Abstracts of talks of the conference: Hamiltonian systems and their applications, June 3 - 8, 2015

Euler International Mathematical Institute, St. Petersburg, Russia

Main speakers

Massimiliano Berti berti@sissa.it
"KAM for PDEs"
Abstract: I will present new existence results of quasi-periodic solution for PDEs like quasi-linearly perturbed KdV equations and water waves.

Sergey Bolotin bolotin@mi.ras.ru
Moscow Steklov Mathematical Institute and University of Wisconsin
"Billiards in the three body problem"
Abstract: We consider the plane three body problem with two of the masses much smaller than the third one. Periodic solutions with near collisions of the small bodies were named by Poincaré second species periodic solutions. The description of such solutions is reduced to a billiard type system with discrete Lagrangian determined by the classical Lambert's problem. In the limit of many revolutions between near collisions the billiard system admits relatively simple description.

Cheng Chong-Qing chengcq@nju.edu.cn
TBA

Benoit Grebert Benoit.Grebert@univ-nantes.fr
"Modified Scattering for cubic NLS on $\mathbb{R} \times \mathbb{T}^d$: the nonresonant case"

Abstract: We consider the cubic nonlinear Schrödinger equation on the spatial domain $\mathbb{R} \times \mathbb{T}^d$, and we perturb it with a convolution potential. Using recent techniques of Hani-Pausader-Tzvetkov-Visciglia, we prove a modified scattering result and construct modified wave operators, under generic assumptions on the potential. As a consequence we prove that the Sobolev norms of small solutions of this nonresonant cubic NLS remain bounded for all time. (joint work with E. Paturel and L. Thomann)

Thomas Kappeler thomas.kappeler@math.uzh.ch
"On the convexity of the KdV Hamiltonian"

Abstract: We show that the KdV Hamiltonian $\mathcal{H}$, defined on the Sobolev space H_0^1 of real valued one-periodic functions with zero mean, is strictly concave near 0 in the strongest possible sense. It means that when expressed in action variables $I = (I_n)_{n\geqslant 1}$, the nonlinear part of $\mathcal{H}$ extends analytically to the positive quadrant ℓ_+^2 of the ℓ^2 sequence space and its Hessian is uniformly negative definite in a neighborhood of 0 in ℓ_+^2. As a consequence, the action to frequency map is a local diffeomorphism near 0 in $\ell_{\mathbb{C}}^2$. In addition, we show that for any $-1/2 \leqslant s \leqslant 0$ and $2 \leqslant p < \infty$, the Fourier Lebesgue space $\mathcal{L}^{s,p}(\mathbb{T}, \mathbb{R})$ admits global

Birkhoff coordinates. In particular, ℓ^2_+ is the space of actions of the underlying phase space $\mathcal{L}^{-1/2,4}(\mathbb{T}, \mathbb{R})$
This is joint work with Alberto Maspero, Jan Molnar, and Peter Topalov.

Konstantin Khanin khanin@math.toronto.edu
"On global solutions for the random Hamilton-Jacobi equation"

Abstract: We shall discuss a problem of existence and uniqueness of global solutions for the random Hamilton-Jacobi equation. While situation in the spatially periodic case is well understood by now the most interesting non-compact translation invariant case is largely open. In this talk we shall present partial results and conjectures in the non-compact setting. A connection with the problem of KPZ universality will also be discussed.

Andreas Knauf knauf@math.fau.de
"New Techniques for the N-Body Problem"

Abstract: This is a report on recent joint work with Jacques Fejoz and Richard Montgomery on classical scattering, in particular for the Coulomb or gravitational interaction.
Topics include - the geometry of scattering kinematics, - existence of Moeller Transformations for n-body scattering, - train correspondences for celestial bodies. Emphasis will be on results of topological and geometrical nature.

V.V. Kozlov Moscow Steklov Mathematical Institute, kozlov@pran.ru
"Homogeneous systems with quadratic integrals, Lie-Poisson quasi-brackets, and Kowalevskaya method"

We study quadratic ordinary differential equations which admit two quadratic first integrals. One of these integrals is supposed to be positive definite. Under some genericity assumptions by a linear transformation such a system can be presented in a canonical form. In this case the system turns out to be divergence free. Moreover, it can be presented in a Hamiltonian form, but the corresponding Lie-Poisson bracket in general does not satisfy the Jacobi identity.
In 3-dimensional case the equations are reduced to the classical equations of the Euler top. In dimension 4 the system turns out to be super-integrable and coincides with the Euler-Poincare equations on some Lie algebra.
As and example we consider Lotka-Volterra quadratic system. The problem of single-valuedness of its solutions (as functions of complex time) was studied by Kowalevskaya.

Lokutsievskiy L.V. lion.lokut@gmail.com
"On new phenomenon of chaotic behavior of non-smooth Hamiltonian systems coming from optimal control."

Abstract: The main tool for solving deterministic optimal control problems is the Pontryagin Maximum Principle (PMP). It allows to reduce the control problem to a two-point boundary value problem of Hamiltonian dynamics. Suppose the control variable u takes values in some set Ω. Then the Hamiltonian function H defining the dynamics is given as the maximum $H(q,p) = \max_{u\in\Omega} \mathcal{H}(q,p,u)$ over the control u of the *Pontryagin function* $\mathcal{H}$, and the optimal control $\hat{u}(q,p)$, if it exists, is found among the maximizers.

In general, the maximizer is unique in an open dense subset of the space of variables q, p and depends smoothly on these variables. On this set, the Hamiltonian H is smooth, whereas on its boundary the derivatives of H experience discontinuities. Usually the Hamiltonian system as a whole is piece-wise smooth on the cotangent bundle. The cotangent bundle is divided into disjoint domains $A_1, \dots, A_k$ on which the Hamiltonian is given by smooth functions $H_1, \dots, H_k$, respectively. The dynamics is described by a system of ODEs with discontinuous right-hand side. We consider situations when the set Ω is a convex polyhedron, and the domains A_i are those regions where the optimal control resides in a particular vertex v_i of the polyhedron. The set of points where the derivatives of H are discontinuous is a stratified manifold, and on each stratum the optimal control is confined to a particular face of the polyhedron Ω.

A trajectory of the Hamiltonian system evolving inside a smoothness domain is called *regular*. If a trajectory passes from one smoothness domain A_i into another one A_j, then the corresponding optimal control will experience a jump from the vertex v_i of the polyhedron Ω to the vertex v_j. This process is called *switching*, and the discontinuity hyper-surface is called *switching surface*. Typically trajectories intersect the switching surface transversally, in which case they are called *bang-bang trajectories*. It may happen, however, that a trajectory moves along the switching surface, in which case one speaks of a *singular trajectory*. Typically uniqueness of the solution does not hold in the vicinity of a singular trajectory, and many regular trajectories can join in the same point on a singular trajectory. This is possible because the right-hand side of the underlying ordinary differential equations experiences a discontinuity. All trajectories ending (or starting) at a fixed singular point form *integral vortex* of the point.

For a singular trajectory lying on a switching hyper-surface one can define an *order*, in dependence on up to what maximum length of the Poisson brackets of the adjoining smooth pieces H_i of the Hamiltonian vanish. If the order of the singular trajectory is even, then a regular trajectory cannot join it in a piece-wise smooth manner. In this case regular trajectories spiral around the singular trajectory and intersect the switching surface in an infinite number of points in finite time, in such a way that the joining point is the accumulation point of switchings. This phenomenon is called *chattering*, and is well-studied for the situation where exactly two smoothness domains meet at the singular trajectory in question.

The situation where three smoothness domains A_1, A_2, A_3 meet at a manifold $\mathcal{S}_{123}$ of codimension 2 will be considered on the talk. This situation is equivalent to an optimal control problem with 2-dimensional control lying in a triangle. We observe an additional phenomenon appearing in this situation, namely, the chaotic behaviour of bounded parts of trajectories. This phenomenon was not yet seen in optimal control problems and is hence entirely new. Our findings are not limited to optimal control problems, but rather hold for a whole class of piece-wise smooth Hamiltonian systems with three smoothness domains joining at surface $\mathcal{S}_{123}$ containing singular trajectories of second order. It appears that integral vortexes of singular point has a chaotic nature: there exists a topological Markov chain Σ_Γ such that the sequence

of control switchings on a trajectory corresponds one to one to the trajectory through a point of Σ_Γ under the right Bernoulli shift. The set of non-wandering trajectories has a fractal structure (as in Smale's horseshoe) and non-integer Hausdorff and box dimensions. Emphasize that all trajectories in an integral vortex fall into the singular point in finite time.

The discovered phenomenon appears in the neighbourhood of a generic singularity. Namely, a theorem on the structural stability is proven.

This phenomenon was discovered in the joined work with Mikhail Il'ich Zelikin and Roland Hildebrand.

Andrey Mironov mironov@math.nsc.ru
"Integrable geodesic flows on 2-torus and the systems of hydrodynamical type"

Abstract: We study quasi-linear system of partial differential equations which describes the existence of the polynomial in momenta first integral of the integrable geodesic flow on 2-torus. We prove that in the case of integrals of degree three and four the system is equivalent to a single equation of order 3 and 4 respectively. Remarkably the equation for the case of degree n=4 has variational meaning: it is Euler-Lagrange equation of a variational principle. This equation for n = 4 has formal double periodic solutions as a series in a small parameter.

A.Neishtadt A.Neishtadt@lboro.ac.uk
"Passages through resonances and capture into resonance in dynamics of charged particles"

Abstract: Small perturbations imposed on an integrable system cause a slow evolution. In the process of this evolution the system may pass through a state of a resonance. The phenomenon of capture into resonance consists in the system starting to evolve in such a way as to preserve the resonance property once it has arisen. The class of perturbations for this phenomenon to occur includes both non-Hamiltonian perturbations and slow change of parameters of Hamiltonian systems. In the talk, a general theory of passages through resonances and captures into resonance, as well as several examples of these phenomena in problems of charged particles acceleration, will be presented.

P.I. Plotnikov plotnikov@hydro.nsc.ru
"Parseval's variational principle and KAM theory"

Abstract: We deal with the problem of persistence of quasi-periodic motions spanning lower dimensional tori in a nearly-integrable Hamiltonian system with the analytic Hamiltonian $H(\boldsymbol{p},\boldsymbol{q}) = H_0(\boldsymbol{p}) + \varepsilon H_1(\boldsymbol{p},\boldsymbol{q})$. For $\varepsilon = 0$ the system is integrable and the phase space is foliated by n-dimensional invariant tori with the frequency vectors $\nabla H_0(\boldsymbol{p})$. An invariant n-dimensional torus of the unperturbed system is said to be resonance if the number of rationally independent components of the frequency vector is $n - k < n$. The resonance invariant torus of the unperturbed system is foliated by invariant $(n - k)$-dimensional tori. The resonance

torus breaks-up under small perturbation and only a few of its constituent invariant $(n-k)$-dimensional tori survive a perturbation. Our goal is to find conditions, which being imposed on the unperturbed Hamiltonian H_0, guarantee the persistence of $(n-k)$- dimensional hyperbolic invariant torus for *every analytic perturbation and for all sufficiently small* ε.

If the frequency vector $\overline{\boldsymbol{\omega}} = \nabla_p H(0)$ has $n-k$ rationally independent components and the Hessian $H_0''(0)$ is nondegenerate, then there is an affine symplectic transformation with rational coefficients such that in new variables we have

$$\boldsymbol{p} = (\boldsymbol{y}, z_2) \in \mathbb{R}^{n-k} \times \mathbb{R}^k, \quad \boldsymbol{q} = (\boldsymbol{x}, z_1) \in \mathbb{T}^{n-k} \times \mathbb{T}^k, \quad \boldsymbol{z} = (z_1, z_2),$$

We assume that the Hamiltonian $H(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = H_0(\boldsymbol{y}, z_1) + H_1(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ satisfies the following conditions:

The function H_0 is analytic in a neighborhood of the origin, and the function H_1 is analytic in the cartesian product of a neighborhood of the origin and the torus $\mathbb{T}^n$ The frequency vector admits the representation $\overline{\boldsymbol{\omega}} = (\boldsymbol{\omega}, 0)$, where

$$\nabla_{\boldsymbol{y}} H_0(0,0) = \boldsymbol{\omega} \in \mathbb{R}^{n-1}, \quad \nabla_{z_2} H_0(0,0) = 0. \tag{0.1}$$

The components of $\boldsymbol{\omega}$ are rationally independent and satisfy the standard diophantine condition. Furthermore, we assume that

$$\partial^2_{z_2} H_0(0,0) \;=\; \mathbf{I}_k.$$

We say that the Hamiltonian H has an analytic $(n-k)$-dimensional invariant torus with the frequency vector $\boldsymbol{\omega}$ if there exists an analytic canonical transformation $\boldsymbol{\vartheta} : (\boldsymbol{\xi}, \boldsymbol{\eta}, \boldsymbol{\zeta}) \mapsto (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$,

$$\boldsymbol{x} = \boldsymbol{\xi} + \boldsymbol{u}(\boldsymbol{\xi}),\ \ \boldsymbol{y} = \boldsymbol{v}(\boldsymbol{\xi}) + O(|\boldsymbol{\eta}|, |\boldsymbol{\zeta}|),\ \ \boldsymbol{z} = \boldsymbol{w}(\boldsymbol{\xi}) + O(|\boldsymbol{\zeta}|), \tag{0.2}$$

which puts H into the normal form

$$H \circ \boldsymbol{\vartheta} = \boldsymbol{\omega} \cdot \boldsymbol{\eta} + \frac{1}{2} \boldsymbol{\Omega}\, \boldsymbol{\zeta} \cdot \boldsymbol{\zeta} + o(|\boldsymbol{\eta}|, |\boldsymbol{\zeta}|^2). \tag{0.3}$$

Here $\boldsymbol{\Omega}$ is a constant symmetric matrix. Without loss of generality we may assume that

$$\boldsymbol{\Omega} = \mathrm{diag}(-\boldsymbol{\lambda}, \mathbf{I}_k), \quad \boldsymbol{\lambda} = (\lambda_i)_{i=1\ldots k} \in \mathbb{R}^k. \tag{0.4}$$

The invariant torus is weakly-hyperbolic if $\lambda_i \geqslant 0$. The following theorem is the main result of this work.

Theorem. *Let* $\det \partial^2_y H_0(0) \neq 0$ *and* $k \geqslant 1$. *Then for every sufficiently small* ε, *there exists a function* $\Psi : \mathbb{S}^k \times [0,1]^k \to \mathbb{R}$ *of the class* C^3 *with the following properties. There is one-to-one correspondence between critical points of* Ψ *and invariant weakly hyperbolic tori in the perturbed system with the frequency vector* $\boldsymbol{\omega}$. *If, in addition,* $k = 1$ *and* 0 *is a saddle point of signature* 1 *of the Hessian, i.e.,*

$$\partial^2_y H_0(0) - \partial_y \partial_{z_2} H_0(0) \,\cdot\, \partial_y \partial_{z_2} H_0(0)^\top \leqslant 0,$$

then the function Ψ *has a critical point in which it takes the absolute minimum. In this case the perturbed Hamiltonian system has a weakly hyperbolic invariant torus for all* H_1 *and for all small* ε.

The second theorem shows that, in general case, the saddle point condition is necessary for the persistence of weakly-hyperbolic tori.

Theorem. *Let $n = 2$ and $k = 1$. Then there are analytic functions $f, g : \mathbb{S}^1 \to \mathbb{R}$ and $\varepsilon_0 > 0$ with the following property. For every $\varepsilon \in (0, \varepsilon_0)$, all invariant tori in the Hamiltonian*

$$H(y, \mathbf{z}) = y + \frac{1}{2}y^2 + \frac{1}{2}z_2^2 + \varepsilon y f(z_1) + \varepsilon g(z_1)$$

are elliptic.

Andrey Sarychev University of Florence, Italy asarychev@gmail.com
"Ensemble Controllability by Lie algebraic methods"

Abstract: Over the last decade there have been a rise of interest and of research regarding controllability of ensembles (parameterized families) of nonlinear control systems

$$\dot{x} = f^{\theta}(x, u), \ \theta \in \Theta \subset \mathbb{R}^{\nu} \tag{0.5}$$

by a single θ-independent control $u(\cdot)$. Such problems arise for example, from a necessity to control a system with "structured uncertainty", when some parameters of the system are subject to "dispersion".

The problems of designing a control, which can compensate the dispersion of the parameters appear for example in NMR spectroscopy. Study of the control of Bloch equation under various types of dispersion has been initiated by S.Li and N.Khaneja, who advocated applications of geometric control tools, (e.g. Campbell-Hausdorff formula). The core of their approach is "generating higher order Lie brackets by use of the control vector fields which carry higher order powers of the dispersion parameters to investigating ensemble controllability". More recent publication by Beauchard-Coron-Rouchon, also dedicated to the Bloch equations with dispersed parameter, invoked *analytic* methods to obtain finer results on ensemble controllability.

In the current presentation (based on joint work with A. Agrachev and Yu. Baryshnikov) we seek to reintroduce in a systematic way the Lie algebraic approach to ensembles of nonlinear systems. Achieving exact controllability of a *continual* ensemble would require, in general, infinite-dimensional set of control parameters. We concentrate instead on *approximate ensemble controllability* by means of controls of fixed finite dimension.

On the contrast to the predecessors we do not use Taylor expansion in the parameter θ, nor do we assume any smoothness of ensembles in θ. Instead we advocate a mixed approach, which combines use of Lie brackets, related to Taylor series in state variables, with Fourier-type series in θ. It turns out that a convenient notion for the formulation of the approximate ensemble controllability criteria is the one of the *frame* in Hilbert space.

We start with finite ensembles to which the Lie rank criteria of *exact controllability* can be applied after proper modification. In particular we prove that the property of global controllability for a finite ensemble of control-linear systems is generic. Besides we establish global controllability property by means of a single scalar control for a finite ensemble of rigid bodies with generic inertial parameters.

Two examples of continual ensembles are studied. First is a model example of an ensemble in $\mathbb{R}^3$. We seek for a control, which generates a loop in $\mathbb{R}^2$ and require the corresponding "holonomy" to approximately trace a prescribed curve (a higher-dimensional surface). Second object of study is a general ensemble of r-dimensioal distributions (control-linear systems) on a manifold.

In both cases we manage to formulate sufficient and necessary approximately controllability criteria in terms of Lie algebraic frame. The second result provides a version of Rashevsky-Chow theorem for ensembles.

Dima Treschev treschev@mi.ras.ru
"Locally linear billiard maps"

Abstract: Can a billiard map be locally linear near a periodic orbit of period 2? This question can be asked in all dimensions beginning from 2. It is more interesting to consider the situation when the linear symplectic map to which the billiard map is locally conjugated, is totally elliptic. Then the billiard is locally integrable. We present some results and open questions concerning this problem.

Andrey Tsiganov andrey-ts@yandex.ru
"On conformally Hamiltonian vector fields"

Abstract: We will discuss integrable non-Hamiltonian vector fields which are the linear combinations of the Hamiltonian vector fields. The main examples are one of the most famous solvable problems in nonholonomic mechanics, describing rolling of a balanced ball over a horizontal surface without slipping, is referred to as the Chaplygin ball, and the nonholomic Veselova system. In classical mechanics the Euler equations describe rotation of a rigid body with a fixed point. In fact, with imposing nonholonomic constraints on the rigid body we make a gentle deformation of the initial Hamiltonian vector field, which preserves many properties of this well-studied Hamiltonian system.

Xiaoping Yuan yuanxiaoping@hotmail.com
"A KAM thorem for some quasi-linear PDEs"

Abstract: In this talk, I will discuss a KAM theorem. In this theorem the second Melnikov conditions can not imposed because of the quasil-linearrity of the considered PDEs. And suppose that the normal frequencies have a finite limit point. As application, by the KAM theorem and a not-too-bad normal form it is proved that the shallow water wave equation has KAM tori.

Jiangong You (Nanjing University) jyou@nju.edu.cn
"Quasi-Periodic Schrödinger Cocycles with Positive Lyapunov Exponent are not Open in the Smooth Topology"

Abstract: One knows that the set of quasi-periodic Schrödinger cocycles with positive Lyapunov exponent is open and dense in the analytic topology. In this paper, we construct cocycles with positive Lyapunov exponent which can be approximated by ones with zero Lyapunov exponent in the space of $\mathcal{C}^l$ $(1 \leqslant l \leqslant \infty)$ smooth quasi-periodic cocycles. As a

consequence, the set of quasi-periodic Schrödinger cocycles with positive Lyapunov exponent is not $\mathcal{C}^l$ open. The talk is based on a joint work with Yiqian Wang

Other speakers

D. Burlakov burlada@mail.ru
"Invariant tori in a neighborhood of a degenerate resonance"

Abstract: Consider a continuous family of invariant curves of an analytic, integrable two-dimensional exact symplectic map, and a resonant curve from this family. Take a perturbed map, which is also exact symplectic and analytic. Generically the resonant curve is destroyed, and in its neighborhood a domain of chaotic motions – the so called stochastic layer – appears. The boundary of the stochastic layer contains a pair of nonresonant curves which are a result of a deformation of curves from the initial family. What is the difference of frequencies on these boundary curves if the perturbation is of order $0 < \varepsilon \ll 1$?

In 1998 D.Treshev and O.Zubelevich proved that in the case of nondegenerate resonant curve difference of frequencies on boundary curves is of order ε. I present some results and conjectures for the same question in the case of degenerate resonant curve.

Hongzi Cong conghongzi@dlut.edu.cn

Abstract: It is proved that the KAM tori (thus quasi-periodic solutions) are long time stable for infinite dimensional Hamiltonian systems generated by nonlinear wave equation, by constructing a partial normal form of higher order around the KAM torus and showing that p-tame property persists under KAM iterative procedure and normal form iterative procedure.

Livia Corsi lcorsi@math.mcmaster.ca
"An abstract KAM result"

Abstract: I'll discuss an abstract KAM result on the existence of invariant tori for possibly infinite dynamical systems. The algorithm is quite general and can be adapted for example to both reversible and Hamiltonian cases. Differently from the classical Moser's approach, I'll show that in principle there is no need to impose the second Mel'nikov conditions but only to invert (in some appropriate norm) the linearized operator in the normal directions. In particular this implies that the serious technical difficulties in small divisors problems are those appearing in forced cases. The latter statement is commonly believed to be true and the main purpose is to prove it under the weakest possible assumptions. The result is obtained in collaboration with R. Feola and M. Procesi from Rome "La Sapienza".

Andrey Dymov adymov88@gmail.com

"Non-equilibrium statistical mechanics of crystals in medium"

Abstract: Investigation of the energy transport in crystals is one of the main problems in the non-equilibrium statistical mechanics. Since it turns out to be extremely difficult, usually one studies toy models, possessing additional ergodic properties. A common idea is to consider a Hamiltonian system of particles where each mode is a subject to stochastic perturbation. Clearly, it is important to study the case when the perturbation goes to zero.

In this talk I will discuss dynamics of a system of weakly interacting oscillators, where each oscillator is weakly coupled with its own stochastic Langevin thermostat. The system can be interpreted as a crystal plugged in medium and weakly interacting with it. I will prove that, under the limit when the couplings of oscillators with each other and with the thermostats go to zero with some precise scaling, behavior of the system is governed by an effective equation which is a rather nice dissipative SDE. Using this asymptotics I will show that under the limit above, dynamics of the local energy satisfies a relation, which resemble the Fourier law (however, which is not).

Marcel Guardia marcel.guardia@upc.edu
"Growth of Sobolev norms for the defocusing analytic NLS"

Abstract: Consider the completely resonant defocusing non-linear Schrödinger equation on the two dimensional torus with any analytic gauge invariant nonlinearity. Fix $s > 1$. We show the existence of solutions of this equation which achieve arbitrarily large growth of H^s Sobolev norms. We also give estimates for the time required to attain this growth. This is a joint work with Emanuele Haus and Michela Procesi.

Jianjun Liu liujj@fudan.edu.cn
"Growth of Sobolev Norms for Nonlinear Schrodinger Equations"

Abstract: I will discuss the growth of high Sobolev norms of solutions of periodic nonlinear Schrodinger equations. Some estimates of polynomial upper bound in time are established, which improve the previously known results.

Alberto Maiocchi alberto.maiocchi@unimi.it
"Time correlations and relaxation times for Hamiltonian systems"

Time correlations of dynamical variables play an important role in defining the relaxation properties of physical systems and the response to external perturbations in the frame of linear response theory. We present some theoretical methods to determine, at least in principle, both the finite time behaviour of time correlations and their long-time asymptotic properties. We comment on the connections to the linear response theory and the problem of thermalization in Hamiltonian dynamics.

Alberto Maspero alberto.maspero@math.uzh.ch
"Freezing of energy of a soliton in an external potential"

Abstract: "We study the dynamics of a soliton in the generalized NLS with a small external potential ϵV of Schwartz class. We prove that there exists an effective mechanical system describing the dynamics of the soliton and that, for any positive integer r, the energy of such a mechanical system is almost conserved up to times of order ϵ^{-r}. In the rotational invariant case we deduce that the true orbit of the soliton remains close to the mechanical one up to times of order ϵ^{-r}. This is a joint work with D. Bambusi.

Jessica Elisa Massetti jessica.massetti@obspm.fr
"MOSER'S NORMAL FORM AND DISSIPATIVE KAM THEORY"

Abstract: In 1967, J. Moser published a powerful normal form theorem for analytic perturbations of analytic vector-fields possessing an invariant quasi-periodic torus with diophantine frequencies. Through Moser's normal form, from the very general case to particular ones, for which we consider a normal form more adapted to the underlying context, it is straightforward to deduce some KAM-type result if the system under consideration depends in an opportune way on a sufficient number of "free parameters" - external or internal to the system -. The persistence result is hence obtained through a technique of elimination of parameters set up by Herman, Russmann and other authors in the 80-90's. In this frame, the dissipative spin-orbit problem in Celestial Mechanics, studied by different authors in the last few years, can more easily be handled and the issue of proving the persistence of quasi-periodic attractors becomes a particular case of low dimension. Additionally, the process of elimination of parameters developed in this context, highlights important relations among dissipation, frequency and perturbation proper to the spin-orbit system and brings out a better understanding of their role, opening the way to a more global study in the parameters' space on the persistence of different kinds of motions under perturbation.

Jan Molnar jan.molnar@math.uzh.ch
"On Two-Sided Estimates for the Nonlinear Fourier Transform of KdV"

Abstract: The KdV-equation $u_t = u_{xxx} + 6uu_x$ on the circle admits a global nonlinear Fourier transform, also known as Birkhoff map, linearizing the KdV flow. The regularity properties of u are known to be closely related to the decay properties of the corresponding nonlinear Fourier coefficients. We obtain two-sided polynomial estimates of all integer Sobolev norms $\|u\|_m$, $m \geqslant 0$, in terms of the weighted norms of the nonlinear Fourier transformed that are linear in the highest order. If time permits, we also discuss the connection with analogous estimates obtained for the defocusing NLS.

Riccardo Montalto riccardo.montalto@math.uzh.ch
"KAM for gravity capillary water waves"

Abstract: I will present a recent result (obtained with M. Berti) concerning the existence and the stability of small-amplitude quasi-periodic solutions for the water waves equations with surface tension. The Proof is based on 1) degenerate KAM theory for infinite dimensional systems in order to verify that the unperturbed frequencies satisfy the Melnikov conditions 2) Reduction of the linearized equation, at any approximate solutions, to constant coefficients, which require Pseudo differential operators theory and a KAM reduction.

Yannick Widmer yannick.widmer@math.uzh.ch
"The periodic complex sine-Gordon equation"

Abstract: In this talk I will present an analysis of the periodic complex sine-Gordon equation in one space dimension in light cone coordinates. I show that on certain parts of phase space, the Hamiltonian of the SG equation, when expressed in appropriate coordinates, can be seen to be an element in the Poisson algebra of the modified Korteweg-de Vries (mKdV) equation and hence by well established properties of the latter equation the corresponding initial value problem can be solved by quadrature. In addition I will also provide smooth initial data for which the (IVP) of the SG-equation has no solution.

Zhiyan ZHAO zyqiao1985@gmail.com
"Ballistic Diffusion in One-Dimensional Lattice Schrödinger Equation"

Abstract: For the solution $q(t) = (q_n(t))_{n\in\mathbb{Z}}$ to one-dimensional lattice Schrödinger equation

$$\mathrm{i}\dot{q}_n = -(q_{n+1} + q_{n-1}) + V_n q_n, \quad n \in \mathbb{Z},$$

we consider the growth rate of the diffusion norm $\|q(t)\|_D := (\sum_n n^2 |q_n|^2)^{\frac{1}{2}}$ for any nonzero $q(0)$ with $\|q(0)\|_D < \infty$. We prove that $\|q(t)\|_D$ grows *linearly* with the time t in the following two cases for the potential $\{V_n\}_{n\in\mathbb{Z}}$:

1) (Periodic) there exists $N \in \mathbb{Z}_+$ such that $V_{n+N} = V_n$, $\forall n \in \mathbb{Z}$.
2) (Quasi-periodic(QP)) $V_n = V(\theta + n\omega)$ for any $\theta \in \mathbb{T}^d$, with $\omega \in \mathbb{R}^d$ Diophantine, and V a sufficiently small real-analytic function on $\mathbb{T}^d$.