# THE BELLOWS CONJECTURE IN ODD-DIMENSIONAL LOBACHEVSKY SPACES

ALEXANDER A. GAIFULLIN

A *flexible polyhedron* in the $n$-dimensional space is an $(n-1)$-dimensional closed polyhedral surface that can be deformed continuously so that every its face remains congruent to itself during the deformation, but the deformation is not induced by an ambient rotation of the space. Intuitively, one may think of a flexible polyhedron as of a polyhedral surface with faces made of some rigid material and with hinges at edges that allow dihedral angles to change continuously. However, this surface may be self-intersected. This definition can be used in all spaces of constant curvature, namely in the Euclidean spaces $\mathbb{E}^n$, in the Lobachevsky spaces $\Lambda^n$, and in the round spheres $\mathbb{S}^n$.

One of the most interesting problems concerning flexible polyhedra is the so-called bellows conjecture stated by Connelly in 1978 that asserts that the volume of any flexible polyhedron (in dimensions greater than or equal to 3) is constant during the flexion. This conjecture was proved in the Euclidean spaces of all dimensions (Sabitov [5] for $n = 3$, and the author [2, 3] for $n \geq 4$). Flexible polyhedra of non-constant volumes were found in all open hemispheres $\mathbb{S}^n_+$ (Alexandrov [1] for $n = 3$, and the author [4] for $n \geq 4$), thus disproving the bellows conjecture in $\mathbb{S}^n$. The talk will be devoted to the following result on the bellows conjecture in the Lobachevsky spaces.

**Theorem.** *The bellows conjecture is true for bounded flexible polyhedra in odd-dimensional Lobachevsky spaces, i. e., the volume of any bounded flexible polyhedron in $\Lambda^n$, where $n$ is odd and $n \geq 3$, is constant during the flexion.*

The proof is based on the study of the analytic continuation of the volume of a simplex in $\Lambda^n$ considered as the function in the hyperbolic cosines of the edge lengths of this simplex.

## References


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Steklov Mathematical Institute, Gubkina str., 8, Moscow, 119991, Russia
*E-mail address*: agaif@mi.ras.ru



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