

# BRANCHED COVERINGS AND HARMONIC AUTOMORPHISMS OF GRAPHS

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In this lecture we give a short survey of old and new results about branched coverings of graphs. This notion was introduced independently by T. D. Parsons, T. Pisanski, P. Jackson (1980), H. Urakawa (2000), B. Baker, S. Norine (2009) and others. The branched covering of graphs are also known as harmonic maps or vertically holomorphic maps of graphs. The main idea of the present talk is to create a parallel between classical results on branched covering of Riemann surfaces and those for graphs. We introduce the notion of harmonic action on a graph and discuss the Hurwitz type theorems for the groups acting harmonically. These results can be regarded as discrete analogues of the well known theorems by Hurwitz and Accola–Maclachlan. They, respectively, give sharp upper and lower bounds for the order of an automorphism group acting on a Riemann surface.

We present discrete versions of theorems by Wiman (1895), Oikawa (1956) and Arakawa (2000), which sharpen the Hurwitz' upper bound for various classes of groups acting on a Riemann surface of given genus.

Then we define a hyperelliptic graph as two fold branched covering of a tree and a  $\gamma$ -hyperelliptic graph as two fold branched covering of a graph of genus  $\gamma$ . A few discrete versions of the well-known results on  $\gamma$ -hyperelliptic Riemann surface will be given.

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