Mathematical Physics: Past, Present, and Future

Euler International Mathematical Institute, March 26-30, 2014

Abstracts of talks:

Global holography

Irina Aref'eva (Steklov Mathematical Institute, Moscow, Russia)

Some particular examples of global holographic effects will be presented.

Virasoro on the lattice: Volterra, Toda–2, and q–Toda

Olivier Babelon (Université Pierre et Marie Curie & CNRS, France)

I will revisit old ideas about putting CFT on the lattice and show that besides the well known Volterra model it is natural to consider the Toda–2 chain i.e. the Toda chain in the second Hamiltonian structure. I will show how this Toda–2 chain is related to the q–Toda chain.

Discrete Liouville equation, discrete minimal surfaces and applications in architecture

Alexander Bobenko (Technische Universität Berlin, Germany)

We present recent results on discrete minimal surfaces. Geometric description is obtained in frames of discrete differential geometry. Analytic aspects of the theory are based on the theory of integrable systems. Applications in free–form architecture will be demonstrated.

A conformal block approach to the connection problem of the third Painlevé equation

Alexander Its (Purdue University Indianapolis, USA)

In the recent series of papers of O. Gamayun, N. Igorov, O. Lisovyy, A. Shchechkin, J. Teschner, and Yu. Tykhyy a novel approach to the analysis of the Painlevé equations has been suggested. The method is based on the Virasoro conformal block representation of the relevant tau-functions, and it allows, in particular, to evaluate the constant term in the asymptotics of the Painlevé V— tau-function. It should be mentioned that the evaluation of the constant terms in the asymptotics of tau–functions is a very serious challenge for the usual Riemann–Hilbert–isomonodromy method. In the talk, the conformal block technique will be extended to the case of the third Painlevé equation. A new feature which needs to be taken care of is the presence of the irregular singularities in the problem. This is the joint work with O. Lisovyy and Yu. Tykhyy.

Fractional and Majorana fermions

Roman Jackiw (MIT, USA)

Localized zero energy modes of Dirac–type equations are signals for novel excitations.

On beta pentagon relations

Rinat Kashaev (University of Geneva, Switzerland)

The beta pentagon relations constitute a special class of five term integral relations satisfied, in particular, by Euler's beta function. I will discuss relationships of the beta pentagon relations with Faddeev's operator pentagon relation for the quantum dilogarithm.

Commuting difference operators and combinatorial Gale transform

Igor Krichever (Columbia University, USA)

Recently difference equations of the form $V + ai1Vi + 1 + ... + aikVi + k + Vi + k + 1 = 0$ with n–periodic coefficients have attracted an additional interest. They provide a unifying framework for problems in the theories of integrable systems, cluster algebras, frieze patterns. It turns out that this equation admits introductions of the spectral parameter and establish a connection with the theory of commuting difference operators and combinatorial Gale transform.

Mandelbrot cascades and their uses

Antti Kupiainen (Helsinki University, Finland)

Mandelbrot cascades are random measures that exhibit nontrivial multifractal behaviour and also a freezing transition as a temperature parameter is varied. I review their construction and properties and their uses in random geometry and disordered systems.

Rogue waves in 1+1 and 2+1 integrable models

Vladimir Matveev (Université de Bourgogne, France)

We discuss the recently discovered phenomenon of the multiple rogue wave generation in a frame of of focusing NLS and KP-I equations making a particular accent on the NLS–KP-I correspondence, which can be fruitfully explored in two senses: first, allowing to isolate the specially interesting subclasses of smooth, real purely rational solutions of the KP-I equation (quite different from well known multi–lums solutions), and second, providing a global vision on the modular dependence of the rank n multi-rogue waves solutions of the focusing NLS equation.

Quantum symmetry approach to solvable lattice models

Tetsuji Miwa (Kyoto University, Japan)

I give a summary of the developments in solvable lattice models using quantum symmetries.

On AKSZ moduli spaces

Pavel Mnev (Steklov Mathematical Institute, St.-Petersburg, Russia)

AKSZ moduli spaces arise as spaces of gauge equivalence classes of solutions of equations of motion (or reduced phase spaces associated to boundaries) in topological AKSZ sigma models. Examples include in particular the moduli spaces of flat connections and the symplectic groupoids of Poisson manifolds. We will discuss natural geometric structures existing on AKSZ moduli spaces, their gluing properties and quantization.

An integral transform for elliptic four-point conformal blocks in Liouville theory

André Neveu (Université Montpellier 2, France)

By applying an integral transformation on a version of the Knizhnik–Zamolodchikov equation, we obtain explicit representations of Liouville theory four–point conformal blocks for values of their external dimensions on a lattice where their expressions were not previously known.

Strings, integrable models and biology — a new frontier?

Antti Niemi (Uppsala University, Sweden & CNRS, France)

The biological function of a protein depends critically on its three dimensional geometry. But at the moment we do not know how the shape of a protein could be deduced from the DNA sequence alone. As a consequence the protein folding problem endures as one of the most important unresolved problems in science, it addresses the origin of life itself. In this talk we shall argue, that the shape of a protein can actually be determined from very general principles, that are also utilized in the context of string theory and integrable models. We shall argue, that there is a universal energy function which relates to the discrete nonlinear Schrodinger equation, that describes all known folded protein structures. We show how to derive this energy function from fundamental geometrical concepts. We show that it supports topological soliton solutions, that describe folded proteins with a precision better than the radius of an atom.

Semiclassical q–6j symbols

Nicolai Reshetikhin (UC Berkeley, USA)

Solutions to the pentagon equation are constructed using symplectic geometry of moduli spaces of flat G–connections. Here G is a compact simple Lie group.

Piecewise linear spaces: Einstein metrics and Ricci flows

Robert Schrader (Freie Universität Berlin, Germany)

Regge calculus has come to play an important role in a formulation of "Lattice gravity". The basic objects are piecewise linear manifolds to which one can associate analogues of the total scalar curvature, of the volume in Riemannian geometry and more generally analogues of all Lipschitz–Killing curvatures. In this talk I will present new results obtained in collaboration with Werner Mueller (Bonn). In particular we provide two types of Einstein spaces and of correspondingly two types of (normalized) Ricci flows. We give examples and show that under the first of these flows the total scalar curvature always decreases strictly, except at an Einstein space of the first type. Under the second flow, a properly defined averaged total scalar curvature also decreases strictly, except at an Einstein space of the second type.

Multidimensional universally typical sets and the Lempel–Ziv algorithm

Rudolf Seiler (Technische Universität Berlin, Germany)

The Lemplel–Ziv algorithm is a constructive algorithm for the construction of typical sets. It is intrinsically one dimensional. A multidimensional version will be discussed.

Poisson geometry of difference Lax operators, and difference Galois theory

Michael Semenov–Tian–Shansky (Steklov Mathematical Institute, St.-Petersburg, Russia & Université de Bourgogne, France)

We discuss the lift of Poisson structures associated with auxiliary linear problems for the differential and difference Lax equations to the space of wave functions. Due to a peculiar symmetry breaking, the corresponding differential and difference Galois groups become Poisson Lie Groups.

Quantum integrable hypergeometric systems

Evgeny Sklyanin (University of York, UK)

The hypergeometric systems form a special subclass of quantum integrable systems. In addition to the general properties that are common to all quantum integrable systems, they have a number of peculiar properties that stem from the hypergeometricity. Those properties include bispectrality, integral representations, ladder operators, resolvent formulae for the integrals of motion. To illustrate the point we use the examples of Calogero–Moser system, Benjamin–Ono equation, quantum nonlinear Schroedinger equation on the infinite interval.

Quantization of gauge fields beyond perturbation theory

Andrey Slavnov (Steklov Mathematical Institute, Moscow, Russia)

New approach to the quantization of gauge fields free of ambiguity in choosing gauge is developed.

One–point functions in sinh–Gordon and three–point–function in Liouville models

Fedor Smirnov (CNRS, France)

Using the fermionic basis we conjecture exact formulae for the one-point function for the sinh-Gordon model on a cylinder. We check the agreement with the Liouville three–point functions in the limit of small radius of the cylinder. Excellent agreement between the two supports both our conjecture and the Zamolodchikov–Zamolodchikov formula for the reflection coefficient in Liouville model.

Elliptic hypergeometric functions, superconformal indices, and integrable systems

Vyacheslav Spiridonov (JINR, Dubna, Russia)

This is a brief survey of interconnections between three objects:
1) elliptic hypergeometric functions and their symmetries,
2) superconformal indices of four dimensional supersymmetric field theories and Seiberg dualities,
3) quantum inverse scattering method developed by Faddeev and his school.

Real Schubert calculus and integrable models

Vitaly Tarasov (Steklov Mathematical Institute, St.–Petersburg, Russia & Purdue University Indianapolis, USA)

I will describe the results on reality of solutions in Schubert calculus problems obtained via the Bethe ansatz for quantum integrable models.

Light–cone lattice, quantum affine algebras, and the modular double

Joerg Teschner (DESY, Hamburg, Germany)

Taylor–made discretizations of affine Toda theories on a light–cone lattice can be constructed using representations of quantum affine algebras of modular double type. The main ingredients of the quantum inverse scattering method (including the R– and Q–operators) are constructed. The results can be obtained from the formal expressions obtained from the known product formulae for the universal R–matrices by means of a natural renormalization procedure.

Loewner–Kufarev evolution and KP hierarchies

Alexander Vasiliev (University of Bergen, Norway)

We consider a homotopic evolution in the space of smooth shapes starting from the unit circle. Based on the Loewner–Kufarev equation we give a Hamiltonian formulation of this evolution and provide conservation laws. The symmetries of the evolution are given by the Virasoro algebra. The "positive" Virasoro generators span the holomorphic part of the complexified vector bundle over the space of conformal embeddings of the unit disk into the complex plane and smooth on the boundary. In the covariant formulation they are conserved along the Hamiltonian flow. The "negative" Virasoro generators can be recovered by an iterative method making use of the canonical Poisson structure. We study an embedding of the Loewner–Kufarev trajectories into the Segal–Wilson Grassmannian, construct the tau-function, the Baker–Akhiezer function, and finally, give a class of solutions to the KP equation which are invariant on Loewner–Kufarev trajectories.

Tetrahedral Y–system

Alexander Volkov (Steklov Mathematical Institute, St.–Petersburg, Russia)

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