List of papers
The best constant of Sobolev inequality

August 14, 2014

[1] H.Yamagishi, K.Watanabe and Y.Kametaka, *The best constant of three kinds of the discrete Sobolev inequalities on the complete graph*, Kodai Mathematical Journal **37** (2014), 383–395.

[2] H.Yamagishi, K.Watanabe and Y.Kametaka, *The best constant of L^p Sobolev inequality corresponding to Dirichlet-Neumann boundary value problem*, Math. J. Okayama Univ. **56** (2014), 145–156.

[3] H.Yamagishi, Y.Kametaka, A.Nagai, K.Watanabe and K.Takemura, *The best estimation corresponding to continuous model of Thomson cable*, JSIAM Letters. **5** (2013), 53–56.

[4] H.Yamagishi, Y.Kametaka, A.Nagai, K.Watanabe and K.Takemura, *Complete low-cut filter and the best constant of Sobolev inequality*, JSIAM Letters. **5** (2013), 33–36.

[5] H.Yamagishi, Y.Kametaka, A.Nagai, K.Watanabe and K.Takemura, *The best constant of three kinds of discrete Sobolev inequalities on regular polyhedron*, Tokyo J. Math. **36** (2013), 253–268.

[6] Y.Kametaka, K.Takemura, H.Yamagishi, A.Nagai and K.Watanabe, *Positivity and hierarchical structure of 16 Green functions corresponding to a bending problem of a beam*, Saitama Math. J. **29** (2012), 1–24.

[7] K.Watanabe, K.Takemura, Y.Kametaka, A.Nagai and H.Yamagishi, *Lyapunov-type inequalities for $2M$th order equations under clamped-free boundary conditions*, J. Inequal. Appl. 2012, 2012:242.

[8] K.Takemura, Y.Kametaka, K.Watanabe, A.Nagai and H.Yamagishi, *Sobolev type inequalities of time-periodic boundary value problems for Heaviside and Thomson Cables*, Bound. Value Probl. 2012, 2012:95.

[9] H.Yamagishi, Y.Kametaka, A.Nagai, K.Watanabe and K.Takemura, *Elliptic theta function and the best constants of Sobolev-type inequalities*, JSIAM Letters **4** (2012), 1–4.

[10] Y.Oshime, H.Yamagishi and K.Watanabe, *The best constant of L^p Sobolev inequality corresponding to Neumann boundary value problem for* $(-1)^M(d/dx)^{2M}$, Hiroshima Math. J. **42** (2012), 293–299.

[11] H.Yamagishi, A.Nagai, K.Watanabe, K.Takemura and Y.Kametaka, *The best constant of discrete Sobolev inequality corresponding to a bending problem of a string*, Kumamoto J. Math. **25** (2012), 1–15.

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[13] K.Watanabe, H.Yamagishi and Y.Kametaka, *Riemann zeta function and Lyapunov-type inequalities for certain high order differential equations*, Appl. Math. Comput. 218 (2011), 3950–3953.

[14] K.Watanabe, Y.Kametaka, A.Nagai, K.Takemura and H.Yamagishi, *The best constants of Sobolev and Kolmogorov type inequalities on a half line*, Far East J. Appl. Math. **52**, No.2 (2011), 101–129.

[15] K.Watanabe, Y.Kametaka, H.Yamagishi, A.Nagai and K.Takemura, *The best constant of Sobolev inequality corresponding to clamped boundary value problem*, Bound. Value Probl. Vol. 2011, Article ID 875057, 17 pages.

[16] K.Takemura, Y.Kametaka, K.Watanabe, A.Nagai and H.Yamagishi, *The best constant of Sobolev inequality corresponding to a bending problem of a beam on a half line*, Far East J. Appl. Math. **51**, No.1 (2011), 45–71.

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C60フラーレンの仲間と離散ソボレフ不等式の最良定数

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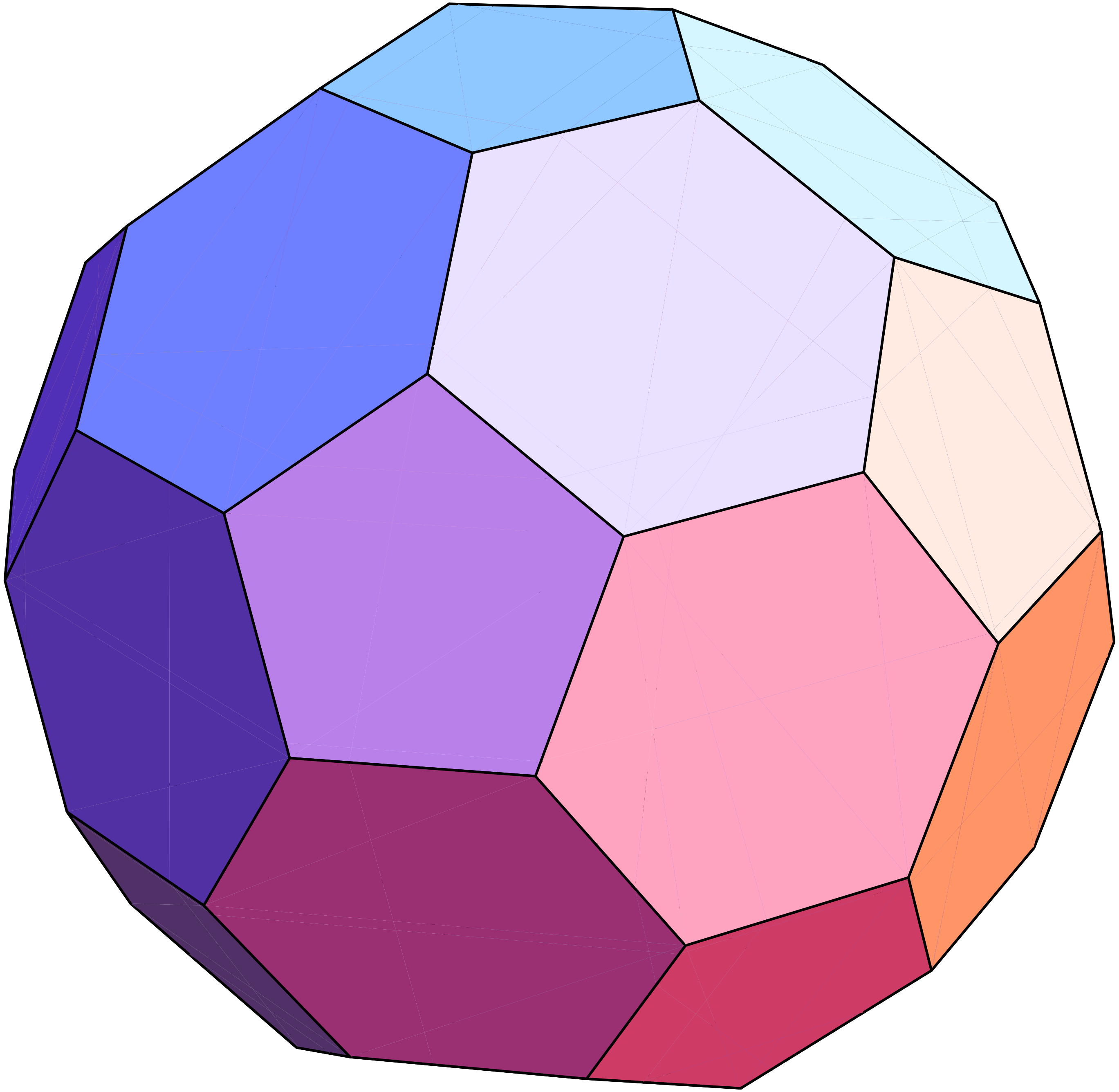
永井 敦 (日大生産工)
山岸 弘幸 (都立産技高専)

The best constant of discrete Sobolev inequality on 13 kinds of the C60 Fullerene

Yoshinori Kametaka
(Osaka Univ.)
Atsushi Nagai
(Nihon Univ.)
Hiroyuki Yamagishi
(Tokyo Metropolitan College)

C60 Fullerene



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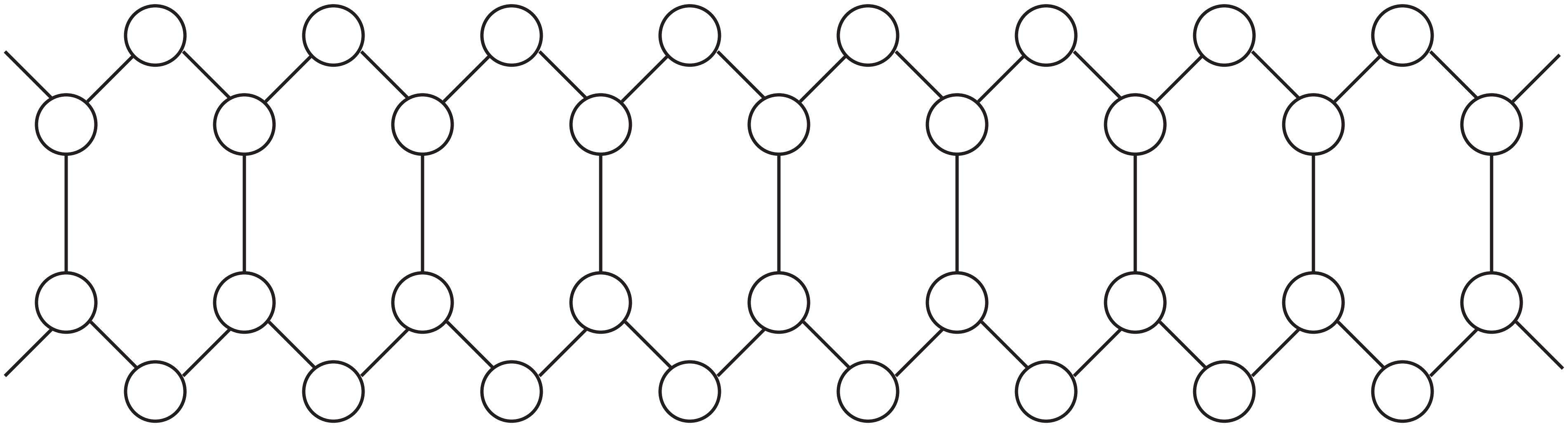

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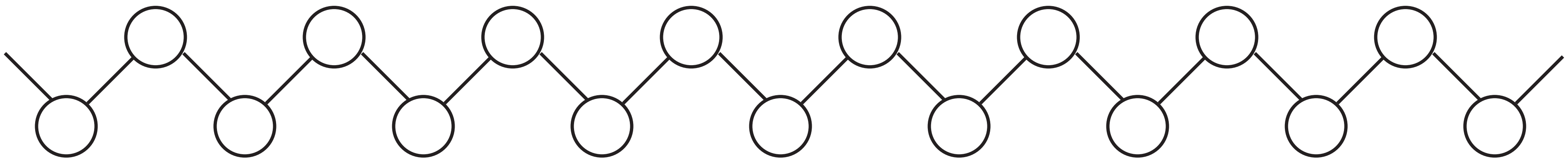

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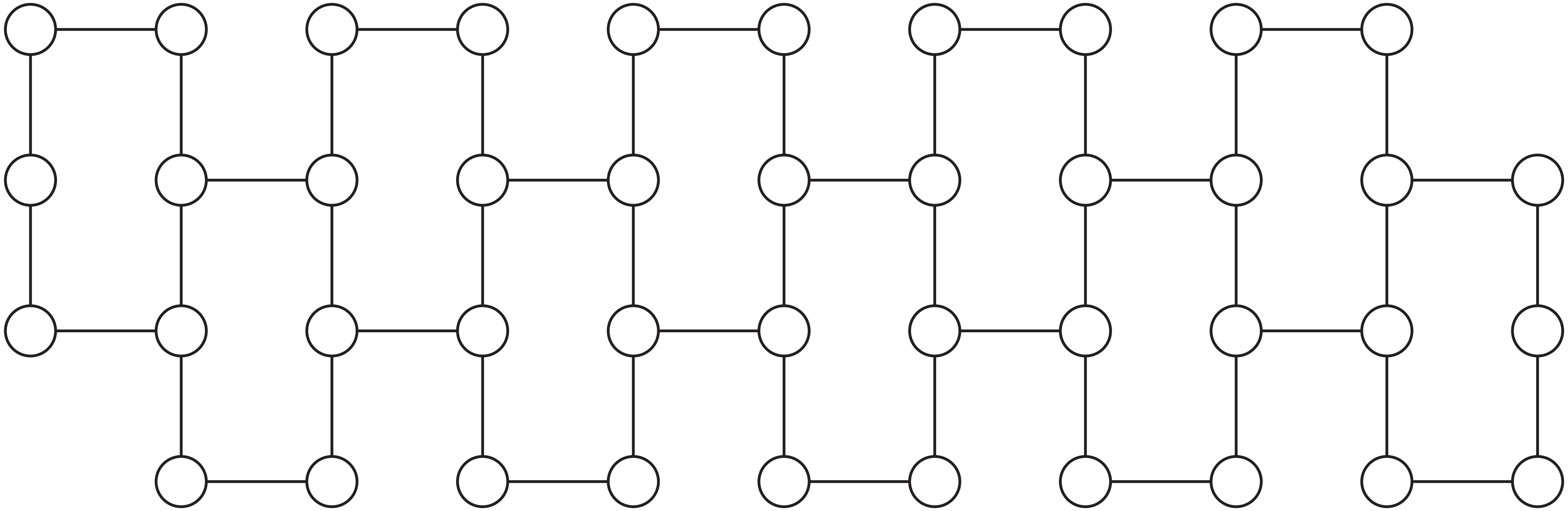

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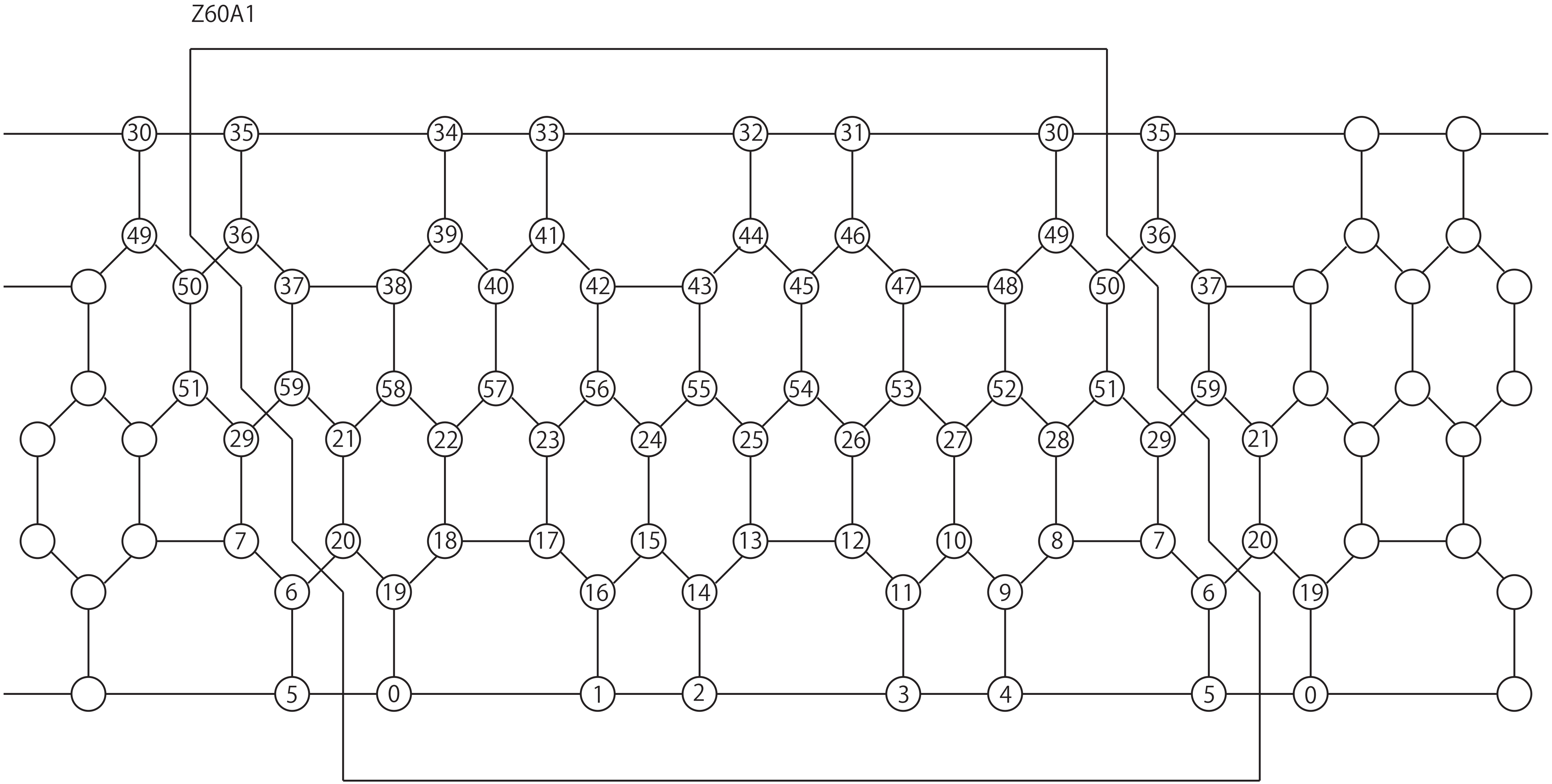

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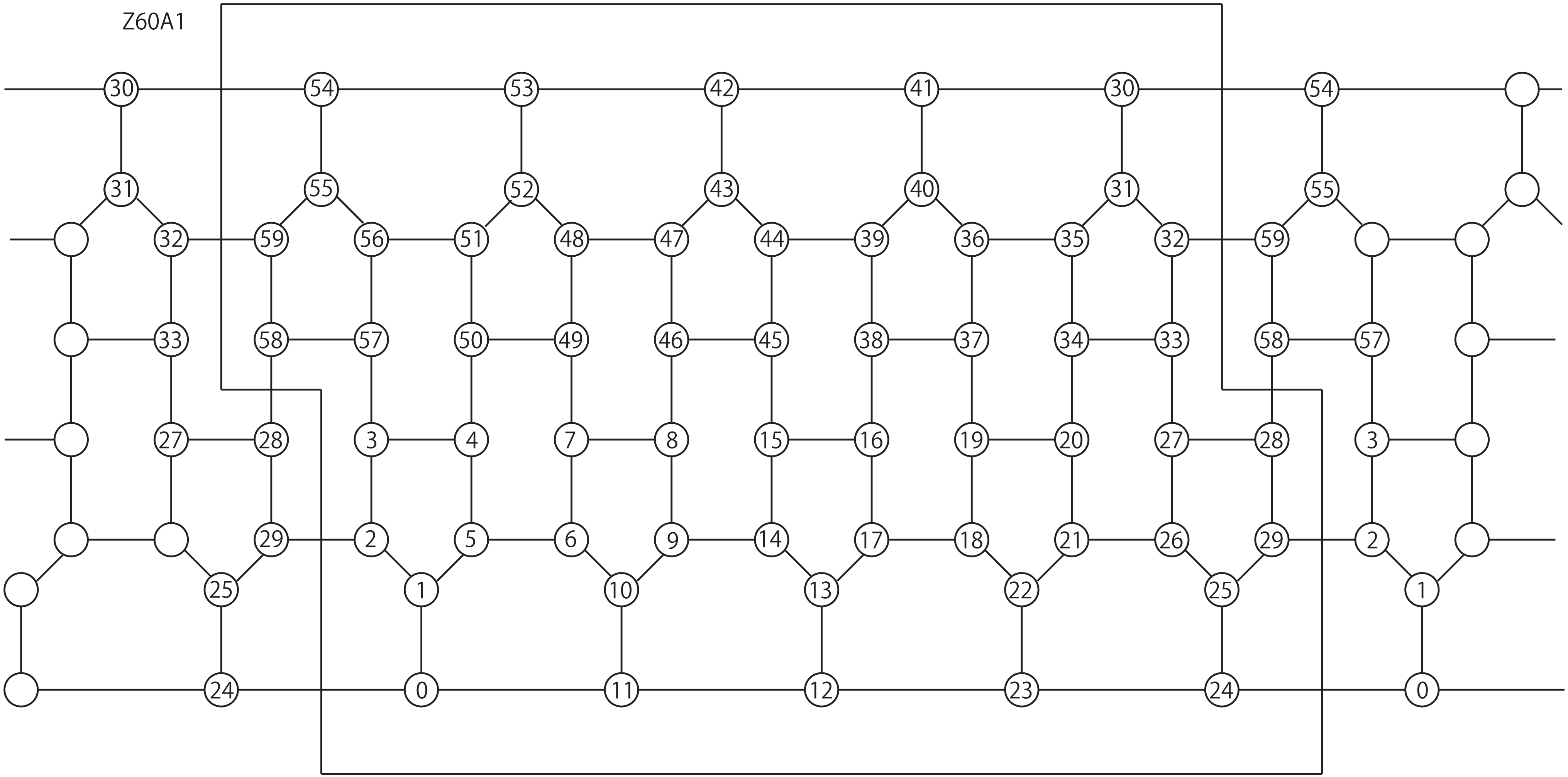

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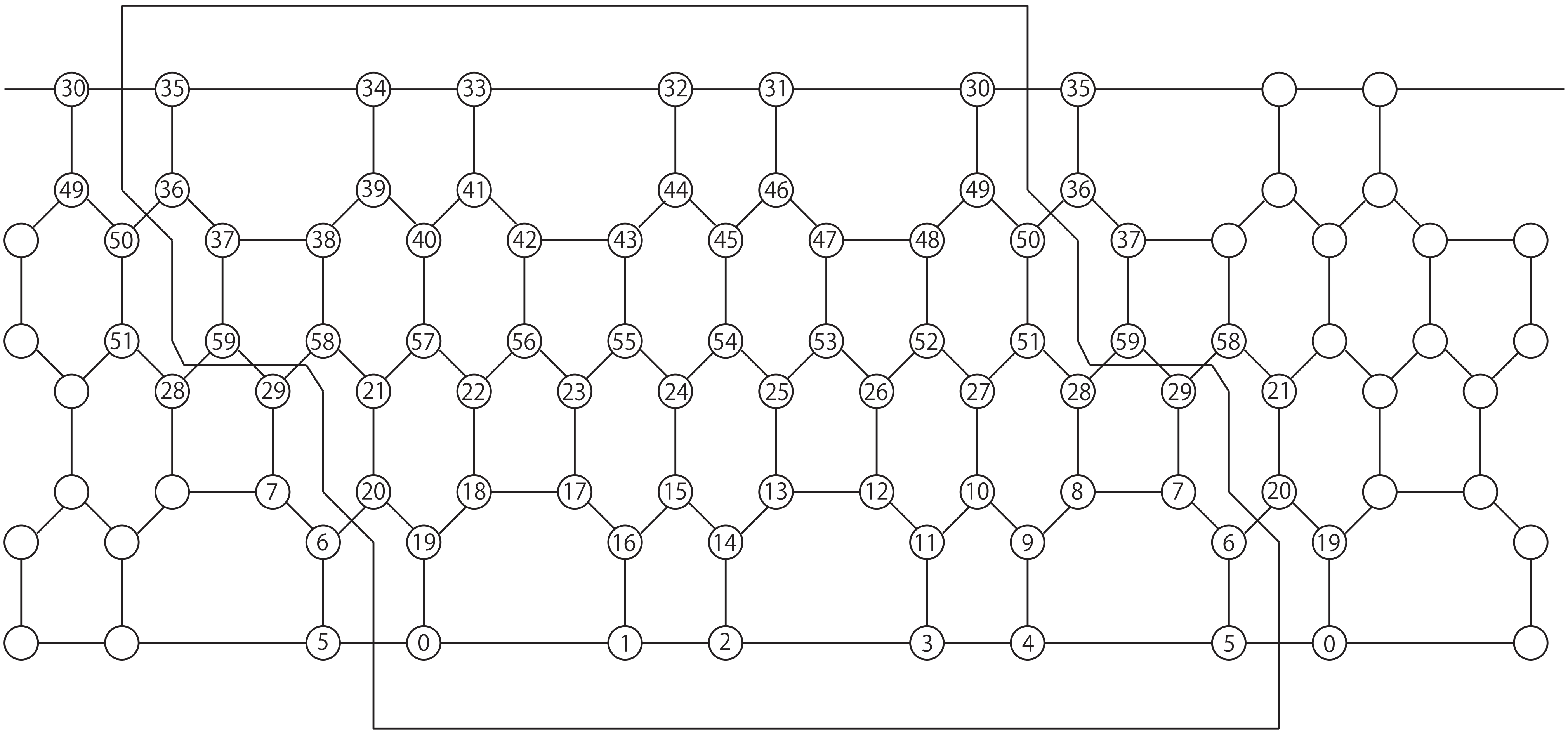
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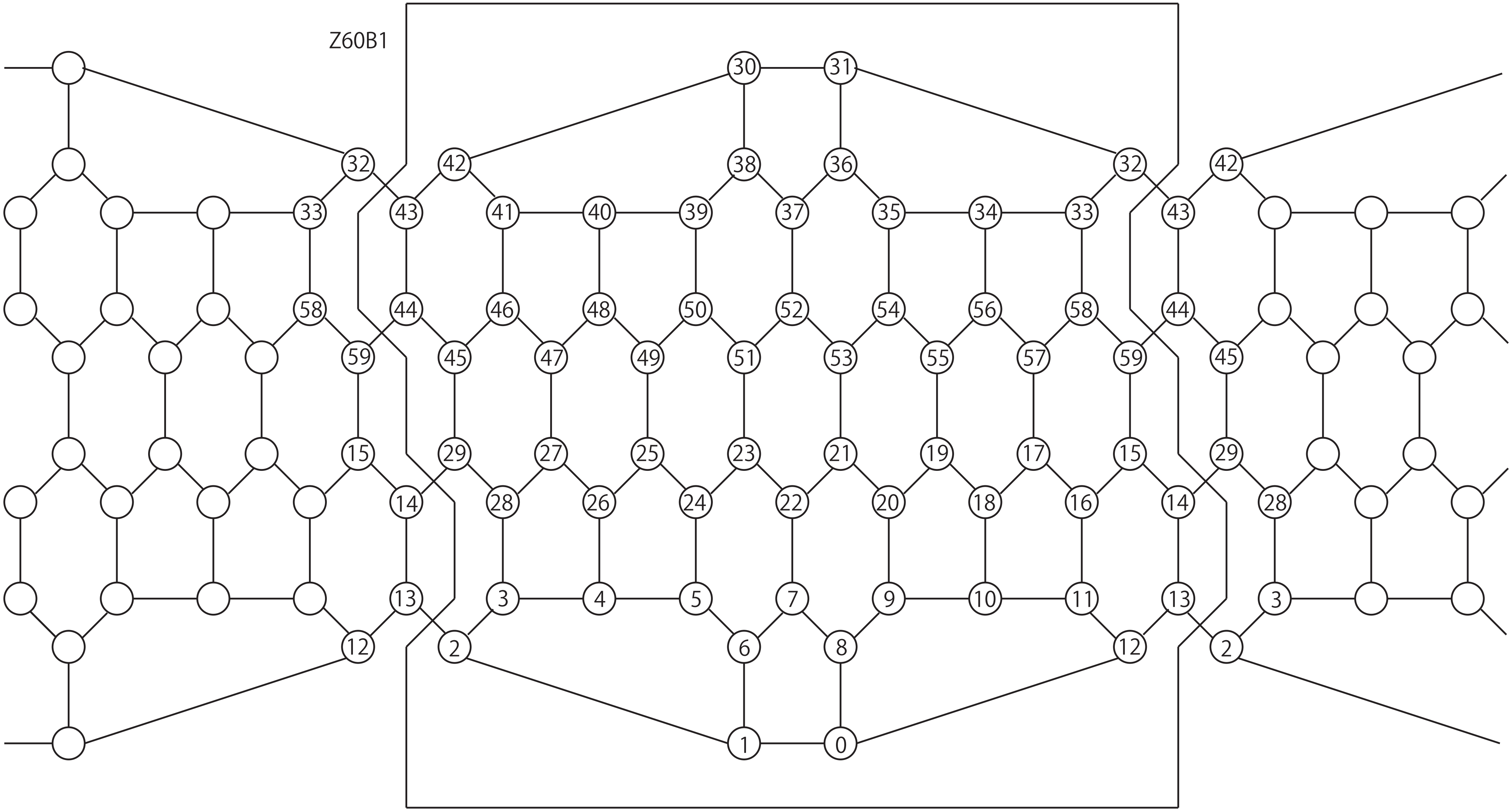
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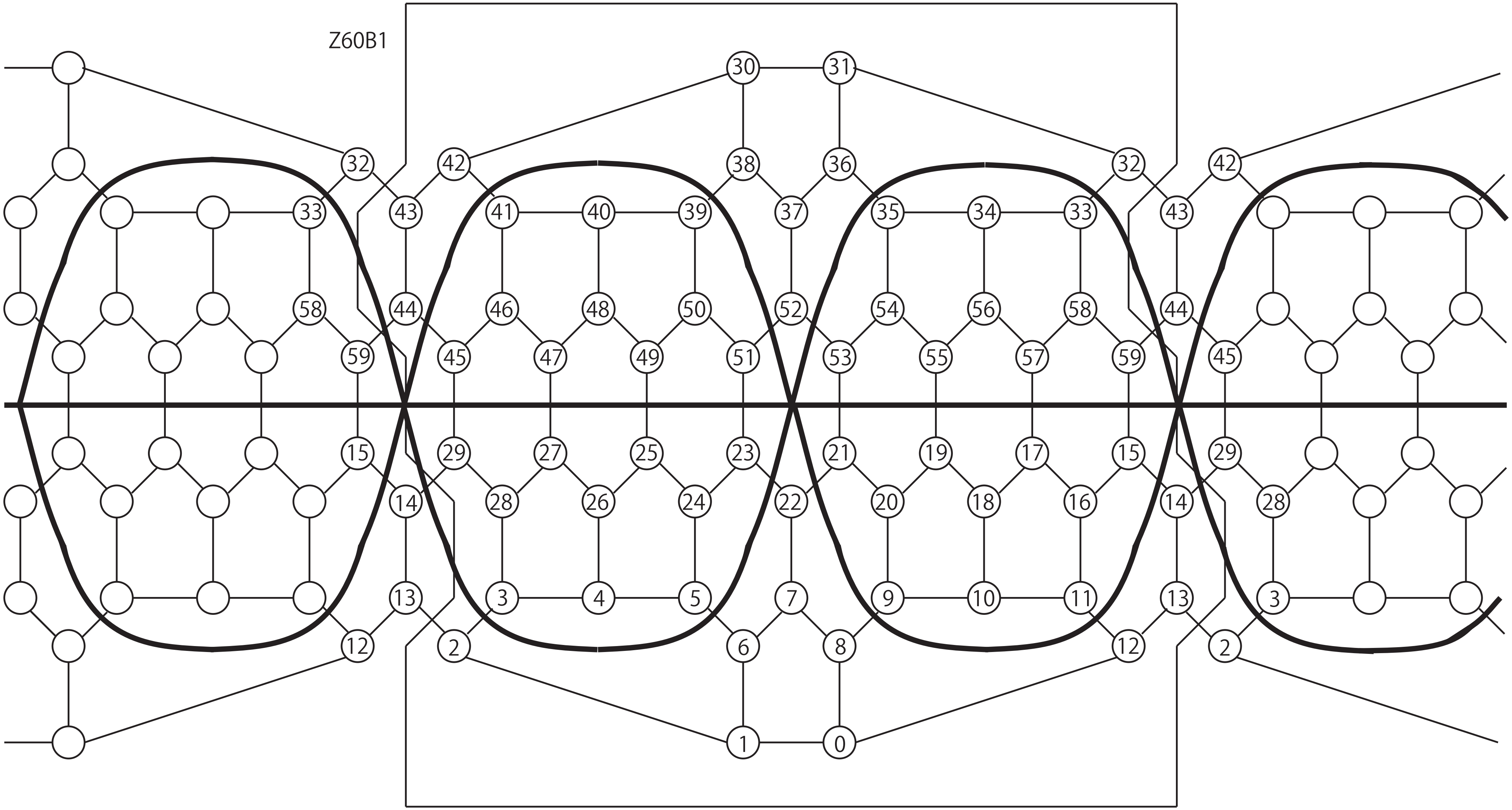
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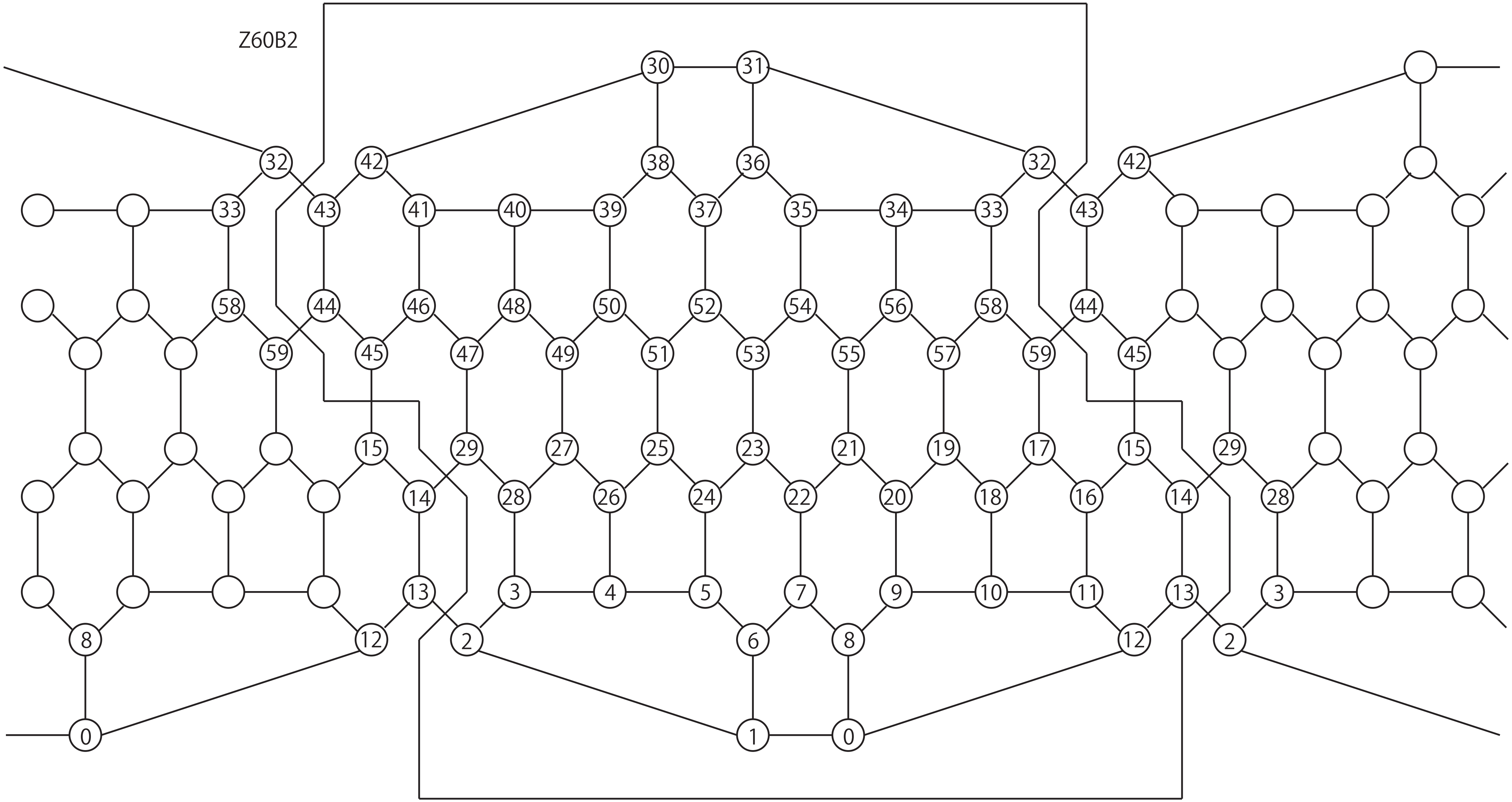

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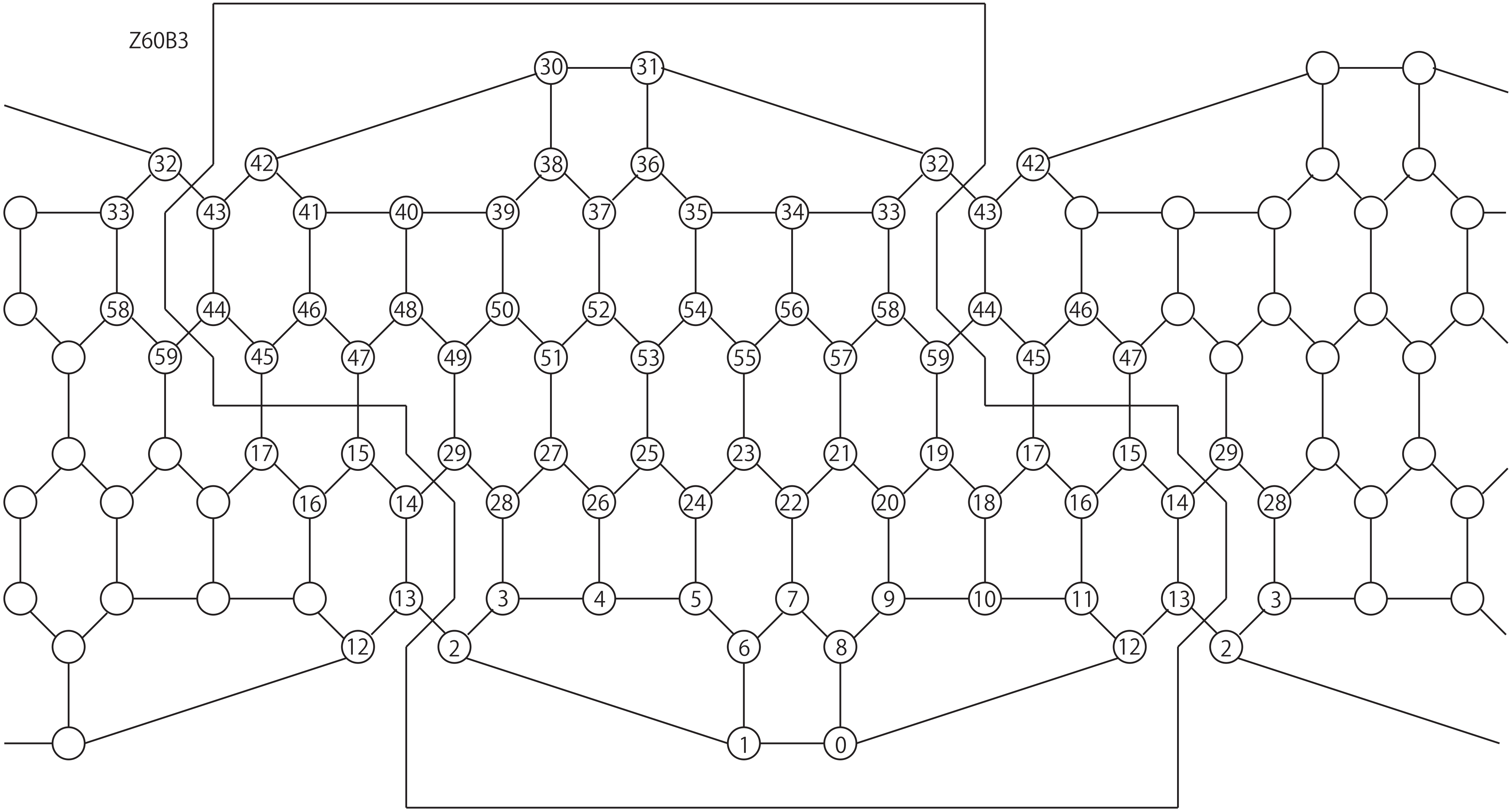

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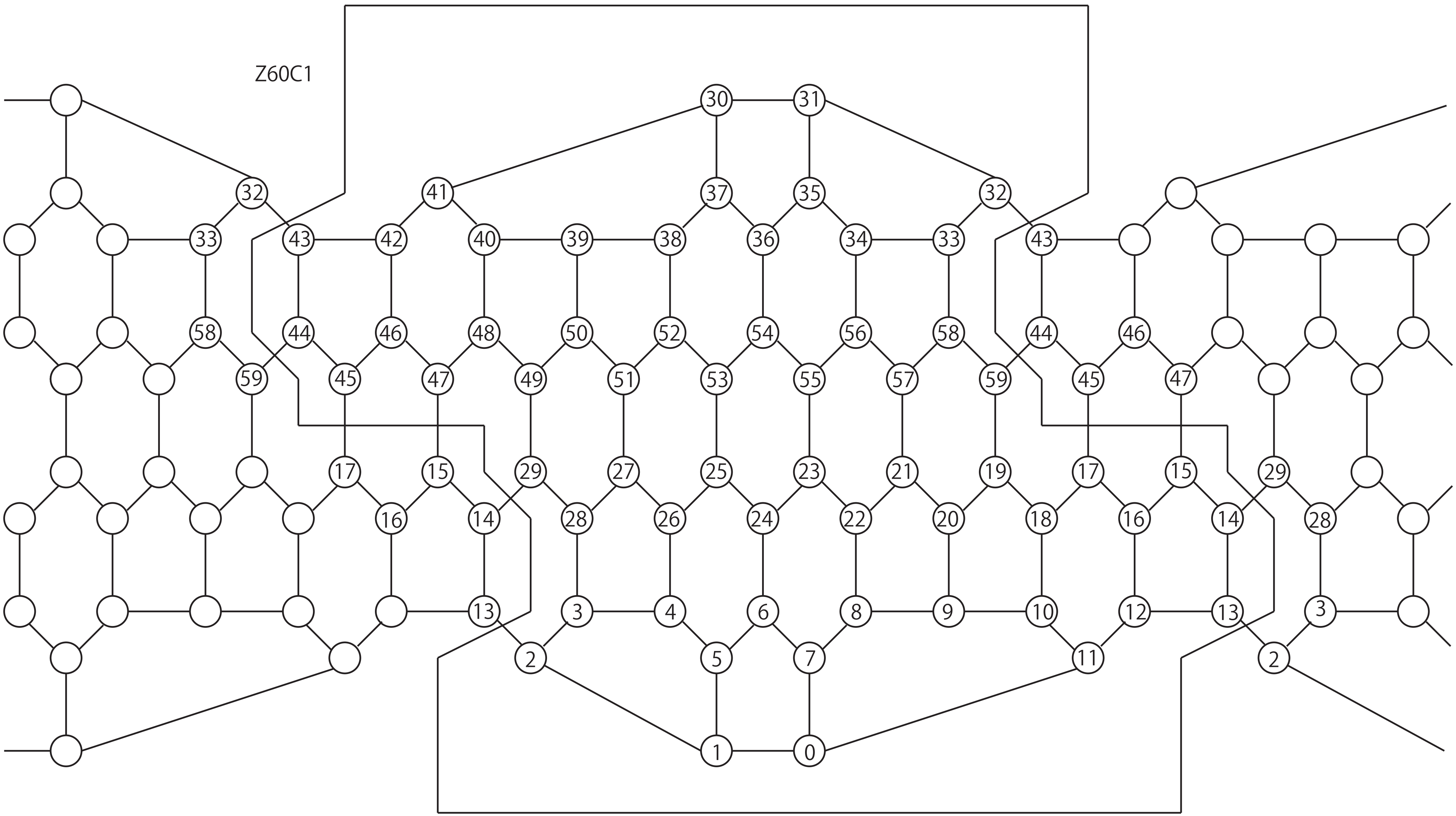
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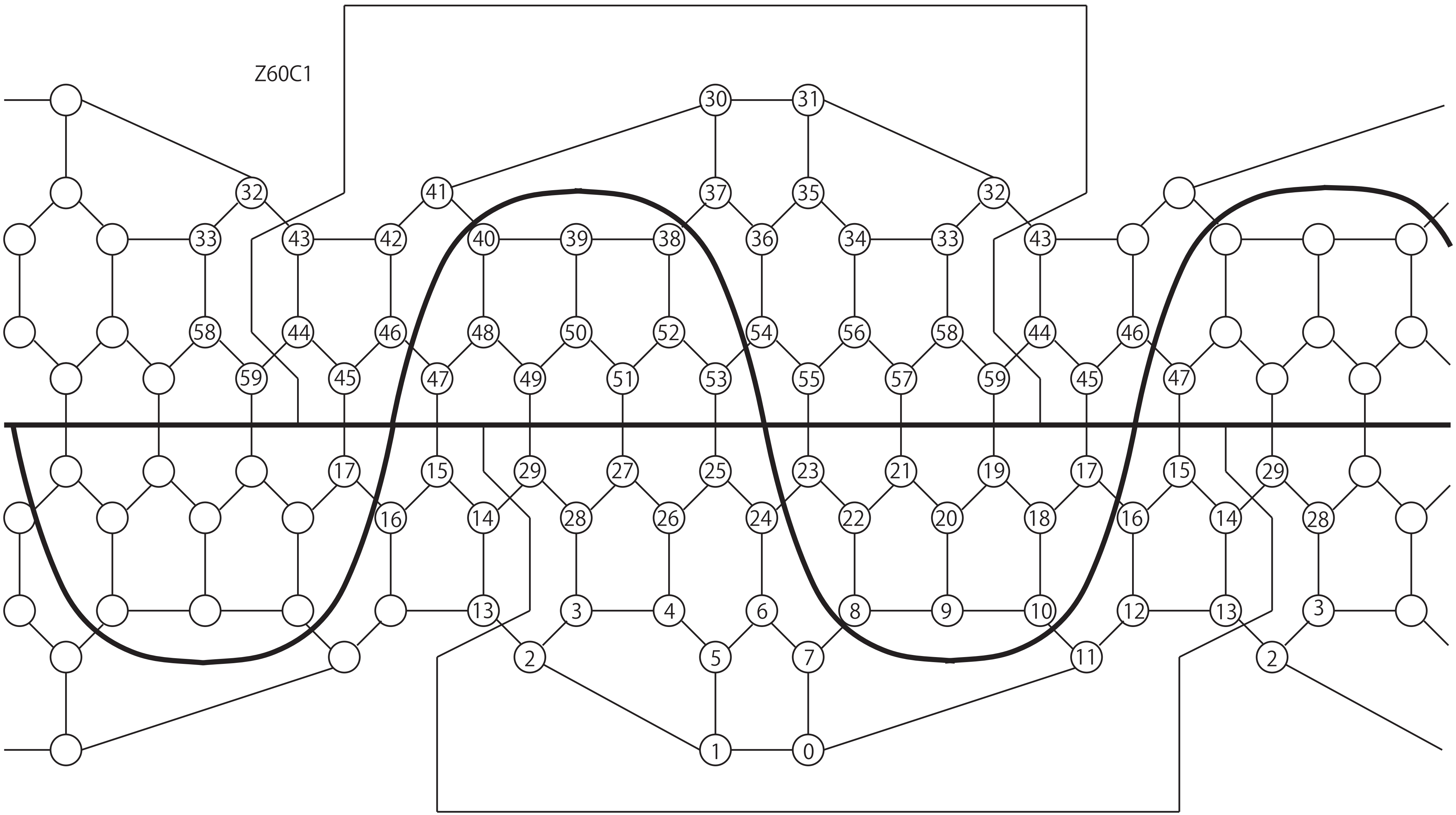

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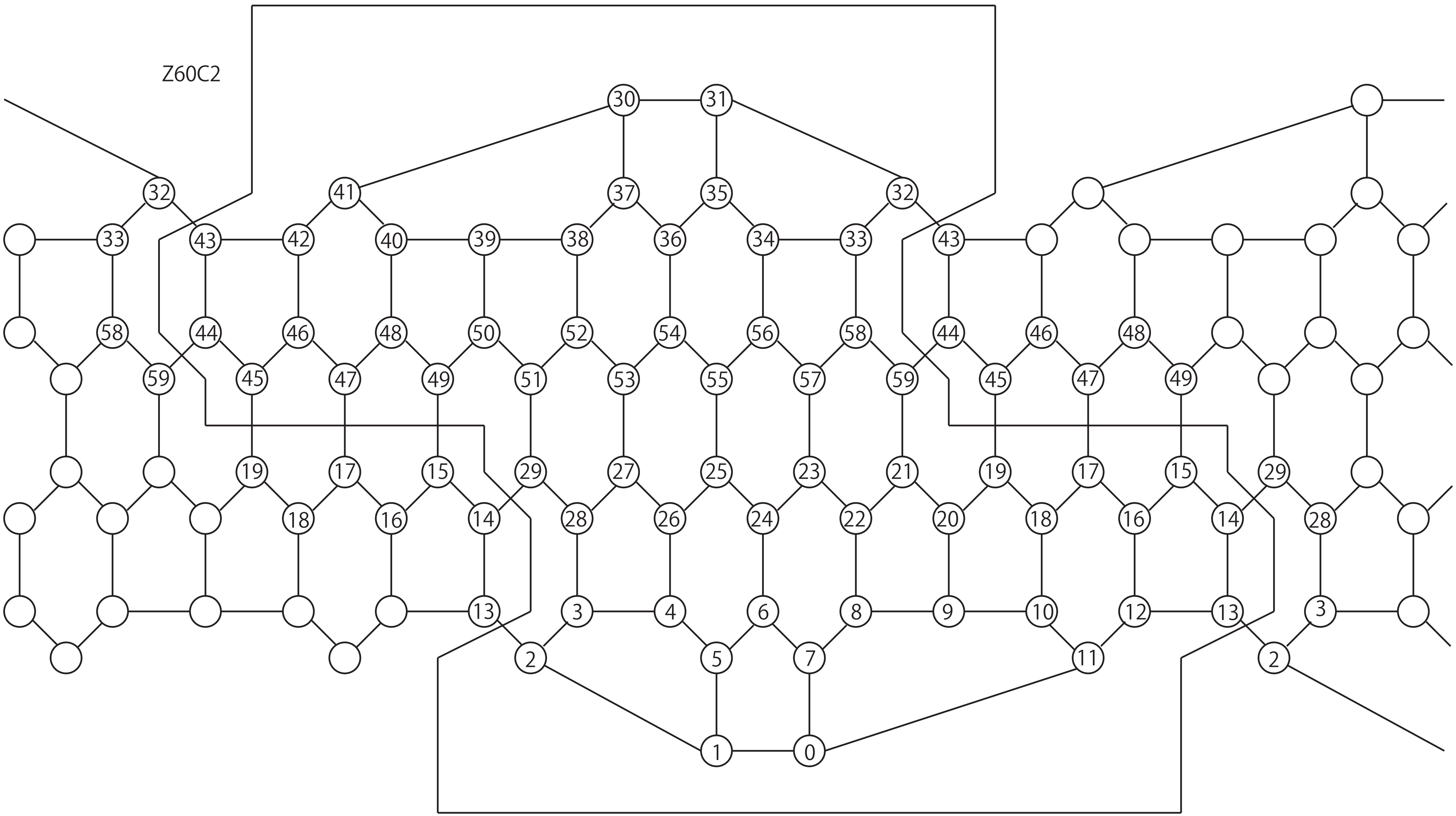
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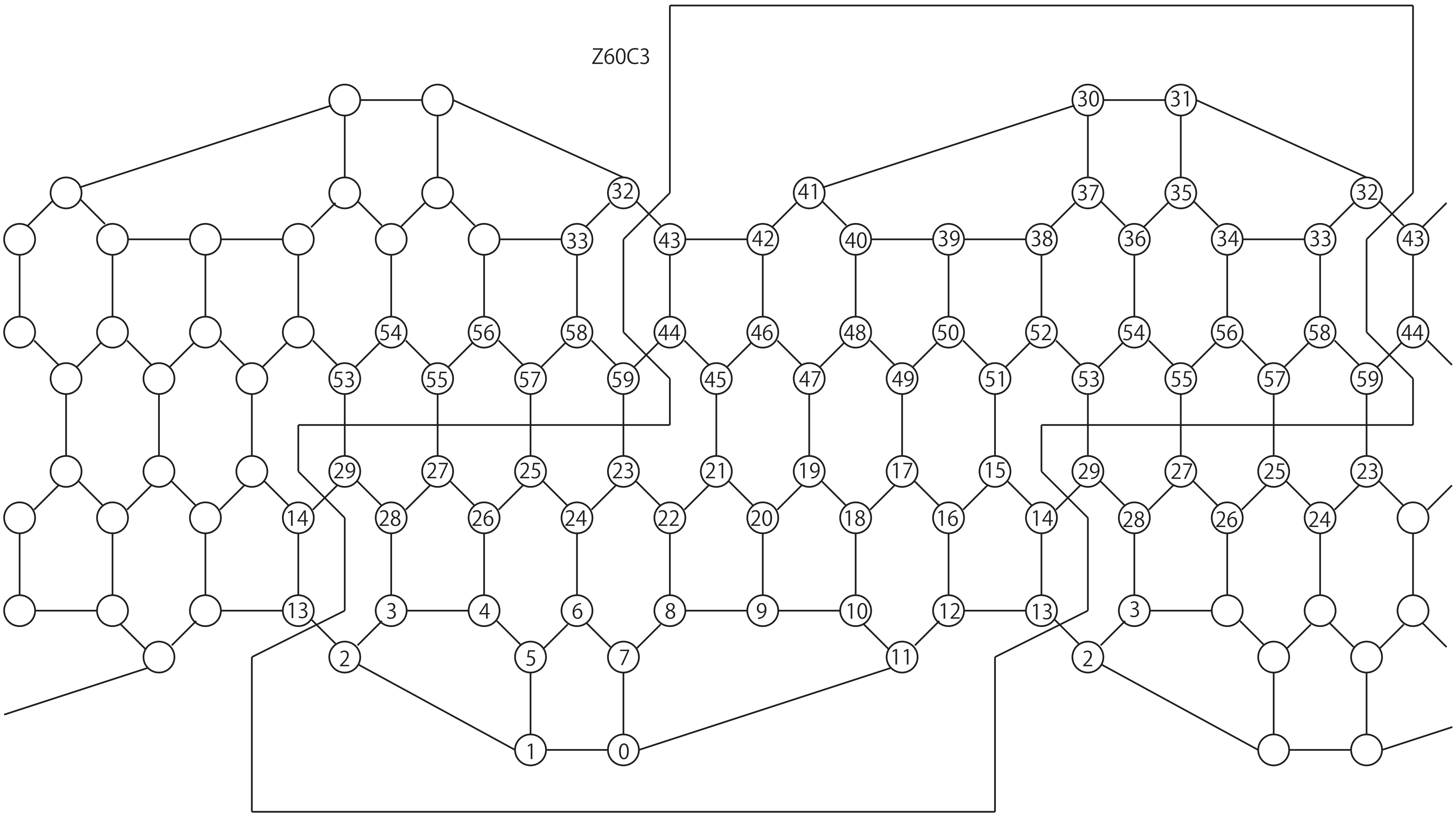

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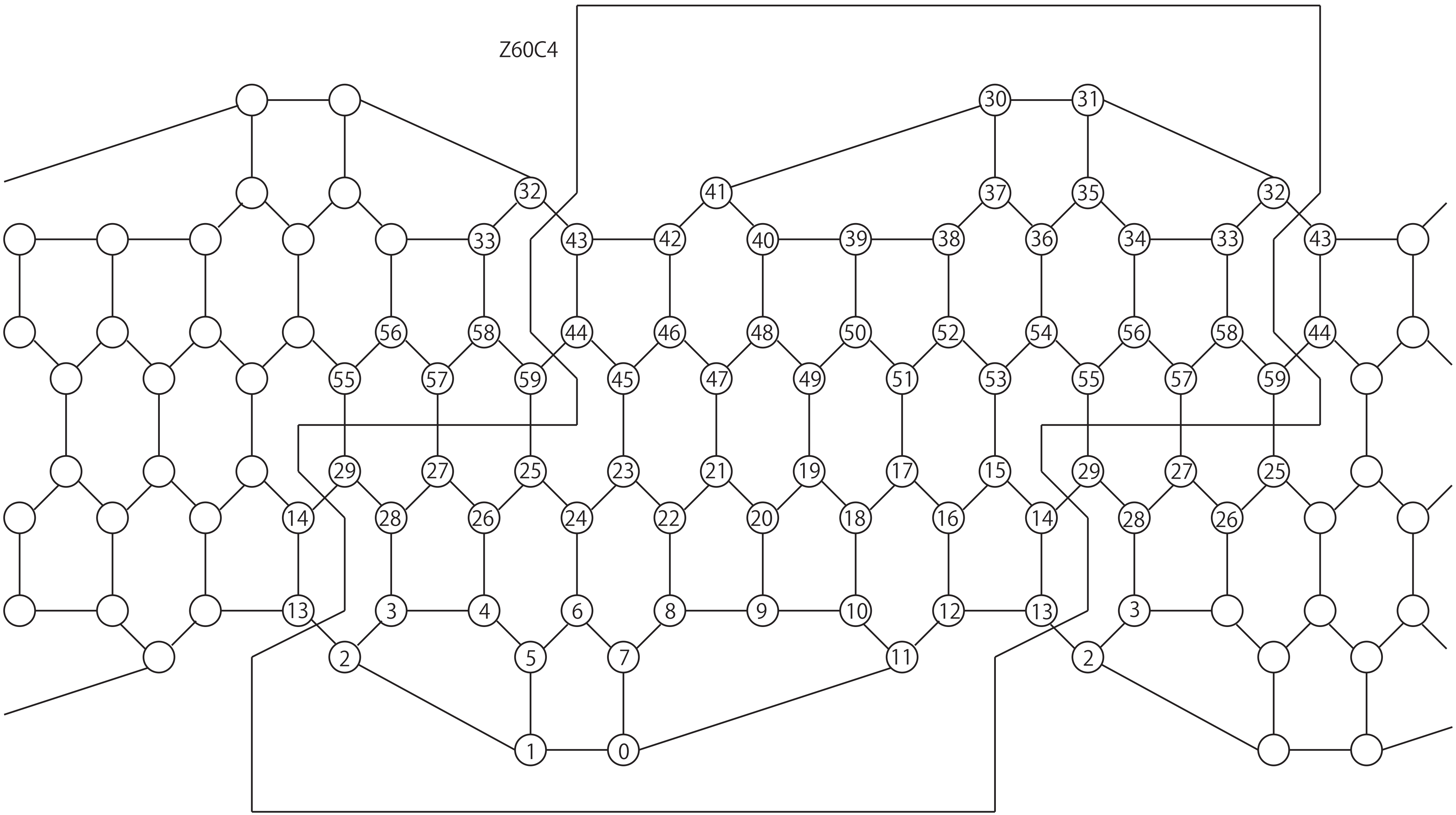

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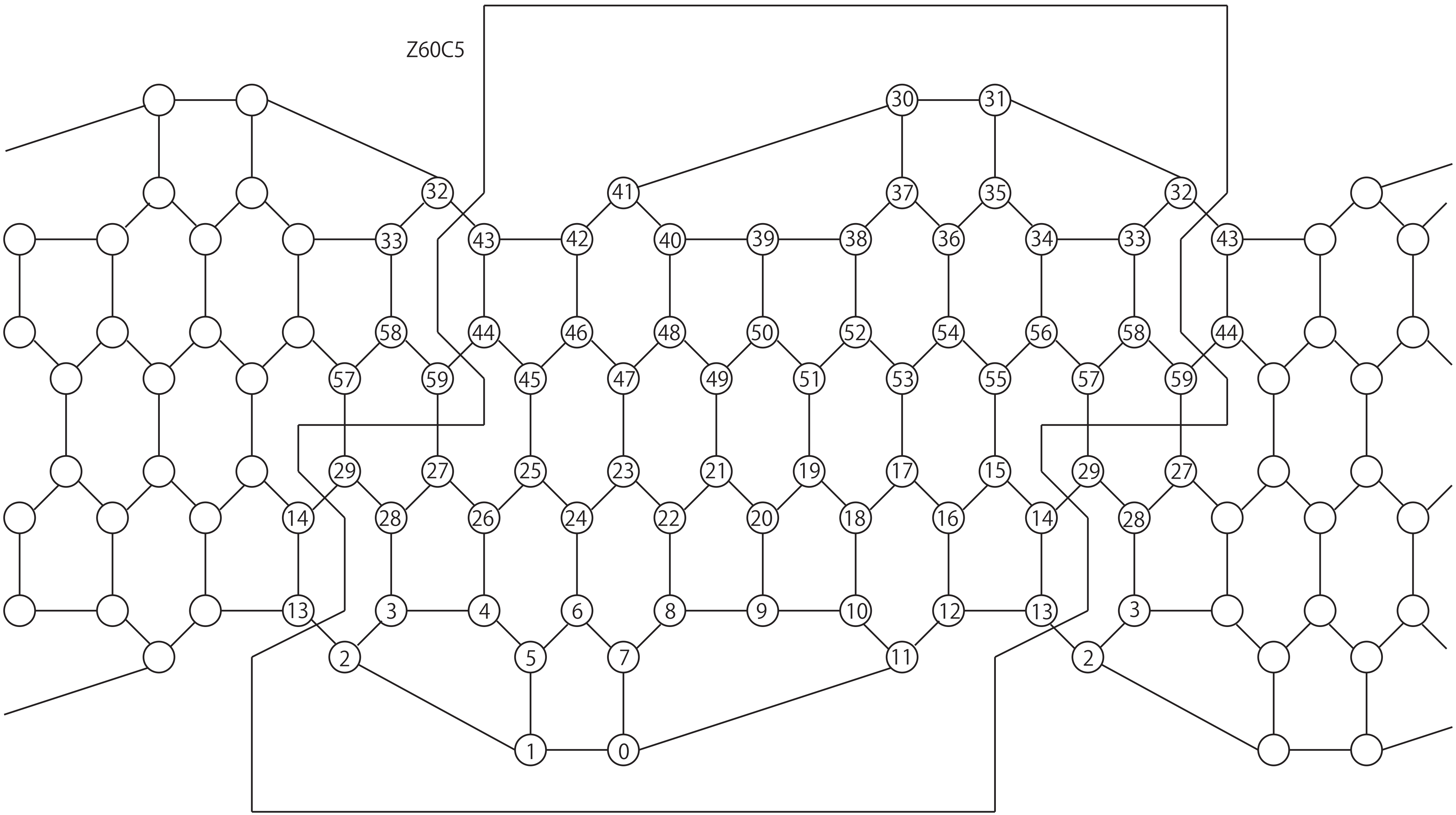
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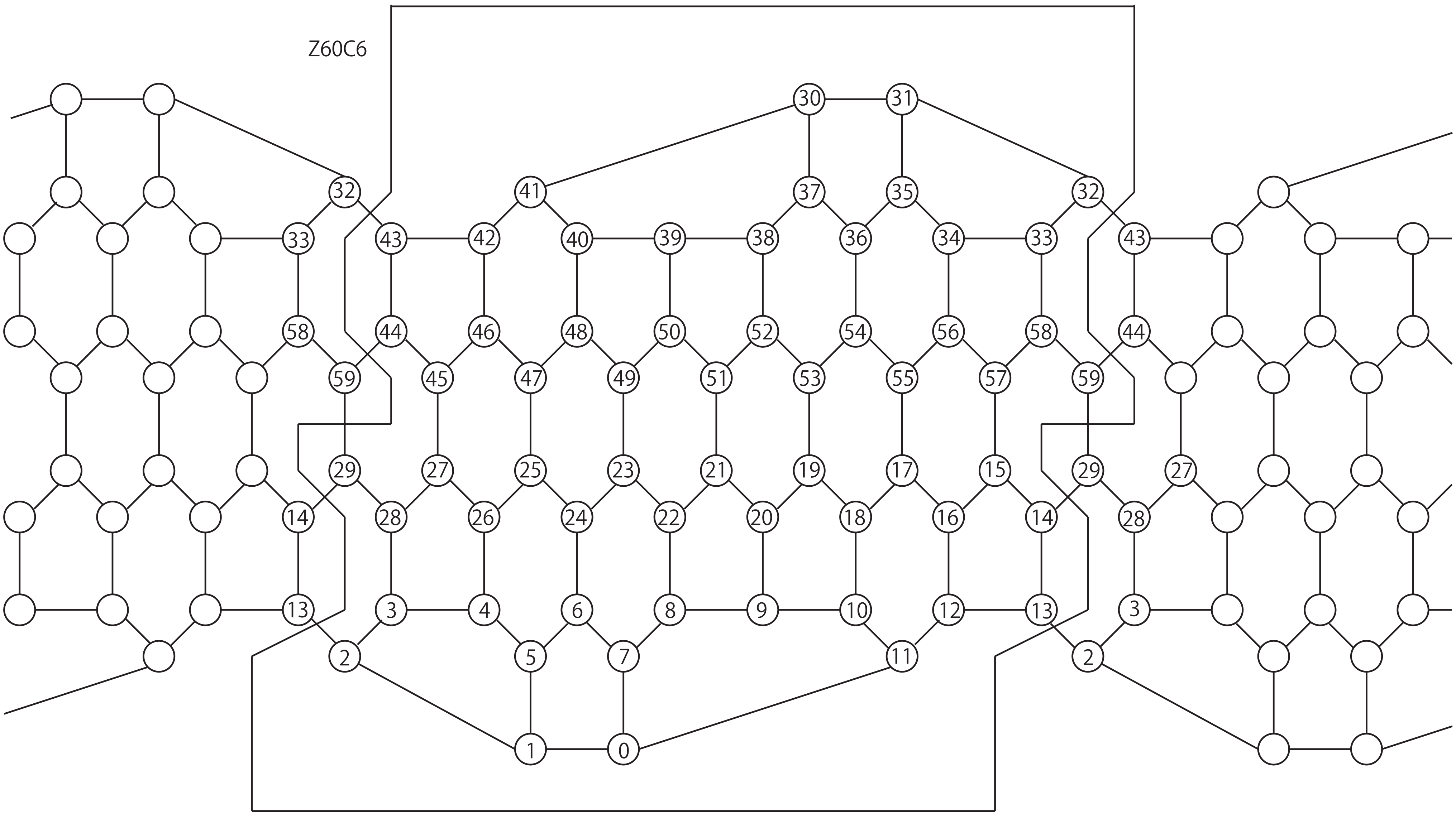

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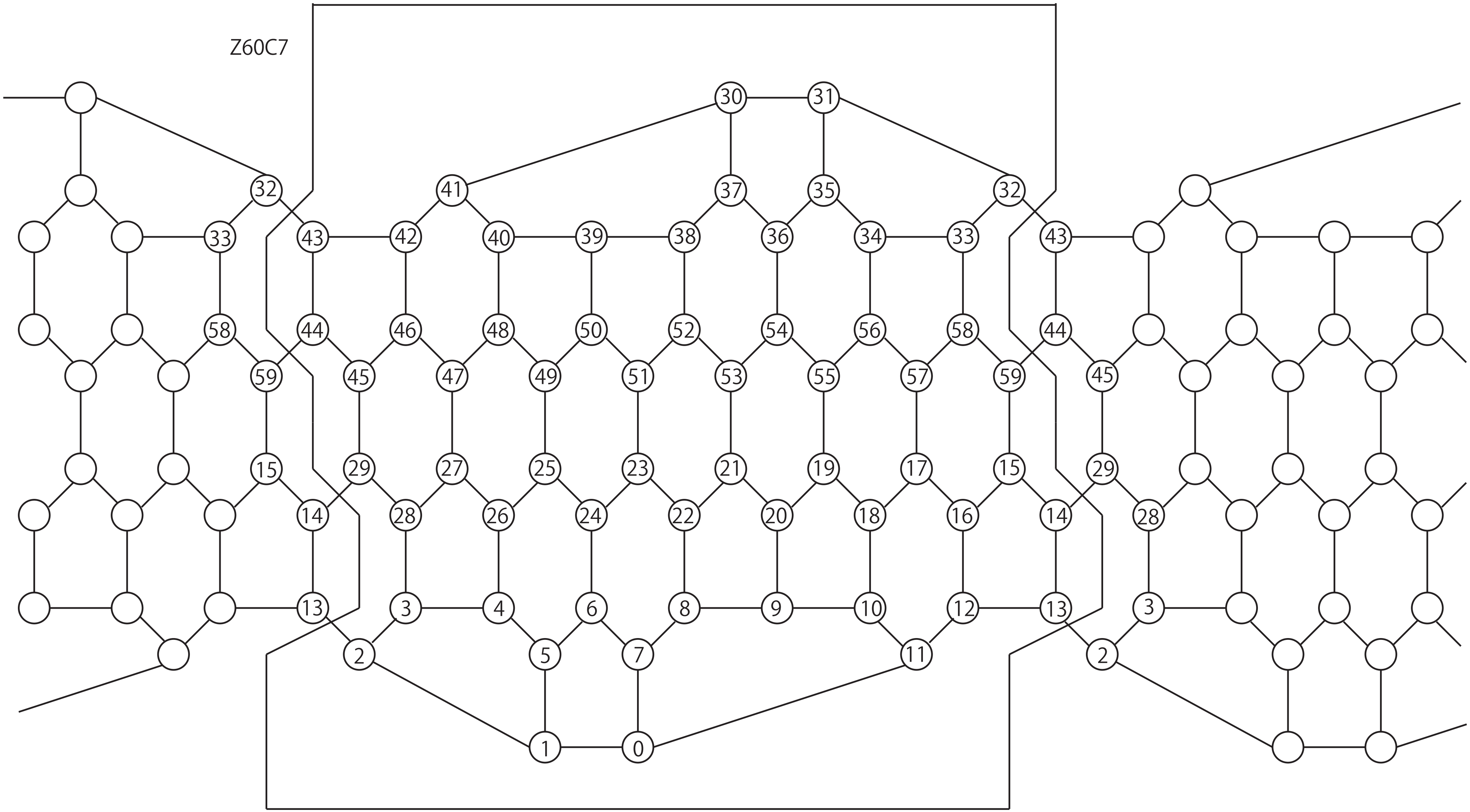
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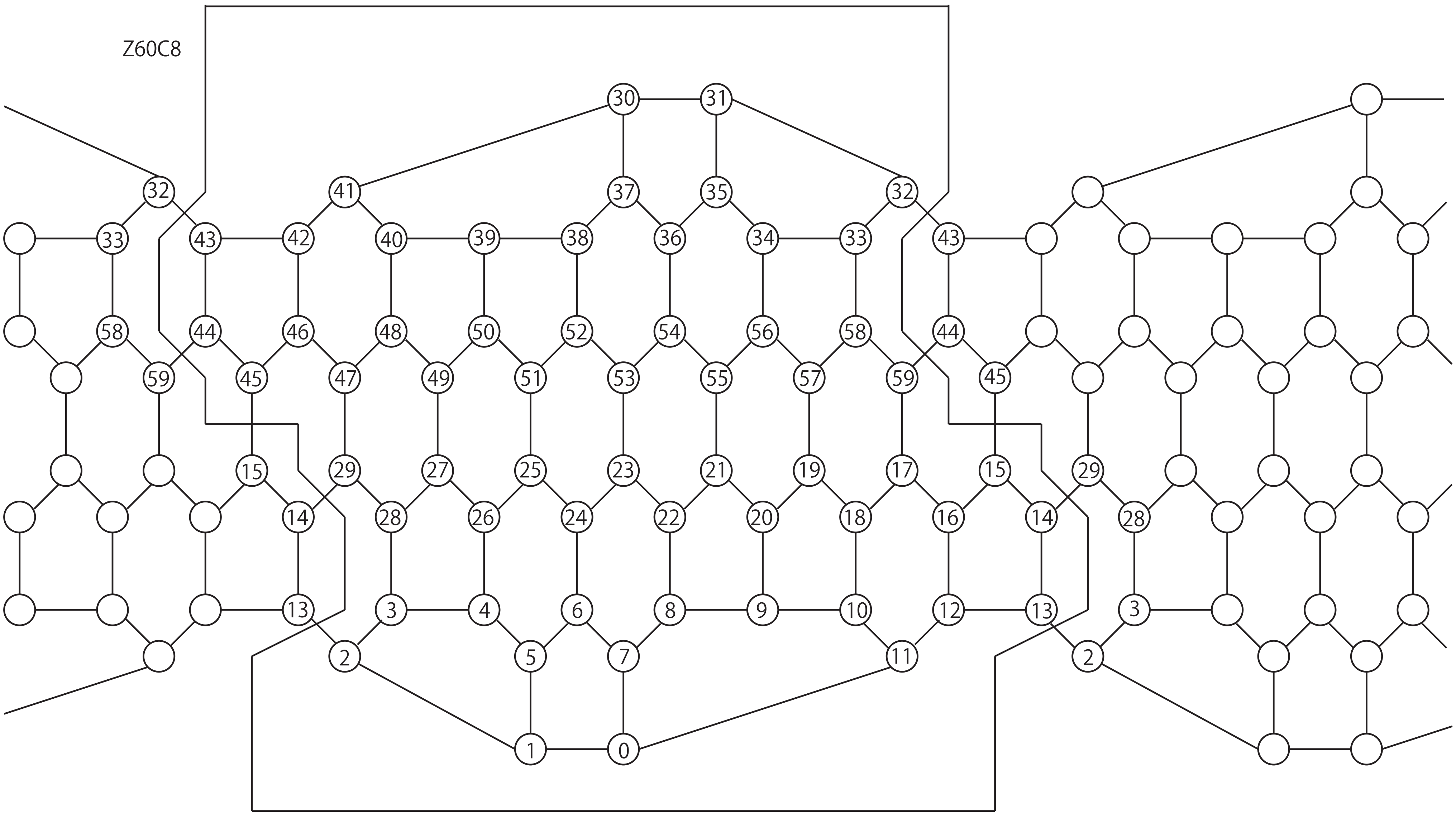
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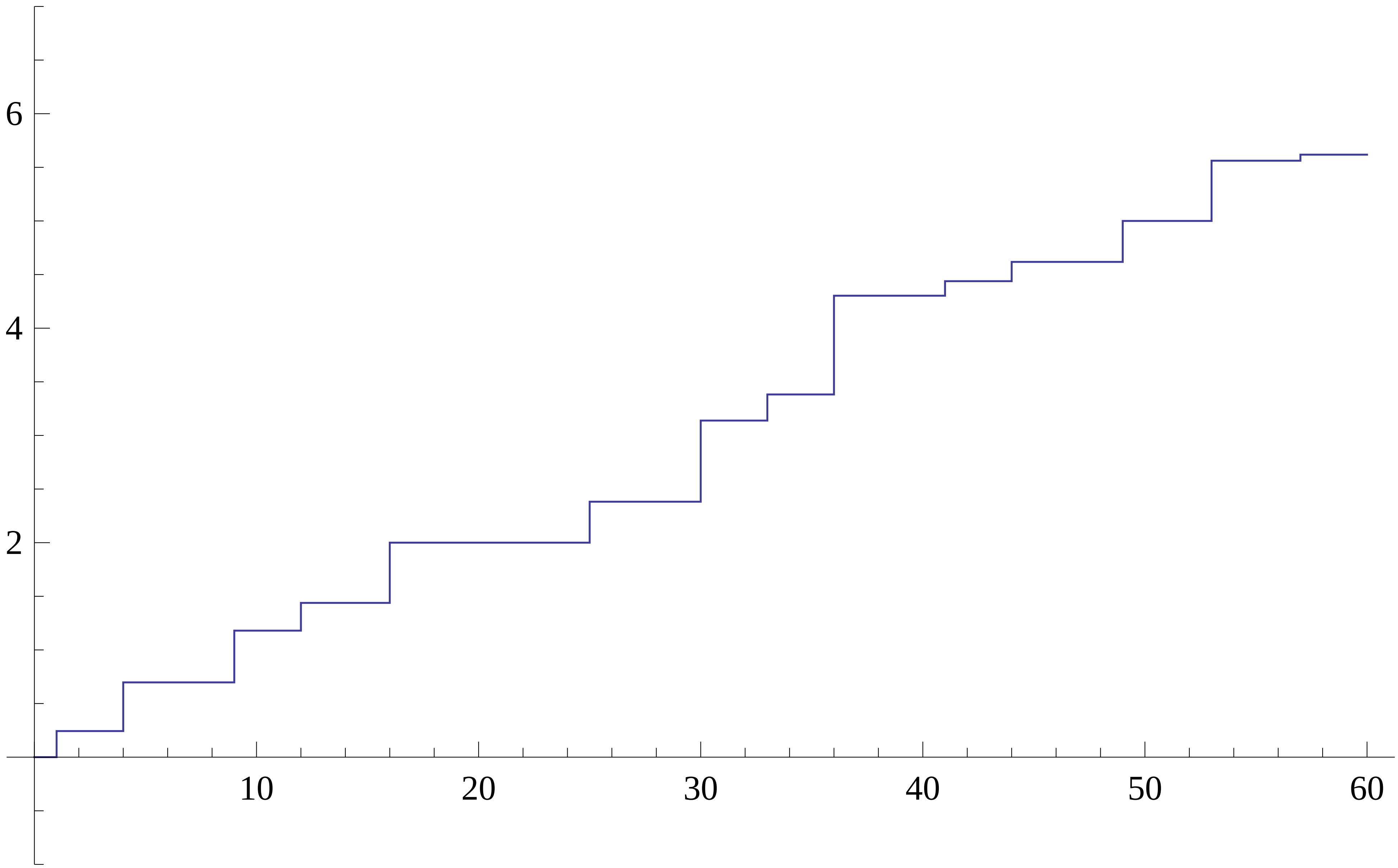
Discrete Laplacian

$$\boldsymbol{A} = \Big(\quad \boldsymbol{a(i,j)} \quad \Big) \qquad (\mathbf{0} \le \boldsymbol{i,j} \le \mathbf{59})$$

$$\boldsymbol{a(i,j)} = \begin{cases} \mathbf{3} & (\boldsymbol{i = j}) \\ \mathbf{-1} & (\boldsymbol{i,j}), (\boldsymbol{j,i}) \in \boldsymbol{e} \\ \mathbf{0} & (\text{else}) \end{cases}$$

Eigenvalue distribution of Z60A1



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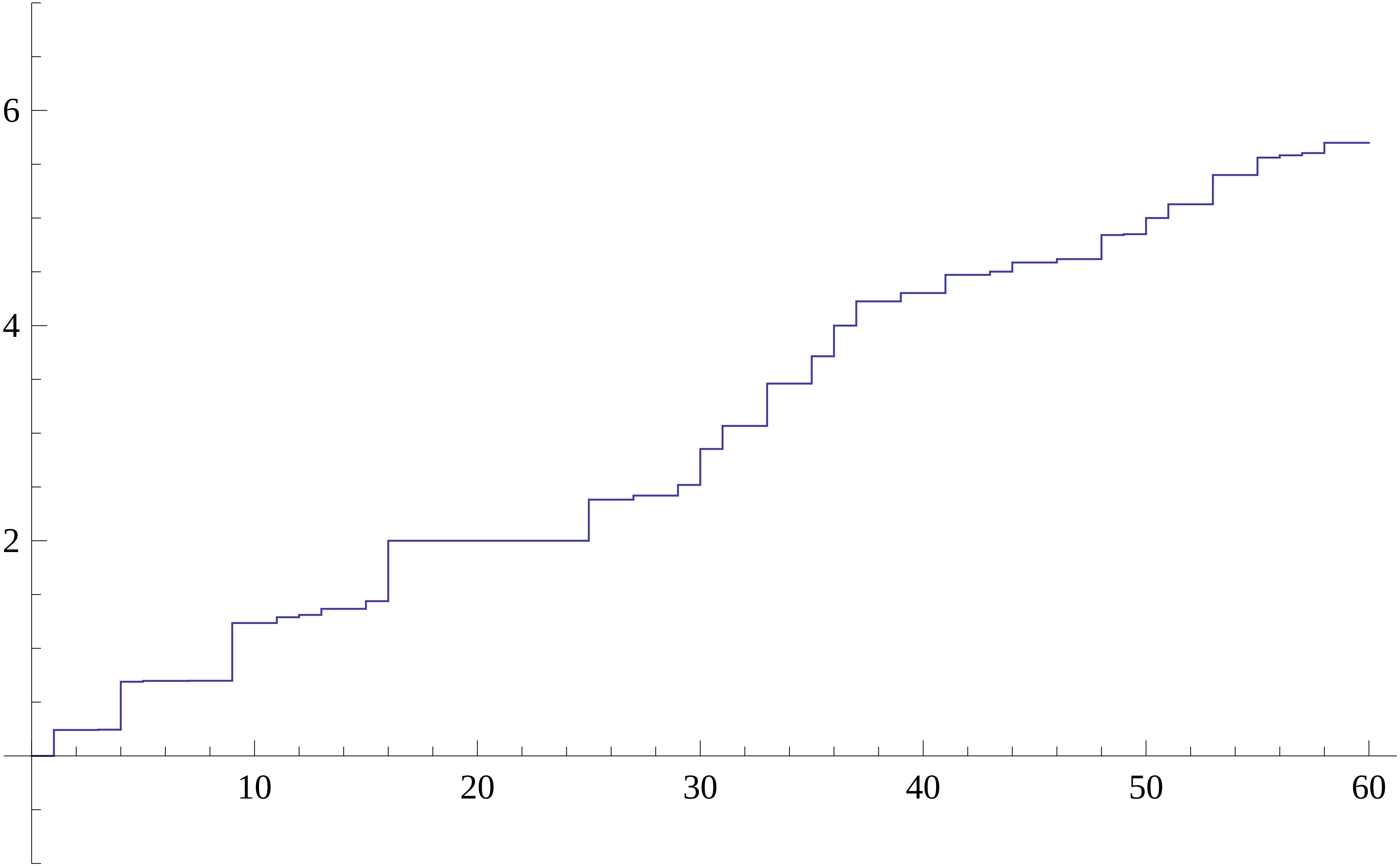
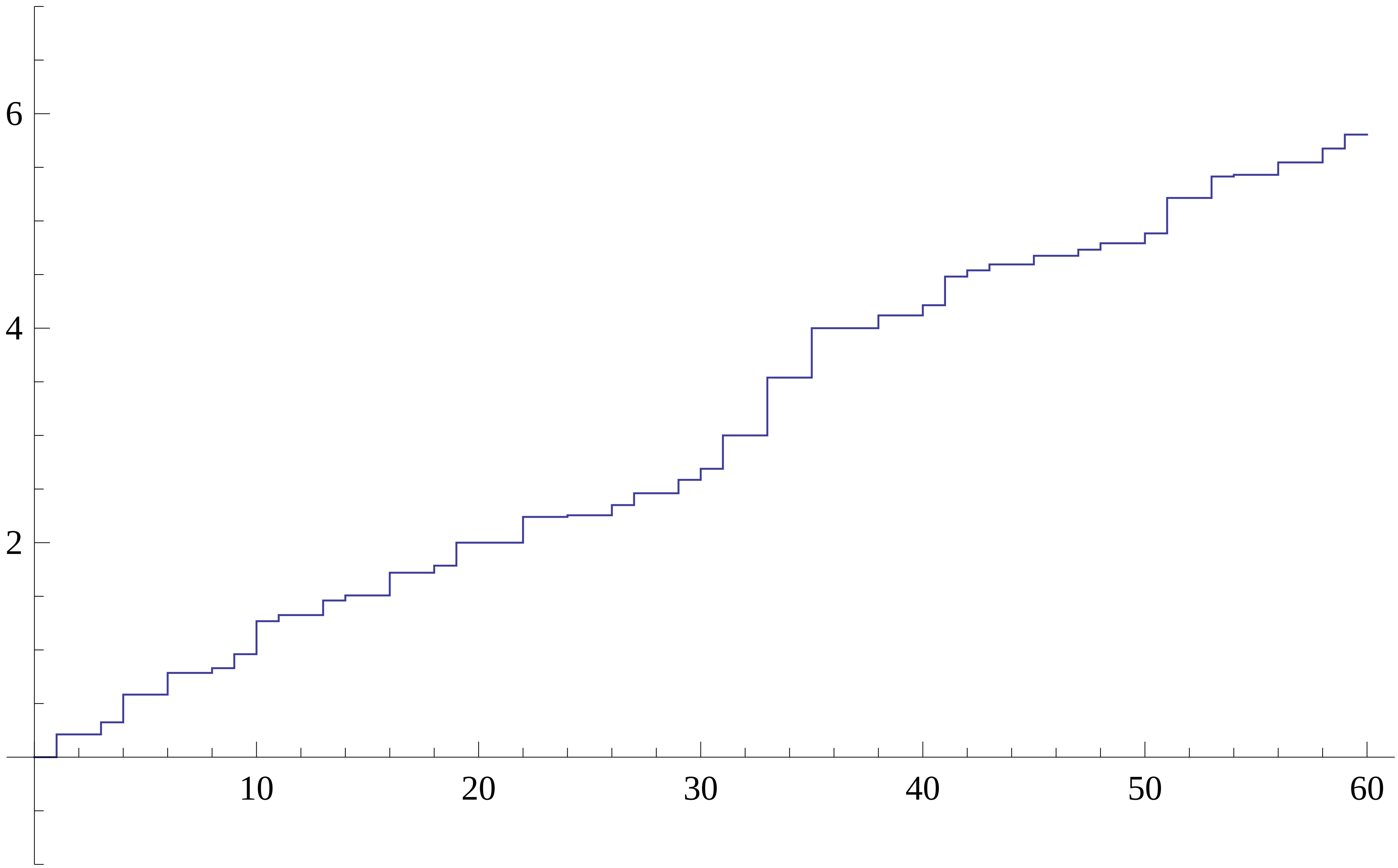


Eigenvalue distribution of Z60B1



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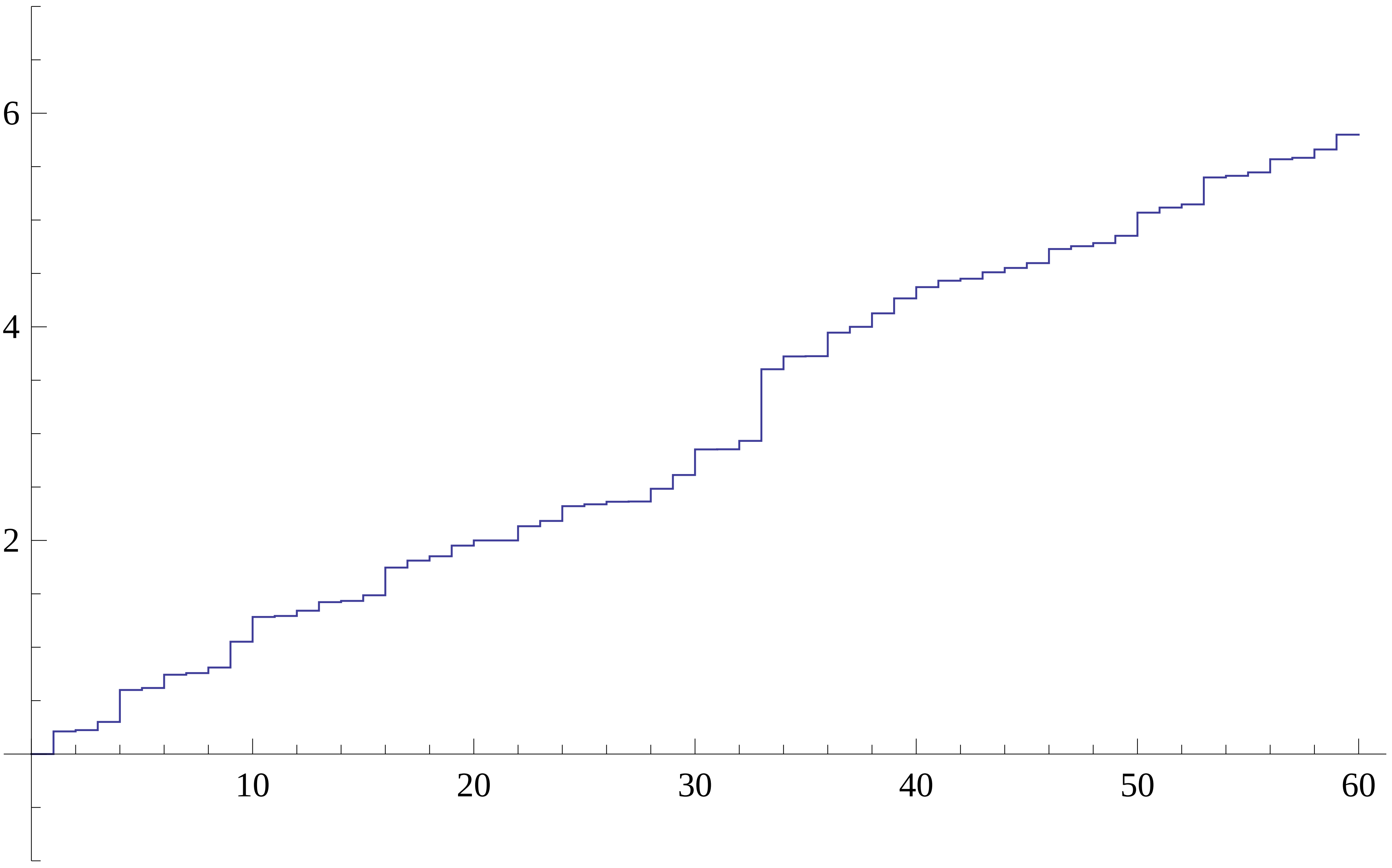

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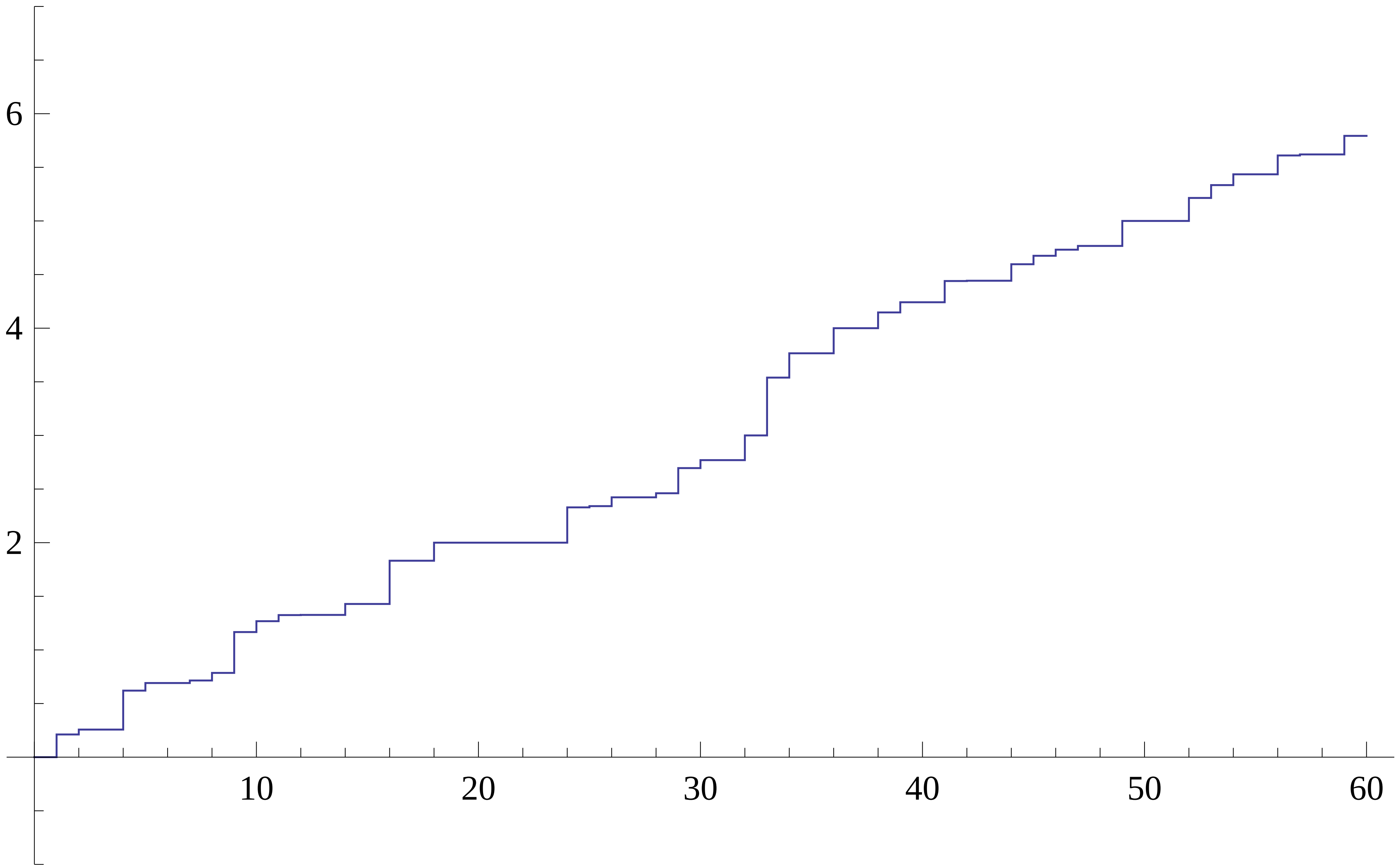
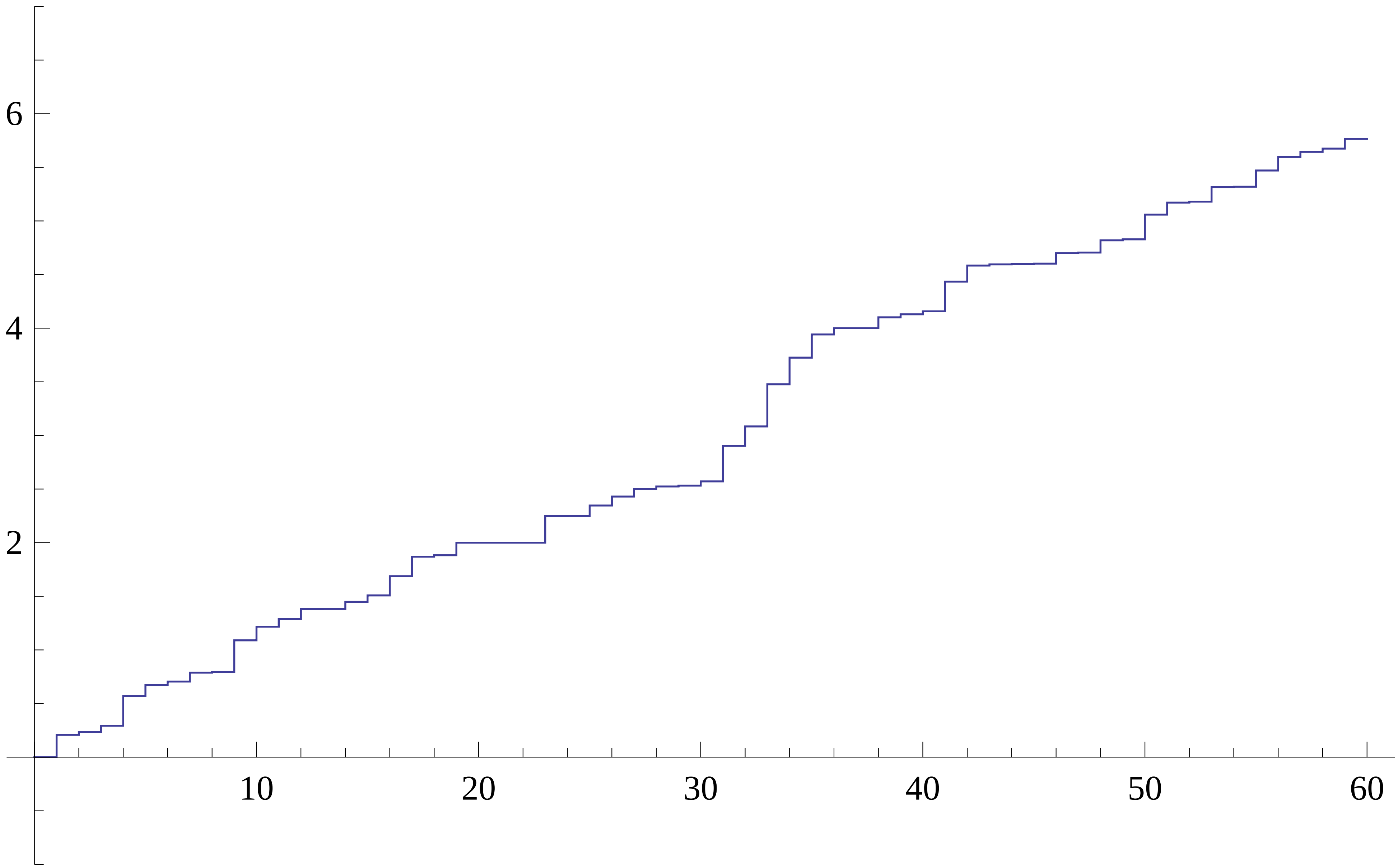


Eigenvalue distribution of Z60C1



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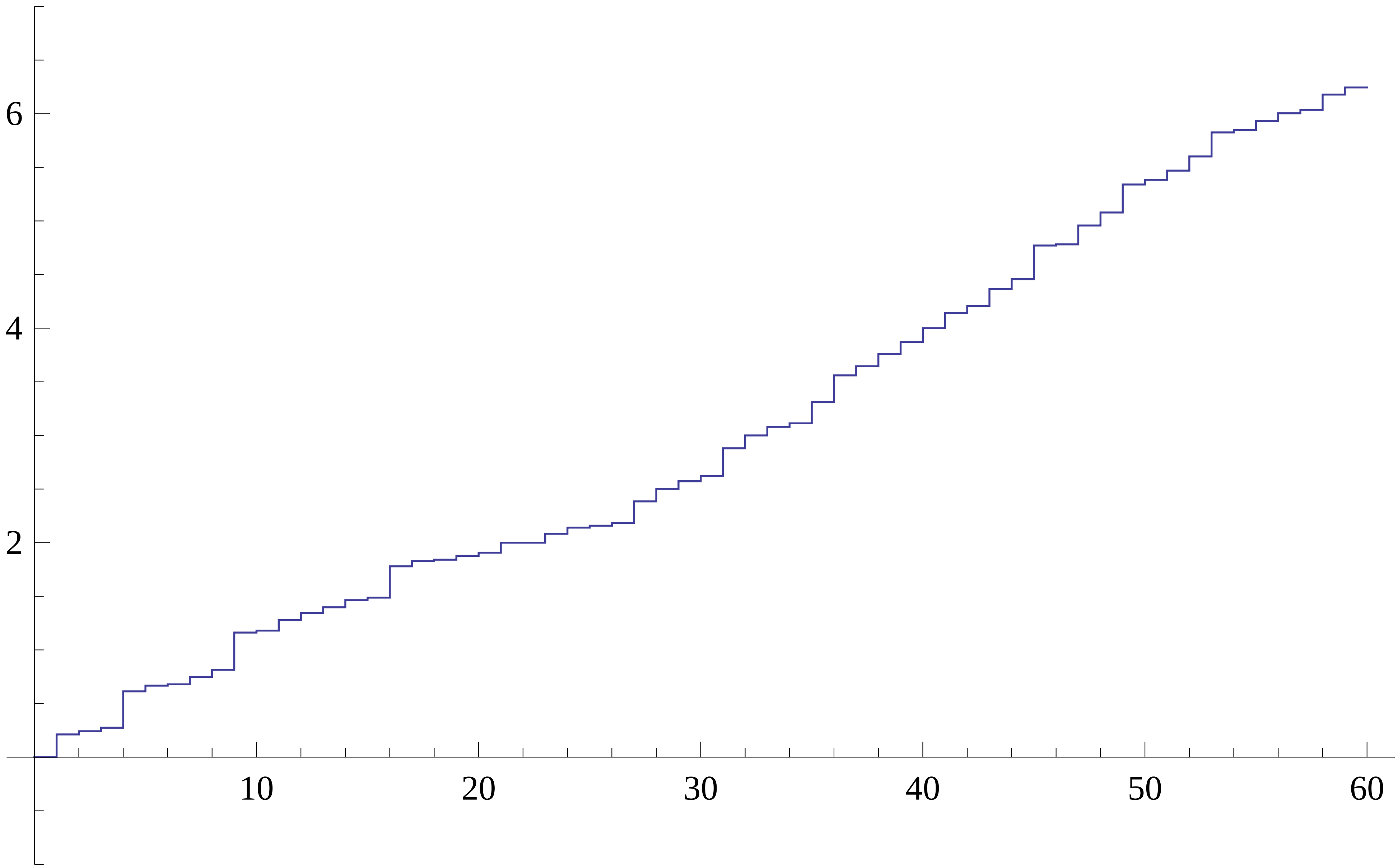

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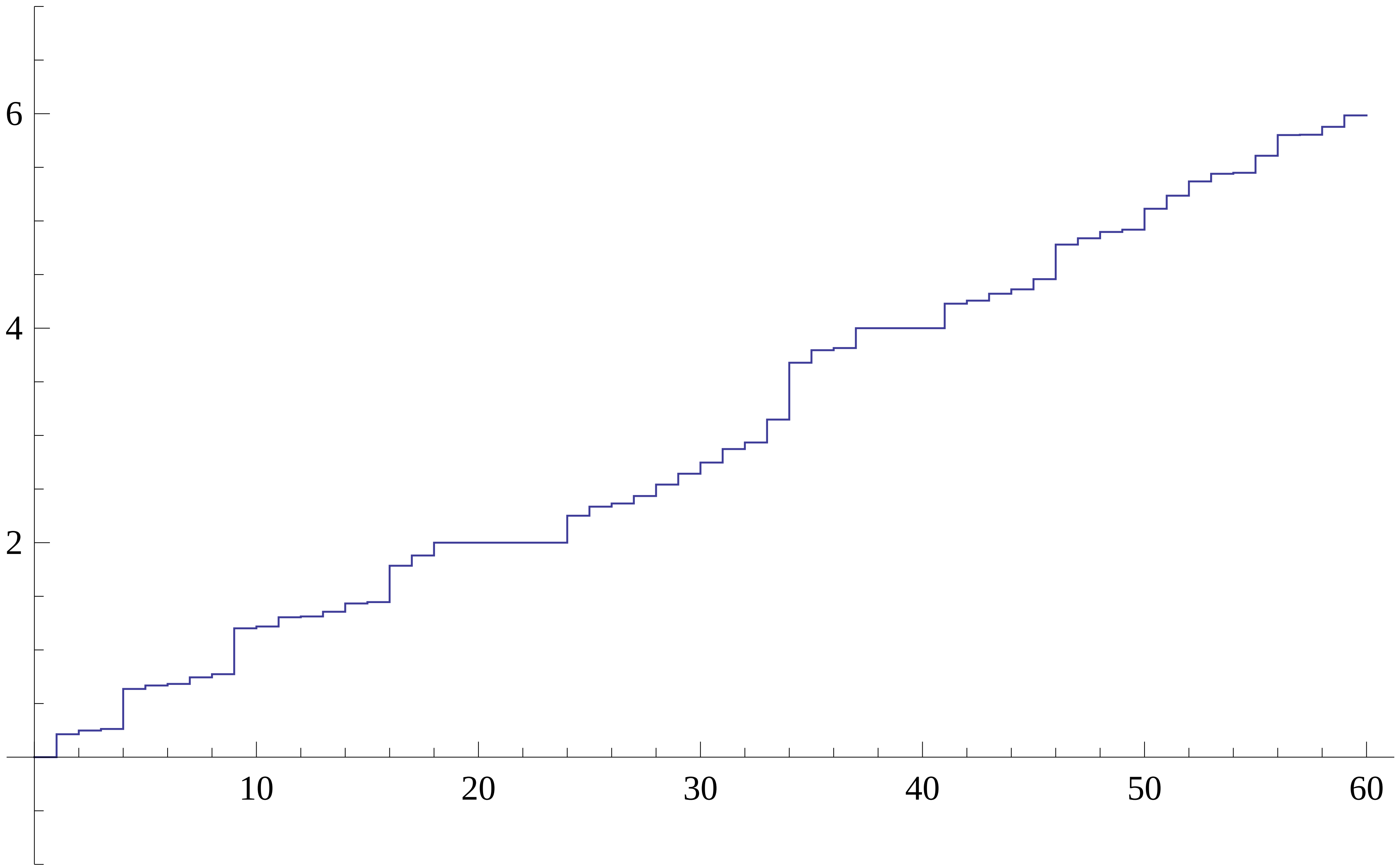
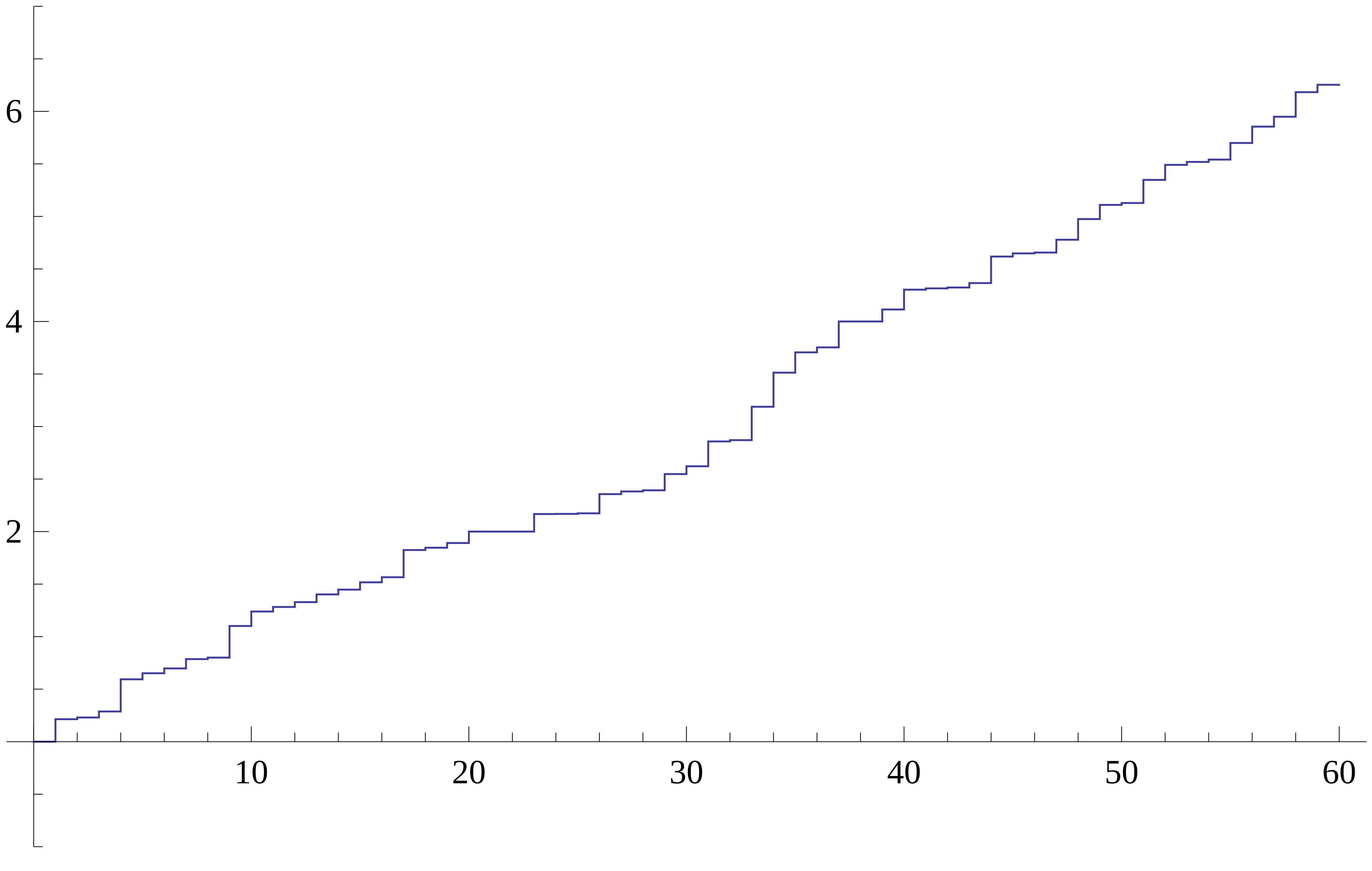
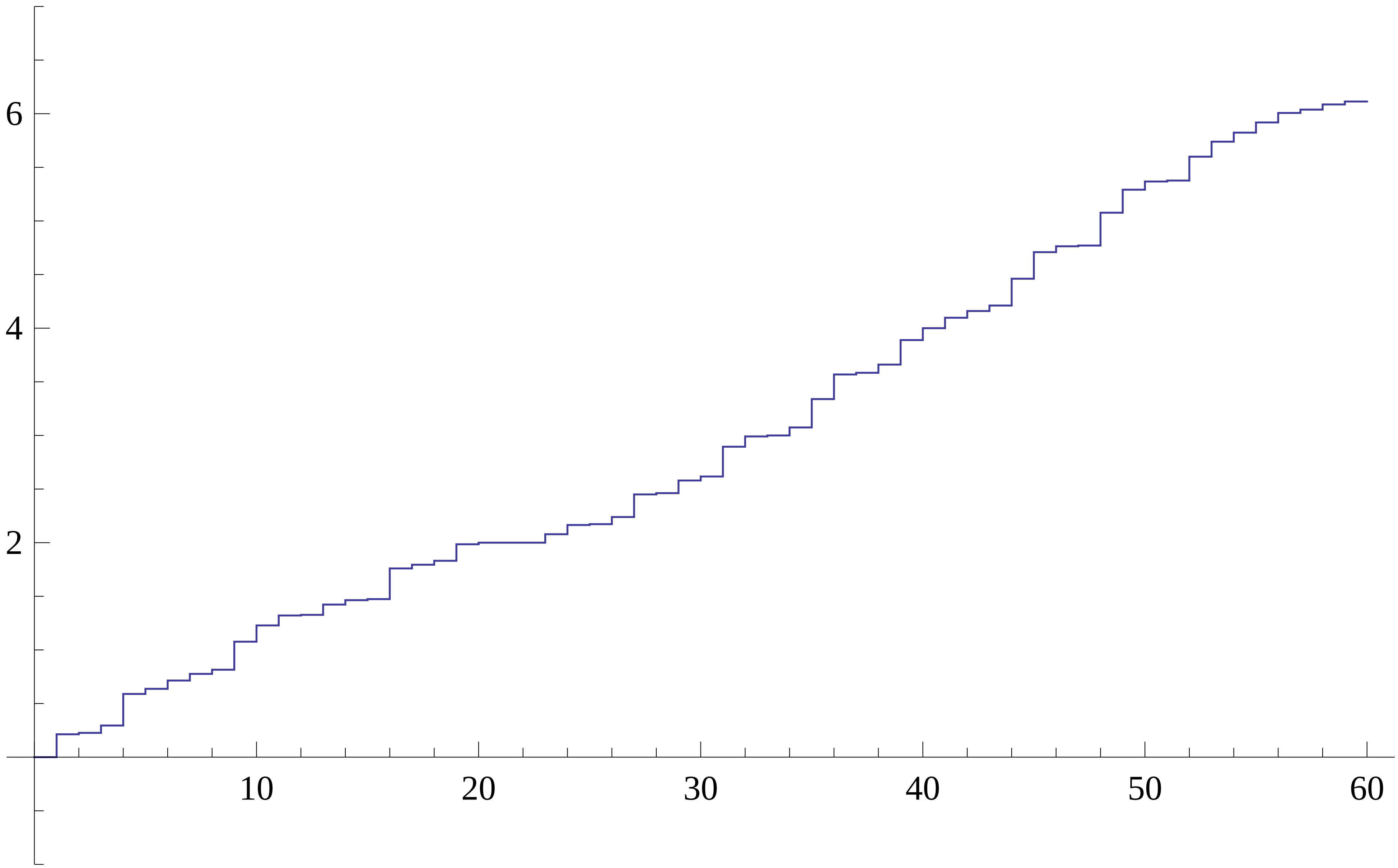


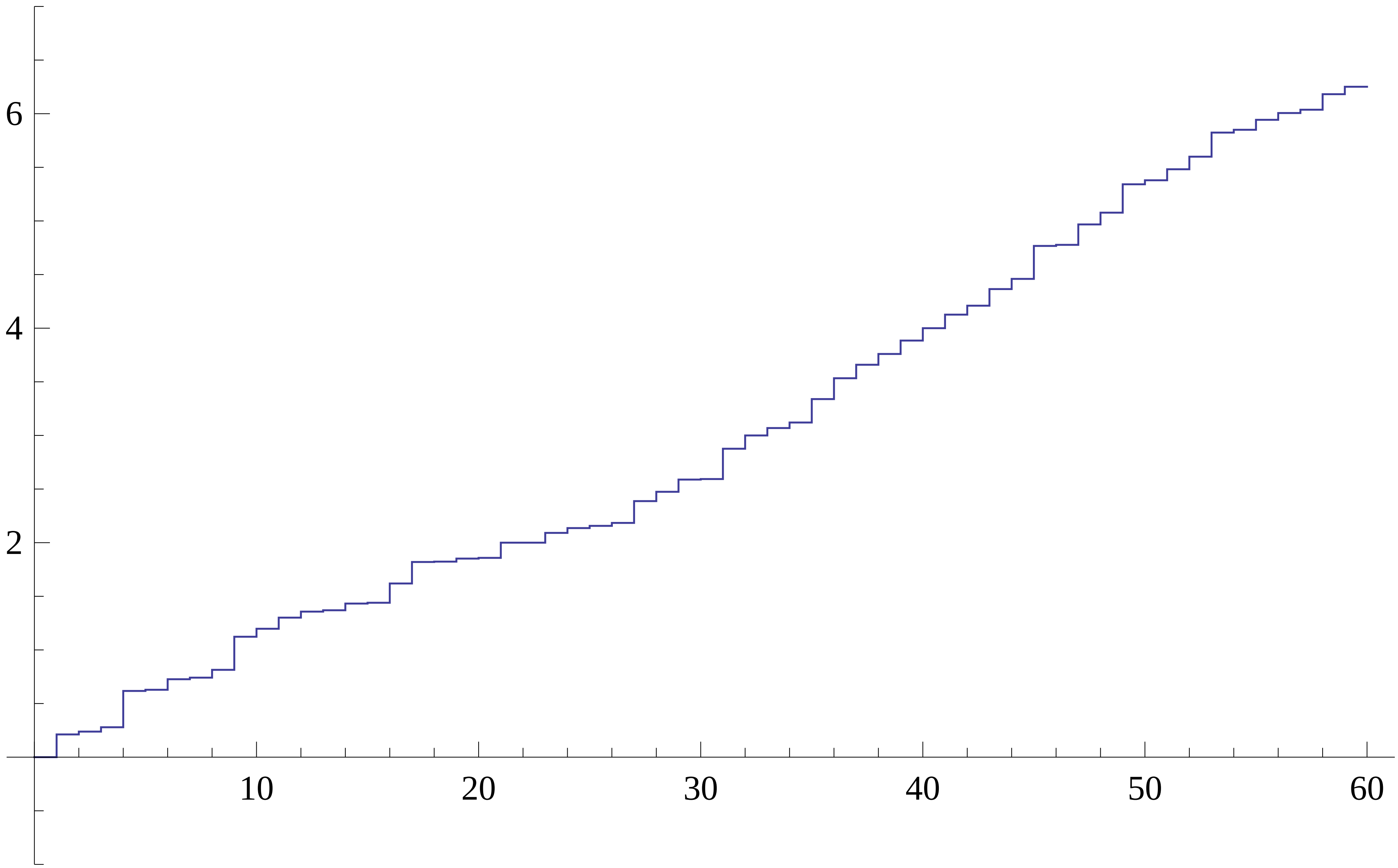
Eigenvalue distribution of Z60C4



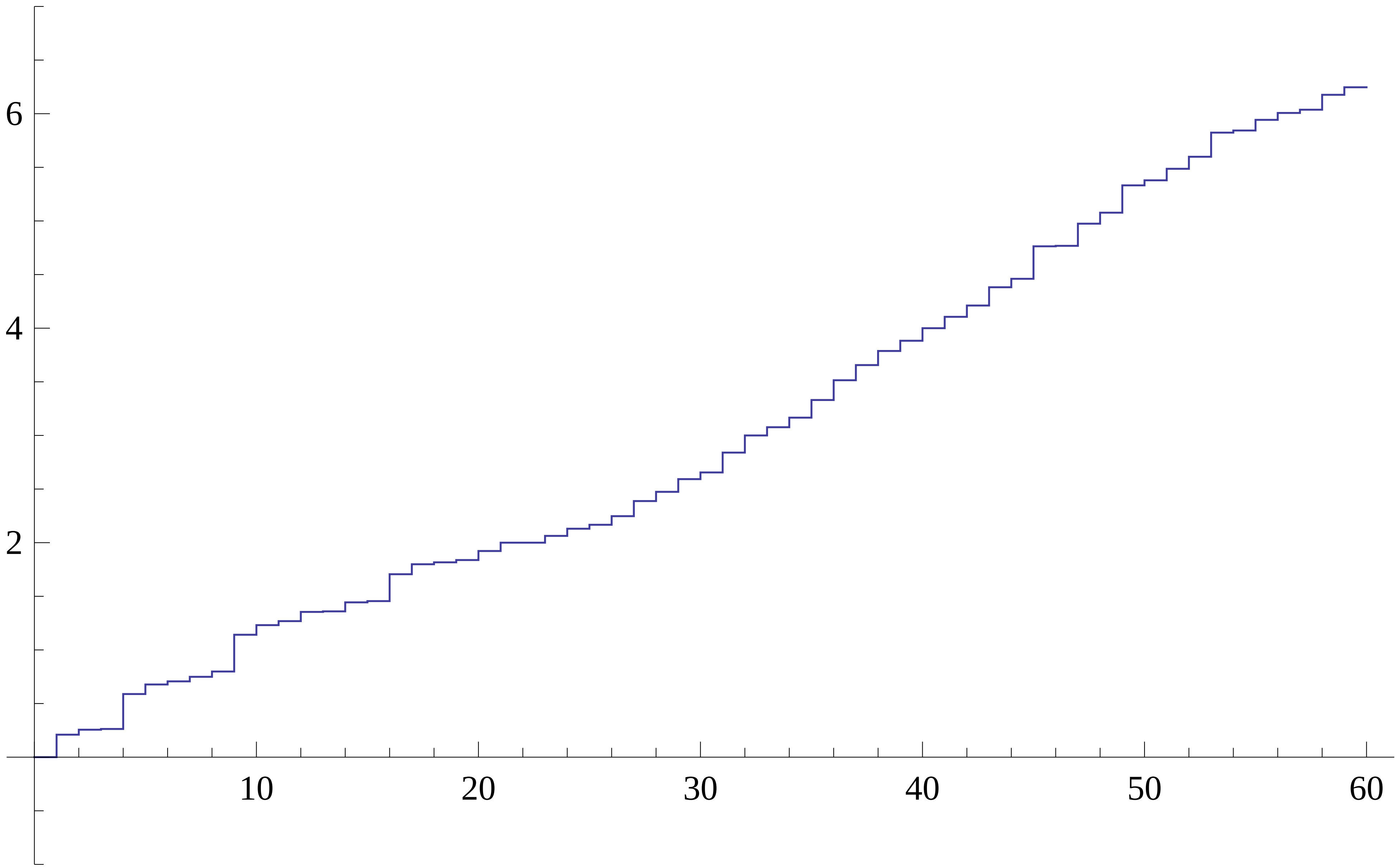
Eigenvalue distribution of Z60C5



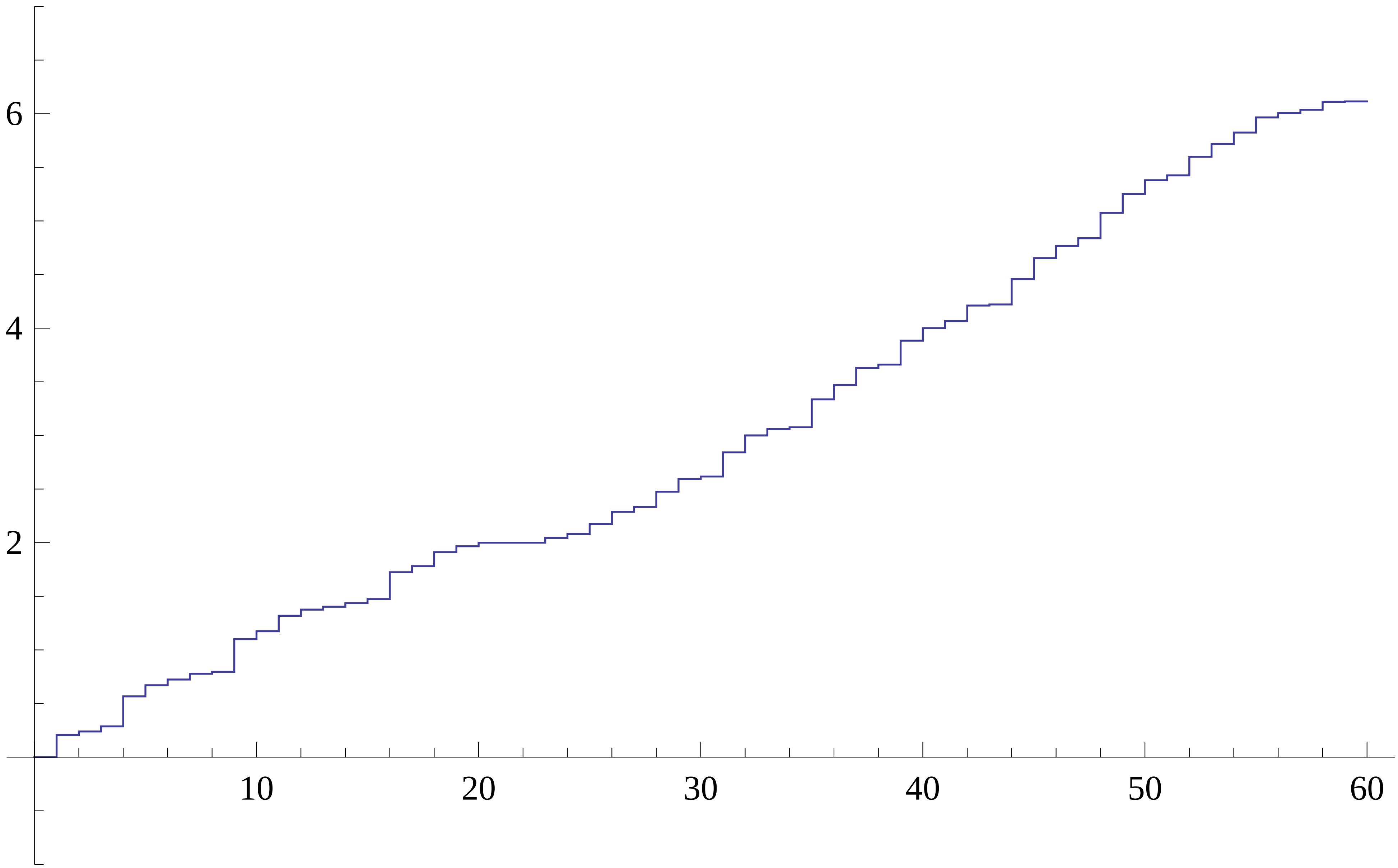
Eigenvalue distribution of Z60C6



Eigenvalue distribution of Z60C7



Eigenvalue distribution of Z60C8



Discrete heat kernel

$$H(t) = \exp(-tA)$$

Green matrix

$$G(a) = (aI + A)^{-1} = \int_0^\infty e^{-at} H(t) dt$$

Pseudo Green matrix

$$G_* = \lim_{a \to +0} \left(G(a) - \frac{1}{a} E_0 \right), \qquad E_0 = \frac{1}{60} 1^t 1$$

Sobolev energy

$$E(u) = \sum_{(i,j) \in e} |u(i) - u(j)|^2 = u^* A u$$

$$E(a, u) = E(u) + a \sum_{j=0}^{59} |u(j)|^2 = u^*(A + aI)u$$

$$u = {}^t(u(0), u(1), \cdots, u(59)) \in \mathrm{C}^{60}, \qquad 0 < a < \infty$$

Theorem 1

$$u \in \mathrm{C}^{60} \quad \text{and} \quad u(0) + u(1) + \cdots + u(59) = 0 \quad \Longrightarrow$$

$$\left(\max_{0 \le j \le 59} |\, u(j)\, | \right)^2 \le C\, E(u)$$

$$C_0 = \max_{0 \le j \le 59} {}^t\delta_j G_* \delta_j = {}^t\delta_{j_0} G_* \delta_{j_0}$$

The equality holds for j_0-th column vector of G_*.

$$C_0(\mathbf{Z60A1}) = \frac{1}{60}\sum_{k=1}^{59}\frac{1}{\lambda_k} = \frac{239741}{376200}$$

$$C_0(\mathbf{Z60A2}) = \frac{36409091911}{55355731200}$$

$$C_0(\mathbf{Z60B1}) = \frac{49616123}{74390400}$$

$$C_0(\mathbf{Z60B2}) = \frac{25524226539887}{38264600989440}$$

$$C_0(\mathbf{Z60B3}) = \frac{3160823}{4737960}$$

$$C_0(\mathbf{Z60C1}) = \frac{\mathbf{642451951335315\,71}}{\mathbf{957462289016877\,00}}$$

$$C_0(\mathbf{Z60C2}) = \frac{\mathbf{3456338284822708922953}}{\mathbf{5157583784730001587600}}$$

$$C_0(\mathbf{Z60C3}) = \frac{\mathbf{156400481511242}}{\mathbf{233327177482275}}$$

$$C_0(\mathbf{Z60C4}) = \frac{\mathbf{2469598657842821}}{\mathbf{3681835419340320}}$$

$$C_0(\mathbf{Z60C5}) = \frac{\mathbf{899130319793743730 3}}{\mathbf{13403692887728666400}}$$

$$C_0(\mathbf{Z60C6}) = \frac{\mathbf{3844278390494454 20497}}{\mathbf{5728204642409805924 00}}$$

$$C_0(\mathbf{Z60C7}) = \frac{\mathbf{964321076346238117}}{\mathbf{1434521140957238400}}$$

$$C_0(\mathbf{Z60C8}) = \frac{\mathbf{501412110751795 13}}{\mathbf{7451957224596780 0}}$$

$C_0(\mathbf{Z60A1}) \fallingdotseq \mathbf{0.63727}$	$\mathbf{0}$
$C_0(\mathbf{Z60A2}) \fallingdotseq \mathbf{0.657729}$	$\mathbf{0.0204593}$
$C_0(\mathbf{Z60B1}) \fallingdotseq \mathbf{0.666969}$	$\mathbf{0.0296994}$
$C_0(\mathbf{Z60B2}) \fallingdotseq \mathbf{0.667045}$	$\mathbf{0.0297753}$
$C_0(\mathbf{Z60B3}) \fallingdotseq \mathbf{0.667127}$	$\mathbf{0.0298573}$
$C_0(\mathbf{Z60C1}) \fallingdotseq \mathbf{0.670995}$	$\mathbf{0.0337245}$
$C_0(\mathbf{Z60C2}) \fallingdotseq \mathbf{0.670147}$	$\mathbf{0.0328767}$
$C_0(\mathbf{Z60C3}) \fallingdotseq \mathbf{0.670305}$	$\mathbf{0.0330354}$
$C_0(\mathbf{Z60C4}) \fallingdotseq \mathbf{0.670752}$	$\mathbf{0.033482}$
$C_0(\mathbf{Z60C5}) \fallingdotseq \mathbf{0.670808}$	$\mathbf{0.0335378}$
$C_0(\mathbf{Z60C6}) \fallingdotseq \mathbf{0.671114}$	$\mathbf{0.0338439}$
$C_0(\mathbf{Z60C7}) \fallingdotseq \mathbf{0.672225}$	$\mathbf{0.034955}$
$C_0(\mathbf{Z60C8}) \fallingdotseq \mathbf{0.67286}$	$\mathbf{0.0355896}$

Theorem 2

$u \in \mathrm{C}^{60}$

$\Longrightarrow$

$$\left(\max_{0 \le j \le 59} |\, u(j) \,| \right)^2 \le C \, E(a,u)$$

$$C_0(a) = \max_{0 \le j \le 59} {}^t\delta_j G(a) \delta_j = {}^t\delta_{j_0} G(a) \delta_{j_0}$$

The equality holds for j_0-th column vector of $G(a)$.

$$C_0(\mathbf{Z60A1}, a) = \frac{1}{60} \sum_{k=0}^{59} \frac{1}{\lambda_k + a} = \frac{N(a)}{D(a)}$$

$$N(a) = 3344 + 160806a + 1153562a^2 + 3594661a^3 + 6334271a^4 + 7104785a^5 + 5406109a^6 + 2893077a^7 + 1109403a^8 + 306415a^9 + 60463a^{10} + 8315a^{11} + 757a^{12} + 41a^{13} + a^{14}$$

$$D(a) = a(2+a)(5+a)\left(3 + 5a + a^2\right)\left(8 + 7a + a^2\right)\left(11 + 7a + a^2\right)\left(19 + 9a + a^2\right)\left(4 + 22a + 25a^2 + 9a^3 + a^4\right)$$

Green matrix

$$(aI + A)u = f$$

$$\Updownarrow$$

$$u = G(a)f$$

$$G(a) = (aI + A)^{-1} = \int_0^\infty e^{-at} H(t)\,dt = \left(\quad g_{ij}(a) \quad \right)$$

Pseudo Green matrix

$$\begin{cases} Au = f \\ E_0 u = 0 \end{cases}$$

$$\Updownarrow$$

$$u = G_* f$$

$$G_* = \lim_{a \to +0} \left(G(a) - \frac{1}{a} E_0 \right) = \left(\quad g_{*ij} \quad \right)$$

Energy form

$$(\boldsymbol{u},\, \boldsymbol{v})_{\boldsymbol{A}} \;=\; (\boldsymbol{A}\boldsymbol{u},\, \boldsymbol{v}) \;=\; \boldsymbol{v}^{*}\boldsymbol{A}\boldsymbol{u},$$

$$\|\, \boldsymbol{u} \,\|_{\boldsymbol{A}}^{2} \;=\; (\boldsymbol{u}, \boldsymbol{u})_{\boldsymbol{A}} \;=\; \boldsymbol{E}(\boldsymbol{u})$$

$$(\boldsymbol{u},\, \boldsymbol{v})_{\boldsymbol{H}} \;=\; ((\boldsymbol{A} + \boldsymbol{a}\boldsymbol{I})\boldsymbol{u},\, \boldsymbol{v}) \;=\; \boldsymbol{v}^{*}(\boldsymbol{A} + \boldsymbol{a}\boldsymbol{I})\boldsymbol{u},$$

$$\|\, \boldsymbol{u} \,\|_{\boldsymbol{H}}^{2} \;=\; (\boldsymbol{u}, \boldsymbol{u})_{\boldsymbol{H}} \;=\; \boldsymbol{E}(\boldsymbol{a}, \boldsymbol{u})$$

$$\boldsymbol{\delta}_{\boldsymbol{j}} \;=\; {}^{\boldsymbol{t}}(\mathbf{0}, \cdots, \mathbf{0}, \overbrace{\mathbf{1}}^{\boldsymbol{j}}, \mathbf{0}, \cdots, \mathbf{0})$$

Reproducing relation

$$u \in \mathrm{C}^N \quad \text{and} \quad u(0) + u(1) + \cdots + u(N-1) = 0 \quad \Longrightarrow$$

$$u(j) = (u,\, G_* \delta_j)_A$$

$$g_{*jj} = \| \, G_* \delta_j \, \|_A^2 = E(G_* \delta_j)$$

$$u \in \mathrm{C}^N \quad \Longrightarrow$$

$$u(j) = (u,\, G(a) \delta_j)_H$$

$$g_{jj}(a) = \| \, G(a) \delta_j \, \|_H^2 = E(a, G(a) \delta_j)$$