Supersymmetric localization and non-AdS/non-CFT correspondence

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A. Buchel, J. Russo, K.Z. 1301.1597

J. Russo, K.Z. 1207.3806, 1302.6968, 1307.####

D. Young, K.Z. 13##.#####

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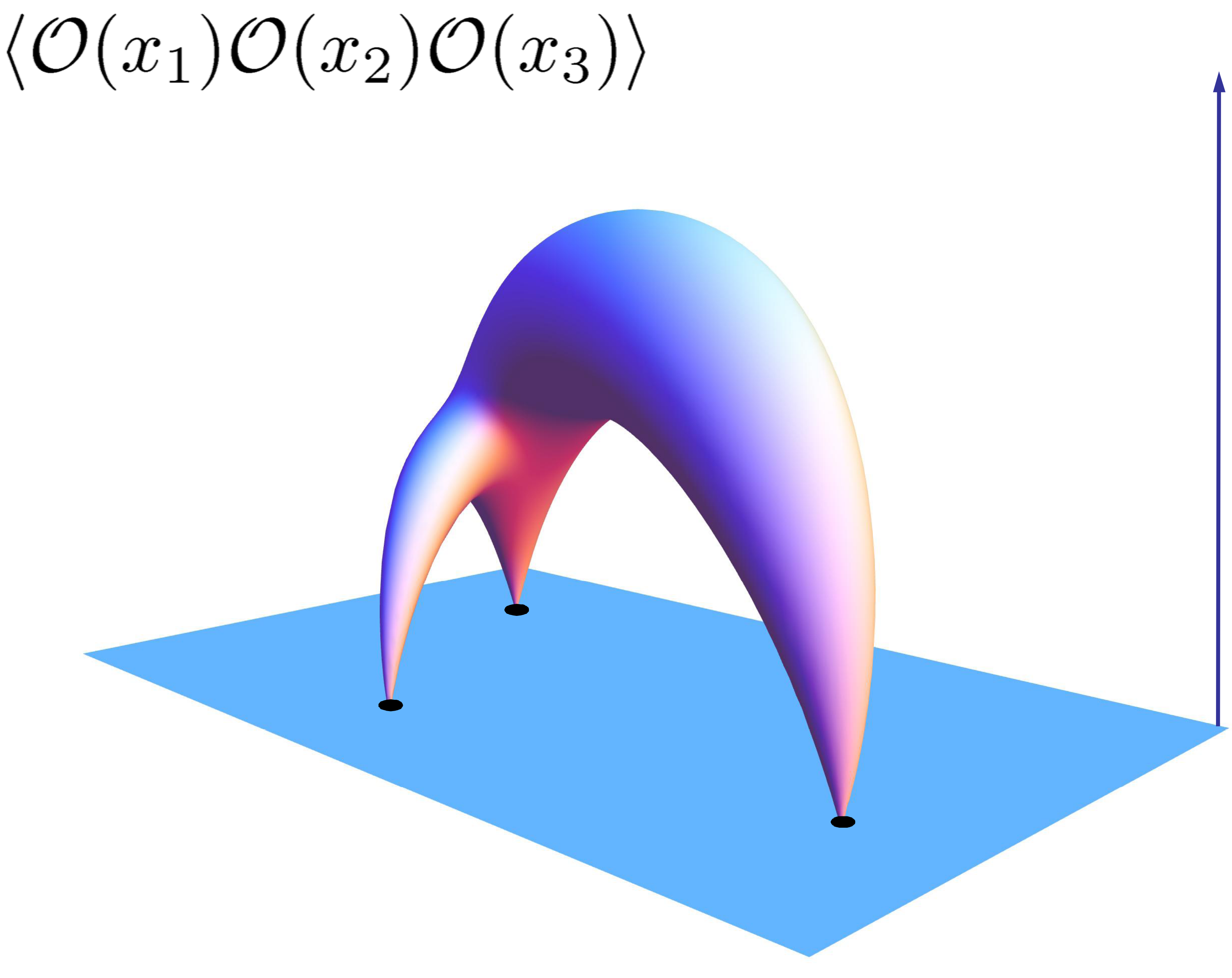
AdS/CFT correspondence

$$ds^2 = \frac{dx^\mu dx_\mu + dz^2}{z^2}$$

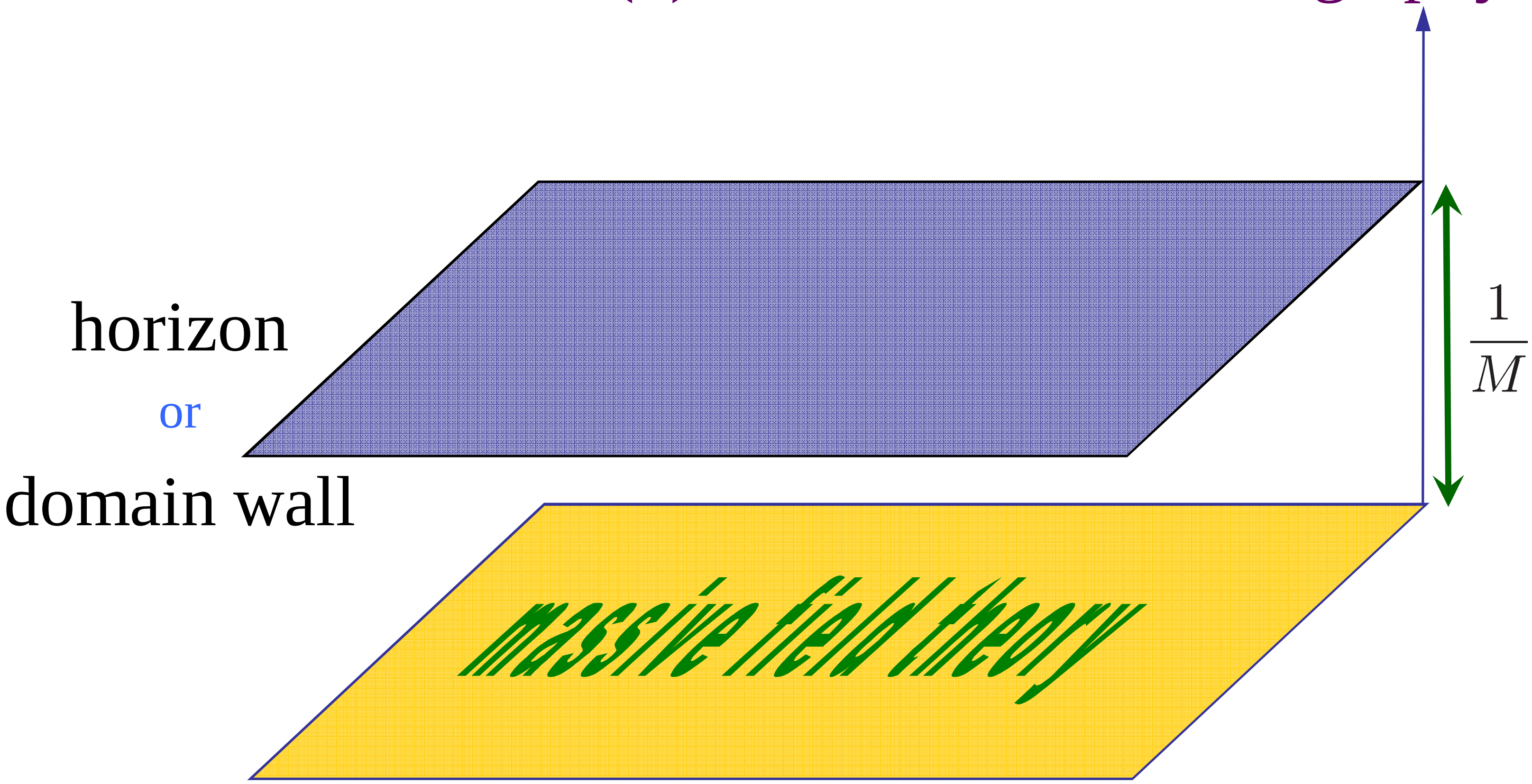

$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3) \rangle$

String Theory (quantum gravity) $\equiv$ 4d CFT

$AdS_5 x S^5$ N=4 super-Yang-Mills

- Still a conjecture…
- Overwhelming number of quantitative tests
- "Experimental" proof

Motivation (1): Non-conformal holography

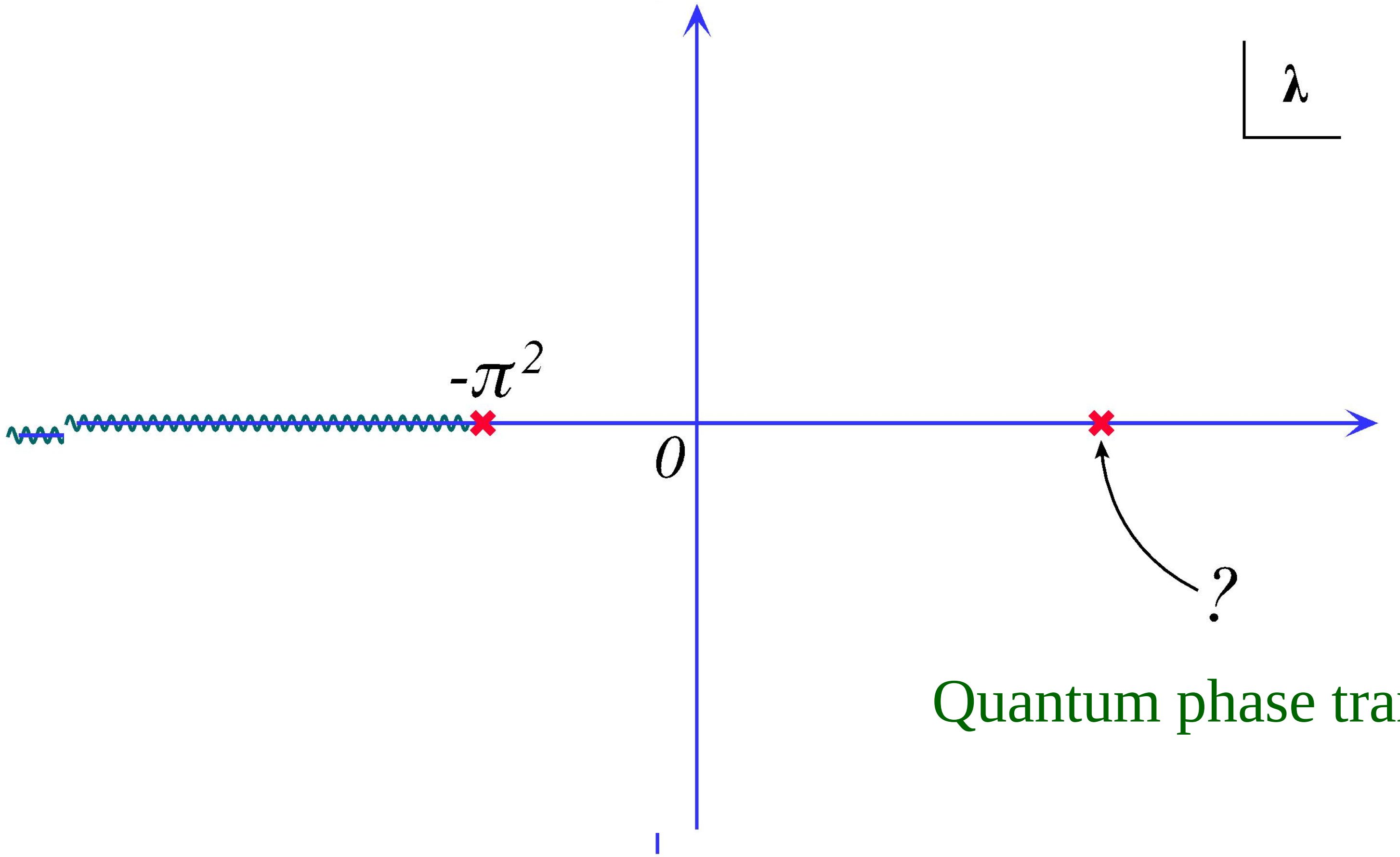


- Routinely used in many contexts
- No quantitative tests so far

Motivation (2): Phase transitions

In this talk:
$N=\infty$
$\lambda = g_{YM} N$

In N=4 super-Yang-Mills:



Motivation (3): Non-perturbative corrections

Dynamically generated scale: Λ

Kinamatic scale: M

$$M \gg \Lambda$$

(perturbative regime)

$$\mathcal{A} = \text{perturbative} + \sum_{n=1}^{\infty} C_n \left(\frac{\Lambda}{M}\right)^{2n}$$

- not calculable in general
- at best, can be parameterized by condensates – ITEP sum rules…
- goal: compute all C_n's in a soluble model

Setup

- CFT <=> short-distances in generic QFT
- QFT = CFT perturbed by relevant operators

- N=4 SYM is the simplest interacting 4d CFT
- The simplest relevant perturbation of N=4 SYM is N=2* theory

N=2* theory

- relevant perturbation of N=4 super-Yang-Mills

$$\mathcal{L}_{\mathcal{N}=2^*} = \mathcal{L}_{\mathcal{N}=4} + M^2\mathcal{O}_2 + M\mathcal{O}_3$$

hypermultiplets

Z

χ $\quad$ $\tilde{\chi}$ $\quad + \text{conj.}$

$\tilde{Z}$

mass = $\pm M$

vector multiplet

A_μ

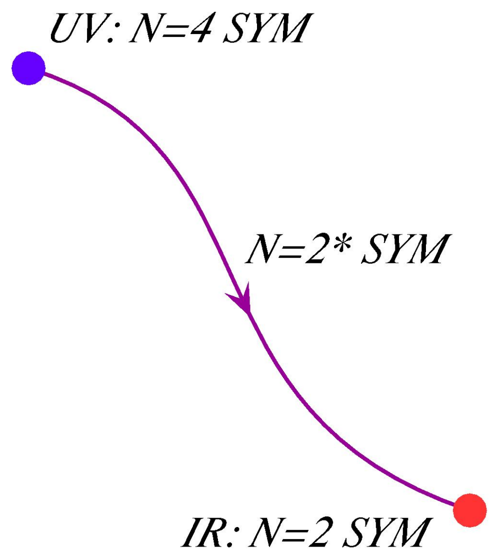
ψ $\quad$ λ

$\Phi + i\Phi'$

mass = 0

At $E \ll M$: integrate out hypermultiplet

- UV regularization of pure N=2 SYM



Dynamically generated scale:

$$\Lambda = M\,\mathrm{e}^{-\frac{4\pi^2}{\lambda}}$$

Low-energy effective action and weak-coupling expansion

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\mathcal{N}=2} + \sum_{d=6}^{\infty} M^{4-d} \mathcal{O}_d$$

OPE:

$$\mathcal{A} = \Lambda^{\Delta} \sum_{d} C_d \, \frac{\Lambda^{d-4}}{M^{d-4}}$$

Weak-coupling expansion in N=2* theory:

$$\mathcal{A} = M^{\Delta} \, \mathrm{e}^{-\frac{4\pi^2 \Delta}{\lambda}} \sum_{n=0}^{\infty} C_n \, \mathrm{e}^{-\frac{8\pi^2 n}{\lambda}}$$

Example:

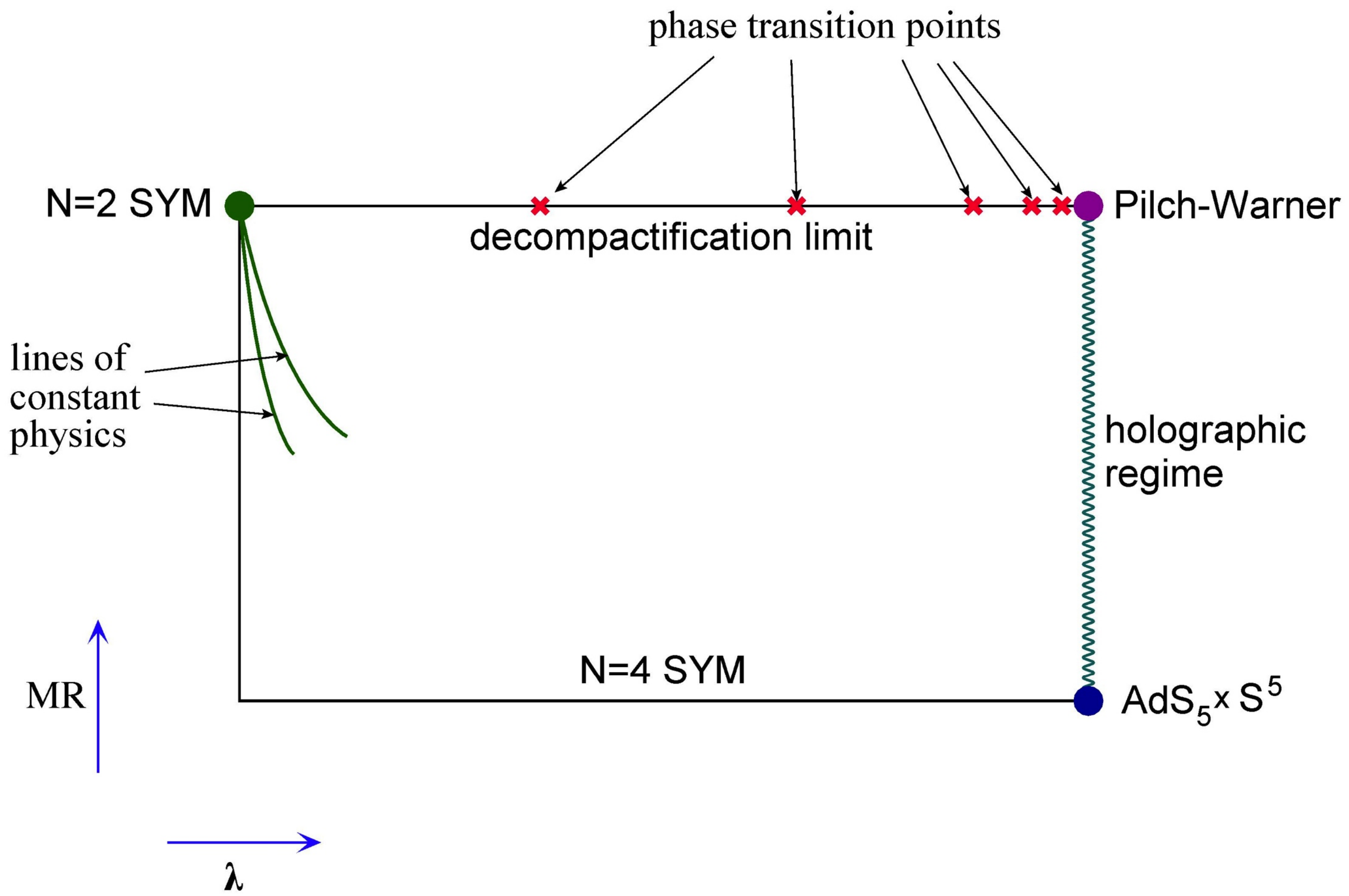
free energy of N=2* SYM

$$f(\lambda) = 2\sum_{n=1}^{\infty} \ln\left(1-(-1)^n\,\mathrm{e}^{-\frac{8\pi^2 n}{\lambda}}\right)$$

OPE:

$$f(\lambda) = 2\,\mathrm{e}^{-\frac{8\pi^2}{\lambda}} - 3\,\mathrm{e}^{-\frac{16\pi^2}{\lambda}} + \frac{8}{3}\,\mathrm{e}^{-\frac{24\pi^2}{\lambda}} - \frac{7}{2}\,\mathrm{e}^{-\frac{32\pi^2}{\lambda}} + \frac{12}{5}\,\mathrm{e}^{-\frac{40\pi^2}{\lambda}} - 4\,\mathrm{e}^{-\frac{48\pi^2}{\lambda}} + \ldots$$

Phase diagram



R: radius of S^4

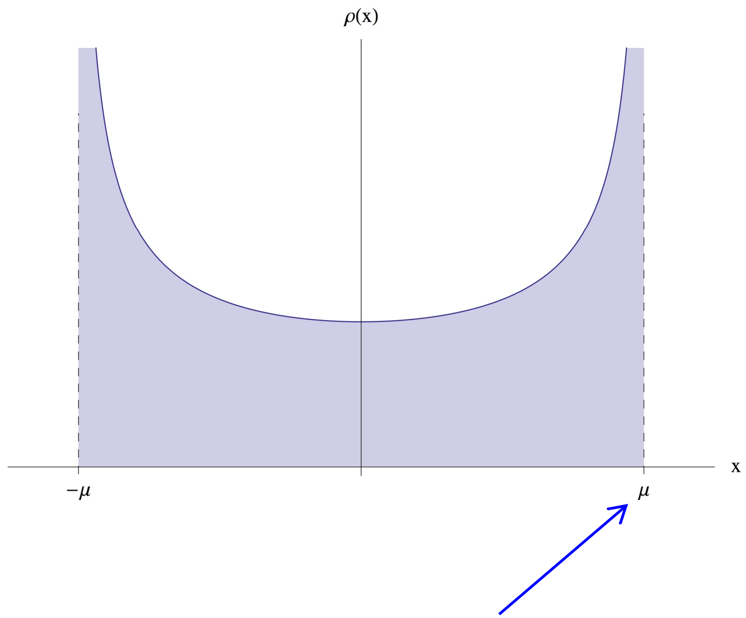
Master field

Hypermultiplet mass triggers symmerty breaking: SU(N) --> $U(1)^{N-1}$

$$\langle \Phi \rangle = \mathrm{diag}\left(a_1, \ldots, a_N\right)$$

Large-N master field:

$$\rho(x) = \frac{1}{N} \sum_{i=1}^{N} \delta\left(x - a_i\right)$$



typical scale of symmetry breaking

Localization

Pestun'07

compactification on S^4 of radius R

$$Z = \int d^{N-1}a \prod_{i<j} \frac{(a_i - a_j)^2 H^2(a_i - a_j)}{H(a_i - a_j + M) H(a_i - a_j - M)} \mathrm{e}^{-\frac{8\pi^2 N}{\lambda} \sum_i a_i^2}$$

$$H(x) = \prod_{n=1}^{\infty} \left(1 + \frac{x^2}{n^2}\right)^n \mathrm{e}^{-\frac{x^2}{n}}$$

$a_i \to a_i R \qquad M \to MR$

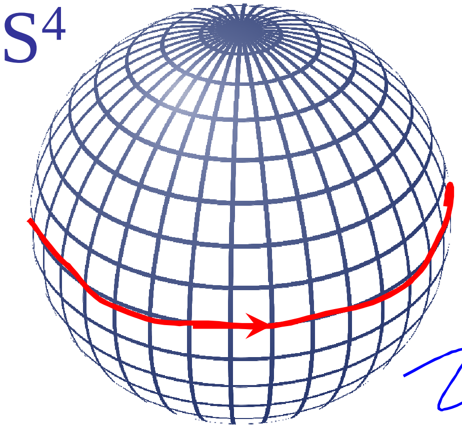
- Matrix model: saddle-point at large N

Wilson loops

$$W(C) = \left\langle \frac{1}{N} \, \mathrm{P} \exp \oint_C ds \, \left(i\dot{x}^\mu A_\mu + |\dot{x}| \Phi \right) \right\rangle$$

coupling to scalar is necessary for SUSY

S^4



localization

$$W(C_{\rm circle}) = \left\langle \frac{1}{N} \sum_{i=1}^{N} \mathrm{e}^{\,2\pi a_i} \right\rangle = \int_{-\mu}^{\mu} dx \, \rho(x) \, \mathrm{e}^{\,2\pi x} \overset{\mu \gg 1}{\Longrightarrow} \mathrm{e}^{\,2\pi\mu}$$

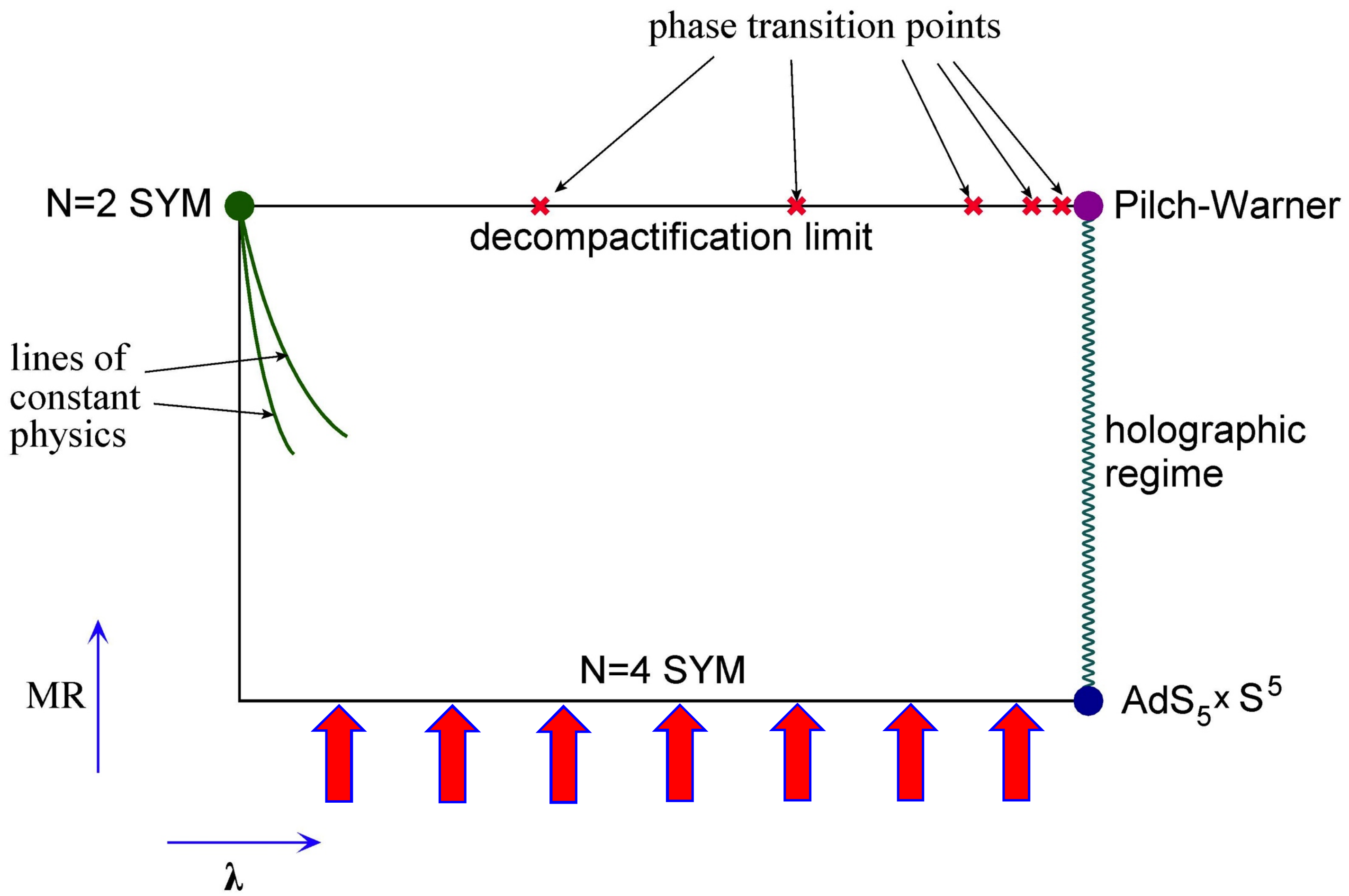
Saddle-point equations

$$\sum_{j\neq i}\left(\frac{1}{a_i-a_j}-K\left(a_i-a_j\right)+\frac{1}{2}K\left(a_i-a_j+M\right)+\frac{1}{2}K\left(a_i-a_j-M\right)\right)=\frac{8\pi^2}{\lambda}a_i$$

$$K(x)=-\frac{H'(x)}{H(x)}=2x\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{n}{n^2+x^2}\right)$$

$$\fint_{-\mu}^{\mu}dy\rho(y)\left(\frac{1}{x-y}-K(x-y)+\frac{1}{2}K(x-y+M)+\frac{1}{2}K(x-y-M)\right)=\frac{8\pi^2}{\lambda}x$$

Conformal limit: N=4 SYM

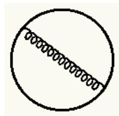
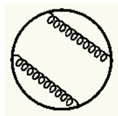


Circular Wilson loop in N=4 SYM

Erickson,Semenoff,Z.'00
Drukker,Gross'00

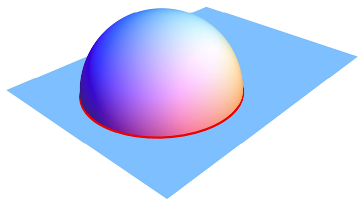
- Gaussian matrix model

$$\fint_{-\mu}^{\mu} dy\,\rho(y)\,\frac{1}{x-y} = \frac{8\pi^2}{\lambda}\,x$$

1 +  +  + …

$$\rho(x) = \frac{1}{\pi\mu^2}\sqrt{\mu^2 - x^2} \qquad \mu = \frac{\lambda}{2\pi}$$

areal law in AdS_5

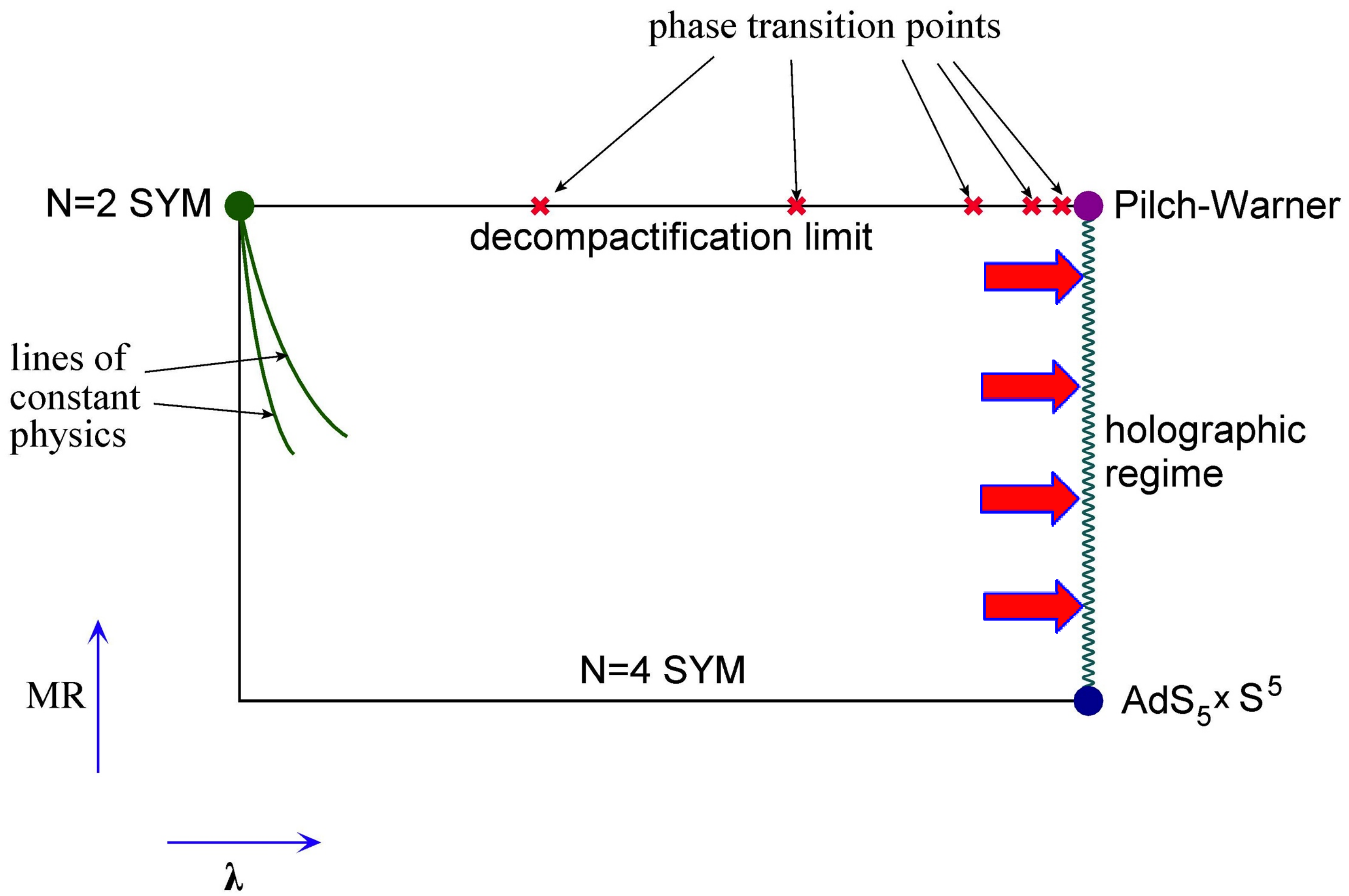


$$W(C_{\text{circle}}) = \frac{2}{\sqrt{\lambda}}\,I_1\left(\sqrt{\lambda}\right) \simeq \sqrt{\frac{2}{\pi}}\,\lambda^{-\frac{3}{4}}\,\mathrm{e}^{\sqrt{\lambda}}$$

string fluctuations

Drukker,Gross'00
Kruczenski,Tirziu'08
Kristjansen,Makeenko'12

Strong coupling



Strong coupling in N=2*

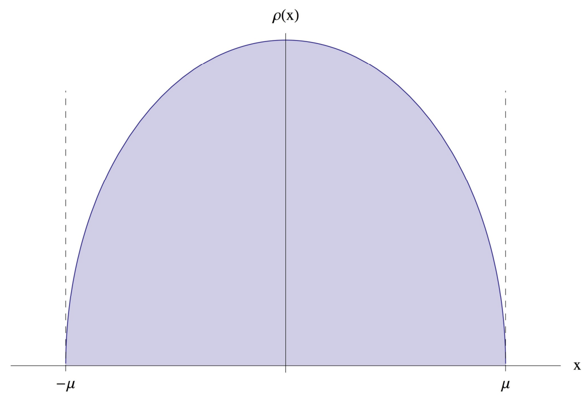
$$\mu \gg M$$

$$\fint_{-\mu}^{\mu} dy\rho(y)\,\frac{1+M^2}{x-y} = \frac{8\pi^2}{\lambda}\,x) + \frac{1}{2}K(x-y+M) + \frac{1}{2}K(x-y-M)\Big) = \frac{8\pi^2}{\lambda}\,x$$

$$\approx \frac{1}{2}K''(x-y)M^2$$

$$\rho(x) = \frac{2}{\pi\mu^2}\sqrt{\mu^2 - x^2}$$

$$\mu = \frac{\sqrt{\lambda\,(1+M^2)}}{2\pi}$$



Perimeter law

$$\mu = \frac{\sqrt{\lambda\left(M^2 + \frac{1}{R^2}\right)}}{2\pi} \overset{R\to\infty}{\Longrightarrow} \frac{\sqrt{\lambda}M}{2\pi}$$

$$W(C) = \left\langle \frac{1}{N} \mathrm{P}\exp \oint_C ds\ \left(i\dot{x}^\mu A_\mu + |\dot{x}|\Phi\right) \right\rangle$$

substitute classical <Φ>

$$W(C) = \frac{1}{N}\sum_i \mathrm{e}^{\,La_i} \overset{L\to\infty}{\Longrightarrow} \mathrm{e}^{\,L\mu}$$

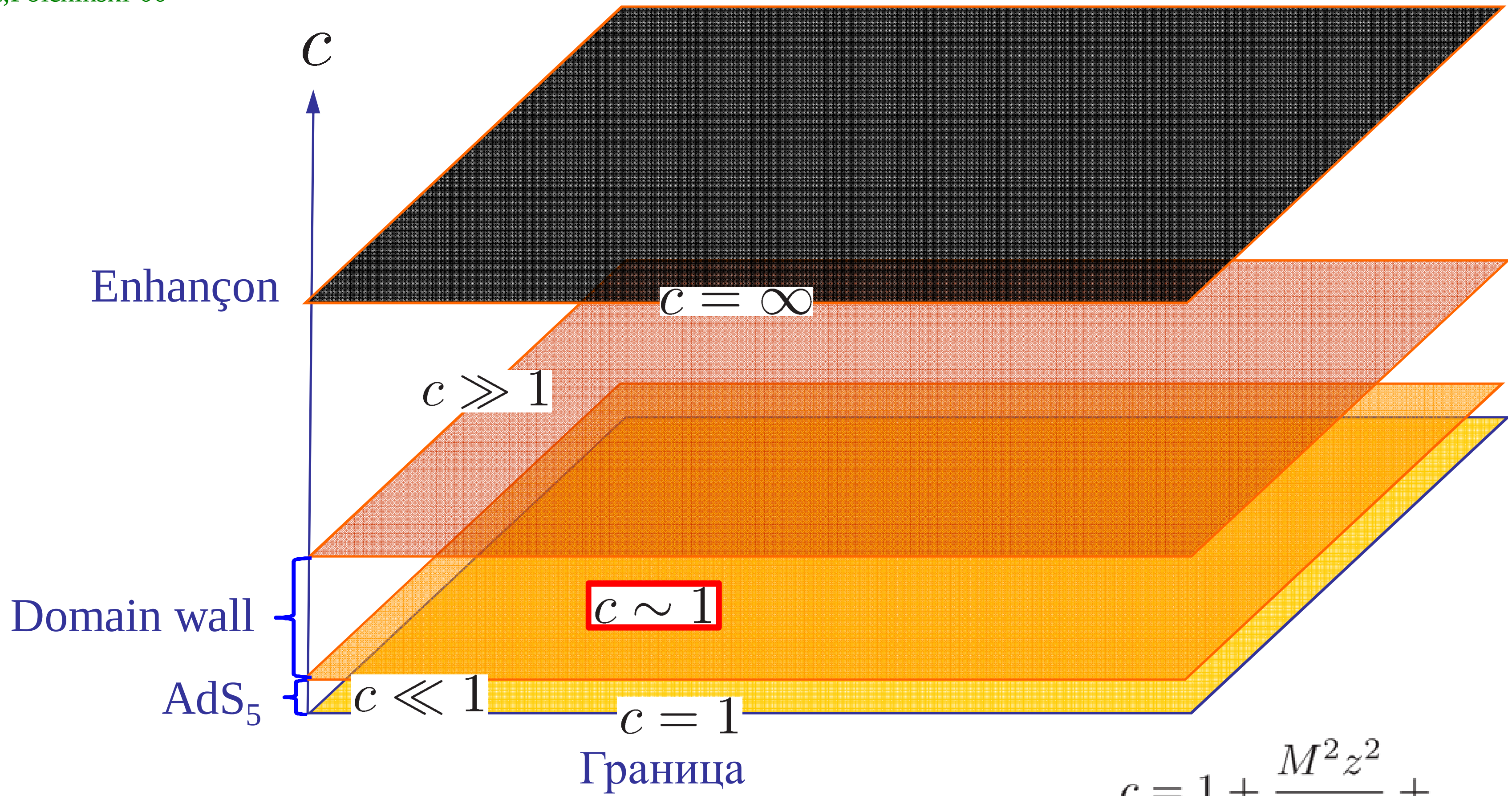
(perimeter law)

$$\ln W(C) \simeq \frac{\sqrt{\lambda}ML}{2\pi} \qquad (\lambda \to \infty,\ ML \gg 1)$$

“AdS$_5$ “x” “S^5”: geometry dual to N=2* SYM

Pilch,Warner’00

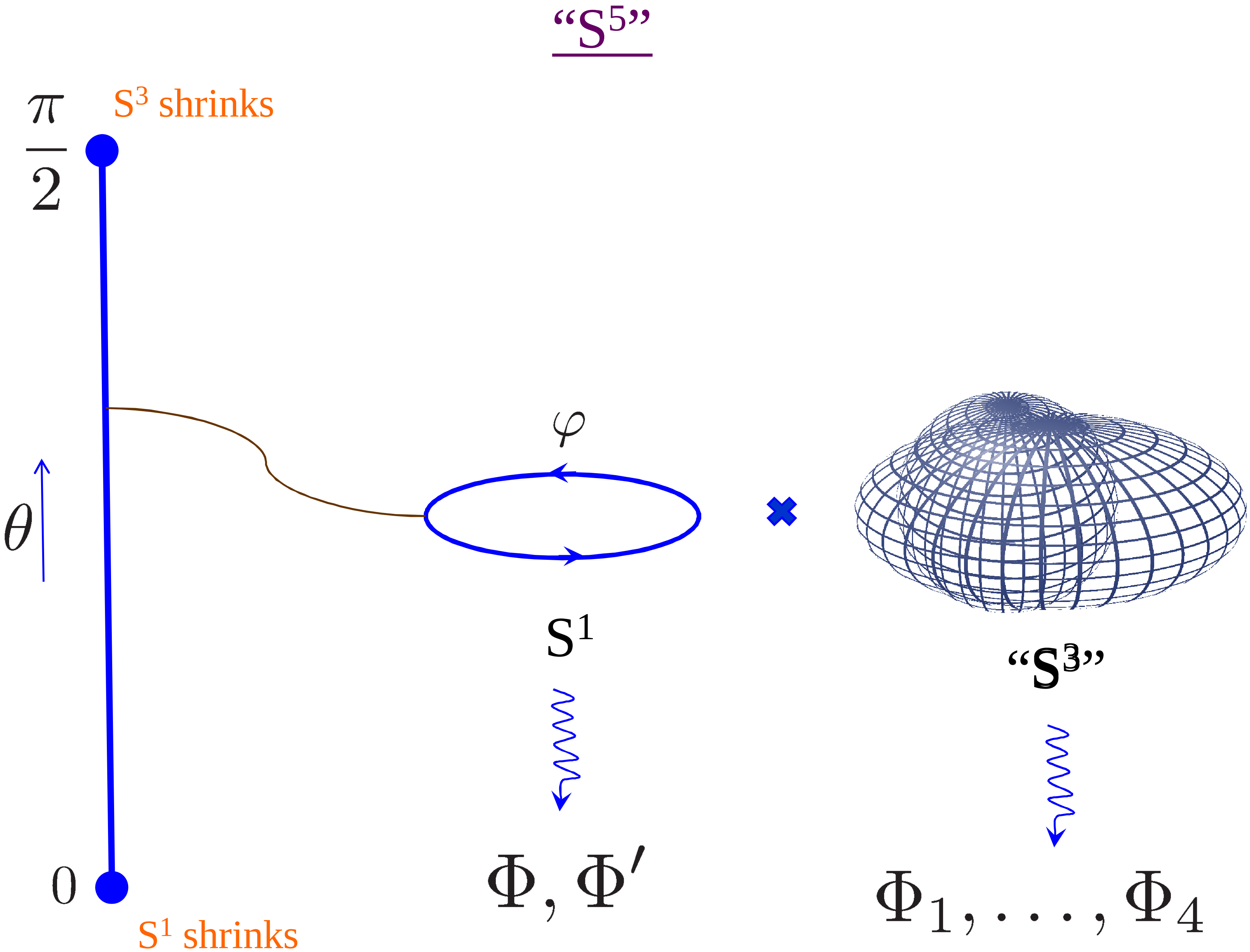
Buchel,Peet,Polchinski’00



$$c = 1 + \frac{M^2 z^2}{2} + \dots$$

$$ds^2 = \frac{\rho^6}{c^2 - 1} M^2 dx_\mu^2 + \frac{1}{\rho^6 \left(c^2 - 1\right)^2} dc^2$$

$$\rho^6 = c + \frac{c^2 - 1}{2} \ln \frac{c - 1}{c + 1}$$


"S^5"
π/2
S^3 shrinks
θ
0
S^1 shrinks
φ
S^1
"S^3"
Φ, Φ′
Φ_1, . . . , Φ_4

“x”

- all supergravity fields appear in the solution
- dilaton runs
- AdS_5 and S^5 are warped

$$g_{MN} = g_{MN}(c, \theta)$$

$$C^n_{\mathrm{RR}} = C^n_{\mathrm{RR}}(c, \theta)$$

$$\Phi = \Phi(c, \theta, \varphi)$$

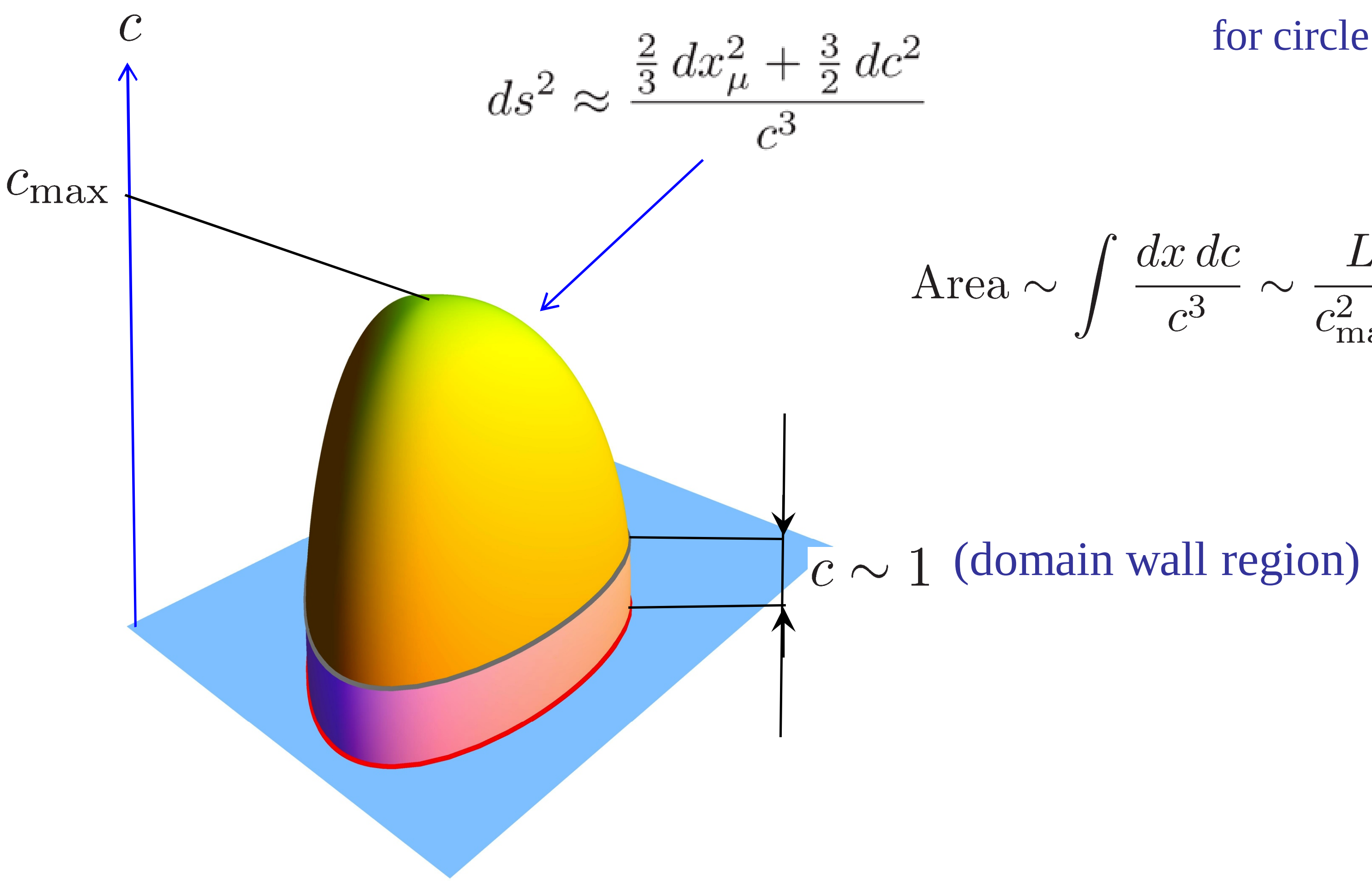
resposible for SU(N) → U(1) symmetry breaking

Minimal surface

$$c_{\mathrm{max}} \propto L$$

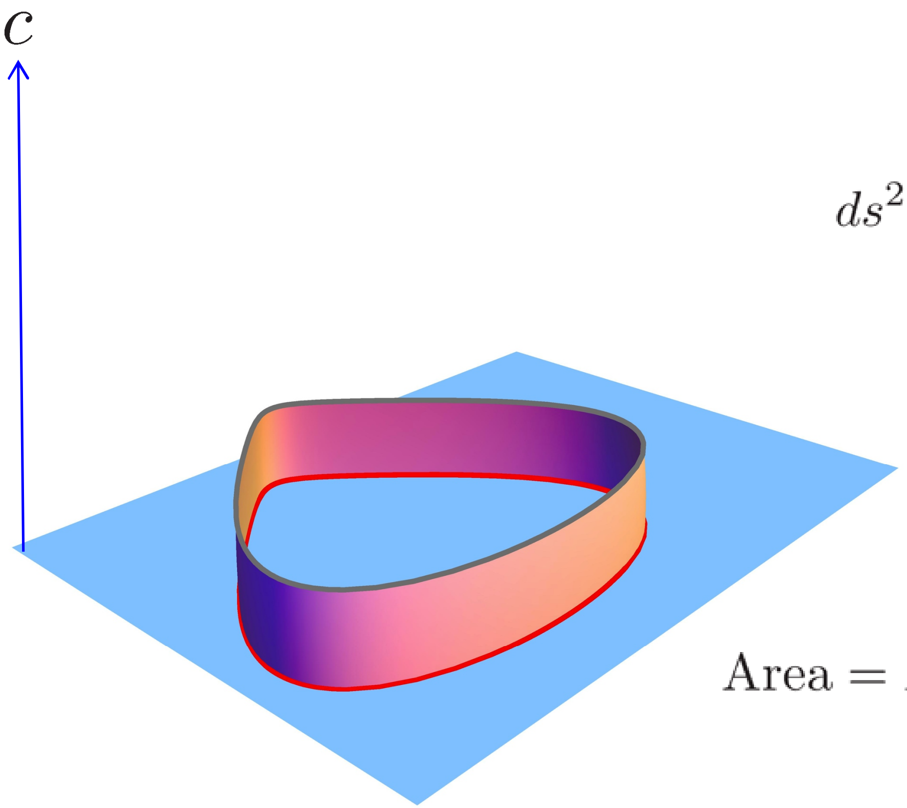
for circle: $c_{\mathrm{max}} = 0.857R$

$$ds^2 \approx \frac{\frac{2}{3}\,dx_\mu^2 + \frac{3}{2}\,dc^2}{c^3}$$



$$\text{Area} \sim \int \frac{dx\,dc}{c^3} \sim \frac{L}{c_{\mathrm{max}}^2} \sim \frac{1}{L}$$

Minimal area



$$ds^2 = \frac{\rho^6}{c^2-1}\,M^2 dl^2 + \frac{1}{\rho^6\left(c^2-1\right)^2}\,dc^2$$

$$\text{Area} = ML\int_{1+\frac{\varepsilon^2 M^2}{2}}^{\infty}\frac{dc}{\left(c^2-1\right)^{\frac{3}{2}}} = \frac{L}{\varepsilon} - ML$$

$\frac{L}{\varepsilon} \to 0$

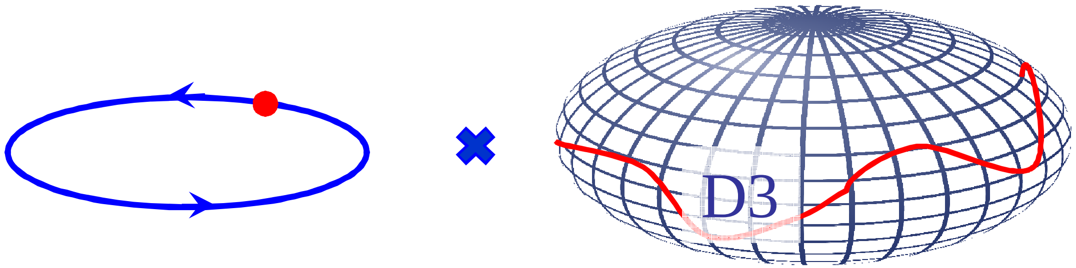
renormalized away

$$\ln W(C) \simeq \frac{\sqrt{\lambda}ML}{2\pi} \qquad (\lambda\to\infty,\ ML\gg 1)$$

agrees with localization!

D3-brane probes

Buchel,Peet,Polchinski'00



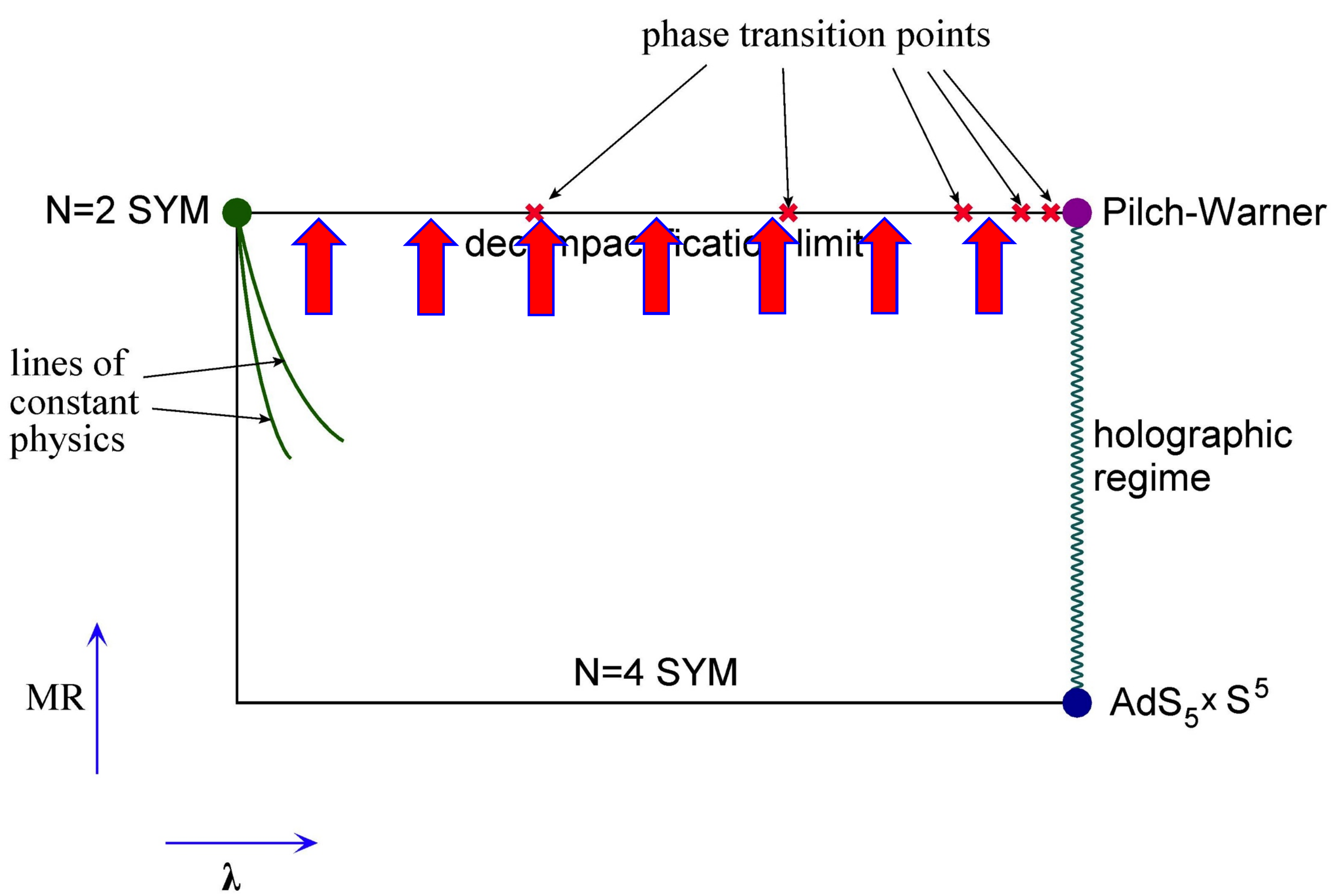
Effective potential on the probe: $\langle \Phi \rangle = \mathrm{diag}\,(a_1, \ldots, a_N)$

$\Updownarrow$

$$\rho(x) = \frac{1}{\pi \mu^2} \sqrt{\mu^2 - x^2} \qquad\qquad \mu = \frac{\sqrt{\lambda} M}{2\pi}$$

Again agrees!

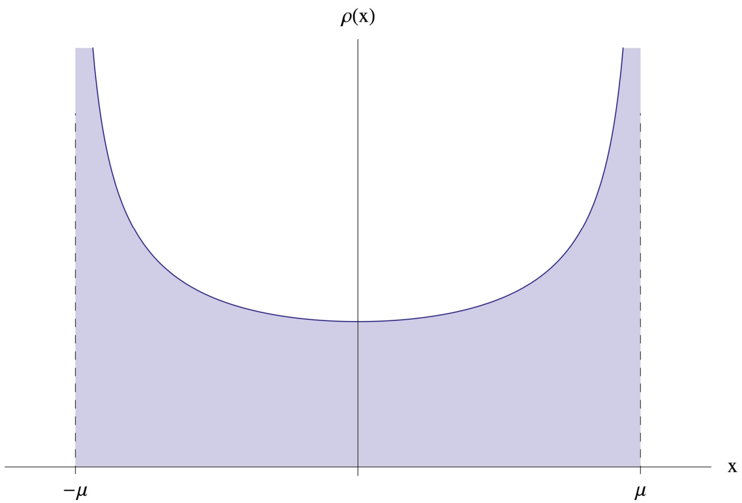
Decompactification limit



$$\fint_{-\mu}^{\mu} dy\, \rho(y) \left(\frac{2}{x-y} - \frac{1}{x-y+M} - \frac{1}{x-y-M} \right) = 0$$

Each pole corresponds to (nearly) massless particle

$$m_{ij}^{\mathrm{v}} = |a_i - a_j| \qquad \text{vector multiplets}$$
$$m_{ij}^{\mathrm{h}} = |a_i - a_j \pm M| \qquad \text{hypermultiplets}$$



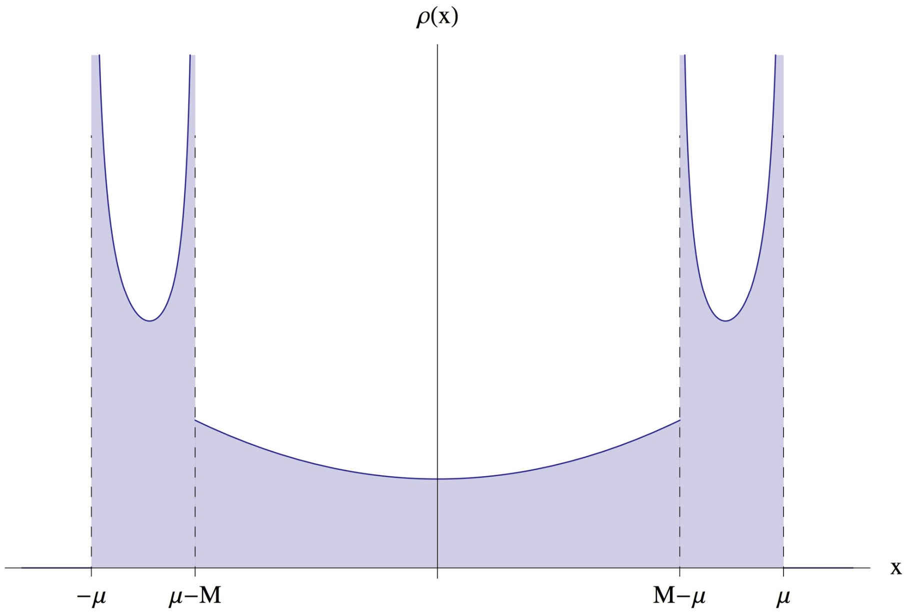
$$2\mu > M$$

⇓

all hypers are heavy

Phase transition

$$\mu(\lambda_c) = \frac{M}{2}$$ ⇒ phase transition: massless hypermutiplet appears in the spectrum



resonance due to massless hypermultiplet

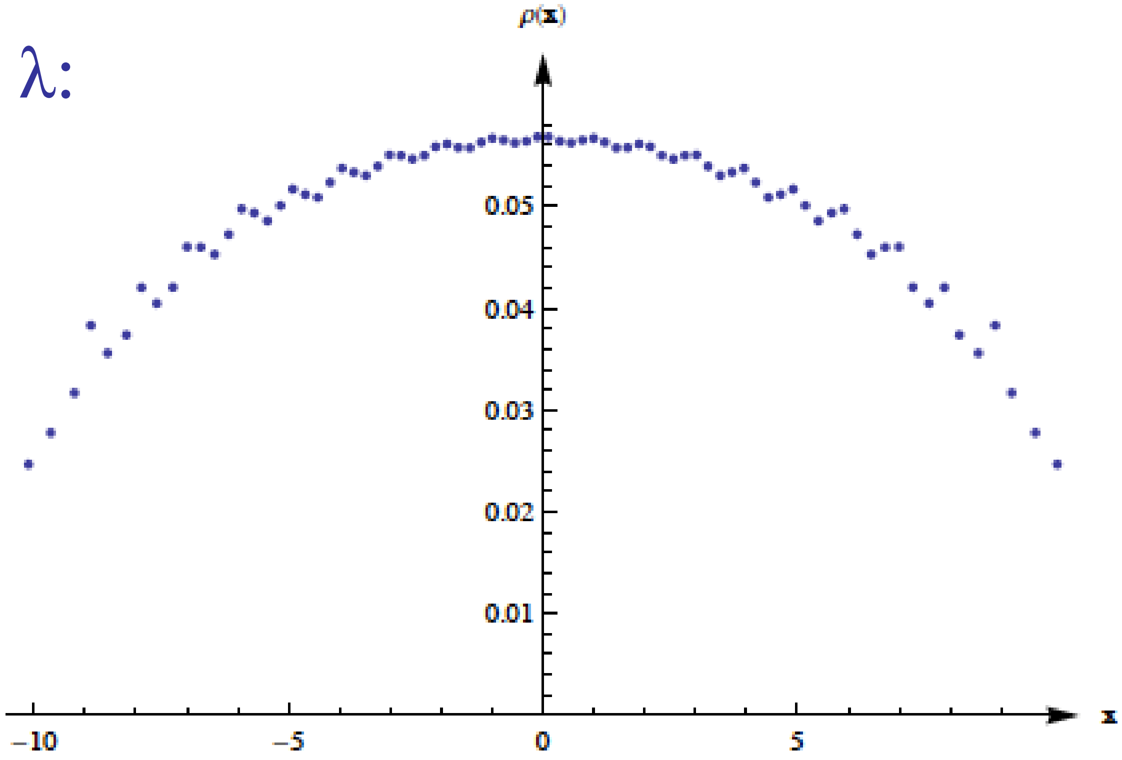
Secondary phase transitions

$$\mu(\lambda_c^{(n)}) = \frac{nM}{2}$$

$$\lambda_c^{(1)} \simeq 35 \qquad \lambda_c^{(2)} \simeq 83$$

$$\lambda_c^{(n)} \simeq \pi^2 n^2 \qquad (n \to \infty)$$

At very large λ:



Conclusions

- N=2* is an interesting theory, with non-trivial phase structure at large-N
- many exact results from localization
- fisrt (?) direct test of non-conformal holography
- what are the implications of the phase transitions AdS/CFT?
- can integrability be used beyond conformal point[#]?

[#]Some hints from conformal perturbation theory that the answer may be “yes”