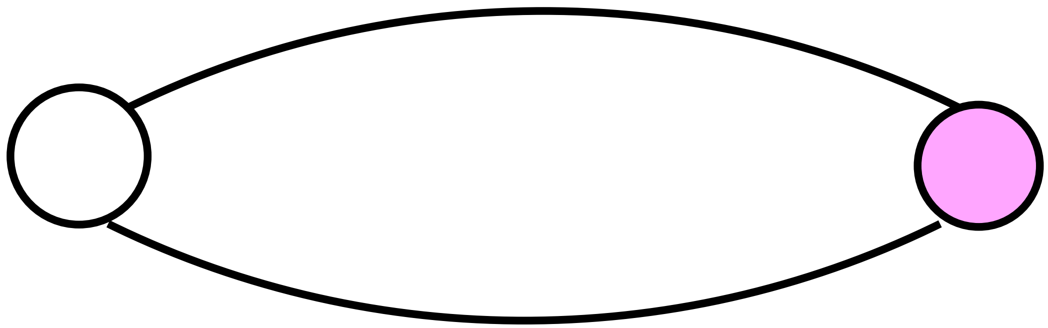
Original theory, $\xi \gg \Lambda^2$ Dual theory, $\xi \ll \Lambda^2$



Screened quarks evolve into monopole-antimonopole mesons

Meson

Constituent quark = monopole



Question: Does these monopole-antimonopole mesons looks like mesons in QCD?

- Correct flavor quantum numbers (adjoint + singlet)
- Lie on Regge tragectories

5 r-Duality at large μ

We need:

$$|\mu| \gg |\sqrt{\xi}|$$

and

$$|\sqrt{\xi_P}| \ll \tilde{\Lambda}_{\mathcal{N}=1}, \qquad P = 1, ..., \tilde{N}\,,$$

where

$$\tilde{\Lambda}^{N-2\tilde{N}}_{\mathcal{N}=1} = \frac{\Lambda^{N-\tilde{N}}_{\mathcal{N}=2}}{\mu^{\tilde{N}}}\,.$$

Infrared-free dual theory is weakly coupled

$$\xi_P = -2\sqrt{2}\,\mu\, e_P, \qquad P = 1, ..., N,$$

First $\tilde{N}$ roots are given by quark masses

$$\sqrt{2}e_I = -m_{I+N},$$

while others are of order of $\Lambda_{\mathcal{N}=2}$.

$\tilde{N}$ non-Abelian dyons have VEV's $\sim \sqrt{\mu m}$

$(N-\tilde{N})$ Abelian dyons have VEV's $\sim \sqrt{\mu \Lambda_{\mathcal{N}=2}}$

Take m_A small.

$\mathrm{U}(1)^{N-\tilde{N}}$ factors of the dual gauge group $\mathrm{U}(\tilde{N}) \times U(1)^{N-\tilde{N}}$ decouple together with Abelian dyons D_J.

We are left at large μ with

$$U(\tilde{N})$$

gauge group and non-Abelian dyons D^{lA}, $l = 1, ..., \tilde{N}$, $A = 1, ..., N_f$

Superpotential

$$\mathcal{W} = -\frac{1}{2\mu}\,(\tilde{D}_A D^B)(\tilde{D}_B D^A) + m_A\,(\tilde{D}_A D^A)$$

Monopole confinement and "instead-of-confinement" phase for quarks/gauge bosons survive.

6 Conclusions

For $r = N$-vacuum at small ξ we have:

- Instead of Seiberg-Witten scenario of quark confinement based on condensation of monopoles we have different scenario:

  "Instead-of-confinement" phase

  Higgs-screened quarks and gauge bosons transform into monopole-antimonopole stringy mesons.

- r-duality survives decoupling of the adjoint matter at large μ

- Large-μ r-dual theory coincides with Seiberg's dual.

7 Generalized Seiberg's duality

Seiberg's duality is formulated for $r = 0$ (monopole) vacua. All other $r \neq 0$ vacua are runaway vacua at $\mu = \infty$

Original theory: integrate adjoint fields at large μ

$$-\frac{1}{2\mu}\,(\tilde{q}_A q^B)(\tilde{q}_B q^A) + m_A\,(\tilde{q}_A q^A)$$

Carlino, Konishi, Murayama, 2000

Generalized Seiberg's dual: $\mathrm{U}(\tilde{N})$ gauge theory with superpotential

$$\mathcal{W}_S = -\frac{\kappa^2}{2\mu}\,\mathrm{Tr}\,(M^2) + \kappa\, m_A\, M_A^A + \tilde{h}_{Al} h^{lB}\, M_B^A,$$

where M_A^B is the Seiberg neutral mesonic M field defined as

$$(\tilde{q}_A q^B) = \kappa\, M_A^B$$

There is a classical vacuum

$$M_A = \frac{\mu}{\kappa}\, m_A, \qquad (\tilde{h}h)_A = 0, \qquad A = 1, ..., N\,,$$
$$(\tilde{h}h)_A = -\kappa\, m_A, \qquad M_A = 0, \qquad A = (N+1), ..., N_f\,,$$

Integrating out the M fields we get

$$\mathcal{W}_S^{\rm LE} = \frac{\mu}{2\kappa^2}\,(\tilde{h}_A h^B)(\tilde{h}_B h^A) + \frac{\mu}{\kappa}\, m_A\,(\tilde{h}_A h^A)\,.$$

Relate Seiberg's dual in this vacuum to our r-dual theory in $r = N$ vacuum:

Both have $U(\tilde{N})$ gauge groups

The change of variables

$$D^{lA} = \sqrt{-\frac{\mu}{\kappa}}\, h^{lA}, \qquad l = 1, ..., \tilde{N}, \qquad A = 1, ..., N_f$$

brings this superpotential to the form

$$\mathcal{W}_S^{\rm LE} = \frac{1}{2\mu}\,(\tilde{D}_A D^B)(\tilde{D}_B D^A) - m_A\,(\tilde{D}_A D^A)\,.$$

This superpotential coincides with the superpotential of our r dual theory

Seiberg's duality and r-duality match for $r = N$ vacuum

Seiberg's "dual quarks" h^{lA} are not monopoles as naive duality suggests. Instead, they are quark-like dyons appearing in the r-dual theory below crossover. Their condensation leads to confinement of monopoles and "instead-of-confinement" phase for the quarks and gauge bosons of the original theory.

8 Towards $\mathcal{N} = 1$ QCD by increasing μ

$r < N$ vacua

Quark and monopole VEVs are determined by

$$\xi_P = -2\sqrt{2}\,\mu\,\sqrt{e_P^2 - \frac{2S}{\mu}}, \qquad P = 1, ..., N$$

$$S = \frac{1}{32\pi^2}\langle \mathrm{Tr}\, W_\alpha W^\alpha \rangle$$

In r vacuum

$$\sqrt{2}\, e_P = -m_P, \qquad P = 1, ..., r$$

To ensure weak coupling we need

$$\sqrt{\xi_P} \ll \Lambda_{\mathcal{N}=2}$$

$$m_P = -\sqrt{2}\, e_P \to -\sqrt{\frac{4S}{\mu}}$$

Argyres-Douglas conformal regime. Strong coupling

Two exceptions: $r = N$ vacuum and zero vacua

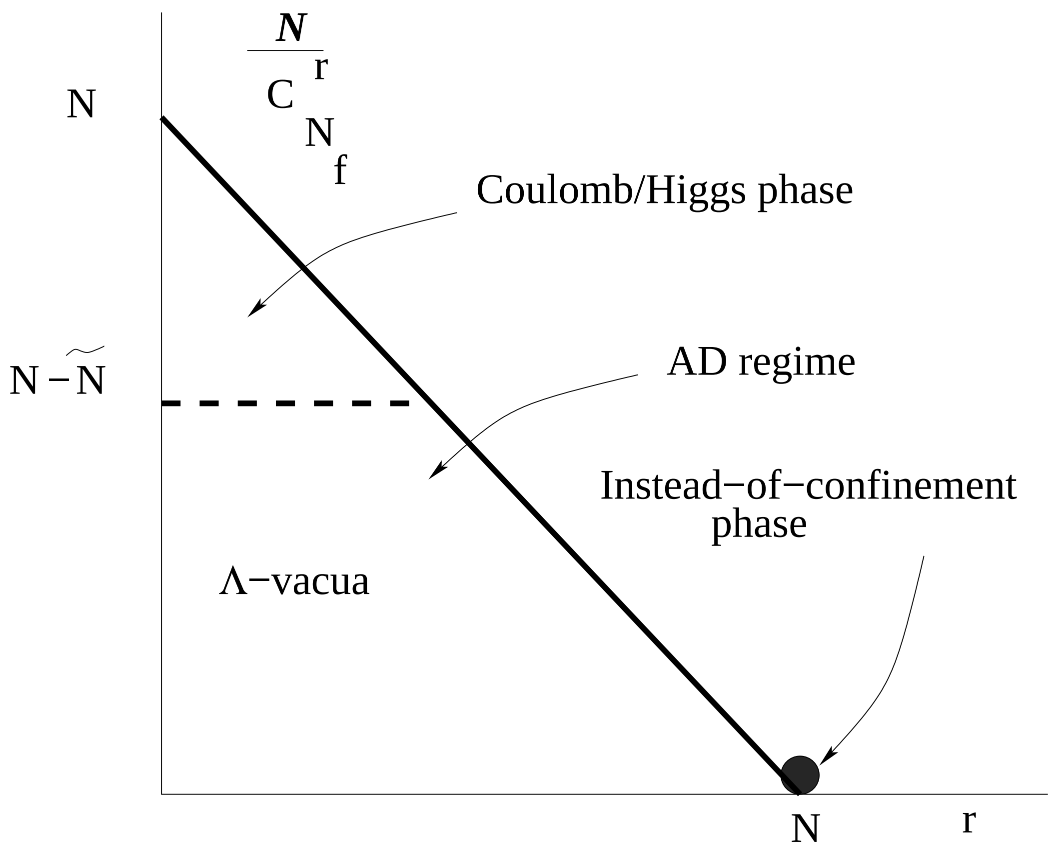
Zero vacua

$$S \approx \mu \, \frac{m^{\frac{N_f - 2r}{\tilde{N} - r}}}{\Lambda_{\mathcal{N}=2}^{\frac{N - \tilde{N}}{\tilde{N} - r}}} \, e^{\frac{2\pi k}{\tilde{N} - r}\, i} \ll \mu\, m^2, \qquad k = 1, ..., (\tilde{N} - r)\,,$$

in the small mass limit

9 Phases of $\mathcal{N} = 1$ QCD

$$N + 1 < N_f < \frac{3}{2}N, \qquad \mu \gg \sqrt{\xi}, \qquad \sqrt{\xi} \ll \tilde{\Lambda}_{\mathcal{N}=1}$$



- Zero vacua. U($\tilde{N}$) gauge group with N_f flavors of quarks r quarks condense. Higgs/Coulomb phase

- Λ vacua

$$S \sim \mu \Lambda^2_{\mathcal{N}=2}$$

Continuation of the Argyres-Douglas conformal strongly coupled regime to large μ

- $r = N$ Vacuum.

Instead-of-confinement phase