The conference at the Euler International Mathematical Institute
Saint-Petersburg, Russia, June 16 - 20, 2014

Stochastic processes and high dimensional probability distributions

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PROGRAM

Monday, 16 June 2014

10:00-10:45	Registration and coffee
10:45-11:15	Ibragimov
11:15-11:55	Bogachev
12:00-12:30	Kwapien
12:30-14:20	Lunch
14:20-15:00	Vershik
15:00-15:40	Zaporozhets
15:40-16:00	Coffee break
16:00-16:40	Neeman
16:40-17:20	Kolesnikov
17:30	Welcome reception

Tuesday, 17 June 2014

10:00-10:40	McCann
10:40-11:00	Coffee break
11:00-11:40	Maas
11:50-12:30	Gozlan
12:30-14:20	Lunch
14:20-15:00	Kühn
15:00-16:30	Poster session and coffee break Livshyts, Melbourne, Wang, Eliseeva, Khartov
17:00	Boat trip from Euler Institute

Wednesday, 18 June 2014

10:00-10:40 Dembo

10:40-11:00 Coffee break

11:00-11:40 M. Meckes
11:50-12:30 Nikitin

12:30-14:00 Lunch
14:00-20:00 Bus excursion to Petergof and Kronstadt

Thursday, 19 June 2014

10:00-10:40 Rio
10:40-11:00 Coffee break

11:00-11:40 Smorodina
11:50-12:30 Borodin

12:30-14:20 Lunch

14:20-15:00 Belopolskaya
15:00-15:40 Gasnikov

15:40-16:00 Coffee break

16:00-16:40 Zaitsev
16:40-17:20 von Weizsäcker

19:00 Conference dinner (bus from Euler Institute)

Friday, 20 June 2014

09:40-10:20 Pivovarov
10:20-11:00 Petrov

11:00-11:10 Coffee break

11:10-11:40 Zaev
11:40-12:10 Islak
12:10-12:40 Renger

12:40-14:20 Lunch

Titles and abstracts

Yana Belopolskaya (Saint Petersburg State University for Architecture and Civil Engineering, Russia)

Title: Diffusion processes associated with nonlinear parabolic equations

We derive stochastic differential equations (both forward and backward) for diffusion processes associated with various types of the Cauchy problem solutions for nonlinear parabolic equations and systems. We state conditions on SDE data which allow to prove the existence and uniqueness of their solutions. Finally, we use the SDE solutions to derive probabilistic representations for classical, generalized and viscosity solutions of the Cauchy problem for a rather general class of nonlinear parabolic equations and systems.

Vladimir I. Bogachev (Moscow State University, Russia)

Title: Gaussian measures: recent progress and open problems

A brief overview will be given of investigations in the theory of Gaussian measures (infinite-dimensional Gaussian distributions) related to research of V.N. Sudakov. Some major achievements in this theory will be recalled along with a discussion of the status of some challenging open problems. Both linear and nonlinear problems will be discussed, including recent results on the structure of measurable polynomials on spaces with Gaussian measures, analytic and asymptotic properties of their distributions and joint distributions.

Andrey N. Borodin (Steklov Mathematical Institute, St.-Petersburg, Russia)

Title: Diffusions with jumps

A wide class of homogeneous diffusions with jumps is considered. The extreme points of this class are homogeneous diffusion processes and the Poisson processes with variable intensity. The diffusions with jumps have many good properties inherited both from classical diffusion processes and from Poisson ones. This class is closed with respect to a composition with invertible twice continuously differentiable functions. A special random time change gives us again a diffusion with jumps. A result on transformation of the measure of the process analogous to Girsanov's transformation is valid for this class. There are an effective results for computation of distributions of certain functionals of diffusions with jumps.

Amir Dembo (Stanford University, USA)

Title: Persistence Probabilities

Persistence probabilities concern how likely it is that a stochastic process has a long excursion above fixed level and of what are the relevant scenarios for this behavior. Power law decay is expected in many cases of physical significance and the issue is to determine its power exponent parameter. I will survey recent progress in this direction (jointly with Sumit

Mukherjee), dealing with stationary Gaussian processes that arise from random algebraic polynomials of independent coefficients and from the solution to heat equation initiated by white noise.

If time permits, I will allude to related joint works with Jian Ding and Fuchang Gao, about persistence for iterated partial sums and other auto-regressive sequences, and to joint work with Mukherjee and Hironobu Sakagawa on persistence probabilities for the height of certain dynamical random interface models.

Yulia Eliseeva (St.-Petersburg University, Russia)

Title: Estimates for the concentration function of weighted sums of independent random variables.

We represent recent results concerning the Littlewood Offord problem. It is about the behavior of the concentration function of weighted sums of independent random variables according to the arithmetic structure of coeffi cients. These results are refinements results of Friedland and Sodin (2007), Rudelson and Vershynin (2009), Vershynin (2011).

Alexander Gasnikov (MIPT, Moscow, Russia)

Title: Limit shapes and concentration of measure phenomenon in application to macrosystem theory

In the work we consider rather general stochastic chemical kinetic Markov dynamic. Such kind of dynamic arises in many different economic applications. We concentrate in this talk on the case when the number of molecules tends to infinity with the number of different type of molecules. So we have to investigate concentration of invariant measure of the described above Markov dynamic. The vector (curve in the limit) of filing numbers (molecules of different types), which contain with small vicinity all the volume of invariant measure, is called equilibrium. We propose a variation principle for finding such equilibriums. Well known example of this theory is Kinetic of social inequality (equilibrium = limit shape = Pareto curve = law of distribution of citizen for the income).

Natael Gozlan (Université Paris Est, Marne-la-Vallee, France)

Title: Transport proofs of variance estimates for some classes of log-concave vectors.

I will present two recent works dealing with the Poincaré inequality. The first one, in collaboration with C. Roberto and P-M Samson, shows that the Poincaré inequality is implied by very weak dimension free concentration properties in Euclidean distance. The second, in collaboration with D. Cordero-Erausquin, shows how transport methods can be used to prove variants of the Poincaré inequality for some classes of log-concave probability measures.

Ildar A. Ibragimov (Steklov Mathematical Institute, Saint-Petersburg, Russia)

Opening lecture

Umit Islak (University of Southern California, Los Angeles, USA)

Title: Concentration inequalities with bounded couplings

Let Y be a nonnegative random variable with mean μ, and let Y^s, defined on the same space as Y, have the Y size biased distribution, that is, the distribution characterized by $\mathbb{E}[Yf(Y)] = \mu\mathbb{E}[f(Y^s)]$ for all functions f for which these expectations exist. Under bounded coupling conditions, such as $Y^s - Y \leq C$ for some $C > 0$, we show that Y satisfies certain concentration inequalities around μ. Further, we discuss that similar results also hold for some related couplings and that such coupling constructions may provide information to analyze other properties of Y such as its entropy.

Alexey Khartov (St.-Petersburg University, Chebyshev Laboratory, Russia)

Title: Approximation complexity of Gaussian random fields of increasing parametric dimension

Consider a sequence of Gaussian random fields $X_d(t)$, $t \in [0,1]^d$, $d \in \mathbb{N}$, with zero mean and the continuous correlation functions $\mathcal{K}^{X_d}$ of the following tensor product-type form:

$$\mathcal{K}^{X_d}(t,s) = \prod_{j=1}^{d} \mathcal{K}^{X_{1,j}}(t_j, s_j), \quad t, s \in [0,1]^d,$$

where $\mathcal{K}^{X_{1,j}}$ are correlation functions of some processes $X_{1,j}(t)$, $t \in [0,1]$. Every X_d admits well known Karhunen-Loève expansion in $L_2([0,1]^d)$ equipped with a norm $\|\cdot\|_{2,d}$. We approximate X_d with finite sum of series corresponding to n maximal eigenvalues of the expansion. Let us denote this sum by $X_d^{(n)}$. The field $X_d^{(n)}$ provides the minimal average quadratic error among all linear approximations to X_d having rank n (see [3]).

We investigate asymptotic behaviour of average approximation complexity

$$n_\varepsilon^{\mathrm{avg}}(X_d) := \min\Big\{n \in \mathbb{N} : \mathbf{E}\,\|X_d - X_d^{(n)}\|_{2,d}^2 \leqslant \varepsilon^2\,\mathbf{E}\,\|X_d\|_{2,d}^2\Big\}$$

and the probabilistic approximation complexity (see [2])

$$n_{\varepsilon,\delta}^{\mathrm{prob}}(X_d) := \min\Big\{n \in \mathbb{N} : \mathbf{P}\left(\|X_d - X_d^{(n)}\|_{2,d}^2 > \varepsilon^2\,\mathbf{E}\,\|X_d\|_{2,d}^2\right) \leqslant \delta\Big\}$$

when the parametric dimension $d \to \infty$, the error threshold $\varepsilon \in (0,1)$ is fixed, and the significance level $\delta = \delta_{d,\varepsilon}$ may go to zero. Our results essentially complement those of M. A. Lifshits and E. V. Tulyakova (see [1]).

References

[1] M. A. Lifshits, E. V. Tulyakova, Curse of dimensionality in approximation of random fields, Probab. Math. Stat. 26 (2006), no. 1, 97–112.

[2] E. Novak, H. Woźniakowski, Tractability of Multivariate Problems. Volume I: Linear Information, EMS Tracts Math. 6, EMS, Zürich, 2008.

[3] K. Ritter, Average-case Analysis of Numerical Problems, Lecture Notes Math. No 1733, Springer, Berlin, 2000.

Alexander Kolesnikov (Higher School of Economics, Moscow, Russia)

Title: Functional inequalities on convex bodies and convex surfaces

We develop applications of geometric instruments to Poincare- and Sobolev-type inequalities on convex bodies and surfaces. We show, in particular, that a number of results can be obtained by equipping our space with various Riemannian metrics and dualizing the corresponding Bochner-Weitzenbock-Lichnerowicz formula. We pay special attention to metrics generated by optimal transportation potentials and spectral properties of the corresponding Monge-Ampere operator. In collaboration with E. Milman and B. Klartag.

Thomas Kühn (Universität Leipzig, Germany)

Title: Small deviations for some smooth Gaussian processes

In the talk some small deviation results for a family of smooth Gaussian processes will be presented. Such processes play a role in diffusion problems as well in the study of zeros of random polynomials. The talk is based on joint work with Frank Aurzada, Frank Gao, Wenbo Li and Qi-Man Shao. We determine the exact logarithmic small deviation rates with respect to the L_2-norm and the sup-norm. The proofs rely on the connection to metric entropy, due to Kuelbs and Li, and use moreover operator-theoretic arguments like interpolation.

Stanislaw Kwapien (University of Warsaw, Poland)

Title: Estimates of moments of order statistic of independent random variables

Joint work with C. Schütt (University of Kiel, Germany).

Definition. A nonnegative random variable ξ is said to be subregular at ∞, 0 with constants D, δ, (or shortly $\xi \in V\Lambda(D,\delta)$) iff $\delta < 1$ and

$$\begin{aligned} P(\xi \le D^{-1}t) &\le \delta P(\xi \le D^{-1}t) \quad \text{for all } t \le \mathrm{Med}(\xi), \\ P(\xi > Ds) &\le \delta P(\xi > s) \qquad \text{for all } s \ge \mathrm{Med}(\xi). \end{aligned}$$

For a sequence $\xi_1, \xi_2, \dots, \xi_n$ of independent, nonnegative random variables, its nondecreasing rearrangement is denoted by $\xi_1^*, \xi_2^*, \dots, \xi_n^*$ and is called the order statistics. Let F_i denote the distribution function of ξ_i, $i = 1, ..., n$.

Theorem. Let $D > 0 < \delta < 1$ and $0 < p < -\ln_D \delta$. Then there are constants c_1, c_2, c_3, $D' > 0 < \delta' < 1$ such that for each n, $1 \le k \le n$ and each nonegative, independent random variables $\xi_1, \xi_2, \dots, \xi_n \in V\Lambda(D,\delta)$ there hold $\xi_k^* \in V\Lambda(D',\delta')$,

$$c_1 \mathrm{Med}(\xi_k^*) \le c_2 \big(E(\xi_k^*)^p\big)^{1/p} \le \inf\left\{ t : \sum_{i=1}^n F_i(t) \ge k - \frac{1}{2} \right\} \le c_3 \mathrm{Med}(\xi_k^*).$$

Galyna Livshyts (Kent State University, USA)

Title: Maximal surface area of a convex set in $\mathbf{R}^n$ with respect to log concave spherically invariant measures.

We obtain a sharp asymptotic bound for the maximal surface area of a convex set in $\mathbf{R}^n$ with respect to log concave spherically invariant measures in terms of the measure's first two moments.

Jan Maas (University of Bonn, Germany)

Title: A discrete curvature-dimension criterion based on optimal transport

In recent years there has been a lot of progress in the analysis on metric measure spaces based on ideas from optimal transport. We discuss how some of these ideas can be applied to discrete spaces, using a discrete analogue of the Kantorovich metric. In particular we present a discrete notion of Ricci curvature based on geodesic convexity of the entropy, which allows us to obtain several discrete functional inequalities. We also discuss a recent refinement, that yields improved bounds depending on the dimension.
This is joint work with Matthias Erbar (Bonn).

Robert McCann (University of Toronto, Canada)

Title: Optimal transport: old and new

The Monge-Kantorovich optimal transportation problem is to pair producers with consumers so as to minimize a given transportation cost. When the producers and consumers are modeled by probability densities on two given manifolds or subdomains, it is interesting to try to understand the analytical, geometric and topological features of the optimal pairing as a subset of the product manifold. This subset may or may not be the graph of a map.

This lecture describes recent developments concerning Monge's original version of this problem, and contrasts them with a capacity constrained variant in which a bound is imposed on the quantity transported between each given producer and consumer. New results concerning the capacity constrained variant are given, including the development of a duality theory for it. We also indicate how the structure of the solution is intimately connected to the differential topology and geometry of the chosen transportation cost.

Mark Meckes (Case Western Reserve University, Cleveland, USA)

Title: Typical marginals of convex bodies

A recent theorem of Klartag asserts that if X is uniformly distributed in a high-dimensional isotropic convex body, then most linear functionals of X are approximately Gaussian in distribution. This phenomenon has connections with observations going back to Maxwell and Archimedes, with modern roots in the work of Sudakov, among others. I will give a survey of the history of Klartag's theorem, partial results which preceded it, and recent developments.

James Melbourne (University of Minnesota, USA)

Title: Hyperbolic measures on infinite dimensional spaces

Localization and dilation procedures are discussed for infinite dimensional α-concave measures on abstract locally convex spaces (following Borell's hierarchy of hyperbolic measures). Joint work with S. Bobkov.

Joe Neeman (University of Texas, Austin, USA)

Title: Gaussian noise stability

Given two correlated Gaussian vectors, X and Y, the noise stability of a set A is the probability that both X and Y fall in A. In 1985, C. Borell proved that half-spaces maximize the noise stability among all sets of a given Gaussian measure. We will give a new, and simpler, proof of this fact, along with some extensions and applications. Specifically, we will discuss hitting times for the Ornstein-Uhlenbeck process, and a noisy Gaussian analogue of the "double bubble" problem.

Yakov Yu. Nikitin (St.-Petersburg State University, Russia)

Title: Exact small ball asymptotics in Hilbert norm for some Gaussian processes appearing in the regression context

We obtain exact small ball asymptotics in quadratic norm for several Gaussian processes appearing in Statistics, namely in the regression problems, as considered by Jandhyala and MacNeill. Such processes can be considered as the "perturbed" Brownian bridges, and small deviation asymptotics for them was not known. In prospect this asymptotics can be useful in statistical problems due to multiple links of Statistics and small deviation theory. Our results also imply the exact small deviation asymptotics for the so-called detrended Brownian motion (improving recent results of Ai, Li and Liu), and for the generalized detrended Slepian process.
Joint work with A.A. Kirichenko.

Fedor Petrov (Steklov Mathematical Institute, Saint-Petersburg, Russia)

Title: Convex subsets of spatial processes

Given a finite subset M on the Eucledian plane, a general question is to choose a large in some sense subset $C \subset M$ in comvex position. Two cases are of special interest: M being a subset of a lattice and M chosen at random (the most natural case is a Poisson process in a region). First question is a traditional subject of geometry of numbers, and it has arithmeti flavour, The second is rather stochastic, but purely geometric aspects and therefore answers are quite similar. We discuss results of Barany, Reitzner, Vu on convex polygons and recent result of Robins, Gravin, Shiryaev and the speaker on convex curves with vertices chosen from points of Poisson processes.

Peter Pivovarov (University of Missouri at Columbia, USA)

Title: A randomized version of the Blaschke-Santalo inequality

I will discuss inequalities for the volume of the polar of random sets, generated for instance by the convex hull of independent random vectors in Euclidean space. Extremizers are given by random vectors uniformly distributed in Euclidean balls. This provides a randomized extension of the Blaschke-Santal inequality which, in turn, can be derived by the law of large numbers. The method involves shadow systems, their connection to Busemann type inequalities, and how they interact with functional rearrangement inequalities. Joint work with D. Cordero-Erausquin, M. Fradelizi, G. Paouris.

Michiel Renger (Weierstrass Institute for Applied Analysis and Stochastics, Berlin)

Title: Connecting particle systems to entropy-driven gradient flows

The statistical mechanics programme has provided us a deep understanding of the connection between stochastic particle systems at the microscopic level and thermodynamics on the macro level. In particular, the entropy functional can be connected to particle systems by its large deviations. I try to extend this principle to the non-equilibrium case, and connect entropic gradient flows to particle systems in a similar fashion. Such connection can reveal a large class of gradient flow structures, among which are the Wasserstein-Entropy structure, as well as a previously unknown structure for discrete Markov chains.

Emmanuel Rio (Université de Versailles, France)

Title: About Vajda's tight lower bound and Hoeffding-McDiarmid inequality

In this talk we give improvements of the Hoeffding McDiarmid function. In particular we replace the usual Gaussian rate function by Vajda's tight information function in the McDiarmid inequality. We also give new lower bounds on Vajda's tight information function for the Pinsker inequality.

Natalya V. Smorodina (St.-Petersburg State University, Russia)

Title: Limit theorems for symmetric random walks and probabilistic approximation of the Cauchy problem solution for Shrödinger-type evolution equations

We discuss a possibility to construct both a probabilistic representation and a probabilistic approximation of the Cauchy problem solution for an equation $\frac{\partial u}{\partial t} = \frac{\sigma^2}{2}\,\Delta u + V(x)u$, where σ is a complex parameter such that $\operatorname{Re}\sigma^2 \geqslant 0$. This equation coincides with the heat equation when $\operatorname{Im}\sigma = 0$ and with the Schrödinger equation when $\operatorname{Re}\sigma^2 = 0$.

Anatolii M. Vershik (Steklov Mathematical Institute, Saint-Petersburg, Russia)

Title: Iteration of Kantorovich metrics and strong convergence of the martingales

We introduce the notion of standard filtration of the sigma-fields which used the iteration of Kantorovich metric and the construction of the tower of the measures. The random processes which has standard filtration of the past can be considered as analogue of Bernoulli processes and in the stationary case looks similar to the class VWB-processes in ergodic

theory (D.Ornstein) which definition is also used Kantorovich metric but not in the same way.

Jing Wang (Purdue University, USA)

Title: Curvature-dimension inequalities on contact manifolds and gradient estimates for associated Markov processes

In this joint work with F. Baudoin, we study the subelliptic diffusion operators that are canonically associated to sub-Riemannian structures. In particular we study the sub-Laplacian L on a contact Riemannian manifold M which appears as the generator of a Markov process on M. We develop a generalized version of curvature-dimension inequality for L, and prove that it implies the spectral gap estimates of L and the convergence to the equilibrium of the associated Markov semigroup. We also obtain weak Bonnet-Myers type theorem and Bishop-Gromov comparison theorem.

Heinrich von Weizsäcker (Technische Universität Kaiserslautern, Germany)

Title: Remarks on Komlos' subsequence theorem

Danila Zaev (Higher School of Economics, Moscow, Russia)

Title: Monge-Kantorovich problem with additional constraints and its application in infinite-dimensional cases

In my talk I will present some results about modified Monge-Kantorovich problems, where sets of admissible transport plans are restricted in some way. An example of such restriction is an invariance with respect to an action of some group, another one is a martingale property. Both examples can be seen as the particular cases of the Monge-Kantorovich problem with additional linear constraint of the following general form: admissible measures should vanish on a given functional subspace. An important application of the invariant Monge-Kantorovich problem is the possibility of a meaningful formulation for the problem on infinite-dimensional spaces. Some results about existence and properties of optimal transport maps in such cases will be also presented.

Andrei Yu. Zaitsev (Steklov Mathematical Institute, Saint-Petersburg, Russia)

Title: Estimates for the rate of strong approximation in the multidimensional invariance principle

We discuss the results on the rate of strong approximation in the multidimensional invariance principle which were published in the recent papers of Zaitsev and Götze and Zaitsev. They may be considered as multidimensional generalizations and improvements of some results of Komlós, Major and Tusnády (1975), Sakhanenko (1985) and Einmahl (1989).

Dmitry N. Zaporozhets (Steklov Mathematical Institute, Saint-Petersburg, Russia)

Title: On applications of a theorem of V. N. Sudakov

In 1973 Sudakov discovered a deep connection between the expected supremum of a Gaussian process and the average width of some convex body corresponding to this process. The aim of the talk is to derive several corollaries (probabilistic and geometric) from this result.