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Abstracts

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Lévy processes with Poisson and Gamma times

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We consider Lévy processes driven by an independent random time, which will be represented by a Poisson or a Gamma process, endowed with a drift. Our attention will be addressed to the semigroups and the infinitesimal generators of these processes. In particular, when the leading process is an α-stable, the governing equation is expressed in terms of new pseudo-differential operators involving the Riesz-Feller fractional derivative of order α, in $(0, 2]$. The special case $\alpha = 2$ is particularly interesting because it concerns the Brownian motion with randomly intermitting times.

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Random intersection graphs

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Vertices of an intersection graph are represented by subsets of a finite auxiliary set: two vertices are adjacent whenever the subsets intersect. Statistical properties of intersection graphs can be learned from random intersection graphs, where vertices select their subsets at random. An interesting and important property of random intersection graphs is that the neighbouring adjacency relations are statistically dependent. Furthermore, the dependence structure is similar to that of real affiliation networks (e.g., coauthorship network, where two authors are adjacent if they have coauthored a publication). This relation to real networks and mathematical tractability of the model makes it an attractive object of analytical study.

In my talk I shall survey some relatively new results about the structure of random intersection graphs. Results are asymptotical (as the number of vertices increase to infinity) and focus on the role played by the egde dependence in defining the structure of the graph: giant component, clique number, connectivity, perfect matching, degree-degree distribution.

Multiple stochastic integrals and random series [1]

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We discuss some approaches to construct multiple stochastic integrals of the form

$$\int_a^b \cdots \int_a^b f(t_1, \ldots, t_m) d\xi(t_1) \cdots d\xi(t_m), \tag{1}$$

where $f(\cdot)$ is a measurable nonrandom function and $\xi(\cdot)$ is a stochastic process. We study both the classical construction of such an integral as the mean-square limit of the corresponding integral sums (see [1]) and nonclassical ones based either on series expansions of the kernel $f(\cdot)$ or on the expansion of the stochastic product-differential in a multiple random series (see [2], [3]).

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On the excess over boundary [1]

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We consider a dynamic version of the Neyman contagious point process that can be used for modelling the spacial dynamics of biological populations, including species invasion scenarios. Starting with an arbitrary finite initial configuration of points in R^d with nonnegative weights, at each time step a point is chosen at random from the process according to the distribution with probabilities proportional to the points' weights. Then a finite random number of new points is added to the process, each displaced from the location of the chosen "mother" point by a random vector and assigned a random weight. Under broad conditions on the sequences of the numbers of newly added points, their weights and displacement vectors (which include a random environments setup), we derive the asymptotic behaviour of the locations of the points added to the process at time step n and also that of the scaled mean measure of the point process after time step $n \to \infty$.

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Convergence in law of infinite-dimensional polynomials and related estimates

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Given two random vectors $F = (F_1, \dots, F_d)$ and $G = (G_1, \dots, G_d)$ whose components are polynomials of a fixed degree k in Gaussian random variables (possibly, infinitely many), we discuss bounds on the total variation distance between the laws of F and G in terms of the Kantorovich distance between them. These bounds provide some quantitative information on convergence in variation which can be derived from convergence in law and improve known recent results due to Nualart, Nourdin, and Polly. Several approaches will be mentioned, one of which is a new estimate generalizing the classical Hardy–Landau–Littlewood inequality $\|f'\|_1^2 \le 2\|f\|_1\|f''\|_1$ on L^1-norms of intermediate derivatives to the multidimensional case in the form

$$\|f\|_1^2 \le C(d)\|Df\|_1\|f\|_K$$

for functions in the first Sobolev class with zero integral, where $\|f\|_K$ is the Kantorovich norm. Similar dimension–free estimates with Gaussian measures will be presented. Remarks about non-Gaussian cases will be made and some simply formulated open problems will be mentioned.

Stability Problems in Variable Selection [1]

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In various research domains one studies a response variable Y depending on some (random) factors $X_1,\dots,X_n$. For instance in medicine Y can describe the health state of a patient ($Y=1$ and $Y=-1$ correspond to the occurrence or not of a disease, respectively) and $X=(X_1,\dots,X_n)$ incudes genetic and non-genetic factors (see, e.g., [1]). Let Y take values in a set $\mathbb{Y}$ and the values of X belong to some set $\mathbb{X}$. It is important to indicate a function $f:\mathbb{X}\to\mathbb{Y}$ to approximate (in a sense) Y by means of $f(X)$. Moreover, since the law of (X,Y) is unknown it is natural to construct approximation of Y (to be able to predict the response variable) using the i.i.d. observations (Y^j,X^j), $j=1,\dots,N$, with the same law as (X,Y).

It would be desirable to identify a significant collection $\alpha=(k_1,\dots,k_r)$ where $1\le k_1<\ldots<k_r\le n$ $(r<n)$ such that Y depends on $X_\alpha:=(X_{k_1},\dots,X_{k_r})$ essentially and then employ the estimate of Y involving X_α. For $\mathbb{X}$ and $\mathbb{Y}$ being finite sets such that $\mathbb{Y}\subset\mathbb{R}$ the problems mentioned above as well as the limit behavior of the proposed regularized estimates were considered in [2]. There we assumed that

$$\mathsf{P}(Y=y|X=x)=\mathsf{P}(Y=y|X_\alpha=x_\alpha) \tag{1}$$

for all $y\in\mathbb{Y}$ and $x\in\mathbb{X}$ whenever $\mathsf{P}(X=x)\neq 0$. Here $x_\alpha=(x_{k_1},\dots,x_{k_r})$ for $x\in\mathbb{X}$. Now we suppose that instead of (1) one admits some specified small dependence of Y for X_k with $k\notin\alpha$. Thus we come to stability problem of inference.

Also we tackle the generalization of MDR method applying the pseudo observations and the Bayesian approach. Special attention is paid to simulation, see, e.g. [3].

[1] This work is supported by the RFBR-grant 13-01-00612.

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Stochastic models of biological problems [1]

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A number of biological problems such as cell growth under inhibition, dynamic of population and many others are modeled by systems of quasilinear parabolic equations [1]. To describe the corresponding phenomena one needs to solve the Cauchy problem or a boundary value problem for these systems.

We are interested in stochastic processes associated with these PDE problems and probabilistic representations of their solutions.

Modeling spatial segregation phenomena of competing species in population dynamics, Shigesada, Kawasaki and Teramoto [2] proposed in 1979 to study some nonlinear parabolic systems which include the following problem

$$\begin{cases} u^1_t = \Delta[(\alpha_1 + \alpha_{11}u^1 + \alpha_{12}u^2)u^2] + u^1(a_1 - b_1u^1 - c_1u^2), \\ u^2_t = \Delta[(\alpha_2 + \alpha_{21}u^1 + \alpha^2_{22})u^2] + u^2(a_2 - b_2u^1 - c_2u^2), \\ u^1(0,x) = u^1_0(x), \quad u^2(0,x) = u^2_0(x). \end{cases} \tag{1}$$

a_q, b_q, c_q – positive constants, α_{ql} – nonnegative constants, $q, l = 1, 2$. This system is a generalization of the famous Lotka-Volterra problem

To obtain the required stochastic processes we consider the system of stochastic equations of the form

$$d\xi^q(\theta) = M^q_u(\xi^q(\theta))dw(\theta), \quad \xi^q(0) = y, \quad q = 1, 2., \tag{2}$$

$$d\eta^q(\theta) = \tilde{m}^q_u(\xi^q_{s,\kappa}(\theta))\eta^q(\theta)d\theta + C^q_u(\xi^q_{s,\kappa}(\theta))\eta^q(\theta)dw(\theta), \quad \eta^q(0) = 1, \tag{3}$$

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where $M_u^q = \sqrt{\alpha_q + \alpha_{q1}u^1 + \alpha_{q2}u^2}, \quad m_u^q = a_q - b_q u^1 - c_q u^2,$

$$\tilde{m}_u^q = m_u^q - \|\nabla M_u^q\|^2, \quad C_u^q = -\nabla M_u^q. \tag{4}$$

Theorem 1. *Assume that there exists a unique regular positive weak solution of the Cauchy problem (1). Then it admits a probabilistic representation of the form*

$$u^q(t) = E[\zeta^q(t) \circ \psi_{0,t}^q], \tag{5}$$

where

$$\zeta^q(t) = \exp\left\{\int_0^t [m_u^q - \frac{1}{2}\|\nabla M^q\|^2] d\theta - \int_0^t \nabla M_u^q \cdot dw(\theta)\right\} u_0^q,$$

and $\psi_{0,t}^q$ *is a stochastic flow generated by the process* $\hat{\xi}(t)$ *time reversal to the stochastic processes* $\xi^q(t)$ *satisfying (2).*

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On the deficiency concept in statistical problems based on the samples with random sizes

Bening V.E.,*

1 Introduction and summary

An interesting quantitative comparison can be obtained by taking a viewpoint similar to that of the asymptotic relative efficiency (ARE) of estimators, and asking for the number $m(n)$ of observations needed by estimator $\delta_{m(n)}(X_1,\ldots,X_{m(n)})$ to match the performance of $\delta_n^*(X_1,\ldots,X_n)$ (based on n observations). Although the differenc $m(n) - n$ seems to be a very natural quantity to examine, historically the ratio $n/m(n)$ was preffered by almost all authors in view of its simpler behaviour. The first general investigation of $m(n) - n$ was carried out by Hodges and Lehmann ([1]). They name $m(n) - n$ the deficiency of δ_n with respect to δ_n^* and denote it as

$$d_n = m(n) - n. \tag{1.1}$$

If $\lim_{n\to\infty} d_n$ exists, it is called the asymptotic deficiency of δ_n with respect to δ_n^* and denote as d. At points where no confusion is likely, we shall simply call d the deficiency of δ_n with respect to δ_n^*.

The deficiency of δ_n relative to δ_n^* will then indicate how many observations one loses by insisting on δ_n, and thereby provides a basis for deciding whether or not the price is too high. If the risk functions of these two estimators are

$$R_n(\theta) = \mathsf{E}_\theta\left(\delta_n - g(\theta)\right)^2, \quad R_n^*(\theta) = \mathsf{E}_\theta\left(\delta_n^* - g(\theta)\right)^2,$$

then by definition, $d_n(\theta) \equiv d_n = m(n) - n$, for each n, may be found from

$$R_n^*(\theta) = R_{m(n)}(\theta). \tag{1.2}$$

In order to solve (1.1), $m(n)$ has to be treated as a continuous variable. This can be done in a satisfactory manner by defining $R_{m(n)}(\theta)$ for non - integral $m(n)$ as

$$R_{m(n)}(\theta) = \big(1 - m(n) + [m(n)]\big)\, R_{[m(n)]}(\theta) + \big(m(n) - [m(n)]\big)\, R_{[m(n)]+1}(\theta)$$

(cf. [1]).

Generally $R_n^*(\theta)$ and $R_n(\theta)$ are not known exactly and we have to use approximations. Here these are obtained by observing that $R_n^*(\theta)$ snd $R_n(\theta)$ will tipically satisfy asymptotic expansions (a.e.) of the form

$$R_n^* = \frac{a(\theta)}{n^r} + \frac{b(\theta)}{n^{r+s}} + o\big(n^{-(r+s)}\big), \tag{1.3}$$

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$$R_n \;=\; \frac{a(\theta)}{n^r} \;+\; \frac{c(\theta)}{n^{r+s}} \;+\; o\big(n^{-(r+s)}\big), \tag{1.4}$$

for certain $a(\theta),\; b(\theta)$ and $c(\theta)$ not depending on n and certain constants $r \;>\; 0,\; s \;>\; 0$. The leading term in both expansions is the same in view of the fact that ARE is equal to one. From (1.1) – (1.4) is now easily follows that (see [1])

$$d_n(\theta) \;\equiv\; \frac{c(\theta) \;-\; b(\theta)}{r\; a(\theta)}\; n^{(1-s)} \;+\; o\big(n^{(1-s)}\big). \tag{1.5}$$

Hence

$$d(\theta) \;\equiv\; d \;=\; \begin{cases} \pm\infty, & 0 \;<\; s \;<\; 1, \\ \dfrac{c(\theta) \;-\; b(\theta)}{r\; a(\theta)}, & s \;=\; 1, \\ 0, & s \;>\; 1. \end{cases} \tag{1.6}$$

A useful property of deficiencies is the following (transitivity): if a third estimator $\bar{\delta}_n$ is given, for which the risk $\bar{R}_n(\theta)$ also has an expansion of the form (1.4), the deficiency d of $\bar{\delta}_n$ with respect to δ_n^* satisfies

$$d \;=\; d_1 \;+\; d_2,$$

where d_1 is the deficiency of $\bar{\delta}_n$ with respect to δ_n and d_2 is the deficiency of δ_n with respect to δ_n^*.

The situation where $s \;=\; 1$ seems to be the most interesting one. Hodges nad Lehmann ([1]) demonstrate the use of deficiency in a number of simple examples for which this is the case.

In the communication, we discuss the number of applications of the deficiency concept in the problems of point estimation and testing statistical hypotheses in the case when number of observations is random.

2 Estimators based on the sample with random size

Consider random variables (r.v.'s) $N_1, N_2, ...$ and $X_1, X_2, ...$, defined on the same probability space $(\Omega,\, \mathcal{A},\, \mathsf{P})$. By $X_1, X_2, ...X_n$ we will mean statistical observations whereas the r.v. N_n will be regarded as the random sample size depending on the parameter $n \in \mathbb{N}$. Assume that for each $n \geq 1$ the r.v. N_n takes only natural values (i.e., $N_n \in \mathbb{N}$) and is independent of the sequence $X_1, X_2, ...$ Everywhere in what follows the r.v.'s $X_1, X_2, ...$ are assumed independent and identically distributed with distribution depending on $\theta \;\in\; \Theta \;\in\; \mathbb{R}$.

For every $n \geq 1$ by $T_n = T_n(X_1, ..., X_n)$ denote a statistic, i.e., a real-valued measurable function of $X_1, ..., X_n$. For each $n \;\geq\; 1$ we define a r.v. T_{N_n} by setting $T_{N_n}(\omega) \;\equiv\; T_{N_n(\omega)}(X_1(\omega), ..., X_{N_n(\omega)}(\omega))$, $\omega \in \Omega$.

Theorem 2.1.

1. If $\delta_n \;=\; \delta_n(X_1, \ldots, X_n)$ is any unbised estimator of $g(\theta)$ that is, it satisfies

$$\mathsf{E}_\theta\; \delta_n \;=\; g(\theta), \quad \theta \;\in\; \Theta$$

and $\delta_{N_n} \;\equiv\; \delta_{N_n}(X_1, \ldots, X_{N_n})$, then

$$\mathsf{E}_\theta\; \delta_{N_n} \;=\; g(\theta), \quad \theta \;\in\; \Theta.$$

2. Suppose that numbers $a(\theta)$, $b(\theta)$ and $C(\theta) > 0$, $\alpha > 0$, $r > 0$, $s > 0$ exist such that

$$\left| R_n^*(\theta) - \frac{a(\theta)}{n^r} - \frac{b(\theta)}{n^{r+s}} \right| \leqslant \frac{C(\theta)}{n^{r+s+\alpha}},$$

where

$$R_n^*(\theta) = \mathsf{E}_\theta \big(\delta_n(X_1, \ldots, X_n) - g(\theta)\big)^2,$$

then

$$\left| R_n(\theta) - a(\theta)\, \mathsf{E}\, N_n^{-r} - b(\theta)\, \mathsf{E}\, N_n^{-r-s} \right| \leqslant C(\theta)\, \mathsf{E}\, N_n^{-r-s-\alpha},$$

where

$$R_n(\theta) = \mathsf{E}_\theta \big(\delta_{N_n}(X_1, \ldots, X_{N_n}) - g(\theta)\big)^2.$$

Corollary 2.1.

Suppose that numbers $a(\theta)$, $b(\theta)$ and $r > 0$, $s > 0$ exist such that

$$R_n^*(\theta) = \frac{a(\theta)}{n^r} + \frac{b(\theta)}{n^{r+s}}$$

where

$$R_n^*(\theta) = \mathsf{E}_\theta \big(\delta_n(X_1, \ldots, X_n) - g(\theta)\big)^2,$$

then

$$R_n(\theta) = a(\theta)\, \mathsf{E}\, N_n^{-r} + b(\theta)\, \mathsf{E}\, N_n^{-r-s},$$

where

$$R_n(\theta) = \mathsf{E}_\theta \big(\delta_{N_n}(X_1, \ldots, X_{N_n}) - g(\theta)\big)^2.$$

Let observations $X_1, \ldots, X_n$ have expectaion

$$\mathsf{E}_\theta\, X_1 = g(\theta)$$

and variance

$$\mathsf{D}_\theta\, X_1 = \sigma^2(\theta).$$

The customary estimator for $g(\theta)$ based on n observation is

$$\delta_n = \frac{1}{n} \sum_{i=1}^{n} X_i. \qquad (2.1)$$

This estimator is unbiased and consistent, and its variance is

$$R_n^*(\theta) = \mathsf{D}_\theta\, \delta_n = \frac{\sigma^2(\theta)}{n}. \qquad (2.2)$$

If this estimator based on the sample with random size we have (see Corollary 1.1)

$$R_n(\theta) = \mathsf{D}_\theta\, \delta_{N_n}(X_1, \ldots, X_{N_n}) = \sigma^2(\theta)\, \mathsf{E}\, N_n^{-1}. \qquad (2.3)$$

If $g(\theta)$ is given, we consider the estimator for $\sigma^2(\theta)$ in the form

$$\bar{\delta}_n = \frac{1}{n} \sum_{i=1}^{n} (X_i - g(\theta))^2. \qquad (2.4)$$

This estimator is unbiased and consistent, and its variance is

$$\bar{R}^*_n(\theta) \;=\; \mathsf{D}_\theta\, \bar{\delta}_n \;=\; \frac{\mu_4(\theta) \;-\; \sigma^4(\theta)}{n}, \quad \mu_4(\theta) \;=\; \mathsf{E}_\theta\; (X_1 \;-\; g(\theta))^4. \tag{2.5}$$

For this estimator with random size one have

$$\bar{R}_n(\theta) \;=\; \mathsf{D}_\theta\; \bar{\delta}_{N_n}(X_1, \ldots, X_n) \;=\; \big(\mu_4(\theta) \;-\; \sigma^4(\theta)\big)\; \mathsf{E}\; N_n^{-1}. \tag{2.6}$$

In the preceeding example, suppose that $g(\theta)$ is unknown but that instead of (2.4) we are willing to consider any estimator of the form (see (2.1))

$$\tilde{\delta}^{(\gamma)}_n \;\equiv\; \tilde{\delta}_n \;=\; \frac{1}{n+\gamma}\sum_{i=1}^{n}\big(X_i \;-\; \delta_n\big)^2, \quad \gamma \;\in\; \mathbb{R}. \tag{2.7}$$

If $\gamma \;\neq\; -1$, this will not be unbiased but may have a smaller expected squared error that the unbiased estimator with $\gamma \;=\; -1$.

One easily find (see [1], (3.6) and [2])

$$\tilde{R}^*_n(\theta) \;=\; \mathsf{E}_\theta\; \big(\tilde{\delta}_n(X_1, \ldots, X_n) \;-\; \sigma^2(\theta)\big)^2 \;=$$

$$= \frac{\sigma^4(\theta)}{n(n \;+\; \gamma)^2}\Big((n \;-\; 1)\; \big((\mu_4(\theta)/\sigma^4(\theta) \;-\; 1)\; (n \;-\; 1) \;+\; 2\big) \;+ n\; (\gamma \;+\; 1)^2\Big) \tag{2.8}$$

and hence

$$\tilde{R}^*_n(\theta) \;=\; \sigma^4(\theta)\; \Big(\frac{\mu_4(\theta)/\sigma^4(\theta) \;-1}{n} \;+$$

$$+ \frac{(\gamma \;+\; 1)^2 \;-\; 2\; (\mu_4(\theta)/\sigma^4(\theta) \;-1) \;+\; 2 \;-2\gamma(\mu_4(\theta)/\sigma^4(\theta) \;-1)}{n^2}\Big) \;+\; O\big(n^{-3}\big). \tag{2.9}$$

Using Theorem 1.1, we have

$$\tilde{R}_n(\theta) \;=\; \mathsf{E}_\theta\; \big(\tilde{\delta}_{N_n}(X_1, \ldots, X_{N_n}) \;-\; \sigma^2(\theta)\big)^2 \;=$$

$$= \sigma^4(\theta)\; \Big((\mu_4(\theta)/\sigma^4(\theta) \;-1)\; \mathsf{E}\; N_n^{-1} \;+$$

$$+ \big((\gamma + 1)^2 - 2\; (\mu_4(\theta)/\sigma^4(\theta) \;-1) \;+\; 2 \;-2\gamma(\mu_4(\theta)/\sigma^4(\theta) \;-1)\big)\; \mathsf{E}\; N_n^{-2}\Big) \;+\; O\big(\mathsf{E}\; N_n^{-3}\big). \tag{2.10}$$

3 Deficiencies of some estimators based on the samples with random size

We now apply the results of section 2 to the three exapmles given in this section. Let M_n be the Poisson r.v. with parameter $n \;-\; 1, \;\; n \;\geqslant\; 2$, i.e.

$$\mathsf{P}\big(M_n \;=\; k\big) \;=\; e^{(1-n)}\;\frac{(n \;-\; 1)^k}{k!}, \quad k \;=\; 0, 1, \ldots$$

Define the random size as

$$N_n \;=\; M_n \;+\; 1,$$

then

$$\mathsf{E}\, N_n \;=\; n$$

and

$$\mathsf{E}\, N_n^{-1} \;=\; \frac{1}{n} \;+\; \frac{1}{n^2} \;+\; o\big(n^{-2}\big). \tag{3.1}$$

The deficiency of δ_{N_n} relative to δ_n (see (2.1)) is given by (2.2), (2.3), (3.1) and (1.6) with $r \;=\; s \;=\; 1$, $a(\theta) \;=\; \sigma^2(\theta)$, $b(\theta) \;=\; 0$, $c(\theta) \;=\; \sigma^4(\theta)$, and hence is equal to

$$d \;=\; 1. \tag{3.2}$$

Similarly, the deficiency of $\bar{\delta}_{N_n}$ relative to $\bar{\delta}_n$ (see (2.4)) is given by (2.5), (2.6), (3.1) and (1.6) with $r \;=\; s \;=\; 1$, $a(\theta) \;=\; c(\theta) \;=\; \mu_4(\theta) \;-\; \sigma^4(\theta)$, $b(\theta) \;=\; 0$, and hence is equal to

$$\bar{d} \;=\; 1. \tag{3.3}$$

Consider now third example (see (2.7)). We have

$$\mathsf{E}\, N_n^{-2} \;\sim\; \frac{1}{n^2}, \quad n \;\to\; \infty. \tag{3.4}$$

Now the deficiency of $\tilde{\delta}_{N_n}$ relative to $\tilde{\delta}_n$ (see (2.7)) is given by (2.9), (2.10), (3.4) and (1.6) with $r \;=\; s \;=\; 1$ and hence is equal to

$$\tilde{d} \;=\; 1 \tag{3.5}$$

and the deficiency of $\tilde{\delta}_{N_n}^{(\gamma_1)}$ relative to $\tilde{\delta}_{N_n}^{(\gamma_2)}$ (see (2.7)) is given by (3.1), (3.4) and (1.6) with $r \;=\; s \;=\; 1$ and hence is equal to

$$\tilde{d}_{\gamma_1,\gamma_2} \;=\; (\gamma_1 \;-\; \gamma_2)\left(\frac{\gamma_1 \;+\; \gamma_2 \;+\; 2}{\mu_4(\theta)/\sigma^4(\theta) \;-\; 1} \;-\; 2\right). \tag{3.6}$$

These examples illustrate the following

Theorem 3.1.

Suppose that numbers $a(\theta)$, $b(\theta)$ and k_1, k_2 exist such that

$$R_n^*(\theta) \;=\; \frac{a(\theta)}{n} \;+\; \frac{b(\theta)}{n^2} \;=\; o\big(n^{-2}\big)$$

and

$$\mathsf{E}\, N_n^{-1} \;=\; \frac{1}{n} \;+\; \frac{k_1}{n^2} \;+\; o\big(n^{-2}\big),$$

$$\mathsf{E}\, N_n^{-2} \;=\; \frac{k_2}{n^2} \;+\; o\big(n^{-2}\big),$$

$$\mathsf{E}\, N_n^{-3} \;=\; o\big(n^{-2}\big),$$

then the asymptotic deficiency of $\delta_{N_n}(X_1,\ldots,X_{N_n})$ with respect to $\delta_n(X_1,\ldots,X_n)$ is equal to

$$d(\theta) \;=\; \frac{k_1\, a(\theta) \;+\; b(\theta)\, k_2 \;-\; b(\theta)}{a(\theta)}.$$

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Positive and Discrete Linnik Distributions revisited

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Continuous *positive Linnik* random variables W_α^λ is defined by their Laplace-Stieltjes transforms

$$\psi_{W_\alpha^\lambda}(u) = \begin{cases} (1+\lambda\, u^\alpha/\beta)^{-\beta} & \text{for } \ 0<\beta<\infty\,, \\ \exp\{-\lambda\, u^\alpha\} & \text{for } \ \beta=\infty\,, \end{cases} \qquad u \geq 0 \qquad (1)$$

and non-negative integer valued *discrete Linnik* random variables L_α^λ by their probability generating functions

$$g_{L_\alpha^\lambda}(z) = \begin{cases} (1+\lambda\,(1-z)^\alpha/\beta)^{-\beta} & \text{for } 0<\beta<\infty\,, \\ \exp\{-\lambda\,(1-z)^\alpha\} & \text{for } \beta=\infty\,, \end{cases} \qquad |z| \leq 1\,, \quad (2)$$

with *characteristic exponent* $\alpha \in (0,1]$*, scale parameter* $\lambda > 0$ *and form parameter* $\beta > 0$, where for $\beta = \infty$ the nonnegative *strictly stable* and the *discrete stable* random variables denoted further by S_α^λ and X_α^λ occur in (1) and (2) as a natural generalizations of both Linnik distributions. See Christoph and Schreiber (2001) and the references therein.

In the mentioned paper we considered some properties of positive Linnik and the discrete Linnik distributions, Among others rates of convergence and **uniform bonds** in asymptotic expansions for $P(n^{-1/\alpha}(W_1 + ... + W_n) \leq x)$ to stable limit distributions $P(S_\alpha^\lambda \leq x) = G_\alpha(x;\lambda)$, where $W_1, W_2, \dots$ are independent and identical distributed copies of positive Linnik random variable W_γ^λ.

The purpose of this paper is to give **non-uniform bounds** for such asymptotic expansions, which may be used also for large deviation problems. Define $G_\alpha^{(k)}(x;\lambda) = \frac{\partial^k}{\partial\lambda^k} G_\alpha(x;\lambda)$, their Laplace-Stieltjes transforms are $(-1)^k u^{k\alpha} \exp\{-\lambda u^\alpha\}$. Since as $x \to \infty$

$$P(W_\gamma^\lambda \leq x) = G_\alpha(x;\lambda) + \frac{\lambda^2 G_\alpha^{(2)}(x;\lambda)}{2\,\beta} + \frac{\lambda^3 G_\alpha^{(3)}(x;\lambda)}{3\,\beta^2} + O(x^{-4\alpha}) \quad (3)$$

and $\quad 1 - G_\alpha(x,\lambda) = \lambda\, c_1 x^{-\alpha} + \lambda^2\, c_2 x^{-2\alpha} + \lambda^3\, c_3 x^{-3\alpha} + O(x^{-4\alpha})$ with $c_k = \frac{1}{\pi k!}(-1)^{k+1}\Gamma(k\alpha)\sin(k\alpha\pi)$ we can make use of the method presented in Christoph and Malevich (2011):

Theorem 1. *For positive Linnik random variable W_α^λ with Laplace-Stieltjes transform (1), $0 < \alpha < 1$ and $\beta < \infty$ we obtain :*

$$\sup_x (1+|x|^{4\alpha})\big|P(n^{-1/\alpha}(W_1+...+W_n) \le x) - G_n(x)\big| = O(n^{-3}), \quad (4)$$

$n \to \infty$ *with* $G_n(x) = G_\alpha(x;\lambda) + \frac{\lambda^2 G_\alpha^{(2)}(x;\lambda)}{2\,\beta\, n} + \frac{\lambda^3 G_\alpha^{(3)}(x;\lambda)}{3\,\beta^2\, n^2} + \frac{\lambda^4 G_\alpha^{(4)}(x;\lambda)}{8\,\beta^2\, n^2}$.

As an application of such non-uniform bounds we investigate random sums occurring e.g. in the Cramér-Lundberg model as the classical risk model or basic insurance risk model:

We consider now a compound sum $S_\nu = W_1 + W_2 + \ldots + W_\nu$, where $\nu \in \{1, 2, 3, \ldots\}$ is a counting random variable, independent of $W_1, W_2, \ldots$ with $Ee^{t\nu} < \infty$ for $|t| < \varepsilon$, $\varepsilon > 0$. Since W_γ^λ is subexponential, we have $\Delta(x) := \frac{P(W_1+W_2+...+W_\nu > x)}{P(W_1 > x)} - E\nu \to 0$ as $x \to \infty$. Using (4) we find

Theorem 2. $\Delta(x) = \frac{\lambda c_2}{c_1 x^\alpha}(E\nu^2 - E\nu) + \frac{\lambda^2}{c_1^2 x^{2\alpha}} C(\beta) + O(x^{-3\alpha})$ *with* $C(\beta) = c_3 c_1\big(E\nu^3 + \frac{3}{\beta^2} E\nu^2 - (1 + \frac{3}{\beta})E\nu\big) - c_2^2\big(E\nu^2 - \frac{E\nu}{\beta}\big)$.

Note that in Theorems 1 and 2 we can get more terms in the asymptotic expansions using more terms in the expansion (3). Similar results may be obtained for discrete Linnik sequences.

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On the extended Gauss-Markov theorem for linear mixed models

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Mixed model became an indispensable statistical tool for analysis of longitudinal and repeated measurements cluster data. Linear mixed model is defined as the set of linear regression models,

$$\mathbf{y}_i = \mathbf{X}_i\boldsymbol{\beta} + \mathbf{Z}_i\mathbf{b}_i + \boldsymbol{\varepsilon}_i, \quad i = 1, 2, ..., N,$$

where i codes the cluster (object, subject, etc.), $\mathbf{y}_i$ is the $n_i \times 1$ vector of observations on the dependent variable, $\mathbf{X}_i$ is the $n_i \times m$ design matrix of independent or explanatory variables, $\boldsymbol{\beta}$ is the $m \times 1$ vector of fixed effect coefficients subject to estimation, $\mathbf{Z}_i$ is the $n_i \times k$ design matrix of random effects, $\mathbf{b}_i$ is the $k \times 1$ vector of normally distributed random effects with zero mean and the covariance matrix $E(\mathbf{b}_i\mathbf{b}_i') = \mathbf{D}$, and $\boldsymbol{\varepsilon}_i$ is the $n_i \times 1$ vector of normally distributed noise with zero mean and covariance matrix $\sigma^2\mathbf{I}$. While observations from different clusters do not correlate, observations within the cluster are dependent and have the covariance matrix $\mathbf{V}_i = \sigma^2\mathbf{I} + \mathbf{Z}_i\mathbf{D}\mathbf{Z}_i$, due to random effect $\mathbf{b}_i$.

As has been studied by Yu. Linnik[1], the problem of efficient estimation of the linear model in small sample, as a special case of mixed model when $N = 1$ and $k = 0$ (no random effects), reduces to linear least squares (the Gauss-Markov theorem).

The extended Gauss-Markov theorem, as the problem of efficient estimation of regression coefficients with unknown covariance matrix, the so called estimated generalized least squares (EGLS), has be posed almost a century ago and is still unsolved. The goal of the talk is to formulate the problem of the EGLS in the framework of mixed model and report advances in this direction. In particular, we discuss five types of unbiased EGLS estimators: (1) minimum norm unbiased quadratic estimator (MINQUE), (2) variance least squares, (3) method of moments, (4) maximum likelihood, and (5) restricted maximum likelihood estimator. The first three estimators are noniterative and use unbiased quadratic estimators of the variance σ^2 and covariance matrix $\mathbf{D}$. It is shown that in all five cases the distribution of EGLS estimator is $\boldsymbol{\beta}$ independent and therefore reduces to the optimal choice of estimators for the variance parameters.

The efficient estimation of linear mixed models is formulated in local (small $\mathbf{D}$) and global (any $\mathbf{D}$) sense. Two types of the efficiency are suggested: the proximity to the lower Cramer-Rao bound in the case of normally distributed errors, and the distribution-free minimum trace, or the maximum eigenvalue, of the covariance matrix of EGLS over the matrices for quadratic variance estimation. As was proven in Demidenko[2], in the case of the balanced random coefficient mixed model, for which $\mathbf{X}_i = \mathbf{Z}_i = \mathbf{Z}$, the maximum likelihood and restricted maximum likelihood estimators for the variance parameters are expressed in closed form, the Gauss-Markov theorem holds true. Thus additional attention is given to unbalanced mixed models which include mixed model with random intercept, growth curve models, and the general mixed model specified above.

The problem of efficient EGLS estimator is one of the oldest statistical problem—it has a great interest from the theoretical as well as the practical standpoint.

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Stochastic calculus for Brownian flows

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The talk is devoted to the flows of Brownian particles. When the flow consists of the solutions to the SDE with the smooth coefficients there exists the Gaussian noise which determines the properties of the flow. In this case such statements as large deviations principle, Girsanov theorem, Krylov-Veretennikov expansion etc can be obtained as a consequence of the corresponding statements for Gaussian measures [1]. When the coalescence can occur there are no, in general, Gaussian noise, which generates the flow [2]. In this case some new technique must be used in oder to get the above mentioned statements [3]. In the talk we propose the unified approach to the investigation of Brownian flows based on the notion of quadratic entropy [4]. In terms of the such entropy one can discuss both smooth and coalescing cases. In particular, the structure of the flow mappings can be described.

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Discrete-time stochastic flow

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We study geometry of $m-$point motion of a stochastic flow with singular interactions. We give an explicit form for the semigroup of $m-$point motion of the Arratia flow [1] in terms of binary forests that correspond to order of trajectories coalescence [2].

The discrete-time approximation of the Arratia flow are considered. This approximations $\{x_k^n(u), k = 0, \dots, n\}$ are given by a difference equation with random perturbation generated by a sequence of independent stationary Gaussian processes $\{\xi_k^n(u),\ u \in \mathbb{R},\ k = 0, \dots, n\}$ with covariance function Γ_n:

$$x_{k+1}^n(u) = x_k^n(u) + \frac{1}{\sqrt{n}}\xi_{k+1}^n(x_k^n(u)),\ x_0^n(u) = u,\ u \in \mathbb{R}.$$

Define the random process $\tilde{x}_n(u, \cdot)$ on $[0, 1]$ as the polygonal line with edges $\left(\frac{k}{n}, x_k^n(u)\right)$, $k = 0, \dots, n$. It was proved in [3] that if the covariance Γ_n approximates in some sense the function $\mathbb{I}_{\{0\}}$ then $m-$point motion of $\tilde{x}_n$ weakly converges to the $m-$point motion of the Arratia flow. We obtain an explicit form of the Ito-Wiener expansion for $f(x_n(u_1), \dots, x_n(u_m))$ with respect to noise that produced by the processes $\{\xi_k^n(u),\ u \in \mathbb{R},\ k = 0, \dots, n\}_{n \geq 1}$. This expansion can be regarded as a discrete-time analogue of the Krylov-Veretennikov representation formula [4].

In contrasts to the flow of Brownian particles on the line, in the discrete-time approximations the order between particles can change in time. We define a measure of disordering for 2-point motion as follows

$$\Phi_n = \int_0^1 \mathbb{I}_{\{\tilde{x}_n(u_2,s) - \tilde{x}_n(u_1,s) < 0\}} ds,$$

where $u_1 < u_2$. If the discrete-time flow approximates the Arratia flow then the following asymptotics holds [5]:

$$\overline{\lim_{n\to\infty}} \frac{2C_n}{n} \ln \mathbb{P}\{\Phi_n > 0\} \le -1$$

$$\underline{\lim_{n\to\infty}} \frac{2C_n}{n} \ln \mathbb{P}\{\Phi_n > \varepsilon\} \ge -K^2,$$

where $C_n = \sup_{\mathbb{R}} \frac{2-2\Gamma_n(x)}{x^2}$ and $K > 0$.

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Quantum Gaussian transition operators: characterization and an optimal property [1]

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1. In the noncommutative probability theory, there is a genuine analog of Gaussian probability measures – the Gaussian states on the algebra of Canonical Commutation Relations. In [1] the symmetry group of the set of Gaussian states was described by showing that any such symmetry is induced by a quasi-free automorphism of the algebra and vice versa. This result was extended in [2] where a characterization was obtained for completely positive maps of the algebra leaving the set of Gaussian states globally invariant. Namely, it was shown that the action of any such map in terms of characteristic functions of the states is described as

$$\chi(\lambda) \to \chi\left(K\lambda\right) \, \exp\left(i\lambda^t l - \frac{1}{2}\lambda^t M\lambda\right), \quad \lambda \in \mathbb{R}^n, \tag{1}$$

where K is a real $n\times n-$matrix, $l\in\mathbb{R}^n$, M is a real symmetric $n\times n-$matrix, satisfying the restriction

$$M \geq \pm\frac{i}{2}(\Delta - K^t\Delta K)\,, \tag{2}$$

where Δ is the real skew-symmetric commutation matrix (the case of [1] corresponds to $M=0, K^t\Delta K=\Delta$). A classical counterpart of this result is a characterization of Feller's transition operators leaving invariant the set of Gaussian probability measures as the maps of the type (1) satisfying $M\geq 0$ instead of (2) .

2. In quantum information theory completely positive "transition operators" describe quantum communication channels. We

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explore the semigroup structure of the quantum Gaussian channels to show that pure Gaussian states, and under certain conditions only they, minimize a broad class of the concave functionals of the output of a gauge-covariant or contravariant channel [3], [4]. A remarkable corollary of this fact is that the key additivity property of the minimal output entropy of the channel, while not valid in general, does hold in this class of quantum Gaussian channels.

This allows us also to show that the classical information capacity of these channels (under the input energy constraint) is additive and is achieved by Gaussian encodings, thus establishing the long-awaited quantum counterpart of the famous Shannon capacity formula.

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Self-intersection local times for Gaussian processes and Hilbert-valued functions

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The construction of renormalization for k-multiple self-intersection local time of Gaussian process $x(t) = ((g(t), \xi_1), (g(t), \xi_2)),\ t \in [0; 1]$, where $g \in C([0; 1], L_2([0; 1])),\ \xi_1,\ \xi_2$ are two independent Gaussian white noises in $L_2([0; 1])$ is equivalent to regularization of divergent integral

$$\int_{\Delta_k} \frac{d\vec{t}}{G(\Delta g(t_1), \ldots, \Delta g(t_{k-1}))} \tag{1}$$

(see [1-5]). Here

$$\Delta_k = \{0 \le t_1 \le \ldots \le t_k \le 1\},\ G(\Delta g(t_1), \ldots, \Delta g(t_{k-1}))$$

is the Gram determinant constructed from increments of function g. In [1-4] for case $g(t) = (I + S)1_{[0;t]}$, where S is a compact operator in $L_2([0; 1])$ with $\|S\| < 1$ we constructed regularization for (1). Since $Ker(I+S) = \{\emptyset\}$, the regularization consist of compensation of impact of diagonals, where integral (1) blow up. In the general case $g(t) = A1_{[0;t]}$, where A is a continuous linear operator in $L_2([0; 1])$ with $KerA \neq \{\emptyset\}$, the denominator of (1) contains additional singularities. The question is does the "old" regularization for (1) hold? We prove that the answer is "yes" for planar Gaussian processes generated by the operator A which satisfies the following conditions

1) $dimKerA < +\infty$

2) The restriction of operator A on orthogonal complement to $KerA$ is continuously invertible operator.

The key moment in the proof is the following low estimate for Gram determinant.

Theorem 1. *Let A satisfies conditions 1)-2). Then there exist partition $0 < s_1 < \ldots < s_N < 1$ and $c(k) > 0$ such that the following relation holds*

$$G(A1_{[t_1;t_2]}, \ldots, A1_{[t_{k-1};t_k]}) \geq$$

$$\geq c(k)G(1_{[t_1;t_2]}, \ldots, 1_{[t_{k-1};t_k]}, 1_{[s_1;s_2]}, \ldots, 1_{[s_{N-1};s_N]})$$

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What are casual stable distributions and why do we need them?

Lev B. Klebanov, Abram A. Zinger

We present an overview of new definitions and notions, closely connected to that of stable distributions.

Introduced classes of distributions are of both theoretical and practical value.

Random braids formed by trajectories of stochastic flows

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In this report we consider braids formed by the trajectories of two-dimensional stochastic flows (the role of the third coordinate of the braid is plaid by time). There exists a full system of invariants for braids that distinguishes them up to homotopy, — the Vassiliev invariants system. We obtained a representation of Vassiliev invariants for braids formed by continuous semimartingales with respect to the common filtraton in a form of the iterated Stratonovich integrals [1].

The mutual winding angles of the braid's strands are Vassiliev invariants of the first order. Asymptotical behaviour of the mutual winding angles of independent planar Brownian motions was studied by M. Yor [2]. Some results about winding angles in Brownian stochastic flows were obtained in [3]. We obtained the following result about asymptotical behaviour (when $t \to \infty$) of the winding angles of particles in Brownian stochastic flows.

Theorem 1. *Let $F_t(x), t \geq 0, x \in \mathbb{R}^2$, be a Brownian stochastic flow defined by equation*

$$dF_t(x) = U(F_t(x), dt),$$

where $\mathbb{E}U(x,t)^k U(y,s)^l = b_{kl}(x-y)t \wedge s$, and b_{kl} has the form

$$b_{kl}(z) = \delta_{kl} b_L(\|z\|).$$

Let us consider trajectories $F_t(x_1), \ldots, F_t(x_k)$ of this flow starting from distinct points $x_1, \ldots, x_k$. Let $\Phi_{kl}(t)$ be the angle wound by $F_s(x_k)$ around $F_s(x_l)$ up to time t. Then

$$\left(\frac{2}{\ln t} \Phi_{kl}(t), k, l = 1, \ldots, n \right) \xrightarrow[t \to \infty]{d} (C_{12}, \ldots, C_{n-1,n}),$$

where $C_{kl}, 1 \leq k < l \leq n$, *are independent random variables with the standard Cauchy distribution.*

In the course of study of asymptotical behaviour of Vassiliev invariants of the 2nd order of the braids formed by independent planar Brownian motions, there arises a need in obtaining the results of the type of Strassen's law of the iterated logarithm for the mutual winding angles of Brownian particles. One of the proofs of the law of the iterated logarithm is based on the large-deviation principle for the Wiener process. We obtain estimates of the large-deviation principle for the family (Φ_ε) of the winding angles of the process $w_\varepsilon(t) = w(\varepsilon t), 0 \leq t \leq 1$, around the point $(0, 0)$. Here w is a two-dimensional Wiener process, $w(0) \neq 0$.

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Random maps and widths of compact sets in Hilbert space

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We consider the images of compact sets in Hilbert space under strong random operators [1] and study the asymptotic behavior of the Kolmogorov widths [2] of such images.
For random operators $T_{s,t}$ $(s \le t)$ related to stochastic flow [3] $\varphi_{s,t}$ as follows

$$(T_{s,t}f)(u) = f(\varphi_{s,t}(u)), \quad f \in L_2(\mathbb{R}), \ u \in \mathbb{R}$$

are obtained such results:

Lemma 1. *$T_{s,t}$ $(s \le t)$ is a bounded random operator if and only if*

$$\sup_{u\in\mathbb{R}} \left(\frac{\partial \varphi_{s,t}(u)}{\partial u}\right)^{-1} < +\infty \qquad a.s.$$

Lemma 2. *Let $\varphi_{s,t}$ be a family of solutions of the stochastic differential equation*

$$dx(t) = a(x(t))dt + b(x(t))dw(t),$$

where $a, b \in \mathcal{C}^1(\mathbb{R}),\ |a'| + |b'| \le L,\ \inf_{y\in\mathbb{R}} b(y) > 0$.
Then $T_{s,t}$ is a strong random operator.

If strong random operator A has a continuous modification on compact set K then image $A(K)$ is compact set.

Lemma 3. *Let A be a Gaussian s.r.o.[4] on a real separable Hilbert space H, $K \subset H$ be a compact set, and N_K be the metric entropy function for K with respect $\|\cdot\|_H$. If*

$$\int_{N^K(u)>1} \left(\ln N^K(u)\right)^{\frac{1}{2}} du < +\infty,$$

then image $A(K)$ is a compact set.

In the next theorem the asymptotic behavior of Kolmogorov n-width of the image of the compact set in Hilbert space under Gaussian s.r.o. is established. Similar statements for semigroups of finite-dimensional random projections were obtained in [5].

Definition[2]. *Kolmogorov n-width of K is*

$$d_n(K) = \inf_{dimL \le n} \sup_{x \in K} \inf_{y \in L} \|x - y\|_H,$$

where $L \subset H$ is a subspace.

Theorem. *Let H be a real separable Hilbert space with an orthonormal basis $\{e_n\}_{n=1}^{\infty}$, $\xi_1, \xi_2, \dots$ be an independent $N(0;1)$. For compact set $K = \left\{x \in H : \quad (x, e_n)^2 \le \frac{1}{n^2}, \text{ for all } n \ge 1\right\}$ and Gaussian strong random operator $Ax = \sum_{n=1}^{+\infty} \xi_n (x; e_n) e_n$, $x \in H$, the following assertions hold*

$$d_n(K) = \sqrt{\sum_{k=n+1}^{+\infty} \frac{1}{k^2}}, \qquad d_n(A(K)) \asymp \frac{1}{\sqrt{n}}, \ n \to \infty \quad a.s.$$

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Asymptotic properties of one-step M-estimators [1]

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Let $X_1, \dots, X_n$ be independent but not necessarily identically distributed observations taking values in an arbitrary measurable space, with the distributions depending on some unknown parameter $\theta \in \Theta$. Let θ_n^* be a preliminary consistent estimator of θ.

We study the so-called one-step M-estimators θ_n^{**} of the form

$$\theta_n^{**} = \theta_n^* - \sum_{i=1}^n M_i(\theta_n^*, X_i) \Big/ \sum_{i=1}^n M_i'(\theta_n^*, X_i), \tag{1}$$

where the functions $M_i(t, x)$, $i = 1, \dots, n$, satisfy the condition $\mathbf{E} M_i(\theta, X_i) = 0$ for all i. The one-step M-estimator θ_n^{**} defines the first step of the Newton procedure starting with the initial point $t_0 = \theta_n^*$ to approximate a consistent M-estimator $\widehat{\theta}_n$, i. e., a consistent solution to the equation (with respect to t)

$$\sum_{i=1}^n M_i(t, X_i) = 0.$$

We study asymptotic behavior of the one-step M-estimators (1) and some their modifications (one-step scoring estimators and one-step weighted M-estimators; see [1], [2]). Sufficient conditions are presented for asymptotic normality of the one-step M-estimators under consideration. As a consequence, for various nonlinear regression models, we consider one-step estimators which are equivalent to the corresponding least-squares, maximum likelihood (MLE), and quasi-likelihood ones. We consider some well-known nonlinear regression models (in particular, the Michaelis-Menton model) where

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the procedure mentioned allows us to construct explicit asymptotically optimal estimators.

For the first time, the idea of one-step estimation was suggested by R. Fisher in the problem of approximate calculation of MLE in the case of identically distributed observations. These Fisher's estimators are asymptotically equivalent to MLE only for n^β-consistent preliminary estimators with $\beta \geq 1/4$. We discuss some new one-step estimators which transform n^β-consistent preliminary estimators for $\beta < 1/4$ into an estimator asymptotically equivalent to MLE (see [3]).

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On the excess over boundary [1]

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We find an asymptotic expansion in the powers of e^{-b} for the distribution of excess over boundary $b \to \infty$ for random walk under one-sided Cramér condition on the distribution of summands. As a corollary, we obtain an asymptotic expansion for renewal function. We also present asymptotic expansion for the distribution of excess over two-sided boundary and give new approximations for the expectation of the first exit time.

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Large deviations for processes with independent increments [1]

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The talk is devoted to the large deviation principles for processes with independent increments. The results include the so-called local and extended large deviation principles that hold in those cases where the "usual" (classical) large deviation principle is inapplicable.

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The geometry of random eigenfunctions [1]

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In this talk we discuss some results on the asymptotic behaviour of random eigenfunctions, in the high-frequency limit. In particular, we focus on the Lipschitz-Killing curvatures of their excursion sets, which include the excursion area, the measure of level curves, and the Euler-Poincaré characteristic; starting from the excursion area, we show how its asymptotic behaviour is dominated by a single term, corresponding to the second-order chaos projection, and how this allows to establish quantitative central limit theorems by means of Stein-Malliavin techniques. We then discuss the extension of this approach to other functionals, reviewing both known results and open problems. Finally, if time permits we will discuss generalizations to further settings, in particular the asymptotic behaviour of band-limited random fields.

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Optimal stopping problem with incomplete information [1]

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The optimal stopping problem has a long history and goes under many names included secretary problem, marriage problem, etc. As the secretary problem [Dynkin], it has many formulations and variations. The number of items may be finite or infinite, the decision-maker may know the actual value of each item as it is presented or may only know its relative rank among the presented items. In some models the items are random variables with known pdf. Some authors assume that the pdf is known but its parameters are unknown [Ano]. Game-theoretic version of this problem was developed in [Gilbert, Mosteller]. Different generalisations of the best-choice games were made in the papers [Enns, Fushimi, Kurano, Sakaguchi, Mazalov]. There are few models devoted to mutual best-choice games [Alpern, McNamara, Mazalov, Falko].

We consider here m-person best-choice game with incomplete information. Assume that m experts observe a sequence of iid random variables (x_i, y_i), $i = 1 \ldots, n$, which represent the quality of incoming objects. The first component is announced to the players and the other component is hidden. We can think that the first component is related with a professional ability of the candidate and the second one is related with his compute skills. Each expert can select at most k candidates and has to maximise the resultant quality $x_i + y_i$ of the selected candidates. In the game-theoretic approach the goal of the player is to select the candidate with the resultant quality which is higher than the resultant qualities of the selected candidates by other players. We illustrate the model considering a popular TV show "The Voice".

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Probabilities related to the cyclic structure of random permutations

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We study a uniform random permutation from the symmetric group $\mathbf{S}_n$ and missing long or short cycles. The goal is to reach the level achieved in the asymptotic theory of natural numbers missing large or small prime factors (see the concise book [1] and more recent papers).

Let $\nu(n, r)$ be the probability that a permutation $\sigma \in \mathbf{S}_n$ has no cycle of length greater than r, where $1 \le r \le n$ and $n \to \infty$. Using the saddle point method and ideas originated in number theory, we obtained asymptotic formulas valid in all specified regions for the ratio n/r. Afterwards, let B be some complex quantity not the same at different places but always bounded by an absolute constant.

Theorem 1. *If $1 \le r \le n$, then*

$$\nu(n, r) = \frac{q(x)}{\sqrt{2\pi\lambda(x)}} \left(1 + \frac{Br}{n}\right).$$

Here

$$q(x) := \frac{1}{x^n} \exp\left\{\sum_{j=1}^{r} \frac{x^j}{j}\right\}, \qquad \lambda(x) := \sum_{j=1}^{r} jx^j,$$

and $x := x(n, r)$ is unique positive solution to the saddle point equation $x^{r+1} - x = n(x - 1)$.

For $r \le \log n$, the result (unfortunately, with frequent misprints) has been circulating in a few papers by other authors. For large r, when Hyman's approach is of no help, the traditional contour integrals have to be combined with relevant Laplace transforms.

Recall that Dickman's function $\rho(v)$ is defined as the continuous solution to the difference-differential equation $v\rho'(v) + \rho(v-1) = 0$ with the initial condition $\rho(v) = 1$ for $0 \le v \le 1$.

Theorem 2. *If* $\sqrt{n \log n} \le r \le n$, $u := n/r$ *and* $n \ge 2$, *then*

$$\nu(n,r) = \rho(u)\Big(1 + \frac{Bu\log(u+1)}{r}\Big).$$

An historical survey and the detailed proofs of Theorems 1 and 2 are exposed in preprint [2]. Analogous results are obtained for the probability $\nu(n,[r])$ of permutations missing cycles of lengths up to r. To formulate one of the results, one needs Buchstab's function $\omega(u)$ defined as a solution to difference-differential equation $(v\omega(v))' = w(u-1)$ for $v > 2$ with the initial condition $\omega(v) = 1/v$ if $1 \le v \le 2$.

Theorem 3. *Let* $u := n/r$. *There exists an absolute constant* $a > 0$ *such that*

$$\nu(n,[r]) = \exp\Big\{\sum_{j\le r} -\frac{1}{j}\Big\}\Big(\mathrm{e}^{\gamma}\omega(u) + B\frac{\mathrm{e}^{-au/\log^2(1+u)}}{r}\Big)$$

for $\sqrt{n\log n} \le r \le n$.

As an application, we establish an asymptotic formula with the remainder term estimate of the total variation distance between the count process of multiplicities of cycle lengths in a random permutation and a relevant independent process.

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The small ball asymptotics in L_2-norm for the Kac-Kiefer-Wolfowitz processes[1]

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We consider the problem of small ball behavior in L_2-norm for some Gaussian processes of statistical interest. The problem is reduced to the spectral asymptotics for some integral-differential operators. To find these asymptotics we construct the asymptotic expansion of several integrals containing slowly varying functions.

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On the ergodicity of mutual service processes in Euclidean space

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Consider a set of objects, abstracted to points of a spatially stationary point process in $\Re^d$, that deliver mutually each other a service at a rate $f(\|x-y\|)$ depending on their distance. Assume that the points arrive as a Poisson process and leave when their service requirements have been fulfilled. In the case of exponential service requirements the system is an infinite spatial birth and death process. We show how such a process can be constructed in this case and establish its ergodicity and a repulsivity property.

Our approach is fully probabilistic. We first construct the process on the positive time axis using an infinitely running algorithm. Next, we build a coupling of two such processes, one with empty initial state and one with a non-empty one, by an algorithm involving three types of points and their interaction rules. The difference of the two initial states is encoded into two types of 'special points', whereas all newborn points are first 'regular points'. Then we derive differential equations governing the time dynamics of Palm expectations of death rates experienced by each point type. With the help of these equations, we show that the special points die out at exponential speed. Finally, this allows a construction of the process on the whole time axis.

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Random flights related to the Euler-Poisson-Darboux equation

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This paper is devoted to the analysis of random motions on the line and in the space $\mathbb{R}^d$ $(d > 1)$ performed at finite velocity and governed by a non-homogeneous Poisson process with rate $\lambda(t)$. The explicit distributions $p(\mathbf{x}, t)$ of the position of the randomly moving particles are obtained solving initial-value problems for the Euler-Poisson-Darboux equation when $\lambda(t) = \frac{\alpha}{t}$, $\alpha > 0$. We consider also the case where $\lambda(t) = \lambda \coth \lambda t$ and $\lambda(t) = \lambda \tanh \lambda t$ where some Riccati differential equations emerge and the explicit distributions are obtained for $d = 1$. We also examine planar random motions with random velocities by projecting random flights in $\mathbb{R}^d$ onto the plane. Finally the case of planar motions with four orthogonal directions is considered and the corresponding higher-order equations with time-varying coefficients obtained.

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On the law of the iterated logarithm for sequences of m-orthogonal random variables

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In [1] a theorem on the upper limit of a sequence of dependent random variables was proved. By means of this theorem some sufficient conditions were found for the applicability of the law of the iterated logarithm to sequences of m-dependent random variables with finite variances. These results were used in [2] where the condition of m-dependence has been replaced by the condition of m-orthogonality introduced in the same paper.

Let m be a nonnegative integer. By definition, a sequence of random variables $\{X_n\,;\; n = 1, 2, \dots\}$ on a probability space is a sequence of m-orthogonal random variables if $\mathbf{E}X_n^2 < \infty$ for every n and $\mathbf{E}(X_k X_j) = 0$ if $|k - j| > m$. In particular, a sequence of 0-orthogonal random variables is a sequence of orthogonal random variables.

Many papers were devoted to limit theorems for sequences of m-dependent random variables. Every sequence of m-dependent random variables with zero means and finite variances is a sequence of m-orthogonal random variables. This statement remains true if we replace the condition of m-dependence by the weaker condition of pairwise m-dependence.

Limit theorems for sequences of m-orthogonal random variables may represent some interest. The following theorem is a generalization of a result in [2].

Theorem. *Let $\{X_n\}$ be a sequence of m-orthogonal random variables with zero means. Put*

$$S_n = \sum_{k=1}^{n} X_k, \quad B_n = \mathbf{E}S_n^2, \quad a_n = (2B_n \log\log B_n)^{1/2}.$$

Suppose that $B_n \to \infty$, $B_n/B_{n+1} \to 1$ $(n \to \infty)$ *and*

$$\sum_{n=1}^{\infty} \mathbf{P}\big(\max_{[c^n] \le k < [c^{n+1}]} S_k \ge (1+\varepsilon) a_{[c^n]}\big) < \infty$$

for every $\varepsilon > 0$ *and every* $c > 1$. *Then*

$$\limsup S_n/a_n \le 1 \quad a.s.$$

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Compound Poisson Processes with alternating intensities and hypo-exponential jumps

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We study the compound Poisson processes based on a two-state self-exciting Markov process with alternating parameters.

The explicit formulae for hypo-exponential distribution with alternating parameters are deduced. Then, bearing in mind financial applications we study in detail the compound Poisson processes with alternating distributions of jumps.

The model with exogenously exited processes is also presented.

Itô-Wiener expansion for functionals from the Arratia's flow $n-$point motion

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The Arratia flow on the real line is a family of random variables $\{x(u,t)\}_{u\in\mathbb{R},t\geq 0}$, such that

1) for every $u\in\mathbb{R}$ $x(u,\cdot)$ is a continuous square integrable martingale with respect to the joint filtration $\mathcal{F}^x_t=\sigma(\{x(v,s):v\in\mathbb{R},s\leq t\})$;

2) $x(u,0)=u$;

3) $<x(u,\cdot),x(v,\cdot)>(t)=(t-\tau_{u,v})_+$, where $\tau_{u,v}=\inf\{t\geq 0: x(u,t)=x(v,t)\}$.

The Arratia flow was constructed in [1]. Informally, it represents the motion of Brownian particles that start from every point of $\mathbb{R}$ and move independently until some of the particles meet each other. Thereafter these particles coalesce and continue their motion as one particle.

Despite the fact that each trajectory $x(u,\cdot)$ in the flow is a Wiener process, the whole flow is a highly non-Gaussian object - it generates black noise in the sense of B. S. Tsirelson [2]. Still, its $n-$point motion $\{(x(u_1,t),\ldots,x(u_n,t))\}_{t\geq 0}$ can be constructed from n independent Wiener processes via certain (non-unique) coalescing procedure [3]. It allows to apply Gaussian analysis to study finite-point motions of the Arratia flow. Such approach has two main limitations. Firstly, it gives results that depend on the coalescing procedure. Secondly, it is inapplicable to describe the whole flow, i.e. when $n\to\infty$.

The aim of the present work is to obtain the intrinsic Itô-Wiener expansion for square-integrable functionals of the Arratia's flow $n-$point motion $\{(x(u_1,t),\ldots,x(u_n,t))\}_{t\geq 0}$, in the sense it will be expressed in terms of stochastic integrals with respect to the trajectories $x(u_i,\cdot)$. Also, it will be calculated explicitly for functionals of the kind $f(x(u_1, t), \ldots, x(u_n, t))$, giving the analogue of the Krylov-Veretennikov formula. The main ingridient of our construction is the intrinsic Itô-Wiener expansion for the stopped Wiener process, obtained in the joint work with A. A. Dorogovtsev.

Let $\{w(t)\}_{t\geq 0}$ be the standard Wiener process in $\mathbb{R}^n$, starting from 0. Given open connected set $G \subset \mathbb{R}^n$, denote $\tau(u)$ the moment when $u + w$ leaves G, and $\alpha(t, u) = \mathbb{P}(\tau(u) > t)$. Let $\Delta_d(T)$ be the $d-$dimensional simplex $\{0 < t_1 < \ldots < t_d < T\}$.

Theorem. *1) For every function $a \in L^2(\Delta_d(\infty), \alpha(t_d, 0)dt)$, following stochastic integral is well-defined:*

$$\int_{\Delta_d(\tau(0))} a(t)dw^{t_d}(t_1)\ldots dw^{t_d}(t_{d-1})dw(t_d),$$

where $w^t(s) = w(s) - \int_0^{s\wedge t} \nabla \log \alpha(t-r, w(r))dr$. Stochastic integrals of different multiplicity are orthogonal in $L^2(w(\tau(0) \wedge \cdot))$.

2) Every random variable $\alpha \in L^2(w(\tau(0) \wedge \cdot))$ has a unique expansion

$$\alpha = \sum_{d=0}^{\infty} \int_{\Delta_d(\tau(0))} a_d(t)dw^{t_d}(t_1)\ldots dw^{t_d}(t_{d-1})dw(t_d).$$

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On a limiting behaviour of a conditional random walk with bounded local times [1]

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We consider a random walk on the integers with i.i.d. jumps taking value 1 and negative values, and with a limited number of visits, say L, to each state. The latter means that the walk stops ("freezes") at any state if it visits the state the $(L+1)$st time. Such a walk freezes at some state with probability one and a probability to hit a large level, say N, tends to zero when N grows to infinity.

We analyse asymptotic properties of the trajectory up to the hitting time of level N given that the hitting time is finite.

Itai Benjamini and Nathanaël Berestycki (2010) considered the symmetric simple random walk and showed, in particular, that the limiting process has a regenerative structure. We generalise their results using different techniques.

We will discuss further a number of extensions of the model.

The talk is based on a joint work with Sergey G. Foss (Heriot-Watt University, Edinburgh and Sobolev Institute of Mathematics, Novosibirsk).

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On the accuracy of the binomial approximation to sums of independent random variables[1]

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We construct an optimal upper bound for the closeness of expectations of smooth functions between standardized sums $\widetilde{S}_n = (S_n - \mathsf{E}S_n)/\sqrt{\mathsf{D}S_n}$, $S_n = X_1 + \ldots + X_n$ of i.i.d. r.v.'s $X_1, \ldots, X_n$ and the normalized symmetric binomial r.v. B_n of the form

$$\zeta_3(\widetilde{S}_n, B_n) \le \frac{\rho A(\rho)}{6\sqrt{n}}, \qquad (*)$$

where ζ_3 is Zolotarev's ideal metric, ρ is the normalized value of the third-order absolute moment of X_1,

$$A(\rho) = \sqrt{\tfrac{1}{2}\sqrt{1+8\rho^{-2}} + \tfrac{1}{2} - 2\rho^{-2}} < 1, \quad \rho \ge 1,$$

$$A(\rho) \le \sqrt{(\rho-1)(\rho+5/3)}, \ \rho \ge 1, \quad A(\rho) \sim \sqrt{\tfrac{8}{3}(\rho-1)}, \ \rho \to 1+,$$

with equality attained in $(*)$ for every value of ρ whenever X_1 takes only two values and $f(x) = x^3/6$.

As a corollary, we derive a sharp upper bound for the accuracy of the normal approximation to $\widetilde{S}_n$ of the form

$$\zeta_3(\widetilde{S}_n, Z) \le \frac{\rho A(\rho)}{6\sqrt{n}} + \frac{0.3}{n}, \quad n \ge 1, \qquad (**)$$

where Z is a standard normal r.v. Inequality $(**)$ improves Tyurin's optimal for $\rho \to \infty$ estimate $\zeta_3(\widetilde{S}_n, Z) \le \rho/(6\sqrt{n})$ for every value of ρ and all sufficiently large n, since $A(\rho) < 1$.

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Fredholm representation of Gaussian processes with applications

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We show that every separable Gaussian process with integrable variance function admits a Fredholm representation with respect to a Brownian motion. We extend the Fredholm representation to a transfer principle and develop stochastic analysis by using it. In particular, we prove an Itô formula that is, as far as we know, the most general Malliavin-type Itô formula for Gaussian processes so far. Finally, we give applications to equivalence in law and series expansions of Gaussian processes.

Our main theorem is the following:

Theorem 1. *Let $X = (X_t)_{t\in[0,T]}$ be a separable centered Gaussian process. Then there exists a kernel $K_T \in L^2([0,T]^2)$ and a Brownian motion $W = (W_t)_{t\geq 0}$, independent of T, such that*

$$X_t = \int_0^T K_T(t,s)\,\mathrm{d}W_s$$

if and only if the covariance R of X satisfies the trace condition

$$\int_0^T R(t,t)\,\mathrm{d}t < \infty.$$

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On asymptotic analysis of symmetric functions [1]

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Most limit theorems, including the central limit theorem in finite dimensional and abstract spaces and the functional limit theorems, admit refinements in terms of asymptotic expansions in powers of $n^{-1/2}$, where n denotes the number of observations, see e.g. [1] and [2]. These expansions are obtained by very different techniques such as expanding the characteristic function of the particular statistic or method of compositions. Alternatively one might use an expansion for an underlying empirical process and evaluate it on a domain defined by a functional of this process. The aim of the talk is to show that for most of these expansions one could safely ignore the underlying probability model and its ingredients (like e.g. proof of existence of limiting processes and its properties). In fact one can obtain expansions in a very similar way based on a simple general scheme reflecting the common nature of these models that is a universal collective behavior caused by many independent asymptotically negligible variables in the distribution of a functional. The following scheme of sequences of symmetric functions is studied. Let $h_n(\varepsilon, ..., \varepsilon_n), n \geq 1$, denote a sequence of real functions defined on $\mathbb{R}^n$ and suppose that the following conditions hold:

$$h_{n+1}(\varepsilon_1, ..., \varepsilon_j, 0, \varepsilon_{j+1}, ..., \varepsilon_n) = h_n(\varepsilon_1, ..., \varepsilon_j, \varepsilon_{j+1}, ..., \varepsilon_n);$$
$$\frac{\partial}{\partial \varepsilon_j} h_n(\varepsilon_1, ..., \varepsilon_j, ..., \varepsilon_n)\Big|_{\varepsilon_j=0} = 0 \quad \text{for all} \quad j = 1, ..., n;$$
$$h_n(\varepsilon_{\pi(1)}, ..., \varepsilon_{\pi(n)}) = h_n(\varepsilon_1, ..., \varepsilon_n) \quad \text{for all} \quad \pi \in S_n,$$

where S_n is a symmetric group.

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This symmetry property follows e.g. from the independence and identical distribution of an underlying vector of random elements X_j (in an arbitrary space) with common distribution P, if $h_n = \mathrm{E}F(\varepsilon_1(\delta_{X_1} - P) + \ldots + \varepsilon_n(\delta_{X_n} - P))$ is the expected value of a functional F of a *weighted* process (based on the Dirac-measures in $X_1, \ldots, X_n$). Here h_n may be regarded as function of "influences" of the various random components X_j. In [3] it was considered limits and expansions for functions h_n of equal weights $\varepsilon_j = n^{-1/2}, 1 \leq j \leq n$. In the talk we present an extension of this scheme to the case of *non identical* weights ε_j, which occurs e.g. for expectations of functionals of weighted i.i.d. random X_j elements in probability theory and mathematical statistics. See details in [4]. The applications of the results to the corresponding examples, e.g. for high order U-statistics, Kolmogorov-Smirnov statistic and Free Central Limit theorem, will be discussed as well.

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A general approach to small deviation via concentration of measures

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Let $(\mathcal{B}, \|\cdot\|_{\mathcal{B}})$ be a Banach-space and let Y be some $(\Omega, \mathcal{B})$-valued random variable. The small deviation problem refers to analysing the probability $\mathbb{P}(\|Y\|_{\mathcal{B}} < \epsilon)$ as ϵ tends to zero.

General small deviation problems have received a lot of attention recently due to their connections to various mathematical topics as well as importance for various applications. Similarly, large deviation theory and concentration of measure phenomena play important role in various topics in mathematics as well as in applications. In general the theory of large deviation and it link to the concentration of measure is better understood than the theory of small deviations. Indeed, the small deviation problems are usually studied only in some particular cases. For example, Gaussian processes with stationary increments and related processes have received a lot of attention. However, while the problem is well-studied in some special cases, it seems there does not exist a unified approach to attack the problem in full generality covering all kind of processes.

In this talk we introduce a general approach to find upper bounds for small deviation probabilities which reveals the connection of small deviation theory to the concentration of measure phenomena; an extensively studied and important topic which is also closely related to large deviation theory. More precisely, we consider small deviation problem for a process $Y = X_1 + X_2$, where X_1 and X_2 are some $(\mathcal{B}, \|\cdot\|_{\mathcal{B}})$-valued random variables, and show how small deviation for Y is linked to the concentration of measure for X_1 and large deviation probability for X_2. The advantages of the presented general approach is that it does not rely on any assumptions of the underlying processes X_1 and X_2 a priori, and it can be used to study different norms. After presenting the general result for Banach-valued random variables, we show how the approach can be used to study small deviation probabilities in different norms for processes on $[0, T]$. Finally, we show how the approach can be used to recover and generalise some existing results in the case of Gaussian processes where the concentration of measure phenomena is well-known.

The family tree for an island model of branching processes [1]

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A critical Galton-Watson branching process $\mathbf{Z}(n) = (Z_1(n), ..., Z_N(n))$ with N types of particles labelled $1, 2, ..., N$ is considered in which a type i parent may produce individuals of types $j \geq i$ only.

Let $Z_i(m,n)$ be the number of type i particles existing in the process at moment $m < n$ and having nonempty number of descendants at moment n. The process $\mathbf{Z}(m,n) = (Z_1(m,n), ..., Z_N(m,n))$, $0 \leq m < n$, can be thought of as the family tree relating the individuals alive at time n. We show that if $\mathbf{Z}(\cdot)$ is a critical process and the variance of the total number of direct descendants of particles of all its types is finite then the finite-dimensional distributions of the conditional process

$$\left\{\mathbf{Z}\left(n^t \log n, n\right), 0 \leq t < 1 | \mathbf{Z}(n) \neq \mathbf{0}\right\}$$

converge, as $n \to \infty$ to finite-dimensional distributions of an $N-$dimensional inhomogeneous branching process $\{\rho(t), 0 \leq t < 1\}$ with step-wise trajectories and which, at any fixed moment consists of particles of a single type only. The phase transition from type i to type $i+1$ happens at moment $t = 2^{-(N-i)}$. This gives a macroscopic view on the structure of the family tree of the process.

On the other hand, for $i \leq N-1$ the conditional process

$$\left\{\mathbf{Z}\left(\left(y + \frac{1}{\log n}\right) n^{1/2^{(N-i)}}, n\right), 0 \leq y < \infty | \mathbf{Z}(n) \neq \mathbf{0}\right\}$$

converges in Skorokhod topology, as $n \to \infty$ to a homogeneous branching process $\{\mu_i(y), 0 \leq y < \infty\}$ which is initiated at moment

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$y = 0$ by a random number of type i particles with probability generating function

$$f_i(s) = 1 - (1 - s)^{1/2^{i-1}}.$$

Each type i particle has an exponential life-length distribution and dying produces either two particles of type i or one particle of type $i + 1$ (each option with probability 1/2). Particles of type $i + 1$ in this process are immortal and produce no offspring. This gives a microscopic view on the structure of the family tree of the process.

Finally, the conditional process

$$\left\{ \mathbf{Z}\left(\left(x + \frac{1}{\log n} \right) n, n \right), 0 \le x < 1 | \mathbf{Z}(n) \neq \mathbf{0} \right\}$$

converges in Skorokhod topology, as $n \to \infty$ to an inhomogeneous branching process $\{\mu_N(x), 0 \le x < 1\}$ which is initiated at moment $x = 0$ by a random number of type N particles with probability generating function

$$f_N(s) = 1 - (1 - s)^{1/2^{N-1}}.$$

The life-length of each initial type N particle is uniformly distributed on $[0, 1]$. Dying such a particle produces exactly two children of type N and nothing else. If the death moment of a parent particle is $x \in (0, 1)$ then the life length of each of its offspring has the uniform distribution on the interval $[x, 1]$ (independently of the behavior of other particles and the prehistory of the process). Dying each particle of the process produces exactly two individuals of type N and so on... .

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Perturbations of singular stochastic flows on the real line

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The main object of studies is one class of stochastic flows of Brownian particles on the real line.

Definition 1. *A family of random variables* $\{X(t,u) \mid t \geq 0, u \in \mathbb{R}\}$ *is called a Harris flow with infinitesimal covariance* φ *if, for any* u, $X(\cdot,u)-u$ *is a standard Wiener process w.r.t. the common filtration of the flow; for any* u, v, $\langle X(\cdot,u), X(\cdot,v)\rangle(t) = \int_0^t \varphi(X(y,u) - X(y,v))dy$ *and, for* $u \leq v$, $X(t,u) \leq X(t,v), t \geq 0$, *a.s.*.

In case φ is Lipshitz outside any neighbourhood of 0, positive definite and its spectral representation is not of pure jump type the existence of a Harris flow is proved in [1]. In case $\varphi(t) = \varphi_0(t) = 1_{\{t=0\}}$ a Harris flow is a system of Wiener processes independent before a collision and coalescing after and is called the Arratia flow [2].

Definition 2.[3] *A Brownian web is a family of random variables* $\{B(s,t,u) \mid 0 \leq s \leq t, u \in \mathbb{R}\}$ *such that, for any* u, t, *the process* $\{B(t,y,u) - u \mid y \geq t\}$ *is a standard Wiener process w.r.t the common filtration; for any* u, v, s, r, $\langle B(s,\cdot,u), B(r,\cdot,v)\rangle(t) = \int_{\max\{s,r\}}^t \varphi_0(B(s,y,u) - B(r,y,v))dy$, *and trajectories inside the web do not cross each other.*

Definition 3.[2] *Let* $a \in Lip(\mathbb{R})$. *A family* $\{X^a(t,u) \mid t \geq 0, u \in \mathbb{R}\}$ *is called the Arratia flow with drift* a *if, for any* u, $X^a(\cdot,u) = u + \int_0^{\cdot} a(X^a(s,u))ds + w_u(\cdot)$, *where* w_u *is a standard Wiener process w.r.t. the common filtration of the flow; given arbitrary* u *and* v *the joint quadratic covariance of the martingale parts of* $X^a(\cdot,u)$ *and* $X^a(\cdot,v)$ *equals* $\int_0^{\cdot} \varphi_0(X^a(y,u) - X^a(y,v))dy$; *for* $u \leq v$, $X^a(t,u) \leq X^a(t,v), t \geq 0$, *a.s.*.

The next result is an analogue to the Trotter formula for interchanging actions of a stochastic flow and a semigroup associated with an ordinary diffefential equation.

Theorem 1.[4] *Suppose* $a \in Lip(\mathbb{R})$. *Define* $A(t,u) = u + \int_0^t a(A(s,u))ds$, *and, for any* u *and* $t \in [\frac{k}{2^n}, \frac{k+1}{2^n})$, *put*

$$X_n(t,u) = B(\frac{k}{2^n}, t, \Big(\circ_{j=1}^{j=k} A\big(\frac{1}{2^n}, B(\frac{j}{2^n}, \frac{j+1}{2^n}, \cdot)\big)\Big)(u)),$$

where $\circ$ *stands for an operation of composition. Then, for any* $u_1, \ldots, u_N$, *the sequence* $\big(X(\cdot, u_1), \ldots, X(\cdot, u_n)\big)$ *weakly converges as* $n \to \infty$ *to* $\big(X^a(\cdot, u_1), \ldots, X^a(\cdot, u_N)\big)$ *in the Skorokhod space.*

The following theorem provides an existence of a Harris web.

Theorem 2. *Suppose* $\varphi(t) = \mathrm{e}^{-\alpha|t|^\beta}, \alpha > 0, \beta \in (0;2)$. *Then there exists a family* $\{X^\varphi(s,t,u) \mid 0 \le s \le t, u \in \mathbb{R}\}$ *such that, for any* u, s, *the process* $\{X^\varphi(s,y,u) - u \mid y \ge s\}$ *is a standard Wiener process w.r.t. the common filtration,* $X^\varphi(t,r,u) = u, r \le t$; *for arbitrary* u, v, s, r

$$\langle X^\varphi(s,\cdot,u), X^\varphi(r,\cdot,v)\rangle(t) = \int_{\max\{s,r\}}^t \varphi(X^\varphi(s,y,u) - X^\varphi(r,y,v))dy,$$

and trajectories $X^\varphi(s,\cdot,u)$ *and* $X^\varphi(t,\cdot,v)$ *never cross each other (coalescence is allowed).*

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Thinning and branching stable point processes

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The concept of stability is central in Probability theory: it inevitably arises in various limit theorems involving scaled sums of random elements. Recall that a random vector ξ (more generally, a random element in a Banach space) is called *strictly α-stable* or StαS, if

$$t^{1/\alpha}\xi' + (1-t)^{1/\alpha}\xi'' \stackrel{D}{=} \xi \qquad \text{for all } t \in [0,1], \tag{1}$$

where ξ' and ξ'' are independent copies of ξ and $\stackrel{D}{=}$ denotes equality in distribution.

Since the notion of stability relies on multiplication of a random element by a number between 0 and 1, integer valued random variables cannot be StαS. Therefore Steutel and van Harn [1] defined a stochastic operation of *discrete multiplication* on positive integer random variables and characterised the corresponding *discrete α-stable* random variables. In a more general context, the discrete multiplication corresponds to the *thinning operation* on point processes. This observation leads to the notion of discrete stable or *thinning stable* point processes (notation: DαS) as the processes Φ which satisfy

$$t^{1/\alpha} \circ \Phi' + (1-t)^{1/\alpha} \circ \Phi'' \stackrel{D}{=} \Phi \qquad \text{for all } t \in [0,1], \tag{2}$$

when multiplication by a $t \in [0,1]$ is replaced by the operation $t\circ$ of *independent thinning* of their points with the retention probability t. The DαS point processes are special Cox processes and they are exactly the processes appearing as a limit in the superposition-thinning schemes, their full characterisation was given in [2].

In a broader context, given an abstract associative and distributive stochastic operation $\bullet$ on point processes, a process Φ is stable with respect to $\bullet$ if and only if

$$\forall n \in \mathbb{N}\ \exists c_n \in [0,1]\ :\ \Phi \stackrel{\mathcal{D}}{=} c_n \bullet (\Phi^{(1)} + ... + \Phi^{(n)}),$$

where $\Phi^{(1)}, ..., \Phi^{(n)}$ are independent copies of Φ. Such stable point processes arise inevitably in various limiting schemes similar to the central limit theorem involving superposition of point processes. It appears that a stochastic operation on point processes satisfies associativity and distributivity if and only if it presents a branching structure: "multiplying" by t a point process is equivalent to let the process evolve for time $-\log t$ according to some general Markov branching process which may include diffusion or general disposition of the points. The thinning is a particular case of this branching operation. We present results of [3], where we characterise branching-stable (i.e. stable with respect to $\bullet$) point processes for some specific choices of $\bullet$, pointing out possible ways to obtain characterisation for general branching operations. To this end, we introduce a stochastic operation in continuous frameworks based on continuous-branching Markov processes and conjecture that branching stability of point processes and continuous-branching stability of random measures should be related in general: the first are Cox processes driven by the second.

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Limit laws of the coefficients of polynomials with only unit roots

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Many interesting problems in probabilistic combinatorics can be reduced to investigation of some discrete random variable X_n taking values from a finite set $\{0, 1, 2, \ldots, n\}$. The probability generating function of such a random variable is a polynomial

$$P_n(z) = \mathbb{E}z^{X_n} = \mathbb{P}(X_n = 0) + \mathbb{P}(X_n = 1)z + \cdots + \mathbb{P}(X_n = n)z^n.$$

We prove that if all the roots ρ of the polynomial $P_n(z)$ are lying on the unit circle $|\rho| = 1$ then the random variable X_n converges to normal distribution $\big(X_n - \mathbb{E}X_n\big)/\sqrt{\mathbb{V}X_n} \to N(0,1)$ as $n \to \infty$ if and only if the centralized and normalized fourth moment of X_n converges to 3. Moreover, we also investigate the class of distributions that can be limits of random variables whose generating functions are polynomials with only unit roots.

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