Workshop on Classical and Quantum Integrable Systems

Euler International Mathematical Institute, Saint-Petersburg, July 11–15, 2016

Flat structures on the deformations of Gepner chiral rings

Alexander Belavin *(Landau Institute, Russia)*

A simple direct way for the computation of the flat coordinates and Saito primitive forms on the Frobenius manifolds of the deformations of Jacobi rings associated with isolated singularities is proposed. This way is based on using the conjecture about integral representations for the flat coordinates and on the Saito cohomology theory. The approach allows to reduce the problem of the computation of the flat coordinates and the Saito primitive form to a simple linear problem. We consider particularly the case of the deformed Gepner chiral rings.

Plane partitions with a "pit": generating functions and representation theory

Mikhail Bershtein *(Landau Institute & Institute for Information Transmission Problems & Higher School of Economics, Russia)*

I will talk about new example of W algebras depending on three integer numbers n, m, k. The category of representations of such algebras is equivalent to the category of representations of quantum groups $U_q(\mathfrak{gl}_{n|m}) \otimes U_{q'}(\mathfrak{gl}_n) \otimes U_{q''}(\mathfrak{gl}_m)$ for certain q, q', q'' (This is an analogue of the fact that the representation theory of Virasoro algebra is related to the representation theory of the modular double of sl_2). Irreducible representations of such W algebras have a basis labeled by plane partition with asymptotic conditions along three coordinate axes. Talk is based on the joint work with B. Feigin and G. Merzon arXiv:1512.08779 and work in progress.

On a discretization of confocal quadrics

Alexander Bobenko *(Technical University of Berlin, Germany)*

I will describe a novel construction of discrete confocal quadrics (discrete confocal coordinate systems). Our discretization respects two crucial properties of confocal coordinates: separability and all two-dimensional coordinate subnets being isothermic surfaces (i.e. they allow a conformal parametrization along curvature lines). The construction is based on an integrable discretization of the Euler-Poisson-Darboux equation.
This is a joint work with Suris, Schief and Techter.

Конформно инвариантные пределы решеточных моделей: алгебра и геометрия (вводная лекция)

Dmitry Chelkak *(Saint-Petersburg Branch of Steklov Mathematical Institute, Russia & University of Geneva, Switzerland)*

Существуют два подхода к описанию пределов критических решеточных моделей на плоскости: "алгебраический", задачей которого является анализ предельных корреляционных функций, и "геометрический", состоящей в описании ансамблей случайных кривых, участвующих в геометрическом представлении данной модели. Основная цель лекции - обсудить эти два подхода и связи между ними на примере модели Изинга. Если позволит время, мы также обсудим недавние результаты о сходимости дискретных корреляций и интерфейсов для модели Изинга и статус аналогичных утверждений для других моделей.

Weyl modules and q-Whittaker functions

Evgeny Feigin *(Higher School of Economics, Russia)*

The characters of the classical Weyl modules are known to be related to the q-Whittaker functions for the quantum Toda difference operators. We briefly describe the classical story and then generalize it to the nonsymmetric case. In particular, we introduce the generalized Weyl modules and describe a link to the theory of nonsymmetric Macdonald polynomials and nonsymmetric q-Whittaker functions. The talk is based on the joint work with I. Makedonskyi and D. Orr.

Cluster coordinates as tau-functions

Vladimir Fock *(Strasbourg University, France)*

TBA

New types of Kulish-Sklyanin type models

Vladimir Gerdjikov *(Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Bulgaria)*

We start with a Riemann-Hilbert problem (RHP) [1]

$$\xi^+(x,t,\lambda) = \xi^-(x,t,\lambda)G(x,t,\lambda), \qquad \lambda^k \in \mathbb{R},$$
$$i\frac{\partial G}{\partial x} - \lambda^k[J, G(x,t,\lambda)] = 0, \qquad i\frac{\partial G}{\partial t} - \lambda^{2k}[J, G(x,t,\lambda)] = 0,$$

related to a BD.I-type symmetric spaces $SO(2r+1)/S(O(2r-1)\otimes O(2))$; in particular $J = \mathrm{diag}\,(1,0,\ldots,0,-1)$. This RHP is equivalent to a Lax pair of the form:

$$L\xi \equiv i\frac{\partial \psi}{\partial x} + U(x,t,\lambda)\xi(x,t,\lambda) - \lambda^k[J,\xi(x,t,\lambda)] = 0,$$
$$M\xi \equiv i\frac{\partial \psi}{\partial x} + V(x,t,\lambda)\xi(x,t,\lambda) - \lambda^{2k}[J,\xi(x,t,\lambda)] = 0,$$

where $U(x,t,\lambda)$ and $V(x,t,\lambda)$ are polynomials in λ of order $k-1$ and $2k-1$ respectively. For $k=1$ this Lax pair leads to the well known Kulish-Sklyanin model whose integrability has been known since 1981 [2]:

$$i\frac{\partial \vec{q}}{\partial t} + \frac{\partial^2 \vec{q}}{\partial x^2} + 2(\vec{q}^{\,\dagger}, \vec{q})\vec{q} - (\vec{q}^{\,T} s_0 \vec{q}) s_0 \vec{q}^{\,*} = 0, \quad s_0 = \sum_{k=1}^{2r-1} (-1)^k E_{k,2r-k}.$$

For applications of this model to Bose-Einstein condensates and detailed analysis for the inverse spectral transform see [3]. We analyze the cases with $k>1$ which give rise to generalizations of the Kulsih-Sklyanin models. We show that these models allow deep reductions and hierarchies of Hamiltonian structures.

[1] V.S. Gerdjikov. Pliska Stud. Math. Bulgar. 21, 201–216 (2012).
[2] P.P. Kulish, E.K. Sklyanin. Phys. Lett. 84A 349–352 (1981).
[3] V.S. Gerdjikov. Discrete and Continuous Dynamical Systems B 4, No. 5, 1181-1197 (2011).

Braided Yangians

Dimitri Gurevich *(Valenciennes University, France)*

It is well known that the Yangian Y(gl(m)), introduced by V. Drinfeld, and associated with the Yang quantum R-matrix, plays the role of a symmetry group for Non-linear Schroedinger model. It is also closely related to W-algebras. It has many interesting properties. However, its q-analogue, called q-Yangian, and defined as a "half" of a quantum affine group, is not properly studied. We suggest a new candidate for the role of the q-analogue of the Yangian Y(gl(m)). We call it "braided Yangian". Its properties are more similar to these of the usual Yangian. Also, we associate "braided Yangians" with other quantum R-matrices, arising from Hecke symmetries (i.e. braidings of Hecke type) via the baxterization procedure. We construct a rich representation theory of these braided Yangians by using an analogue of the evaluation morphism and discuss "braided bosonization" of these objects. We compare this bozonization with approach based on the Zamolodchikov-Faddeev algebra.

Bethe subalgebras in affine Birman–Murakami–Wenzl algebras and flat connections for q–KZ equations

Alexey Isaev *(Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna, Russia)*

Commutative sets of Jucys–Murphy elements for affine braid groups of A(1), B(1), C(1), D(1) types were defined. Construction of R-matrix representations of the affine braid group of type C(1) and its distinguish commutative subgroup generated by the C(1)-type Jucys–Murphy elements are given. We describe a general method to produce flat connections for the two-boundary quantum Knizhnik–Zamolodchikov equations as necessary conditions for Sklyanin's type transfer matrix associated with the two-boundary multicomponent Zamolodchikov algebra to be invariant under the action of the C(1)-type Jucys–Murphy elements. We specify our general construction to the case of the Birman–Murakami–Wenzl algebras. As an application we suggest a baxterization of the Dunkl–Cherednik elements Y's in the affine Hecke algebra of type A.

Уравнение Янга–Бакстера и инварианты узлов (вводная лекция)

Rinat Kashaev *(University of Geneva, Switzerland)*

Исходя из обратимого решения уравнения Янга–Бакстера, у которого частично транспонированная матрица также обратима, я опишу процедуру построения инварианта узлов принимающего значения в матричной алгебре.

New integrable 3D and 4D QFTs from strongly twisted N=4 SYM

Vladimir Kazakov *(Ecole Normale Superieure, Paris, France)*

TBA

Hamiltonian geometry of skew-mean-curvature flows

Boris Khesin *(University of Toronto, Canada)*

We show that an approximation of the hydrodynamical Euler equation describes the skew-mean-curvature flow on vortex membranes in any dimension. This generalizes the classical binormal, or vortex filament, equation in 3D. We present a Hamiltonian framework for dynamics of higher-dimensional vortex filaments and vortex sheets as singular 2-forms (Green currents) with support of codimensions 2 and 1, respectively.

Orthogonal and symplectic Yangians

Roland Kirschner *(Leipzig University, Germany)*

We consider Yang-Baxter relations symmetric with respect to orthogonal or symplectic algebra representations, in particular the RLL relations involving the fundamental R-matrix. The generators of the related Yangian algebra are obtained from the expansion of the L-operator in inverse powers of the spectral parameter. In the case when this expansion is truncated the involved algebra generators have to obey additional conditions which can be fulfilled in distinguished representations only.
Based on work done in collaboration with A.P. Isaev and D. Karakhanyan.

Elliptic q-KZ equation and the weight functions

Hitoshi Konno *(Tokyo University of Marine Science and Technology, Japan)*

It is know that the representation theory of the elliptic quantum group $U_{q,p}(\mathfrak{g})$ yields a systematic construction of the integral expressions of the solutions to the face type (dynamical) elliptic q-KZ equation. There the vertex operators and the elliptic (half) currents (screening operators) play important roles. On the other hand there are a lot of significant works by Felder, Tarasov and Varchenko [FTV] on the same solutions for the case $\mathfrak{g} = \widehat{\mathfrak{sl}}_2$. However no systematic study on a comparison between the two. In this talk we address this issue. We will present a simple rule of deriving the weight functions, whose transition property with the transition matrix given by the elliptic dynamical R matrix is manifest, as well as their dual functions w.r.t the hypergeometric pairing in the terminology by FTV. We also recover the recursive construction of the weight functions which is the basis in the works by FTV. As examples, we present the expressions of the weight functions associated with the representations of $U_{q,p}(\widehat{\mathfrak{sl}}_2)$ of level k ($\in \mathbb{Z}_{>0}$) as well as of $U_{q,p}(\widehat{\mathfrak{sl}}_N)$ of level 1. The case $U_{q,p}(\widehat{\mathfrak{sl}}_2)$ of level k with the finite dim. rep. being the tensor product of the $k+1$-dim.reps. coincides with the one obtained by FTV. The higher rank result is new. If time allows we will make some comments on the relation between the weight functions and the stable envelopes by Okounkov as well as on a connection of the solution to the elliptic q-KZ eq. with the conjectural Nekrasov partition function of the 6-dim. SUSY gauge theory.

Symplectic geometry of Schroedinger equation on a Riemann surface

Dmitry Korotkin *(Concordia University, Montreal, Canada)*

We study symplectic properties of monodromy map of second order linear equation with meromorphic potential with simple poles on a Riemann surface. We show that the canonical symplectic structure on the cotangent bundle of the moduli space of punctured Riemann surfaces implies the Goldman-Wolpert Poisson brackets between monodromy matrices. A new system of Darboux coordinates on the cotangent bundle is constructed in terms of periods of a quadratic differential. Generating functions between various systems of Darboux coordinates are computed.
The talk is based on joint work with M. Bertola and C. Norton.

Free fields, W-algebras and isomonodromic deformations

Andrey Marshakov *(Lebedev Physical Institute & Institute for Theoretical and Experimental Physics & Higher School of Economics, Russia)*

I consider the twist-field representations of W-algebras at (half-)integer central charges and study their relations with the representations of Kac-Moody algebras and isomonodromy deformation problem. The corresponding characters and conformal blocks are discussed from these two perspectives.

Hamiltonian reduction and duality for the BC_n Ruijsenaars model

Ian Marshall *(Higher School of Economics, Russia)*

TBA

Integrability of evolution equation for higher twist B-meson distribution amplitude

Alexander Manashov *(Hamburg University, Germany)*

It was discovered recently that the evolution equations for heavy mesons (baryons) distribution amplitudes are integrable and can be solved exactly. I will describe the corresponding integrable models and discuss their properties.

Multi-state extension of the asymmetric simple exclusion process

Chihiro Matsui *(University of Tokyo, Japan)*

There are few far-from-equilibrium systems which are analytically solvable. One of those examples is the asymmetric simple exclusion process (ASEP). The ASEP is an integrable two-state stochastic process in one dimension. The integrability of the model lies in the $U_q(sl_2)$-invariance of the bulk part. We consider the multi-state extension of the ASEP based on the fact that the Markov matrix of this process satisfies the Temperley–Lieb algebra. Besides the construction of steady states, we derive the exact expressions of particle-density profiles and currents on the steady states under the closed boundary condition. Although strong restrictions are imposed on hopping rates to keep integrability, we show that they are simplified in the limit $q \to 0$.

Generators of affine W–algebras

Alexander Molev *(University of Sydney, Australia)*

We construct explicit generators of the affine W-algebra of type A associated with a nilpotent matrix whose Jordan blocks are of the same size. By applying the quantum Miura transformation we recover the construction of the principal W-algebra of Fateev and Lukyanov. In the classical limit and the critical level we get generators of the respective classical W-algebras. The construction extends to the classical W-algebras associated with the simple Lie algebras of all classical types and the exceptional Lie algebra of type G_2. The generators are given by determinant formulas in the context of the Poisson vertex algebras.

Equivariant star product on a sphere of even dimension

Andrey Mudrov *(University of Leicester, UK)*

We construct an equivariant local star product on an even dimensional sphere regarded as a conjugacy class of the orthogonal Poisson group equipped with the standard factorizable r-matrix. Such a quantization is known for homogeneous spaces with Levi stabilizers since 2003, thanks to dynamical twist or, equivalently, Shapovalov form. We demonstrate how to extend that approach to the simplest example of non-Levi conjugacy classes.

Cherednik algebras and Zhelobenko operators

Maxim Nazarov *(University of York, UK)*

This is a joint work with Sergey Khoroshkin. We study canonical intertwining operators between modules of the trigonometric Cherednik algebra, induced from the standard modules of the degenerate affine Hecke algebra. We show that these operators correspond to the Zhelobenko operators for the affine Lie algebra of series A. To establish the correspondence, we use the functor of Arakawa, Suzuki and Tsuchiya which maps certain modules of the affine Lie algebra to modules of the Cherednik algebra.

Diagonal reduction algebra and reflection equation

Oleg Ogievetsky *(CTP Marseille, France)*

Rings of h-differential operators appear naturally in the study of reduction (or Mickelsson) algebras. I will introduce the necessary notions and discuss structure, applications and generalizations of rings of h-differential operators.

BKP and Hurwitz numbers

Aleksander Orlov *(Institute of oceanology, Russia)*

TBA

Bethe ansatz and stochastic systems of interacting particles

Aleksander Povolotsky *(Bogoliubov Laboratory of Theoretical Physics, JINR & Higher School of Economics, Russia)*

The models of interacting particles were shown to be efficient tool in finding universal distributions, characterizing broad classes of non-equilibrium stochastic processes. When the Markov transition matrix governing the time evolution of these models possesses a special mathematical structure of integrable system, on can advance very far in obtain the exact results, which turn out to be applicable in a much wider context of universality classes. In the talk we describe a construction of a family of stochastic particle models solvable by the Bethe ansatz and discuss how they are used to study the systems within the Kardar-Parisi-Zhang universality class and beyond.

Quantisation of Kadomtsev-Petviashvili equation

Evgeny Sklyanin *(University of York, UK)*

A quantisation of the KP equation on a cylinder is proposed that is equivalent to an infinite system of one-dimensional bosons carrying masses m=1,2,... The Hamiltonian is Galilei-invariant and includes the cubic split/merge (m1,m2)↔(m1+m2) terms for all combinations of particles with masses m1, m2 and m1+m2, with a special choice of coupling constants. The Bethe eigenfunctions for the model are constructed. The consistency of the coordinate Bethe ansatz, and therefore, the quantum integrability of the model is verified for the sectors up to the total mass M=8.
The talk is based on joint work with K. Kozlowski and A. Torrielli.

Form factors of the monodromy matrix entries in the models with gl(2|1) symmetry

Nikita Slavnov *(Steklov Mathematical Institute, Russia)*

We apply the nested algebraic Bethe ansatz to the models with $gl(2|1)$ symmetry. We obtain explicit representations for the Bethe vectors and their scalar products. In some particular cases a we find determinant formulas for the scalar products. Starting from these formulas and using the zero modes method we obtain compact determinant representations for the form factors of the monodromy matrix entries.

Эллиптический бета–интеграл, лемма Бейли и соотношение звезда–треугольник (вводная лекция)

Vyacheslav Spiridonov *(Bogoliubov Laboratory of Theoretical Physics, JINR, Russia)*

В этих вводных лекциях я кратко представлю ключевые элементы теории эллиптических гипергеометрических функций и обрисую одно и ее приложений. Затрагиваемые темы:
– тэта-функция Якоби и эллиптические функции,
– конечно-разностные уравнения первого порядка с эллиптическими коэффициентами,
– эллиптическая гамма–функция,
– эллиптический бета–интеграл,
– интегральная лемма Бейли и ее следствия,
– операторное соотношение звезда–треугольник и соотношения Кокстера для группы перестановок.

Superconformal indices and integrable systems

Vyacheslav Spiridonov *(Bogoliubov Laboratory of Theoretical Physics, JINR, Russia)*

Seiberg duality (1995) assumes an equivalence of certain 4d N=1 supersymmetric nonabelian gauge field theories. Superconformal indices of such theories (2006) are described by elliptic hypergeometric integrals introduced by the speaker in 2000. Symmetry transformations of these integrals give rigorous confirmations of the duality hypotheses. Many of such symmetries are found from the Bailey lemma yielding the star-triangle relation. The latter leads to solutions of the Yang-Baxter equation in terms of the integral operators with elliptic hypergeometric kernels and relates the Seiberg duality to integrability of 2d lattice systems.

Generalized Wick theorems in conformal field theory and the Borcherds identity

Taichiro Takagi *(Department of Applied Physics, NDA, Japan)*

We consider the well-known generalized Wick theorem for interacting fields in two dimensional CFT, and present a new formula for the operator product expansion of a normally ordered operator and a single operator on its right hand. Quite similar to the original Wick theorem for the opposite order operator product, it expresses the singular part of the operator product expansion as a contour integral of only two terms. We discuss the relationship between these formulas and the Borcherds identity satisfied by the quantum fields associated with the theory of vertex algebras.

Q-operators for higher spin eight vertex models

Takashi Takebe *(Higher School of Economics, Russia)*

The Q-operator for generalised eight vertex models associated to higher spin representations of the Sklyanin algebra is constructed, following Baxter's 1973 paper. As an application, we prove the sum rule for the Bethe roots.

Представления янгиана $\mathbf{Y}(gl_n)$ и комбинаторика весовых функций (off-shell векторов Бете) (вводная лекция)

Vitaly Tarasov *(St.-Petersburg Branch of Steklov Mathematical Institute, Russia & Indiana University–Purdue University Indianapolis, USA)*

Янгиан $Y(gl_n)$ – это ассоциативная алгебра, которая возникла при изучении квантовых интегрируемых моделей. В лекции будет рассказаны некоторые результаты про представления этой алгебры и интегрируемые модели, полученные с помощью исследования определенных векторнозначных рациональных функций, известных как весовые функции (или off-shell векторы Бете). В основном, будет рассматриваться случай янгиана $Y(gl_2)$.

Supersymmetric quantum spin chains and classical integrable systems

Anton Zabrodin *(Institute of biochemical physics, Russia)*

For integrable inhomogeneous supersymmetric spin chains (generalized graded magnets) constructed employing $gl(N|M)$-invariant R-matrices in finite-dimensional representations we introduce the master T-operator which is a sort of generating function for the family of commuting quantum transfer matrices. Any eigenvalue of the master T-operator is the tau-function of the classical mKP hierarchy. It is a polynomial in the spectral parameter which is identified with the 0-th time of the hierarchy. This implies a remarkable relation between the quantum supersymmetric spin chains and classical many-body integrable systems of particles of the Ruijsenaars-Schneider type. As an outcome, we obtain a system of algebraic equations for the spectrum of the spin chain Hamiltonians.

Topological strings, Dotsenko-Fateev integrals and gauge theory

Yegor Zenkevich *(Institute for Theoretical and Experimental Physics, INR, Russia)*

We investigate the connection between refined topological strings on toric Calabi-Yau manifolds and Dotsenko-Fateev type matrix models. We show how different choices of preferred direction in the topological string correspond to different averages in the matrix model and provide some explicit formulas for them. We also interpret the preferred direction in terms of gauge theories and spectral dualities between them. Using spectral duality we construct an explicit R-matrix acting on topological string states and write down the RTT relations for it.

Equivariant K-theory of the Grassmannian and quantum integrability

Paul Zinn-Justin *(University Paris 6, France)*

In this talk we shall discuss two things. First we shall recall how the general connection between equivariant cohomolgy and quantum integrable systems allows to describe the (quantum or classical) equivariant K-theory of the Grassmannian in terms of the so-called 5-vertex model. We shall then show how an extension of this model produces new formulae for the structure constants (generalized Littlewood-Richardson rule).

Associative Yang-Baxter equation as a link between classical and quantum integrable systems

Andrei Zotov *(Steklov Institute, Russia)*

We discuss applications of the associative Yang-Baxter equation (AYBE) to integrable systems. Solutions of the equation are certain quantum R-matrices (in fundamental representation) including the Baxter-Belavin elliptic one. Such R-matrices can be considered as matrix-valued generalizations of the Kronecker function, while AYBE provides noncommutative analogues of the elliptic function identities. On the one hand the latter identities lead to constructions of the classical Lax pairs for the Calogero type systems, the Painleve equations, integrable tops and their matrix extensions. On the other hand, due to the same identities we get compatibility of the Knizhnik-Zamolodchikov-Bernard connections and the quantum Lax pairs for the long-range spin chains of the Haldane-Shastry-Inozemtsev type. Spectral parameter and the Planck constant entering quantum R-matrix play different roles from case to case. We also discuss dualities related to different interpretations of these parameters.