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Abstracts

Andrey ALPEEV(St. Petersburg State University)

Independence of sofic entropy on sofic approximation for a class of measure-preserving systems

Dependence of sofic entropy on the sofic approximation forms a widely open problem. I will show that for some class of measure-preserving systems sofic entropy is the same for any sofic approximation.

Grigori AMOSOV (Steklov Institute of Mathematics, Moscow) On the noncommutative deformation of the Klein group associated with a quantum channel.

This is joint work with I. Yu. Zhdanovskiy.

Denote $\mathfrak{B}(H)$, $\mathfrak{T}(H)$ and $\mathfrak{S}(H)$ the algebra of all bounded operators, the Banach space of all trace class operators and the convex set of positive unit-trace operators (quantum states) in a Hilbert space H, respectively. A completely positive trace-preserving linear map $\Phi : \mathfrak{T}(H) \to \mathfrak{T}(H)$ is said to be a quantum channel. Given a quantum channel Φ one can pick up its Kraus decomposition of the form

$$\Phi(\rho) = \sum_{n} V_n \rho V_n^*,\tag{1}$$

where $V_n \in \mathfrak{B}(H)$, $\sum_n V_n^* V_n = I$, *I* is the identity operator in *H*, $\rho \in \mathfrak{S}(H)$.

Let us consider the Kraus decomposition (1), then the noncommutative graph $\mathcal{G}(\Phi)$ is the operator subspace

$$\mathcal{G}(\Phi) = \overline{Lin(V_j^* V_k)}.$$
(2)

It was shown that the operator subspace S is associated with some quantum channel in the sense (2) (T.S.Cubitt, J.Chen, A.W.Harrow, R.Duan' 2009) iff $I \in S$ and $S^* = S$.

Consider the operator subspace $\mathcal{L}_{\theta} \subset M_n(\mathbb{C})$ (M.E. Shirokov' 2014):

$$\begin{pmatrix} a & b & c\theta & d \\ b & a & d & c/\theta \\ c/\theta & d & a & b \\ d & c\theta & b & a \end{pmatrix}$$
(3)

for $\theta \in \mathbb{C}^*$. This graph is associated with so called pseudo-diagonal quantum channels and plays an important role in quantum information.

We shall show that \mathcal{L}_{θ} can be considered as a image of representation π_{θ} of the ring generated by the group G with three generators x, y, z satisfying the relations

$$x^{2} = y^{2} = z^{2} = 1, \ xz = zx, \ yz = zy.$$
 (4)

Notice that adding to (4) the additional relation

$$xy = yx = z \tag{5}$$

we obtain the Klein group K_4 .

The algebra \mathcal{A}_{θ} which is a quotient of $\mathbb{C}G$ with respect to the relation

$$(xy + yx)z = (\theta + \theta^{-1}) \cdot 1$$

is such that the matrix representation ϕ of \mathcal{A}_{θ} results in the algebra \mathcal{M}_{θ} generated by \mathcal{L}_{θ} . In the case of $\theta = \pm 1 \phi$ is degenerated to the exact representation of $\mathbb{C}K_4$, where K_4 is the Klein group. Thus, \mathcal{L}_{θ} can be considered as a noncommutative deformation of the graph associated with the Klein group.

Victor ARZUMANIAN (Armenian Institute of Mathematics, NAS of RA) and Suren GRIGORYAN (Kazan Federal University)

Graded Operator Algebras and Crossed Products

Crossed products are among the most fruitful notions of the general theory of C^* -algebras. While crossed products associated with the group dynamical systems are studied sufficiently, the theory of irreversible systems have not yet fully established. In recent years a huge number of publications devoted to various kinds of generalizations appeared. Almost obviously that as a first step towards the creation of an irreversible theory one should refer to inverse semigroup dynamical systems, arising when the actions on C^* -algebras by isometries and partial isometries are examined (corresponding to endomorphisms or polymorphisms of the phase space).

In the report we present a canonical construction of a C^* -algebra associated with a unital C^* -algebra and an inverse semigroup. As a main tool in realization of the project, we use a notion of group-graded system which was indirectly present in many previous constructions. The concrete algebras are obtained choosing an appropriate representation.

Vladimir BOGACHEV (Moscow State University and High School of Economics)

On the equality of the values in the Monge and Kantorovich problems.

Given two Radon probability measures μ and ν on a topological space X and a bounded continuous cost function h on $X \times X$, we discuss the corresponding Monge and Kantorovich problems. The first one is minimization of the integral of h(x, Tx) with respect to μ over all measurable transformations of μ into ν . The second problem is minimization of the integral of h with respect to all probability measures on $X \times X$ with projections μ and ν on the factors. The Kantorovich problem is always solvable unlike the Monge problem. However, if μ can be transformed into ν , then

we can consider the infimum in place of minimum in the Monge problem, and it turns out surprisingly that its value coincides with the Kantorovich minimum under rather broad conditions on μ and ν (for example, this is true for non-atomic measures on Souslin spaces). The talk will be concerned with conditions of this sort, and an example will be presented in which the equality fails, although there are transformations of non-atomic μ into ν . Some candidates for optimal conditions will be conjectured. If time permits, also Vershik's admissible metrics will be briefly discussed and a generalization of a measurability result of Vershik–Zatitskiy–Petrov will be proved.

Nikolay BOGOLIUBOV and CYRIL MALYSHEV (PDMI) Combinatorial aspects of correlation functions of Heisenberg chain in the limiting cases.

We discuss the connection between quantum integrable and some aspects of enumerative combinatorics and the theory of partitions. As a basic example, we consider the spin XXZ Heisenberg chain in the limiting cases of zero and infinite anisotropy. The representation of the Bethe wave functions via the Schur functions allows to apply the well-developed theory of the symmetric functions to the calculation of the thermal correlation functions as well as of the form-factors. The determinantal expressions of the form-factors and of the thermal correlation functions are obtained. We provide a combinatorial interpretation of the formula for the correlation functions in terms of nests of the self-avoiding lattice paths. The interpretation proposed is in turn related to enumeration of the boxed plane partitions. The asymptotical behavior of the thermal correlation functions is studied in the limit of small temperature provided that the characteristic parameters of the system are large enough. The leading asymptotics of the correlation functions are found to be proportional to the squared numbers of boxed plane partitions.

Alexander BUFETOV (Steklov IITP NRU-HSE CNRS I2M) Quasi-Symmetries of Determinantal Point Processes

The classical De Finetti Theorem (1937) states that an exchangeable collection of random variables is a mixture of Bernoulli sequences.

The first result of the talk is that determinantal point processes on Z induced by integrable kernels are quasi-invariant under the action of the infinite symmetric group. The Radon-Nikodym derivative is a regularized multiplicative functional on the space of configurations. A key example is the discrete sine-process of Borodin, Okounkov and Olshanski.

The second result is a continuous counterpart of the first: namely, it is proved that determinantal point processes with integrable kernles on R, a class that includes processes arising in random matrix theory such as Dyson's sine-process, or the processes with the Bessel kernel or the Airy kernel studied by Tracy and Widom, are quasi-invariant under the action of the group of diffeomorphisms of the line with compact support.

While no analogues of these results in higher dimensions are known, in joint work with Yanqi Qiu it is shown that for determinantal point processes corresponding to Hilbert spaces of holomorphic functions on the complex plane C or on the unit disk D, the quasi-invariance under the action of the group of diffeomorphisms with compact support also holds.

Alexander GAMBURD (The Graduate Center, CUNY) Markoff Triples and Strong Approximation

We study the connectedness of the set of solutions (mod p) of the Markoff equation $x^2 + y^2 + z^2 = 3xyz$. under the action of the group of morphisms generated by coordinate permutations and Vieta involutions. In particular, it is shown that for almost all primes the induced graph is connected. Similar results for composite moduli enable us to establish certain new arithmetical properties of Markoff numbers, for instance the fact that almost all of them are composite. Joint work with J. Bourgain and P. Sarnak.

Vadim GORIN (Massachusetts Institute of Technology and Institute for Information Transmission Problems)

Central Limit Theorem for discrete log-gases

A log-gas is an ensemble of N particles on the real line, for which the probability of a configuration is the power of the Vandermonde determinant times the product of a weight w(x) over the positions of particles. Such ensembles are widespread in the random matrix theory, while their discrete counterparts appear in numerous statistical mechanics models such as random tilings and last passage percolation, and also in the asymptotic representation theory. I will explain a new approach which gives Central Limit Theorems for global fluctuations of discrete log-gases for a wide class of the weights w(x). The approach is based on novel discrete equations, which are analogues of the loop equations known in the continuous settings.

Boris GUREVICH (Moscow State University)

Toward history of dynamical entropy: comparing two definitions

For an automorphism T of a Lebesgue space (M, \mathcal{M}, μ) , the measuretheoretic entropy h(T) can be defined as follows [1]. Let F be family of finite measurable partitions $\xi = (C_1, \ldots, C_k)$ of M and, for $\xi \in F$,

$$H(\xi) = -\sum_{i=1}^{k} \mu(C_i) \log_2 \mu(C_i), \quad h(T,\xi) = \lim_{n \to \infty} n^{-1} H(\xi T \xi \lor \cdots \lor T^{n-1}).$$

Then $h(T) := \sup_{\xi \in F} h(T, \xi)$. As is well known, the concept of entropy for dynamical systems was put forward by A.N. Kolmogorov, while the above definition of h(T) was suggested by Ya.G. Sinai [2].

D.Z. Arov introduced in his thesis [3] a notion of ε -entropy $h_{\varepsilon}(T)$ by $h_{\varepsilon}(T) := \sup_{\xi \in F(\varepsilon)} h(T,\xi)$, where $0 < \varepsilon \leq 1/2$ and $F(\varepsilon)$ is subfamily of those partitions $\xi \in F$ with atoms of measure $\geq \varepsilon$.

Remark 1. Strictly speaking, only the continuous time dynamical systems were considered in [3], but the definition of $h_{\varepsilon}(T)$ was implicitly contained there as well. Besides, it was not assumed in [3] that (M, \mathcal{M}, μ) was a Lebesgue space, but this is of little importance for the problem at hand.

For many years the definition of $h_{\varepsilon}(T)$ remained unpublished, but now it is available (see [4]), where some events related to this notion are also described), and one has a possibility to compare $h_{\varepsilon}(T)$ with h(T). The results are as follows.

Theorem 1. Let T be an ergodic automorphism with $0 < h(T) < \infty$. Then $h_{\varepsilon}(T) = h(T)$ for $\varepsilon \leq ([2^{h(T)}]+1)^{-1}$ and $h_{\varepsilon}(T) = \log_2[1/\varepsilon]$ otherwise. Moreover, sup in the definition of $h_{\varepsilon}(T)$ can be taken over the partitions ξ all atoms of which have the same measure $[1/\varepsilon]^{-1}$.

Corollary 1. If S and T are ergodic automorphisms with $h(S) = h(T) < \infty$, then $h_{\varepsilon}(S) = h_{\varepsilon}(T)$ for every ε .

However in the class of all automorphisms the situation is different.

Theorem 2. There exist non-ergodic automorphisms S, T and a number $\varepsilon \in (0, 1/2]$ such that $0 < h(S) = h(T) < \infty$ and $h_{\varepsilon}(S) \neq h_{\varepsilon}(T)$.

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References

- I.P. Cornfeld, S.V. Fomin, Ya.G. Sinai. Ergodic theory. Springer NY, 1982.
- [2] Ya.G. Sinai. On the notion of entropy of a dynamical system (Russian). Dokl. Akad. Nauk SSSR, 124 (1959), 768–770.
- [3] D.Z. Arov. The theory of information and its transmission through channels (Russion, unpublished), Odessa State University, magister thesis, 1957, 1–125.
- [4] D.Z. Arov. The influence of P.V. Potapov and M.G. Krein on my scientific work. Operator Theory: Adv. and Appl., **72** (1994), 1–16.

Ben HAYES (Vanderbilt University) Fuglede-Kadison Determinants and Sofic Entropy

Let G be a countable, discrete group, an algebraic action of G is an action by automorphisms of a compact, abelian, metrizable group X. The data of an algebraic action is equivalent, via Pontryagin duality, to a countable Z(G)-module A, where Z(G) is the integral group ring. There has been a long history of relating the dynamical properties of the action of G on X (ignoring the structure of X as a group and thinking of this as an action on a compact space or an action on a probability space) to the functional analytic properties of the Z(G)-module A. A particular case of interest is as follows: fix f in Z(G), and let X_f be the Pontraygin dual of Z(G)/Z(G)f (as an abelian group). This is called a principal algebraic action. There has been a long history of connecting the entropy of the action of G on X_f to the Fuglede-Kadison determinant (defined via the von Neumann algebra of G) of f. In the amenable case, this was studied by Lind-Schmidt-Ward, Deninger, Deninger-Schmidt, Li and completely settled by Li-Thom. We study the entropy of such actions when G is sofic. Entropy for actions of sofic groups is an exciting new field first developed by Bowen in 2009, and defined in full generality by Kerr-Li in 2009. Generalizing work of Bowen, Kerr-Li, Bowen-Li (as well as the amenable case) we completely settle the connection between Fuglede-Kadison determinants and sofic entropy of principal algebraic actions. We will comment on the techniques, which differ from the amenable case and are the first to avoid approximating the Fuglede-Kadison determinant of f by finite-dimensional determinants. No knowledge of sofic groups, sofic entropy or von Neumann algebras will be assumed.

Vadim KAIMANOVICH (Ottawa University) Stopping times and Poisson boundaries

The Poisson boundary of a random walk on a group is defined as the space of ergodic components of the time shift in its path space. In this talk I will discuss how stopping times applied to the path space (in a way similar to time changes in the classical dynamical setup) give rise to new random walks with the same Poisson boundary.

Konstantin KOKHAS (St. Petersburg State University) Urban renewal revisited.

This talk is about our joint work with V. Aksenov.

We describe a new combinatorial-algebraic transformation on graphs that we call "chip removal". It generalizes the well known Urban Renewal trick of Propp and Kuperberg. Shortly speaking, given a graph G we remove its induced subgraph H, that we call a chip, and connect some vertices in the remaining part of G with new weighted edges; denote the resulting graph by G'. Then

$$f(G) = f(H)f(G'),$$

where f is the determinant of the adjacency matrix or pfaffian or matching number.

Chip removal is useful in calculations of determinants of adjacency matrices, pfaffians and matching numbers of graphs. A beautiful example of this technique is the theorem about removing four-contact chips. Though this technique has in fact linear algebraic background, we describe all necessary details for arbitrary chips in a pure combinatorial way via pfaffians. This generalizes Kuo's graphical condensation method and resent Cuicu's results. Numerous examples are given.

Alla KUZNETSOVA (Kazan Federal University)

On the structure of C^* -algebra generated by a family of partial isometries and multipliers

This report concerns an operator algebra \mathfrak{M}_{φ} generated by a family of partial isometries and by multipliers. The starting point is a self-mapping φ of a countable set X with finite numbers of preimages of each point. The mapping gives rise to family \mathcal{U} of partial isometries.

Operator algebra \mathfrak{M}_{φ} is a subalgebra of $B(l^2(X))$ generated by the algebra $\mathcal{M}(X)$ of multipliers by bounded functions on X, and by partial isometries from \mathcal{U} . We suppose that there is no cyclic element in X, i.e. an element that $\varphi^n(x) = x$ for some $n \in \mathbb{N}$.

We discuss the main structure properties of the algebra \mathfrak{M}_{φ} .

(i) We can equip \mathfrak{M}_{φ} with a semi-saturated circle action so \mathfrak{M}_{φ} is a \mathbb{Z} -graded algebra, where the grading corresponds to the so called monomial' index determined by powers of partial isometries. Obviously, $M(X) \in \mathfrak{M}_{\varphi,0}$, the fixed point (or, equivalently index zero) subalgebra of \mathfrak{M}_{φ} .

(ii) In the case φ is a surjective mapping, there is an inner endomorphism β and the fixed point subalgebra is invariant under β . If φ is a surjective mapping and the number of preimages is uniformly bounded then \mathfrak{M}_{φ} is the crossed product of the fixed point subalgebra with the endomorphism β_U (in sense of Stacey).

Andrei MALYUTIN (PDMI)

Eigenfunctions of Laplace operators and exit boundaries

Given a regular graph G, we consider the corresponding graded dynamic graph D(G) and the simplex $\Delta(D)$ of invariant (central) Markov measures on (the path space of) D. This simplex has several important subspaces: the exit boundary of the simple random walk on G is the set of extreme points of Δ ; the Martin boundary of D can be regarded as a subspace in Δ ; the space of positive eigenfunctions of the averaging Laplace operator on G embeds in Δ . We will discuss results concerning relative positions, in Δ , of the mentioned subspaces. The talk is based on joint research with A. M. Vershik.

Aleksei MINABUTDINOV (Higher School of Economics, St. Petersburg)

The limiting curves for the Pascal adic transformation

Let $X = \{0, 1\}^{\infty}$ be an infinite-dimensional unit discrete cube, $(\mu_p)_{0$ be a one-parameter family of Bernoulli measures on X and P be thePascal-adic transformation. In 2004, É. Janvresse, T. de la Rue and Y. Velenik considered random fluctuations of ergodic sums for some cylindricalfunctions non-cohomologous to a constant, viewed upon as graphs of functions of an integer argument on a segment of integers. These sums, linearly interpolated and properly renormalized, can be naturally consideredas bridges and converge to some limiting curve. For a certain subspaceof cylindrical functions it was shown that the limiting function belongs to $Takagi-type functions (and in the special case <math>p = \frac{1}{2}$ it is the well-known Takagi function). In the above-mentioned work, a question has been raised whether this is true for a wider classes of functions.

In my talk, I will tell about new results obtained in collaboration with A. Lodkin that describe the limiting curves for arbitrary cylindrical functions. In particular, the limiting curve is a certain Takagi-type curve μ_p -almost surely. I will describe this curve and its self-similarity properties. I will show that the ergodic sums of the Pascal adic transformation can be naturally expressed in terms of the Krawtchouk polynomials. The proof is based on the analysis of the asymptotic behavior of the Krawtchouk polynomials extending the classical result on the convergence to the Hermite polynomials.

Yuri NERETIN (Institute for Theoretical and Experimental Physics) Infinite symmetric group and combinatorial constructions of topological field theory type.

We discuss train constructions for infinite symmetric groups and related groups. For certain pairs (a group G, a subgroup K), we construct categories, whose morphisms are two-dimensional surfaces tiled by polygons and colored in a certain way. A product of morphisms is a gluing of combinatorial bordisms. For a unitary representation of G we assign a functor from the category of bordisms to the category of Hilbert spaces and bounded operators. The construction has numerous variations, instead of surfaces there arise also one-dimensional objects of Brauer diagram type, multi-dimensional pseudomanifolds, bipartite graphs.

Nikolay NESSONOV (Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine)

Stable spin-representations of the infinite symmetric group

The infinite symmetric group \mathfrak{S}_{∞} possesses nontrivial central extension $\widetilde{\mathfrak{S}_{\infty}}$. Recently, A. Vershik and N. Nessonov introduced the concept of the stable representations of a group. They obtained the full classification of the stable representation of \mathfrak{S}_{∞} . In this talk I will present the full discription the stable representation of group $\widetilde{\mathfrak{S}_{\infty}}$.

Pavel NIKITIN (Aix Marseille University)

Two semigroups connected with the infinite symmetric group, their representations and random walk on Bratteli diagrams.

Representation theory of the infinite partial bijections semigroup and the infinite Brauer semigroup naturally leads to the two operations on Bratteli diagrams. This operations are connected with the random walk on these diagrams. We will show how to describe the ergodic central measures for the resulting class of diagrams. Based on a joint work with A. Vershik.

Grigori OLSHANSKI (Institute for Information Transmission Problems)

The extended Gelfand-Tsetlin graph, its *q*-boundary, and *q*-hypergeometric determinantal measures.

The conventional Gelfand-Tsetlin graph encodes the classical branching rule for the irreducible characters of the unitary groups U(N). The works of Stratila and Voiculescu (in the 70's) and of Vershik and Kerov (in the 80's) showed that the Gelfand-Tsetlin graph plays a fundamental role in the representation theory of the infinite-dimensional unitary group U(infinity). In particular, the indecomposable characters of U(infinity)are parametrized by an infinite-dimensional topological space which is in a certain sense the boundary of the graph. Next it was found that the problem of harmonic analysis on U(infinity) leads to a family of remarkable probability measures on the boundary called the zw-measures; these measures in turn give rise to determinantal random point processes (Borodin and Olshanski, 2005).

In my talk, which is based on recent joint paper with Vadim Gorin (arXiv:1504.06832, to appear in J. Funct. Anal.), I will describe a q-version of the above picture. The first result in this direction is due to Gorin (2012). He introduced a natural q-deformation of the canonical cotransition probabilities on the levels of the Gelfand–Tsetlin graph and described the corresponding "q-boundary". However, when we tried to find q-analogs of the zw-measures in the framework of Gorin's formalism, we encountered with unexpected obstacles.

A non-obvious way to solve the problem consists in extending the basic object, the Gelfand-Tsetlin graph (the extended graph contains two copies of the initial one). This is a key point of our work. By making use of the extended graph it becomes possible to construct the desired qanalogs of the zw-measures. We also obtain an explicit expression for their correlation kernels in terms of q-hypergeometric functions.

Gaiane PANINA and Alena ZHUKOVA (SPIIRAS and St. Petersburg State University)

Discrete Morse theory for moduli spaces of flexible polygons, or solitaire game on the circle

Robin Forman's discrete Morse theory [2] is a very powerful technique (at least as powerful as the smooth Morse is): it allows to compute homologies, cup-product, Novikov homologies, develop Witten's deformation of the Laplacian, etc.

In the talk we demonstrate how it works: we build a perfect discrete Morse function on the moduli space of a flexible polygon. Should be mentioned that not all the manifolds possess a perfect Morse function. Even if it is the case, it is difficult to find it. In particular, in the discrete setting it is an NP-hard problem.

The starting point of our construction is a cellulation of the moduli space of a planar polygonal *n*-linkage [4]. Its cells are labeled by some cyclically ordered partitions of the set $[n] = \{1, ..., n\}$. The number of cells is big: it exceeds the sum of Betti numbers very much.

The discrete perfect Morse function is constructed in two steps. On the first step, we introduce some natural pairing on the cell complex which substantially reduces the number of critical cells. However, this number is not yet minimal possible.

The rules of manipulating with the cells, and the rules describing gradient paths resemble the solitaire game.

On the second step we (following once again R. Forman) apply *path* reversing technique, which gives a perfect Morse function. This technique is the discrete version of the Milnor's "First Cancelation Theorem" [3].

References

- M. Farber and D. Schütz, *Homology of planar polygon spaces*, Geom. Dedicata, 125 (2007), 75-92.
- [2] R. Forman, Morse Theory for Cell Complexes, Adv. in Math. Vol. 134, 1(1998), 90-145.

- [3] J. Milnor, Lectures on the h-cobordism theorem, Princeton Univ. Press, 1965.
- [4] G. Panina, Moduli space of a planar polygonal linkage: a combinatorial description, arXiv:1209.3241

Fedor PETROV (PDMI)

Duality for virtual continuous functions and optimal transportation

The space $VC^1(X \times X)$ of virtual continuous functions on the standard Lebesgue–Rokhlin space (X, μ) is a predual to the space of quasi-doubly stochastic measures on $X \times X$. On the other hand, bistochastic measures may be considered as Kantorovich' plans in the transportation problem. We discuss virtual continuity itself and especially the interplay between two dualities.

The talk is based on joint works with A. M. Vershik and P. B. Zatitskiy.

Sasha SKRIPCHENKO (Higher School of Economics, Moscow) Systems of isometries: dynamics, combinatorics, topology

Systems of partial isometries of the interval represent a simple combinatorial object which appears in topology in connection with measured foliations on a surface (orientable or non-orientable), in dynamics as a nice model to study billiards in rational polygons and in geometric group theory as a way to describe actions of free groups on R-trees.

We will discuss several classes of systems of isometries (interval exchange transformations, interval exchange transformations with flips, linear involutions, band complexes) and compare their basic dynamical properties: minimality, ergodicity, invariant measures etc.

Tatiana SMIRNOVA-NAGNIBEDA (Geneva University) Substitutional dynamics associated with some self-similar groups

We shall discuss substitutions and subshifts associated with Grigorchuk's group of intermediate growth and some other self-similar groups. They appear naturally in recursive presentations of these groups by generators and relators but also happen to describe the dynamics of the action of the group on the boundary of the rooted tree that defines the self-similar structure of the group. We will also discuss how the spectral theory of Schroedinger operators on the substitutional subshifts can be used to study the spectra of the random walks associated with the group actions. The talk is based on joint works with I. Bondarenko, D. D'Angeli, R. Grigorchuk and D. Lenz.

Jean-Paul THOUVENOT (Université Paris 6) The \bar{f} metric revisited

We are going to describe some ancient results on the \bar{f} metric, and also some new ones. We are also going to show how \bar{f} techniques have had some bearing on questions which seemed unrelated.

Natalia TSILEVICH (PDMI)

The serpentine representation of the infinite symmetric group and the basic representation of the affine Lie algebra $\widehat{\mathfrak{sl}_2}$

We introduce and study the so-called serpentine representations of the infinite symmetric group, which turn out to be closely related to the basic representation of the affine Lie algebra $\widehat{\mathfrak{sl}_2}$ and representations of the Virasoro algebra (joint work with A.M.Vershik)

Evgeny VERBITSKIY (Leiden University) Spanning Trees, Sandpiles, and Algebraic Dynamics

In 1993, Burton and Pemantle posed a question about the relation between uniform spanning forests on lattices \mathbb{Z}^d and algebraic dynamical systems with equal entropy. It has been conjectured that uniform spanning forests form symbolic covers of the corresponding algebraic systems. In the talk, I will review some partial progress in understanding this intriguing relation and will discuss a new example: uniform spanning trees on ladder graphs. The talk is based on joint works with K. Schmidt (Vienna) and T. Shirai (Fukuoka).

Anatoly VERSHIK (PDMI)

Standard and non-standard filtrations in dynamics and combinatorics. Attempt of classificaton

1. Filtration in measure theory and combinatorics of paths of graph;

2.General notion of standard filtration (fixed central measure) and standard graph (all standard measures);

3. Main theorem about standard Markov filtration;

4. Criteria of standardness and intrinsic metric;

5.Nonstandardness — noton of "cloud in the past" and vmm-spaces (vmm=virtual measure metric);

6. Conjecture about classification of the filtrations and invariants.

Pavel ZATITSKIY (PDMI and Chebyshev Lab) On scaling entropy sequence of a metric dynamical system.

We discuss definition and basic properties of the scaling entropy sequence. It is a metric invariant of dynamical system which characterises the asymptotics of the so-called epsilon entropy of averaged metrics on a fixed Lebesgue–Rokhlin space. Systems with zero Kolmogorov–Sinai entropy are of most interest in this context. We are going to discuss a continual family of examples with pairwise distinct scaling entropy sequences.

Ilya ZHDANOVSKII (Moscow Institute of Physics and Technology) Mutually unbiased bases in dimension 6.

Mutually unbiased bases in Hilbert space C^n are two orthonormal bases $\{e_i\}, \{f_i\}$ such that $|(e_i, f_j)|^2 = \frac{1}{n}$ for any i, j. This notion was introduced by physicist Schwinger in 1960. Classification of mutually unbiased bases is unsolved problem in dimension more than 5.

I will talk about mathematical version of mutually unbiased bases. Using this version, I will construct 4-dimensional family of mutually unbiased bases in dimension 6. Also, I will propose conjecture about complete classification of mutually unbiased bases in dimension 6. This talk is based on joint work with A. Bondal.

Peter ZOGRAF (PDMI and Chebyshev Lab) Map enumeration revisited

Counting problems for maps (ribbon graphs) and hypermaps (Grothendieck's dessins) on orientable surfaces will be discussed. It will be shown that the generating functions associated with map and hypermap count satisfy a number of remarkable integrability properties, like Virasoro constraints, evolution equation, KP hierarchy and topological recursion in the sense of Eynard-Orantin.