ON THE NUMBER OF NON-HYPERBOLIC KNOTS

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Computations show that the overwhelming majority of the simplest prime knots are hyperbolic knots. The following table gives the number of hyperbolic, satellite, and torus prime knots of n crossing for $n = 3, \dots, 16$ (see the sequences A002863, A052408, A051765, and A051764 in the Sloane encyclopedia of integer sequences).

type \ $n =$	3	4	5	6	7	8	9	10	11	12	13	14	15	16
all prime	1	1	2	3	7	21	49	165	552	2 176	9 988	46 972	253 293	1 388 705
hyperbolic	0	1	1	3	6	20	48	164	551	2 176	9 985	46 969	253 285	1 388 694
satellite	0	0	0	0	0	0	0	0	0	0	2	2	6	10
torus	1	0	1	0	1	1	1	1	1	0	1	1	2	1

TABLE 1. Number of knots

This gives rise to the following conjecture (see, e. g., [1, p. 119]).

Conjecture A**.** *The percentage of hyperbolic knots amongst all of the prime knots of* n *or fewer crossings approaches* 100 *as* n *approaches infinity.*

It turns out that Conjecture A contradicts the (120-year-old) conjecture on the additivity of the crossing number (see, e. g., [1, p. 69]).

Conjecture B**.** *The crossing number of knots is additive under connected sum.*

Moreover, it can be shown that Conjecture A contradicts the following (much weaker than Conjecture B but still open)

Conjecture B'**.** *The crossing number of a composite knot is not less than the crossing numbers of its summands.*

Furthermore, we show that Conjecture A contradicts the following (weaker than Conjecture B')

Conjecture B''**.** *The crossing number of a composite knot is not less than* $\frac{2}{3}$ *of (the maximum of) the crossing numbers of its summands.*

Theorem. *At least one of Conjectures* A *and* B'' *is false.*

The proof of this theorem uses results of W. B. R. Lickorish [2] on prime tangles.

There are additional arguments showing that the following conjecture is plausible.

Conjecture C**.** *The percentage of satellite knots amongst all of the prime knots of* n *or fewer crossings approaches* 100 *as* n *approaches infinity.*

REFERENCES

[1] C. C. Adams, *The Knot Book: An Elementary Introduction to the Mathematical Theory of Knots*, New York: W. H. Freeman, (1994).
[2] W. B. R. Lickorish, "Prime knots and tangles", *Trans. Amer. Math. Soc.*, Vol. 267, No. 1, 321-332 (1981).

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