

Arithmetic geometry, Chow groups and rational points
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Scientific/organizing committee :

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Abstracts

Ivan Arzhantsev (Higher School of Economics, Moscow)

Title : Infinite transitivity on universal torsors

Abstract : Let X be an algebraic variety covered by open charts isomorphic to the affine space and $q : X' \rightarrow X$ be the universal torsor over X . We prove that the automorphism group of the quasiaffine variety X' acts on X' infinitely transitively. Also we find wide classes of varieties X admitting such a covering. This is a joint work with Alexander Perepechko and Hendrik Süß.

Mikhail Bondarko (St-Petersburg State University)

Title : On Chow-weight (co)homology and mixed motivic decomposition of the diagonal (joint work with V. Sosnilo)

Abstract : The seminal result of Bloch and Srinivas on the "decomposition of the diagonal" (of a smooth projective P) is closely related to the following statement: an effective Chow motif M is "divisible" by the Lefschetz one (i.e., it can be presented as $M' \otimes L$ where M' is effective; we say that M is 1-effective) if and only if $Chow_0(M) = 0$ (if we take rational coefficients and a "large" algebraically closed base field). We generalize this result to (effective) Voevodsky motives. Several criteria are formulated in terms of the so-called Chow-weight homology (a new homology theory defined in terms of the weight complex functor); in particular, we generalize all the previously known general "effectivity" statements (for Chow motives). So, Chow-weight homology allows to calculate the "effectivity" of a motif M , its "weights", and its "connectivity" (in the sense of the Voevodsky's homotopy t -structure for motivic complexes), whereas the dual Chow-weight cohomology bounds the "dimension" of M . We also relate the effectivity of motives to that of the weight factors of their singular and étale (co)homology (using weight spectral sequences generalizing Deligne's ones).

Ulrich Derenthal (Leibniz Universität, Hannover)

Title : Cox rings over nonclosed fields

Abstract: Universal torsors, introduced by Colliot-Thélène and Sansuc, can be used to study rational points on algebraic varieties. Cox constructed homogeneous coordinate rings for toric varieties, and generalizations of Cox's construction have been studied for varieties over algebraically closed fields. In this talk, we present a new definition of Cox rings and Cox sheaves for varieties over nonclosed fields. We discuss their existence and classification, and we make their relation to universal torsors precise. (Joint work with Marta Pieropan.)

David Harari (Université Paris-Sud)

Title : Arithmetic duality theorems and rational points

Abstract : We present some results on Galois cohomology of finite modules and tori over various fields of cohomological dimension 2 and 3. We also give some applications to local-global principles and approximation properties.

Yonatan Harpaz (École Normale Supérieure, Paris)

Title : The descent-fibration method for integral points

Abstract : In 1995 Swinnerton-Dyer initiated a powerful method for establishing (under suitable hypothesis) the existence of rational points on certain surfaces fibered into curves of genus 1, by combining the fibration method with a sophisticated form of descent. This method, later extended and generalized by several authors including Swinnerton-Dyer, Colliot-Thélène and Skorobogatov, is currently the only method whose domain of application includes non-rationally connected varieties of dimension greater than 1. In this talk we will describe an adaptation of Swinnerton-Dyer's method to the setting of integral points, where curves of genus 1 are replaced by affine conics. This adaptation can be applied, in particular, to certain log K3 surfaces, a class of surfaces whose arithmetic is considered largely unknown in general.

Johannes Nicaise (KU Leuven)

Title : The specialization index of varieties over discretely valued fields

Abstract : The index of an algebraic variety is one of the basic obstructions to the existence of a rational point. In this talk, which is based on joint work with Lore Kesteloot, I will present a refinement of this obstruction for proper varieties over a discretely valued field. The rough idea is that, for all models of the variety over the valuation ring, we only consider zero-cycles that specialize to a single point in the special fiber. Already in the case of curves, there are many examples where the index is one while the specialization index is strictly larger, and thus explains the absence of a rational point. I will also prove that, in equal characteristic zero, the specialization index of a smooth and proper variety with trivial coherent cohomology equals one. The proof is based on Hodge theory and the Woods Hole trace formula.

René Pannekoek (Imperial College, London)

Title : Rational points on Kummer varieties and the Brauer–Manin obstruction

Abstract: Let B be an abelian variety over a number field k and let X be the Kummer variety $\text{Km}(B)$ of B , which is a smooth projective model of the quotient of B by the antipodal involution. Roughly speaking, rational points on X correspond to rational points on quadratic twists of B . We use this to show that if the Brauer–Manin obstruction controls the failure of weak approximation on all Kummer varieties, then for every positive-dimensional abelian variety A over a number field, the ranks of quadratic twists of A are unbounded. This is joint work with David Holmes (Leiden University).

R. Parimala (Emory University, Atlanta)

Title : Reduced Whitehead groups of central simple algebras over function fields of p -adic curves

The question whether every norm one element of a central simple algebra is a product of commutators was formulated in 1943 by Tannaka and Artin in terms of the reduced Whitehead group $SK_1(D)$. For central simple algebras of degree 4, it is a theorem of Merkurjev/Rost that $SK_1(D)$ is trivial over fields of cohomological dimension 3. This is a consequence of an injection of $SK_1(D)$ into a subquotient of degree 4 Galois cohomology. This leads Suslin to ask whether $SK_1(D)$ is trivial for algebras of indices l^2 for a prime l over fields of cohomological dimension 3. In this talk I report on the recent work of Nivedita Bhaskhar on the triviality of $SK_1(D)$ for period l algebras over function fields of p -adic curves with l not equal to p .

Alena Pirutka (CNRS/École Polytechnique, Palaiseau)

Title : On the integral aspects of the Tate conjecture.

Abstract : We will discuss some examples of algebraic varieties over finite fields, such that the cycle class map to the étale cohomology with Z_l -coefficients is not surjective. These examples come from studying the classifying spaces of algebraic groups and motivic cohomology operations.

Kanetomo Sato (Chuo University)

Title: p -adic étale Tate twists with moduli

Abstract : In a recent work of Binda and Saito, they defined cycle complexes defining higher Chow groups with moduli. Motivated by their work, I will talk about the following topics:

- (1) construction of p -adic cohomology playing the role of Tate twists with moduli.
- (2) Poincaré duality for such cohomology groups.
- (3) cycle class maps, Chern class maps with values in Tate twists with negative log poles.
- (4) application to p -adic regulator maps.

Alexander Schmidt (Universität Heidelberg)

Title : Class field theory for singular varieties over finite fields (jt. wk. with T. Geisser)

Abstract : The abelian tame fundamental of a smooth variety over a finite field can be described by using zero-cycles via Suslin homology (Kato/Saito/Schmidt/Spieß). This fails for singular varieties, the reciprocity map is neither injective nor surjective in general. In the talk we explain how to modify Suslin homology to Weil-Suslin homology in a similar style like Weil-étale cohomology modifies étale cohomology. Using Weil-Suslin homology instead, tame class field theory extends to singular varieties.

Michael Stoll (Universität Bayreuth)

Title: Uniform bounds for the number of rational points on curves of small Mordell-Weil rank.

Abstract : We show that there is a bound $N(d,g,r)$ for the number of K -rational points on curves C of genus g when the degree of the number field K is d and the Mordell-Weil rank r of the Jacobian of C is at most $g-3$. The proof uses an extension of the method of Chabauty-Coleman, based on p -adic integration on (p -adic) disks and annuli covering the p -adic points of the curve. Our original result was for hyperelliptic curves; by now there is a generalization to arbitrary curves due to E. Katz, J. Rabinoff and D. Zureick-Brown. We also deduce a uniform version of the result (due to Poonen and the speaker) that "most" hyperelliptic curves of odd degree over \mathbb{Q} have only one rational point, where "uniform" refers to families of curves defined by congruence conditions.

Yuri Tschinkel (Courant Institute and Simons Foundation, New York)

Title : Rational points on compactifications of linear algebraic groups

Abstract : I will discuss some recent results concerning height zeta functions of equivariant compactifications of linear algebraic groups.

Olivier Wittenberg (CNRS/ École Normale Supérieure, Paris)

Title: A restriction isomorphism for cycles of relative dimension zero

Abstract : I will report on a joint work with Moritz Kerz and Hélène Esnault on the Chow group of 1-cycles of a regular projective scheme over an excellent henselian discrete valuation ring. We study a restriction map from this group to a cohomological version of the Chow group of 0-cycles on the special fiber. Our main result extends to general perfect residue fields the work of Saito and Sato on the étale cycle class map for 1-cycles on regular projective schemes over the ring of integers of a p -adic field.

Fei Xu (Capital Normal University, Beijing)

Title: Strong approximation with Brauer–Manin obstruction for varieties containing a connected linear algebraic group with compatible action.

Abstract : There are several difficulties to extend strong approximation with Brauer–Manin obstruction for toric varieties to varieties containing a connected linear algebraic group with compatible action. For example, one has not the same type affine covering even over an algebraic closed field. One can not establish the local approximation property by using affine pieces like toric varieties, which is the first step towards strong approximation with Brauer–Manin obstruction. Instead of explicit construction for toric varieties, we apply the descent theory developed by Colliot-Thélène and Sansuc and the rigidity of torsors under the groups of multiplicative type by Colliot-Thélène to prove such results. This is a joint work with Yang Cao.

Yuri Zarhin (Penn State/Weizmann Institute)

Title: Families of hyperelliptic jacobians with large ℓ -adic monodromy .

Abstract : We study the monodromy of a certain class of semistable hyperelliptic curves over the rationals that was introduced by Shigefumi Mori forty years ago in his Master Thesis. Using ideas of Chris Hall, we prove that the corresponding ℓ -adic monodromy groups are (almost) "as large as possible". We also discuss an explicit construction of two-dimensional families of hyperelliptic curves with big monodromy over an arbitrary global field.