Embedded Graphs

Abstracts of talks

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Weighted trees with primitive edge-rotation groups

We consider bicolored maps on the sphere with only one face of degree greater than one. In some situations it is convenient to represent these maps as plane trees with weighted edges, so we call them *weighted trees*. The theory of *dessins d'enfants* introduces the action of absolute Galois group $Aut(\overline{Q}/Q)$ on the maps on surfaces. An important invariant of this action is the *edge rotation group*, which is also the monodromy group of a ramified covering corresponding to the Belyi function.

All special primitive monodromy groups of polynomials (and hence primitive edge rotation groups of ordinary trees) were described by P. Müller 20 years ago, using a theorem by W. Feit. A recent result due to G. A. Jones lets us to extend this classification to the case of weighted trees. Both Feit's and Jones's theorems are based on the classification of the finite simple groups.

We use GAP, a system for computational group theory, to find a complete list of special monodromy groups of weighted trees and their generators.

In a more general setup we are able to classify special primitive monodromy groups of rational functions on a complex sphere with only one multiple pole. Almost all of them have at most four critical values. The rational functions with three critical values are Belyi's function, they can be drawn as maps. The rational functions with four critical values are represented by cacti. Such functions form one-dimensional families (curves) in the Hurwitz spaces with naturally defined Belyi functions on them. We present the pictures for these cacti and corresponding dessins d'enfants.

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Polyhedral model of the fibers of a period map

We study a period mapping from the moduli space of real hyperelliptic curves with marked point on an oval to euclidian space. This mapping arises in the analysis of Chebyshev construction used in the constrained optimization of uniform norm of polynomials (and rational functions). Cell decomposition of the moduli space described in terms of planar graphs allows to reconstruct the global topology of (low dimensional) fibers of the period mapping.

[1] A. B. Bogatyrev, *Extremal Polynomials and Riemann Surfaces*, Springer Monographs in Mathematics, 2012.

[2] A. B. Bogatyrev, *Fibers of periods map are cells?* J. Comp. Appl. Math., 153:1-2 (2003), 547–548.

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Limit theorems for translation flows

The talk is devoted to limit theorems for translation flows on flat surfaces. Consider a compact oriented surface of genus at least two endowed with a holomorphic one-form. The real and the imaginary parts of the one-form define two foliations on the surface, and each foliation defines an area-preserving translation flow. By a theorem of H. Masur and W. Veech, for a generic surface these flows are uniquely ergodic. The first result of the talk, which extends earlier work of A. Zorich and G. Forni, is an asymptotic formula for time integrals of Lipschitz functions. One of the main objects of the talk is the space of finitely-additive Hölder transverse invariant measures for our foliations. These measures are classified and related to G. Forni's invariant distributions of Sobolev regularity -1 for translation flows. Time integrals of Lipschitz functions are then shown to admit an asymptotic expansion in terms of the finitely-additive measures. Next, given a Lipschitz function of average zero on the surface, we consider its time integrals as random variables and study the limit behaviour of their probability distributions. Informally, the main result states that the probability distributions of time integrals converge to an orbit of an ergodic dynamical system in the space of random variables with compactly supported distributions. The argument relies on a symbolic representation of translation flows as suspension flows over Vershik's automorphisms, a construction developing one proposed by S. Ito.

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On the geometry of decomposition of the cyclic permutation into the product of a given number of permutations

In this talk I will speak about the proof of a formula for the number of genus 0 decompositions of the cyclic permutation into the product of no more than m non-identical permutations. Formula itself is a specialization of Bousquet-Mélou–Shaeffer's formula [BMS]. Let $\mathcal{P}$ be the space of polynomials $f(x) = x^n + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$. The proof is based on methods from the paper [LZ] by S. Lando and D. Zvonkine. It is a consequence of the fact that one can find the degree of the restriction of the Lyashko–Looijenga mapping on some discriminant strata of the space $\mathcal{P}$. Such a stratum is defined by polynomials with fixed set of degeneracies of their critical values. These results suggest that our methods can be extended to the cases of positive genera.

[BMS] M. Bousquet-Mélou, G. Schaeffer, *Enumeration of planar constellations*, Advances in Applied Math., V. 24, I. 4, P. 337–368 (2000).

[LZ] D. Zvonkine, S. K. Lando, *On multiplicities of the Lyashko-Looijenga mapping on discriminant strata*, Functinal Analysis and its Appl., V. 33, I. 3, P. 178–188 (1999).

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Hypergeometric Hurwitz numbers: KP tau-functions and matrix models

We begin with constructing a matrix-model representation for the generating function of numbers of Belyi morphisms, clean Belyi morphisms, and two-profile Belyi morphisms. These generating functions can be reformulated un terms of fat graphs on Riemann surfaces and are correspondingly described by Hermitian one-matrix model with logarithmic addition to the potential, by the Kontsevich–Penner matrix model, and by the Generalized Kontsevich matrix model thus being tau-functions of the KP hierarchy. We extend these results to hypergeometric Hurwitz numbers with arbitrary (but fixed) number n of branching points and two fixed profiles. Under some restrictions on the corresponding generating functions, we were able to construct a representation of their generating functions in the form of a special chain of matrices, which admits solution in terms of the topological recursion procedure. As an example, I present the spectral curve calculation for the case $n = 4$.

Based on two recent joint works with J. Ambjorn, NBI, Copemhagen.

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Minimum genus embeddings of vertex-transitive graphs

By a theorem of Škoviera and Nedela (1989), almost all vertex-transitive graphs are 'upper-embeddable', in that they have 2-cell embeddings on orientable surfaces of maximum conceivable genus, with just one or two faces. (The only exceptions are 3-valent examples of girth 3 and order 18 or more.) In particular, every finite connected Cayley graph is upper-embeddable.

In contrast, relatively little is known about the *minimum* genus of vertex-transitive graphs. Finding the minimum genus of a given connected graph is a notoriously difficult problem, except in some very special circumstances (such as when the graph is planar, or is a Cayley graph for some quotient of the $(2,3,7)$ triangle group).

In this talk, I will describe two recent developments on this topic. One is some work with Ricardo Grande in 2012/13, on finding the minimum genus of families of connected circulants (Cayley graphs for cyclic groups), including a complete determination of all such graphs that have minimum genus 0, 1 or 2. The second is joint work in 2014 with Klara Stokes, on exploiting symmetries to find the smallest genus of embedding of the Hoffman-Singleton graph, in both the orientable and non-orientable cases, with 69 faces and 70 (pentagonal) faces respectively.

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Minor theory for surfaces and divides of maximal signature

Divide graphs are peculiar graphs embedded in the disc that have been introduced by A'Campo and Gusein-Zade for studying plane curve singularities. To every divide graph is associated a certain surface with boundary embedded in the three-sphere, which corresponds to the Milnor fiber of the singularity. In this talk, we introduce a natural partial order on such surfaces, called "surface minority". Our main result is a Robertson–Seymour type theorem, namely that the restriction of surface minority to fiber surfaces of divides is a well-quasi-order. It implies that every property of fiber surfaces of divides that is preserved by surface minority is characterized by a finite number of prohibited minors. For the signature to be equal to the first Betti number is such a property. We explicitly determine the corresponding prohibited minors. As an application we establish a correspondance between divide links of maximal signature and Dynkin diagrams.

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On the circular maps, bipartite maps and hypermaps which are self-equivalent with respect to reversing the colors of vertices

A *map* (S, G) is a closed Riemann surface S with an embedded graph G such that $S \setminus G$ is homeomorphic to a disjoint union of open disks. Two maps (S, G) and (S_1, G_1) are called *equivalent* whenever there exists an orientation-preserving homeomprphism $h : S \to S_1$ with $h(G) = G_1$. We introduce the concept of circular maps and establish its equivalence to the concept of map admitting a coloring of the faces in two colors. A relation between bipartite maps and circular maps will be demonstrated through the concept of a duality of maps. A *hypermap* is a map whose vertices are colored in black and white in such a way that every edge connects vertices of different colors. A *hypermap is self-equivalent with respect to reversing the colors of vertices*, if the given hypermap and hypermap which is obtained by reversing colors of vertices of the given hypermap are equivalent.

The main results are enumeration formulas for the numbers of circular maps, of bipartite maps and of hypermaps which are self-equivalent with respect to reversing the colors of vertices.

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Topological recursion and a quantum curve for monotone Hurwitz numbers

Take a permutation and count the number of ways to express it as a product of a fixed number of transpositions — you have calculated a Hurwitz number. By adding a mild constraint on such factorisations, one obtains the notion of a monotone Hurwitz number. We have recently shown that the monotone Hurwitz problem fits into the so-called topological recursion/quantum curve paradigm. This talk will attempt to explain what the previous sentence means.

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The map asymptotic constants and Wiener index of trees

Let $M_g(n)$ be the number of n-edge rooted maps on an orientable surface of genus g. It is known that, for each fixed g and as $n \to \infty$,

$$M_g(n) \sim t_g n^{5(g-1)/2} 12^n$$

for some positive constants t_g. It is also known that t_g is related to Painlevé I equation $y'' = 6y^2 - x$.

In this talk I will describe how t_g is related to the moments of a random variable defined in terms of path length in a random rooted binary tree. I will also describe how t_g appears in the asymptotic formulas for the number of n-vertex graphs which are embeddable in an orientable surface of genus g.

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Generalizing Cayley maps

An orientable map is said to be regular if its full orientation preserving automorphism group acts regularly on its set of darts. A Cayley map $M = CM(G, X, p)$ is an embedding of a Cayley graph $C(G, X)$ for which every left-mulitplication permutation by an element of G induces a map automorphism of M. Thus, every Cayley map admits a group of automorphisms acting regularly on its set of vertices, and the regularity of a Cayley map is equivalent to the existence of an automorphism fixing a vertex and mapping a dart to its neighbor. For these reasons, Cayley maps have proved to be a rich source of regular maps. In order to better understand the position of Cayley maps within the class of all orientably regular maps, as well as to find other rich families of regular maps, we propose to generalize the concept of Cayley maps in ways that would retain some of their desirable properties. We will review several of the equivalent definitions of Cayley maps in order to find the most promising of the generalizations, and report some preliminary results along these lines.

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Reflections in vertex- and edge-transitive maps

I shall consider how many conjugacy classes of reflections a map can have, under various transitivity conditions. For vertex- and for face-transitive maps there is no restriction on their number or size, whereas edge-transitive maps can have at most four classes of reflections. Examples are constructed, using topology, covering spaces and group theory, to show that various distributions of reflections can be achieved.

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Virasoro constraints and topological recursion for Grothendieck's dessin counting

We compute the number of Belyi coverings of the Riemann sphere with a given monodromy type over infinity and given numbers of preimages of 0 and 1. We show that the generating function for these numbers enjoys several remarkable integrability properties: it obeys the Virasoro constraints, an evolution equation, the KP (Kadomtsev–Petviashvili) hierarchy, and satisfies a topological recursion in the sense of Chekhov–Eynard–Orantin.

This is a joint work with P. Zograf.

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Deformations of Grothendieck's dessins d'enfants and isomonodromy deformations

We intorduce a general notion of Special Functions of the Isomonodromy Type and explain how dessins d'enfants and their deformations help to solve various problems for these functions. Particular examples concern Gauss hypergeometric and the sixth Painlevé functions.

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Computation of the first Stiefel–Whitney class of $\overline{\mathcal{M}^{\mathbb{R}}_{0,n}}$

We compute the Poincaré dual class to the first Stiefel–Whitney class of the Deligne–Mumford compactification of the real moduli space of algebraic curves with n marked and numbered points. The computation is given in terms of the natural cell decomposition of the variety under consideration.

The talk is based on our joint results with N. Ya. Amburg.

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On the signed number of circuits of even length in nonoriented graphs

A chord diagram is a one-face map. To a chord diagram, a simple graph can be associated, which is the intersection graph of the diagram. The chord diagram structure allows one to assign signs to circuits of even length in this graph in a natural way. The difference between the number of positive and negative circuits of given length $2k$ is a graph invariant. This invariant is closely related to the

weight system (in other words, to the Vassiliev knot invariants) associated to the Lie algebra $\mathfrak{sl}_2$. It happens that this invariant admits a natural extensions to arbitrary graphs, including those that are not intersection graphs of chord diagrams. This fact leads to a number of questions concerning the existence of more graph invariants related to $\mathfrak{sl}_2$ and other Lie algebras.

The talk is based on a joint paper with E. Kulakova, T. Mukhutdinova and G. Rybnikov (2014).

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Some arithmetic functions in counting unrooted topological maps

In this talk I survey briefly enumerative formulae for unrooted maps and non-equivalent coverings of surfaces with the emphasis on the multiplicative arithmetic functions taking part in these formulae. Obvious and frequent is the presence of the Möbius function $\mu(n)$ and the Euler totient function $\phi(n)$. Such are in particular formulae for unrooted planar maps of diverse classes. Some more special enumeration functions (in particular, ones for coverings of the torus and the Klein bottle) contain classical functions 'the sum of divisors' $\sigma(n)$, 'the number of divisors' $\delta(n)$ and their generalizations. Last years more and more important role belongs to the Jordan totient functions $\phi_k(n)$ (together with some of their special modifications) starting from my old formula for the number of subgroups of the free group up to conjugacy (what in the case of rank 2 gives rise to the number of arbitrary non-isomorphic dessins d'enfants). An important new multivariate multiplicative function $E(m_1, ..., m_r)$ called 'orbicyclic' has been introduced recently in connection with the enumeration of unrooted maps and hypermaps on orientable surfaces. Its properties will be discussed in more detail, in particular the non-vanishing conditions for $E(m_1, ..., m_r)$, which were shown to coincide with familiar Harvey's conditions (1966) on branching data of finite cyclic groups acting on Riemann surfaces.

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Automorphism groups and branch coverings of graphs

In this lecture we give a short survey of the results about branched coverings of graphs. This notion was introduced independently by many authors. See, for example, paper [1] for one of the first expositions and paper [2] for the list of references. The branched covering of graphs are also known as harmonic maps or vertically holomorphic maps of graphs. The main idea of the present talk to is create a parallel between classical results about branched coverings of Riemann surfaces and those for graphs. We introduce the notion of harmonic automorphism for graph and discuss the upper and lower bounds for the number of harmonic automorphisms acting on a graph of a prescribed genus. We give a few discrete versions of the Wiman, Oikawa and Arakawa theorems for graphs. We present also some statements of the Lefschetz fixed point theorem for graphs.

[1] T. D. Parsons, T. Pisanski, P. Jackson, *Dual imbeddings and wrapped quasi-coverings of graphs*, Discrete Mathematics, Vol. 31, No. 1, 43–52 (1980).

[2] B. Baker, S. Norine, *Harmonic morphisms and hyperelliptic graphs*, Int. Math. Res. Notes, Vol. 15, 2914–2955 (2009).

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How to calculate rational coverings efficiently

A powerful tool for investigating these non-congruence subgroups was introduced by Kulkarni in 1991 and is now known as Farey-Symbols. A complete and efficient implementation of it became available only recently. With the help of the Farey-Symbols it is possible to substantially extend methods from analytic number theory. We will discuss three methods. The first is a generalization of a numerical algorithm originally due to Hejhal. The second approach is based on series of papers by Rademacher and Zuckerman which turned out to be very useful in the recent studies the properties of Mock modular forms. A third surprisingly simple method was recently discovered by us. We will present some interesting non-congruence subgroups with interesting Galois groups.

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Enumeration of embedded surface graphs and quantum curves

The theory of quantum curves has been rapidly developed in the last few years since its first appearance in physics 10 years ago. This talk is aimed at reporting the current status of rigorous mathematical theory of quantum curves as of now. In particular, its direct relations to certain graph counting problems will be presented. The talk is based on my joint papers with Dumitrescu, Dunin-Barkowski, Norbury, Shadrin, Sulkowski, and others.

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Symmetric solutions of the dispersionless $2D$ Toda hierarchy, Hurwitz numbers and conformal dynamics

Integrable system dispersionless $2D$ Toda hierarchy first emerged in the theoretical physics in connection with models of gravity. Later it turned out that special solutions of this system are related to the two completely different classical problems: the calculation of the Hurwitz numbers, and the construction of biholomorphic functions mapping an arbitrary domain in the complex plane into the standard disc. In both cases, the desired solutions belong to a special class of important solutions of hierarchy. In report we will show how to find all solutions of this class, how to distinguish among the solutions related to the Hurwitz numbers and conformal maps, and with their help significantly to progress in solving these classical problems.

The report is based on a joint work with A. V. Zabrodin.

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Abelian and nilpotent regular maps and dessins

A map is a 2-cell decomposition of a surface. In our talk we will consider exclusively closed orientable surfaces. A dessin is a bipartite map with a fixed 2-colouring of the vertex-partition. A map is regular if its automorphism group acts regularly on its set of darts (arcs). A dessin is regular if its automorphism group acts regularly on its set of edges. A regular map or a dessin is abelian, or nilpotent, if its automorphism group is, respectively, abelian or nilpotent. We present a classification of abelian regular maps and dessins. Further, we consider a more complex class of nilpotent regular maps and dessins. We present some fundamental results and partial classifications for small nilpotency classes.

New results were obtained in collaboration with S. F. Du, M. Conder, Kan Hu and Naer Wang.

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Knotted graphs with framed vertices

The subject of my talk is an isotopic classification of embeddings of graphs in 3-sphere. For connected finite graphs with 2-*chord framings* we reduce the isotopic classification problem of embeddings of graphs *with framed* (= *oriented ridged*) *vertices* in 3-sphere to the isotopic classification problem of *tangles*. As a corollary we get an isotopic classification of the *rational* embeddings of the connected finite cyclic rank two graphs with framed vertices in 3-sphere.

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Abel pairs and modular curves

An *Abel pair* is a pair (X, α), where X is a complete algebraic curve and α is a rational function on it, whose divisor has the form $\operatorname{div}(\alpha) = nA - nC$. Such pairs appear in connection with the calculation of quasi-elliptic integrals in Abel's work [1].

Similarly to the case of Belyi pairs, an embedded graph $\Gamma_{X,\alpha}$ is associated to an Abel pair; it provides a combinatorial-topological description of Abel pairs. This description will be applied to families of Abel pairs, essentially parametrized by modular curves. The Abel–Belyi pairs will be introduced and counted and their Galois orbits described.

In the current literature similar questions are studied; for example, [3] is devoted to the p-adic reduction of Belyi–Abel pairs. In [2], similar considerations are undertaken.

[1] N. H. Abel, *Über die Integration der Differential-Formel $\rho dx/\sqrt{R}$, wenn R und ρ ganze Funktionen sind*, J. für Math., **1** (1826), 185-221.

[2] F. B. Pakovitch, *Combinatoire des arbres planaires et arithmétique des courbes hyperelliptiques*, Ann. Inst. Fourier, **48**, N2, 323–351 (1998).

[3] L. Zapponi, *Lame curves with bad reduction*, Preprint (2006).

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Embedded graphs and trigonal curves

We discuss an approach to the study of real algebraic and real pseudoholomorphic trigonal curves on ruled surfaces via graphs embedded into Riemann surfaces.

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On gluing a surface of genus g from one and two bicolored polygons

We denote by $B_g(n, k)$ the number of ways to glue together k bicolored polygons $D_1, D_2, \ldots, D_k$, that have (together) $2n$ edges, into connected orientable surface of genus g. Any polygon has even number of vertices, which are properly colored in black and white color, its edges are oriented counterclockwise and one edge, passing from white to black vertex, is marked. We can glue together these edges in such a way that we get a compact oriented surface of genus g, but we can glue together only vertices of the same color. There are a lot of equivalent descriptions of these numbers in terms of maps, fatgraphs, chord diagrams, permutations etc.

These numbers for $k = 1$ and similar numbers for one not necessary bicolored polygon were thoroughly studied. The main result on gluing surfaces from one polygon is the famous Harer-Zagier formula [4]. An analogue of this formula for one bicolored polygon was independently proved by Jackson [5] and Adrianov [1]. Some results on gluing surfaces from 2 and 3 polygons could be found in [2, 3, 6, 7]. But we know only one result about gluing from more then one bicolored polygon: it is an explicit formula for number of gluings of the sphere from two bicolored polygons [6].

Using the similar notations, like in papers [2, 3], we consider a generating function

$$\mathcal{B}_g^{[k]}(z) = \sum_{n\geq 0} B_g(n,k)z^n.$$

For $k = 1$ and $g > 0$ we prove that

$$\mathcal{B}_g(z) = \frac{Q_g(z)}{(1-4z)^{3g-\frac{1}{2}}},$$

where $Q_g(z)$ is a polynomial with integer coefficients of degree $4g-1$, divisible by z^{2g+1}, and $Q_g(\frac{1}{4}) > 0$. The polynomial $Q_g(z)$ can be calculated by a recursion.

Fore $k = 2$ we prove

$$\mathcal{B}_g^{[2]}(z) = \frac{Q_g^{[2]}(z)}{(1-4z)^{3g-\frac{1}{2}}},$$

where $Q_g^{[2]}(z)$ is a polynomial with integer coefficients of degree at most $4g+2$, divisible by z^{2g+2}, and $Q_g^{[2]}(\frac{1}{4}) > 0$. This polynomial can be calculated by formula

$$Q_g^{[2]}(z) = z^{-1}Q_{g+1}(z) - \sum_{h=1}^{g} Q_h(z)Q_{g+1-h}(z).$$

As a consequence we obtain an explicit formulas for $B_1(n,2)$ and $B_1(n,3)$.

[1] N. M. Adrianov, *An analogue of the Harer-Zagier formula for one-celled bicolored maps*, Functs. Anal. Prilozh., **31** (1997), no. 3, p. 1-9.

[2] J. E. Andersen, R. C. Penner, C. M. Reidys, M. S. Waterman, *Enumeration of linear chord diagrams*, (2010), `arXiv:1010.5614`.

[3] J. E. Andersen, R. C. Penner, C. M. Reidys, R R. Wang, *Linear chord diagrams on two intervals*, (2010), `arXiv:1010.5857`.

[4] J. Harer, D. Zagier, *The Euler characteristic of the moduli space of curves*, Inv. Math., **85** 1986, no.3, p. 457-485.

[5] D. M. Jackson, *Some combinatorial problems associated with products of conjugacy classes of the symmetric group*, J. Combin. Theory Ser. A, **49** (1988) 363-369.

[6] A. V. Pastor, O. P. Rodionova, *Some formulas for the number of gluings*, Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov. (POMI) (2012) **406**, p.117-156.

[7] A. V. Pastor, *On gluing a surface of genus g from two and three polygons*, Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov. (POMI) (2013) **417**, p.128-148.

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Calculating and drawing Belyi pairs

The talk will contain an overview of the known methods of constructing explicit correspondence between Belyi pairs and dessins d'enfants. It will start with a brief description of this correspondence in terms of the equivalence of the appropriate categories; the dream of visualizing the objects of arithmetic geometry will be discussed in this context.

Some pre-Grothendieck constructions (platonian solids, Cayley graphs, Klein quartic) will be mentioned. Several first examples of Belyi pairs calculated after the appearance of *Esquisse d'un programme* will be presented together with the general problems motivated by them.

Certain further calculations up to the most recent ones will be presented, illustrating various methods elaborated during the last decades – using symmetry, differential equations satisfied by Belyi functions and their inversions, including Belyi pairs into Fried families, etc. A special attention will be paid to Galois orbits and to the primes of bad reduction.

Several methods of the approximate calculation (circle packings, discrete periods, PL-Beltrami equations, . . .) will be metioned. The modern possibilities of drawing the *true shapes* of dessins d'enfants will be discussed.

A list of open problems will be suggested.

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Vertex-transitive embeddings of graphs related to the degree-diameter and degree-girth problems

A number of constructions of largest (smallest) currently known graphs of given degree and diameter (girth) produce vertex-transitive graphs. In the talk we will address the question of which of these graphs admit vertex-transitive embeddings.

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Non-orientable regular maps of Euler characteristic equal to the negative of an odd prime power

Let G be the automorphism group of a non-orientable regular map of Euler characteristic equal to the negative of an odd prime power and let O_G be the largest odd-order normal subgroup of G. It turns out that if G is not solvable, then G/O_G is isomorphic to $PSL(2, F)$ or $PGL(2, F)$ for some finite field F of odd characteristic. We will give a classification of all such groups G together with the corresponding regular maps in the case when $G/O_G \cong PSL(2, F)$. The case $G/O_G \cong PGL(2, F)$ appears to be much harder; we will present infinite families of groups in this category.

This is a joint work with M. Conder, N. Gill and I. Short.

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A classification of link maps of graphs to $\mathbb{R}^3$ and polyhedra to $\mathbb{R}^m$

Let P and Q be connected graphs. A *link map* is a map $f : P \sqcup Q \to \mathbb{R}^3$ such that $f(P) \cap f(Q) = \emptyset$. A *link homotopy* is a homotopy through link maps.

Linking coefficients define a 1–1 *correspondence between the set of link homotopy classes of link maps and* $\mathbb{Z}^{(\chi(P)+1)(\chi(Q)+1)}$*, where* χ *is Euler characteristic.*

Although this result is simple (and, for this reason, may be folklore), the proof involves 3-dimensional visualization of the celebrated 4-dimensional Casson's finger moves.

Main Theorem (a particular case). [1] *If* P *and* Q *are closed orientable* 2- *and* 3-*manifolds, then linking coefficients define a* 2–1 *map between the set of link homotopy classes of link maps* $f : P \sqcup Q \to \mathbb{R}^5$ *and* $H_1(P) \oplus H_2(Q)$*, where* H_* *is the homology group with* $\mathbb{Z}$*-coefficients.*

The proof involves higher-dimensional generalizations of Whitney trick and Casson's finger moves.

[1] A. Skopenkov, *On the generalized Massey–Rolfsen invariant for link maps*, Fund. Math. 165 (2000), 1–15.

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Discrete complex analysis: convergence results

Various discretizations of complex analysis have been actively studied since 1920s because of applications to numerical analysis, statistical physics, and inegrable systems. This talk concerns complex analysis on quadrilateral lattices tracing back to the works of J. Ferrand. We solve a problem of S. K. Smirnov on convergence of discrete harmonic functions on planar nonrhombic lattices to their continuous counterparts under lattice refinement. This generalizes the results of R. Courant–K. Friedrichs–H. Lewy, L. Lusternik, D. S. Chelkak–S. K. Smirnov, P. G. Ciarlet–P.-A. Raviart. We also prove convergence of discrete period matrices and discrete Abelian integrals to their continuous counterparts (this is a joint work with A. I. Bobenko). The proofs are based on energy estimates inspired by electrical network theory.

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Locally maximal embeddings of graphs in orientable surfaces

Departing from the classical concept of the maximum genus of a graph we introduce the concept of a locally maximal embedding as a 2-cell embedding of a graph in an orientable surface whose genus cannot be raised by moving any arc within its local rotation. We show that locally maximal embeddings are characterised by the property that every vertex is incident with at most two different faces and derive a natural lower bound on the minimum genus of a locally maximal embedding of a graph in terms of its Betti number and the maximum number of disjoint cycles. We further investigate relationships between the minimum genus, maximum genus, and the locally-maximal genus of a graph and prove that every locally maximal embedding that is not maximal can be transformed into an embedding of higher genus by a sequence of edge moves that never decrease genus.

This is a joint work with Michal Kotrbčík.

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Intrinsic metric on the space of levels of the infinite graded graphs and the notion of standardness

1. Description of the intrinsic metric.
2. The standard graded graph as the graph which has the uniform compact spaces of the levels in the intrinsic metric.
3. Applicaions to the problem of invariant measures.
4. Examples: Concrete epsilon net for various graphs – Pascal graphs of arbitrary dimension, Young graph, etc.

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Fundamental groups of spaces of nonsingular trigonal curves

For the space of $PGL(2,\mathbb{C})$-orbits of the space of complex trigonal curves on Hirzebruch surface Σ_k, a cell structure has been constructed. The cell structure is described via Grothendiek's *dessins d'enfants*. For the space of nonsingular complex trigonal curves on the Hirzebruch surface Σ_k and for its subspace of the curves with the simple roots of the discriminant of the curve equation, the fundamental groups (for $k = 1$) and their images in the spherical braid group (for any k) have been calculated.

This is a joint work with Stepan Orevkov.

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Hurwitz numbers for real polynomials

There are n^{n-3} (properly normalized) complex degree n polynomials with $n-1$ fixed critical values. This can be found by establishing a one-to-one correspondence between these polynomials and marked trees, which are enumerated by the Cayley formula. The number of (properly normalized) real degree n polynomials with $n-1$ fixed real critical values is equal to the n-th Euler-Bernoulli number. This can be found by establishing a one-to-one correspondence between these polynomials and alternating permutations. The problem above can be generalized by allowing multiple critical values and fixing their ramification profiles. In the complex case this problem is solved; in the real case, however, the answer depends on the order of the critical values on the real line. Thus the question arises whether it is possible to attribute a sign to every real polynomial in such a way that the number of polynomials counted with signs is invariant under permutations of critical values. We construct a sign with this property and study the invariant thus obtained.

This is a joint work with Ilia Itenberg.