ON FUNDAMENTAL PROPERTIES OF SKEIN ALGEBRAS OF SURFACES

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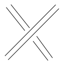
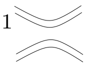
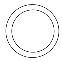
For an oriented surface F, let $\mathcal{S}(F)$ be the Kauffman bracket skein algebra of F over a ring R with a distinguished invertible element A, cf. eg. [1, 2]. That is $\mathcal{S}(F)$ is a quotient of the free R-module spanned by the set of isotopy classes of framed unoriented links in $F \times [0,1]$, including $\emptyset$, modulo the Kauffman bracket skein relations:

(1)  $- A$  $- A^{-1}$  ,  $+ (A^2 + A^{-2})\emptyset.$

The product of two links, $L_1 \cdot L_2$, given by a union of these two links when L_1 is positioned above L_2.

In a continuation of our joint work with J.H. Przytycki, we are going to prove the following two fundamental properties of skein algebras, announced (without proof) in [2]:

Theorem 1. *If R has no zero divisors then $\mathcal{S}(F)$ has no left nor right non-zero zero divisors.*

Theorem 2. *If $A^n - 1$ is not a zero divisor in R for any $n > 0$ then the center of $\mathcal{S}(F)$ is generated by knots parallel to components of ∂F.*

We are going to see that our proofs of these results generalize to analogous statements for relative skein algebras built of framed tangles in $F \times [0,1]$ with endpoints on $\partial F \times [0,1]$.

The proofs relay on pants decompositions of surfaces and the associated Dehn-Thurston classification of multi-curves in surfaces. We will see that any pants decomposition induces a filtration on $\mathcal{S}(F)$. And although an explicit algebraic description of the multiplication in $\mathcal{S}(F)$ appears very difficult, it is possible to describe the multiplication explicitly in the graded algebra $\mathcal{GS}(F)$ associated with the above filtered one. We are going to see that these graded algebras have very similar properties to skein algebras.

References

[1] J.H. Przytycki, Fundamentals of Kauffman bracket skein modules, arXiv:math/9809113

[2] J. H. Przytycki, A. S. Sikora, On Skein Algebras And $Sl_2(\mathbb{C})$-Character Varieties, *Topology*, Vol. 39, No. 1, 115–148, (2000), arXiv:q-alg/9705011

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