Lamé Chair conference

The Mathematics of Quantum Disordered System

December 15 - 19, 2014

Euler International Mathematical Institute

St. Petersburg, Russia

Monday, 15/12/2014

09h30 Registration
09h55 Welcome
10h00 L. Pastur
11h00 Coffee break
11h30 P. Hislop
12h30 Lunch
14h30 C. Rojas-Molina
15h30 Coffee break
16h00 C. Shirley
18h00 Welcome party

Tuesday, 16/12/2014

10h00 A. Soshnikov
11h00 Coffee break
11h30 A. Klein
12h30 Lunch
14h30 C. Sadel
15h30 Coffee break
16h00 M. Vogel

Wednesday, 17/12/2014

10h00 L. Pastur
11h00 Coffee break
11h30 D. Chafaï
12h30 Lunch
14h00 An afternoon in St Petersburg

Thursday, 18/12/2014

10h00 A. Elgart
11h00 Coffee break
11h30 F. Nakano
12h30 Lunch
14h30 M. Tautenhahn
15h30 Coffee break
16h00 F. Hoecker-Escuti
19h30 Conference dinner

Friday, 19/12/2014

10h00 L. Pastur
11h00 Coffee break
11h30 M. Gebert
12h30 Lunch
14h30 F. Klopp

Around the circular law

Djalil Chafaï

Abstract

Among the basic results in Random Matrix Theory, the Circular Law is probably the simplest to state and the hardest to prove. We will provide a gentle introduction to this topic. We will also take the time to discuss recent progresses, extensions, and open questions.

Trimmed Anderson model - localization and its breakup

Alexander Elgart

Abstract

We will discuss properties of discrete random Schrödinger operators in which the random part of the potential is supported on a sublattice. For these models, in the strong disorder regime, one can trace out the onset of the localization breakup, at least for some examples. This is a joint work with Sasha Sodin.

On the exact asymptotics of the orthogonality catastrophe in Fermi gases

Martin Gebert

Abstract

We consider the asymptotics of the scalar product of the ground states of two non-interacting Fermi gases in the thermodynamic limit. We recall recent results deduced in [GKM14] and [GKMO14] on upper bounds on this scalar product and on the related asymptotics of products of spectral projections for rather general Schrödinger op- erators. In the following we focus on the special case of a zero-range perturbation in 3-dimensional Euclidean space for which we compute the exact asymptotics of the ground-state overlap. Our result confirms the asymptotics Anderson claimed in [And67]. On the other hand it shows that the the upper bound on the ground state-overlap deduced in [GKMO14] does not provide the exact asymptotics in general.

References

[And67] P. W. Anderson, *Ground state of a magnetic impurity in a metal*, Phys. Rev. 164, 352359 (1967).

[GKM14] M. Gebert, H. Küttler, and P. Müller, *Anderson's Orthogonality Catastrophe*, Comm. Math. Phys., 329, 979998 (2014).

[GKMO14] M. Gebert, H. Küttler, P. Müller, and P. Otte, *The exponent in the orthogonality catastrophe for Fermi gases*, arXiv:1407.2512 (2014).

EIGENVALUE STATISTICS FOR RANDOM SCHRÖDINGER OPERATORS WITH HIGHER-RANK PERTURBATIONS

PETER D. HISLOP

Abstract. We prove that certain natural random variables associated with the local eigenvalue statistics for generalized lattice Anderson models constructed with finite-rank perturbations are compound Poisson distributed. This distribution is characterized by the fact that the Lévy measure is supported on at most a finite set determined by the rank. The proof relies on a Minami-type estimate for finite-rank perturbations. For Anderson-type continuum models on $\mathbb{R}^d$, we prove a similar result for certain natural random variables associated with the local eigenvalue statistics. We prove that the compound Poisson distribution associated with these random variables has a Lévy measure whose support is at most the set of positive integers. This is joint work with M. Krishna.

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On the ground state energy of a random Schrödinger operator on finite rooted trees

Francicso Hoecker-Escuti

Abstract

In this talk we will consider the Anderson model on finite, symmetric, rooted trees, with constant degree except for the root and the leaves. We will discuss the behavior of its ground state energy in function of the length of the tree. Understanding the asymptotic behavior of the ground state energy is one of the crucial steps of our recent proof of the existence of Lifshits tails on the infinite regular tree with no loops (the Bethe lattice). This is joint work with C. Schumacher.

LOCALIZATION FOR MULTI-PARTICLE CONTINUOUS ANDERSON HAMILTONIANS

ABEL KLEIN
UNIVERSITY OF CALIFORNIA, IRVINE

We extend the bootstrap multiscale analysis developed by Germinet and Klein to the multi-particle continuous Anderson Hamiltonian, obtaining Anderson localization with finite multiplicity of eigenvalues, decay of eigenfunction correlations, and a strong form of dynamical localization. We do not require a covering condition. The initial step for this multiscale analysis, required to hold for energies in a nontrivial interval at the bottom of the spectrum, is verified for multi-particle continuous Anderson Hamiltonians. We also extend the unique continuation principle for spectral projections of Schrödinger operators to arbitrary rectangles, and use it to prove Wegner estimates for multi-particle continuous Anderson Hamiltonians without the requirement of a covering condition. (Joint work with Son Nguyen.)

References

Abel Klein and Son T. Nguyen: Bootstrap multiscale analysis and localization for multi-particle continuous Anderson Hamiltonians, J. Spectr. Theory (to appear). Preprint at arXiv:1311.4220.

Interacting one dimensional electrons in a Poisson random potential

Frédéric Klopp

Abstract

In this talk, we consider the one dimensional Schödinger operator with a repulsive Poisson random potential. We consider n interacting electrons located in this random background and restricted to an interval of length L. We study the limit of the ground state and of the ground state energy (per particle) of this quantum system when n and L go to infinity in such a way that n/L converges to a fixed positive density, say, ρ. The density of particles ρ is our main parameter to control the thermodynamic limit; it will be assumed to be small.

Level statistics for 1-dimensional Schrödinger operator and beta-ensemble

Fumihiko Nakano

Abstract

A part of this talk is based on joint work with Prof. Kotani. We consider the following two classes of 1-dimensional random Schrödinger operators :

1. operators with decaying random potential, and
2. operators whose coupling constants decay as the system size becomes large.

Our problem is to identify the limit ξ_∞ of the point process consisting of rescaled eigenvalues. The result is :

1. for slow decay, ξ_∞ is a clock process ; for critical decay ξ_∞ is the $Sine_\beta$ process,
2. for slow decay, ξ_∞ is a deterministic clock process ; for critical decay ξ_∞ is the Sch_τ process.

As a byproduct of 1., we have a proof of coincidence of the scaling limits of circular and Gaussian beta ensembles.

Analogs of Szegö Theorem for Ergodic Operators and Related Topics of Quantum Informatics

L. Pastur

Institute for Low Temperatures, Kharkov, Ukraine

St Petersburg, 15 – 19 December 2014

Abstract

In the first part of the mini-course we consider an asymptotic setting for ergodic operators generalizing that for the Szegö theorem on the determinants of finite-dimensional restrictions of the Toeplitz operators. The setting is motivated by certain problems of quantum mechanics and quantum informatics and formulated via the asymptotic trace formula determined by a triple consisting of an ergodic operator and two functions, the symbol and the test function. In the frameworks of this setting we analyze two important examples of ergodic operators: the one dimensional discrete Schroedinger operator with random i.i.d. potential and the same operator with quasiperiodic potential. In the random case we find that for smooth symbols the corresponding asymptotic formula contains a new subleading term, which is random and proportional to the square root of the length of the interval of restriction. The origin of the term are the Gaussian fluctuations of the corresponding trace, i.e, in fact, the Central Limit Theorem for the trace. We also present an example of a non-smooth symbol for which the subleading term is the sum of two ergodic processes bounded with probability 1, while for the convolution operators and the same symbol the subleading term grows logarithmically in the length of the interval. In the quasiperiodic case and for smooth symbols the subleading term is bounded as in the Szegö theorem but unlike the theorem, where the term does not depend on the length, in the quasiperiodic case the term is the sum of two ergodic processes in the length of the interval of restriction.

In the second part of the mini-course we discuss one of the problems of quantum informatics, which can be formulated as a version of the Szegö theorem. We give a brief description of the corresponding notions and basic results on the entanglement entropy of macroscopic systems and then present recent results on the entanglement entropy of the d-dimensional quasifree fermions whose one body Hamiltonian is the discrete Schroedinger operator with random potential. Using basic facts on Anderson localization, we show first that the disorder averaged entanglement entropy $\langle S_L \rangle$ of the d dimension cube of side length L admits the area law asymptotic scaling $\langle S_L \rangle \sim L^{(d-1)}$, $L \gg 1$ even in the gapless case, thereby manifesting the area law in the mean for our model. For $d = 1$ and $L \gg 1$ we obtain then asymptotic bounds for the entanglement entropy of typical realizations of disorder and use them to show that the entanglement entropy is not selfaveraging, i.e., has non vanishing random fluctuations even if $L \gg 1$.

Ergodicity and localization for the Delone-Anderson model

Constanza Rojas Molina

Abstract

Delone-Anderson models arise in the study of wave localization in random media, where the underlying configuration of impurities in space is aperiodic, as for example, in disordered quasicrystals. The lack of translation invariance in the model yields a break of ergodicity, and the loss of properties linked to it. In this talk we will present recent results on the ergodic properties of such models, namely, the existence of the integrated density of states and the almost-sure spectrum. We use the framework of coloured Delone dynamical systems, which allows us to retrieve properties known for the ergodic Anderson model, under some geometric assumptions on the underlying configuration of impurities. In the particular case of a Delone-Anderson perturbation of the Laplacian, we can prove that the integrated density of states exhibits a Lifshitz-tail behavior, which allows us to study localization at low energies. This is joint work with F. Germinet (U. de Cergy-Pontoise) and P. Müller (LMU Munich).

Anderson transition at 2D volume growth for the completely shell connecting graph

Christian Sadel

Abstract

We consider the following graph: The graph vertices consist of countably many finite sets, $S_0, S_1, ...$ consisting of $s_0, s_1, ...$ elements where s_n is any sequence of positive integers. We connect each point in S_n with each point in S_{n+1} and normalize the weight of the edge by $1/\sqrt{s_n s_{n+1}}$ and let A be the corresponding weighted adjacency operator. The set S_0 can be considered as set of roots and S_n is the set of vertices of graph distance n. Therefore, we say the graph has d-dimensional volume growth if $s_n \sim n^{d-1}$, and it has at least d-dimensional volume growth if $s_n > cn^{d-1}$. The volume growth is uniform if s_n/n^{d-1} has a positive limit as $n \to \infty$. We consider the Anderson model given by $A + \lambda V$ where V is an i.i.d. compactly supported potential. For small disorder, in a certain energy region the spectrum is purely absolutely continuous if the volume growth is at least d dimensional for $d > 2$ and it is pure p oint if the volume growth is uniform d-dimensional for any $d < 2$. The special structure of the graph allows a description with transfer matrices, it can be seen as a hybrid between one and multi-dimensional graphs.

Spectral Statistics of one dimensional random Schrödinger operators

Christopher Shirley

Abstract

The talk is devoted to the spectral stastictics of one-dimensional random Schrödinger operators in the localized regime, and in particular to the decorrelation estimates of eigenvalues, which are for instance used to prove the convergence to a Poisson process of the local level statistics. The decorrelation estimates of close eigenvalues, better known as Minami estimates, or the decorrelation of distant eigenvalues were essentially known for the Anderson model, in any dimension. In dimension one, we are now able to prove these results for many models. We will present the connections bewteen spectral statistics and decorrelation estimates and recent advances for one- dimensional models.

Products of Independent Elliptic Random Matrices

Alexander Soshnikov

Abstract

For fixed $m > 1$, we study the product of m independent $N \times N$ elliptic random matrices as N tends to infinity. Our main result shows that the empirical spectral distribution of the product converges, with probability 1, to the m-th power of the circular law, regardless of the joint distribution of the mirror entries in each matrix. This leads to a new kind of universality phenomenon: the limit law for the product of independent random matrices is independent of the limit laws for the individual matrices themselves.

This is a joint work with Sean O'Rourke, David Renfrew, and Van Vu.

Quantitative unique continuation and Wegner estimate for the random breather model

Martin Tautenhahn
(joint work with Ivica Nakić, Matthias Täufer and Ivan Veselić)

Abstract

In this talk we present a scale free and quantitative unique continuation principle for linear combinations of eigenfunctions of Schrödinger operators. Let $\Lambda_L = (-L, L)^d$ and $H_L = -\Delta + V$ be a Schrödinger operator on $L^2(\Lambda_L)$ with a bounded potential $V : \mathbb{R}^d \to [-K, K]$ and Dirichlet boundary conditions. Our main result is of the type

$$\|\phi\|_{\Lambda_L}^2 = C_{\mathrm{sfuc}} \|\phi\|_{W_\delta(L)}^2,$$

where $\phi = \sum_{E_k \in [a,b]} \alpha_k \phi_k$ is a complex linear combination of eigenfunctions corresponding to eigenvalues in $[a, b]$, $W_\delta(L)$ is some union of equidistributed δ-balls in Λ_L and $C_{\mathrm{sfuc}} = C_{\mathrm{sfuc}}(d, a, b, \delta, K)$ some constant. In particular, the constant C_{sfuc} is independent of L and the dependence on the other parameters is known explicitely.

This result finds application, e.g., in control theory of the heat equation and Wegner estimates for random operators. In particular, we sketch the proof of a Wegner estimate for the random breather model and compare our result with earlier results on Wegner estimates for the random breather model.

Eigenvalue statistics for a class of non-selfadjoint operators under random perturbations

Martin Vogel

Abstract

We consider a class of non-selfadjoint h-differential operators P_h in the semiclassical limit ($h \to 0$) subject to small random perturbations with a small coupling constant $\delta > 0$. Results by M. Hager and W. Bordeaux-Montrieux show that under suitable conditions on δ, there is with probability close to 1, a Weyl law for the eigenvalues in the interior of Σ. We will give a precise description of the average density of eigenvalues in all of Σ and show that two eigenvalues of P_h^δ in the interior of Σ exhibit close range repulsion and long range decoupling.