ON MODULAR TQFT-REPRESENTATIONS OF MAPPING CLASS GROUPS

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The celebrated Witten-Reshetikhin-Turaev quantum invariants of 3-manifolds fit into a Topological Quantum Field Theory (TQFT) as axiomatized by Atiyah and Segal. This means in particular that they give rise to finite-dimensional unitary representations of mapping class groups of surfaces. I will explain that in some cases, one can also get modular representations (that is: representations in finite characteristic), using some integrality properties of the TQFT.

These representations behave differently depending on whether the characteristic ℓ of the finite field is equal to the order (assumed to be a prime number) p of the root of unity used to construct the TQFT or not. Accordingly, we speak of modular TQFT-representations in equal or in unequal characteristic according as $\ell = p$ or $\ell \neq p$.

I plan to discuss two applications of these representations. The first application is joint work with Reid. We use modular TQFT-representations in unequal characteristic to answer a question of Hamenstädt about finite index subgroups of the mapping class group. I will explain why the answer to this question is interesting from the point of view of comparing mapping class groups to arithmetic groups.

The second application is joint work with Gilmer where we compute the irreducible factors of these representations in the case of equal characteristic. We find explicit dimension formulas for these irreducible factors which are similar to the celebrated Verlinde formula for the dimension of the TQFT-representations over the complex numbers. But our formulas also involve cosines where the Verlinde formula only has sines. We then use these dimension formulas to find previously unknown dimensions of certain simple highest weight modules for symplectic groups in finite characteristic.

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