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BOOK OF ABSTRACTS

Author index

Characterizations of Distributions by Properties of Ordered Random Variables

M. Ahsanullah

Rider University, Lawrenceville, NJ, USA

E-mail: ahsan@rider.edu

V.B. Nevzorov

St-Petersburg State University, St-Petersburg, Russia

E-mail: vanev@mail.ru

Let $X_1, X_2, \ldots$ be a sequence of independent random variables (r.v.'s) having a common continuous distribution function. Let also U_1, U_2 be uniformly U([0,1])-distributed and ξ_1, ξ_2 be exponentially E(1)-distributed r.v.'s . Ordered r.v.'s , such as order statistics $X_{1,n} \le X_{2,n} \le \ldots \le X_{n,n}$, $n = 1, 2, \ldots$, and upper record values $X(1) < X(2) < \ldots$, based on the sequence of X's are considered.

Characterizations of different distributions based on properties of ordered random variables are obtained. Among others the following types of equalities are investigated:

$$E\left(X_1 \mid X_{k,n} = x\right) = E\left(X_1 \mid X_{r,m} = x\right), \quad X(n-1) + \xi_1 \overset{d}{=} X(n+1) - \xi_2, \quad X(n-1)/U_1 \overset{d}{=} X(n)U_2$$

Some characterizations of distributions based on linear regression relations, which include order statistics and record values, also are obtained.

Estimation of Odds Ratio in Group Testing

Tathagata Bandyopadhyay
Indian Institute of Management (Ahmedabad, India)

Group testing method is employed to screen individuals on the basis of pooled samples to reduce cost and time of testing. Recently it has been used in epidemiological studies to estimate the probability of incidence of a disease. It is observed that if the test for the detection of a disease is not confirmatory i.e., it may lead to false positive and false negative outcomes, the group testing may result in a better estimate than the estimate obtained by individual testing. In this talk we will consider the estimation of Odds ratio when the exposure status is available at individual level, the disease status is known only at the pool level and the test outcome is not 100% accurate. The maximum likelihood method of estimation is used but the estimate is found to be biased especially when the disease is rare. Firth's bias correction is applied to improve the estimate. Efficacy of group testing method is justified through numerical studies.

Characterization of Markov Processes via Martingale Problems

Abhay G. Bhatt

Indian Statistical Institute, Delhi

The Stroock-Varadhan theory of martingale problems is widely used and important tool in the study of Markov processes. It characterizes a Markov process X as the unique process for which the associated processes $M_t^f = f(X_t) - \int_0^t Af(X_s)ds$ are martingales for every function f in the domain of a suitable operator A. This is indeed satisfied when A is the generator L of the semigroup associated with the Markov process. In general, typically L is an extension of A.
Here we give a characterization of all operators A for which the martingale problem is well-posed.

Distributions of Functionals of Switching Diffusions

A. N. Borodin
St. Petersburg Department of Steklov Mathematical Institute
E-mail: borodin@pdmi.ras.ru

There is an increasing interest to diffusions with switching in view of the applications in financial engineering and wireless communication. Many investigations are devoted to the stability properties of switching diffusions (see [1]). We consider the switching from one diffusion to another according to the Poisson time moments. We are interested in results, allowing one to compute the distributions of various functionals of switching diffusions. The general approach to the computation of the distributions of functionals of classical diffusions can be found in [2].

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Quasi-Symmetries of Determinantal Point Processes

Alexander I. Bufetov
Steklov Mathematical Institute, RAS;
CNRS, France;
Institute for Information Transmission Problems, RAS;
National Research University Higher School of Economics

The classical De Finetti Theorem (1937) states that an exchangeable collection of random variables is a mixture of Bernoulli sequences. Markov measures with full support and, more generally, Gibbs measures, on the space of binary sequences are easily seen to be quasi-invariant under the natural action of the infinite symmetric group. The first result of the talk is that determinantal point processes on Z induced by integrable kernels are also quasi-invariant under the action of the infinite symmetric group. A key example is the discrete sine-process of Borodin, Okounkov and Olshanski. The Radon-Nikodym derivative is a regularized multiplicative functional on the space of configurations. The formula for the Radon-Nikodym derivative can be seen as the analogue of the Gibbs property for our processes. The discrete sine-process is very different from a Gibbs measure: for example, the rigidity theorem of Ghosh and Peres shows that the number of particles in a bounded interval is almost surely determined by the configuration outseide the interval. The quasi-invariance can then informally be understood as the statement that there are no other invariants except the number of particles. The second result is a continuous counterpart of the first: namely, it is proved that determinantal point processes with integrable kernels on R, a class that includes processes arising in random matrix theory such as Dyson's sine-process, or the processes with the Bessel kernel or the Airy kernel studied by Tracy and Widom, are quasi-invariant under the action of the group of diffeomorphisms of the line with compact support (rigidity for the sine-process has been established by Ghosh, for the Airy and the Bessel by the speaker).

Conditional Limit Theorems Related to the Feature Selection

A.V. Bulinski

Lomonosov Moscow State University

E-mail: bulinski@yandex.ru

Feature selection problems are intensively studied on the borderline of mathematical statistics and machine learning, see, e.g., [1] and [2]. The main problem in this research domain can be described as follows. There is a response variable which depends on a collection of factors. One has to identify a subcollection of relevant (in a sense) factors. Such problem is of great importance for applications, e.g., in medicine and biology.

We concentrate on conditional central limit theorem arising in the framework of nonlinear regression analysis. The goal is to compare the response variable predictions involving different subcollections of factors. Some extensions of recent results [3] are obtained. We also discuss certain problems concerning the MDR-EFE method developed in [4].

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Stochastic Dynamics of Intermolecular Hydrogen Bonds in a Liquid Solution Driven by a Dichotomous Markovian Noise

N.F. Bunkin, G.A. Lyakhov, A.V. Shkirin*

Prokhorov General Physics Institute, Russian Academy of Sciences, Moscow, Russia

*E-mail: avshkirin@mephi.ru

We have applied a model of dichotomous Markovian noise, which was first accepted in radio physics, to describe the dynamics of solute-solvent hydrogen bonds. In this model, it is assumed that the process of formation and breaking of hydrogen bonds between solvent and solute molecules is driven by a dichotomous Markovian noise in the binary interaction energy $u_{12} = u_{12}^0 + 2T_c\xi(t)$, where u_{12}^0 is the energy of the pair interaction of non-bonded molecules, T_c is the critical temperature, $\xi(t)$ is a random function, having a magnitude of either 0 or v, where v is a dimensionless (normalized to $2T_c$) hydrogen-bond energy. On the basis of Landau − Khalatnikov kinetic equation, it is possible to find the stationary probability distribution function $p(C)$ for the solute concentration C, which describes deterministic stable states. This function has a dependency on the three model parameters: the normalized hydrogen-bond energy v, the probability of hydrogen-bond formation Q and the ratio of the characteristic diffusion time to the hydrogen-bond lifetime γ. The diagram of the extremums of $p(C)$ in temperature-concentration coordinates shows that in the area between binodals of the solution there can be discerned three new sub-domains, which are affected by the dichotomous noise of forming/breaking of hydrogen bonds. To illustrate this approach, we have obtained a numerical solution to the inverse problem aimed at the determination of the parameters v, Q and γ, which make $p(C)$ best fit the experimental data measured in tetrahydrofuran-water mixtures.

This study was supported by the RFBR Projects № 15-02-07586 and № 16-52-540001.

Pseudo-Likelihood Estimation and Bootstrap in Binary Regression in Presence of Nuisance Parameters

A. Chatterjee

Theoretical Statistics and Mathematics Unit, Indian Statistical Institute, Delhi

E-mail: cha@isid.ac.in

Logistic regression is a commonly used method for handling binary responses. In this talk, we study a situation when a portion of the responses have been misclassied. Pseudo-likelihood is used to estimate the regression parameter and we study the asymptotic properties of the estimator. We also show that the bootstrap is consistent in this set up. A small numerical study is presented to illustrate the theoretical results.

Convergence Rates in Nonparametric Regression with Infinite Dimensional Covariates

Probal Chaudhuri
Indian Statistical Institute, Kolkata
E-mail: probal@isical.ac.in

It is well-known that nonparametric regression estimates converge at a rate slower than the usual parametric rate (which is square root of the sample size), and the rate of convergence depends on the dimension of the covariate. In the 80's, many interesting results were reported in the literature on convergence rates of nonparametric regression estimates and how the dimension of the covariate adversely affects those rates. In the recent past, there is considerable interest in nonparametric regression analysis with functional data. When the covariate is infinite dimensional, as it happens in the case of functional data, the derivation of convergence rates for nonparametric regression estimates is a problem that is not yet adequately solved. Main theoretical hurdle in the problem arises from the complex behavior of small ball probabilities in infinite dimensional spaces. These small ball probabilities play a critical role in determining the asymptotic variance of nonparametric regression estimates. In this talk, I shall discuss why, unlike what happens in the case of finite dimensional covariates, the bias variance trade off in the asymptotic mean square error of a nonparametric regression estimate is a much trickier issue when the covariate is infinite dimensional. I shall also present the best possible rates of convergence for kernel regression estimates when some specific stochastic models hold for functional covariates. This talk will be based on joint research with Joydeep Chowdhury, who is a PhD student in Indian Statistical Institute, Kolkata.

Random Flights in Nonhomogenious Poissonian Environment

Yu. Davydov

University of Lille 1,

E-mail: youri.davydov@math.univ-lille1.fr

V. Konakov

National Research University Higher School of Economics

S. Menozzi

Uinversity of Evry

We consider the process $X(t)$ in R^d which is defined as the position of a particle which starts from zero and moves in the direction θ_1 up to the moment T_1: It then changes direction to θ_2 and moves on within the time interval $T_2 - T_1$; etc. The directions θ_k are i.i.d. and have a common distribution concentrated on the unit sphere. The moments $(T_k) = f(\Gamma_k)$, represent a transformed standard homogeneous Poisson point process on R_+ independent of (θ_k).

We are interested in the global behavior of the process $X = \{X(t),\ t \in R_+\}$, namely, we are looking for conditions under which the processes $\{Y_T,\ T > 0\}$; $Y_T = \frac{1}{B(T)} X(tT)$, weakly converges in $C[0,\ 1]$:

$$Y_T \Rightarrow Y, \quad T \to \infty.$$

It is clear that in homogeneous case the process X is a conventional random walk, and then a limit process is Brownian motion. In an inhomogeneous case, it was possible to distinguish three modes that determine the type of limiting process.

If the function f has power growth, $f(t) = t^{\alpha}$, $\alpha \geq 1$ the behavior of the process is analogous to the uniform case and then in the limit we obtain a linearly transformed Brownian motion.
In the case of exponential growth $f(t) = e^{t\beta}$, $\beta > 0$, the limiting process is piecewise linear with an infinite number of units.

Finally, with the super exponential growth of f, the process degenerates: its trajectories are linear functions.

On a Class of Multitype Critical Branching Processes in Random Environment

Dyakonova E.E.
Steklov Mathematical Institute, Moscow
E-mail: elena@mi.ras.ru

Let

$$\mathbf{Z}(n) = \left(Z_1(n), \ldots, Z_p(n)\right), \quad n = 0, 1, \ldots$$

be a p-type critical branching process in an i.i.d. random environment. Suppose that the mean matrices of this process have a common positive right eigenvector

$$\mathbf{u} = \left(u_1, \ldots, u_p\right), \qquad \sum_{i=1}^{p} u_i = 1.$$

Assume that the associated random walk of the process belongs to the domain of attraction of a stable law with parameter $\alpha \in (0,2]$.

We show (under some additional technical conditions) that there exists a slowly varying sequence $l(0), l(1), l(2), \ldots$ such that the conditional law

$$\mathcal{L}\left(\left\{\frac{\ln\left(\mathbf{Z}(nt), \mathbf{u}\right)}{n^{1/\alpha} l(n)}, \ 0 \le t \le 1\right\} \middle| Z_1(n) + \ldots + Z_p(n) > 0\right)$$

weakly converges, as $n \to \infty$; to the law of the meander of a strictly α–stable Levy process.

Remarks on the Stochastic Integral

Rajeeva L Karandikar

Chennai Mathematical Institute, India

We will make some interesting observations on the stochastic integral that give an insight into the theory. First one concerns characterization of the class $\mathbb{L}(X)$ (where X is a semimartingale) of integrands f for the stochastic integral $\int f\, dX$ as the class of predictable processes h such that $|h|$ serves as the dominating function in the dominated convergence theorem for stochastic integral.

The second remark is on the vector stochastic integral $\int < f, dX >$ (where f is $\mathbb{R}^d$ valued predictable process, X is $\mathbb{R}^d$ valued semimartingale and completeness of the range of the integral $f \mapsto \int < f, dX >$ when M is $\mathbb{R}^d$ valued martingale. The later is an important step for the second fundamental theorem of asset pricing.

The third is on Jacod's countable extension - a result which underscores that the non-anticipating stochastic integral is truly a *pathwise* integral - a property not shared by other anticipating integrals.

Approximation Complexity of Random Fields with Large Parametric Dimension

A. A. Khartov

Saint-Petersburg State University

E-mail: alexeykhartov@gmail.com

We study approximation properties of centered second order random fields $X_d(t)$, where $d \in \mathbb{N}$ is a dimension of the parameter *t*. We focus on tensor product-type random fields, which have covariance operators of corresponding tensor product form. The *average case approximation complexity* $n^{X_d}(\varepsilon)$ is defined as the minimal number of evaluations of arbitrary linear functionals that is needed to approximate X_d with normalized 2-average error not exceeding a given threshold $\varepsilon \in (0,1)$. We consider the quantity $n^{X_d}(\varepsilon)$ as a function of two variables $d \in \mathbb{N}$ and $\varepsilon \in (0,1)$, and investigate its growth. There exist two natural settings for this multivariate problem (see [1]). The first setting is obtaining upper bounds for $n^{X_d}(\varepsilon)$ for arbitrary ε and *d*. Here *tractability* questions are rather actual now (see [2], [3]). The second setting is an asymptotic analysis of $n^{X_d}(\varepsilon)$ for fixed ε and $d \to \infty$. Here rather general results were recently obtained (see [4]).

In the talk we will review important existing results for two above-mentioned settings and illustrate these results by applying to well known tensor product-type random fields.

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Gap Probabilities for Stationary Gaussian Processes and for Point Processes in the Plane

Manjunath Krishnapur
Indian Institute of Science (Bangalore)

For a stochastic process indexed by integers, what is the probability that it stays above zero for a long time? For a point process on the plane, what is the probability that it puts no points in a large disk of radius r? By gap probabilities (also called persistence probabilities or hole probabilities), we mean such quantities. In this lecture we shall review known results, old and new, for two situations: (a) For a stationary, centered Gaussian process on integers and (b) For the infinite Ginibre ensemble, a translation invariant point process in the plane. New results on the point processes will be based on works of Kartick Adhikari and Nanda Kishore Reddy and the work on stationay Gaussian processes is based on joint works with Krishna Maddaly and with Rajesh Sundaresan.

Energy Saving Approximation for Random Processes

Lifshits M.A.
St. Petersburg State University, Russia and Linköping University, Sweden
E-mail: mikhail@lifshits.org

We consider a stationary process or a process with stationary increments (with either discrete or continuous time) as a target and find an approximating process from the same class combining good approximation properties and, appropriately understood, small expense of energy.
Our aim is to solve the problem in terms of spectral characteristics of approximated process. If there is no extra adaptivity assumptions on the approximating process, the problem is easy and admits a closed universal solution, which is however non-obvious even for approximation of i.i.d. sequences.

Under adaptivity assumption, the problem has very much in common with classical prediction problems and solution construction depends on the spectrum of the approximated process. In this direction, we also extend classical spectral criteria for regularity and singularity of second order stationary processes due to Kolmogorov and Krein.

This is a joint work with I.A. Ibragimov, Z. Kabluchko, and E. Setterqvist.

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On the structure of UMVUEs

Yaakov Malinovsky

University of Maryland, Baltimore County

E-mail: yaakovm@umbc.edu.

Abram Kagan

Department of Mathematics, University of Maryland, College Park,

E-mail: amk@math.umd.edu.

In all setups when the structure of UMVUEs is known, there exists a subalgebra $\mathcal{U}$ (MVE-algebra) of the basic σ-algebra such that all $\mathcal{U}$-measurable statistics with finite second moments are UMVUEs. It is shown that MVE-algebras are, in a sense, similar to the subalgebras generated by complete sufficient statistics. Examples are given when these subalgebras differ, in these cases a new statistical structure arises.

Factorial Regression under Sparsity Assumption

M.B. Malyutov

Northeastern University, Boston, MA, USA

E-mail: m.malioutov@neu.edu

The Response Surface Methodology (RSM) due to R. Fisher, G.E.P. Box et al, is presently a part of Statistical Quality Improvement. Its aim is maximizing the numerical response described by a regression function of multiple inputs using noisy independent measurements of the response. The Box-Jenkins Methodology (BJM) takes into account also temporal dependence between measurements. A certain revision of BJM−m−Markov Chains with anisotropic sparse memory structure, is explored in Chapter3 of [1]. Let us outline our revision of RSM [2] based on sparsity of active inputs of the regression model. We use a number (which usually is much less than the total number of parameters) of repeated random samples from binary Complete Factorial Design (CFD). We prove that the main effects and interactions are mutually independent. Thus, the factorial model on CFD is an extended linear model. Our analysis consists of two steps. Step 1 is choosing effects with maximal Empirical Shannon Information with the response as active and step 2: Least Squares estimate of parameters of the reduced model consisting of active effects. The asymptotic optimality of this model fit is proved under small noise. We effectively use the parallel computing for large dimensions of the model and compare the performance of our method with that based on the Linear Programming relaxation by simulation.

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Extremes of Multi-type Branching Random Walks: Heaviest Tail Wins

Krishanu Maulik

Indian Statistical Institute

Branching random walk is a very important model with wide applications in Probability and Statistical Physics, in particular, to Gaussian free fields, tree polymers, first passage percolation and so on. The model was introduced by Hammersley (1974), Kingman (1975) and Biggins (1976), who considered the asymptotic behaviour of the scaled extremum of the branching random walk. Durrett (1979, 1983) considered the same problem when the displacements are heavy tailed. Brunet and Derrida (2011) considered the walk as a point process. They conjectured the walk with suitable scaled points to have a non-degenerate limit with a particular structure, which they called a decorated point process decorated by another point process. Madaule (2015) proved the conjecture when the displacements have exponentially decaying tails. Bhattacharya et al. (2016) proved the conjecture when the displacements are heavy-tailed. They further did not require the displacements to be independent, but allowed the displacements of a particle in a generation to be multivariate regularly varying. However, in all these articles, there were only one type of particles and hence all the displacements have same distribution. We extend the results in the heavy-tailed setup for multitype branching random walk, when the distribution of the displacement depends on the type of the offspring.

We consider a branching process with finitely many Q types of offsprings. We assume that the process is super critical and satisfies the generalized Kesten-Stigum conditions. Given the genealogical tree $\mathbb{T}$, each particle in $\mathbb{T}$ associates, with its offsprings, an independent copy of $(\boldsymbol{X}^{(1)}, \ldots, \boldsymbol{X}^{(Q)})$, where $\boldsymbol{X}^{(q)} = (X_1^{(q)}, X_2^{(q)}, \ldots)$ and $X_i^{(q)}$ denotes the random displacement of the i-th offspring of q-th type. We assume that the displacements corresponding to each type of offspring have same marginal distribution and that of the Q-th type offspring has the heaviest tail. The displacement vector of the Q-th type offsprings of a particle need not be independent, we assume them to have regular variation in the space $\mathbb{R}^{\mathbb{N}} \setminus \{\mathbf{0}\}$, index $-\alpha$ and limiting measure λ.

Given a vertex $\boldsymbol{v}$ in the tree $\mathbb{T}$, let $I_{\boldsymbol{v}}$. denote the path from the root to the vertex $\boldsymbol{v}$ and for each particle $\boldsymbol{w}$, let $X_{\boldsymbol{w}}$. denote the displacement associated with the particle, when it was born. Then the position of the vertex $\boldsymbol{v}$ is given by $S_{\boldsymbol{v}} = \sum_{\boldsymbol{w} \in I_{\boldsymbol{v}}} X_{\boldsymbol{w}}$. We further denote $|\boldsymbol{v}|$ to be the generation of the vertex $\boldsymbol{v}$ in the genealogical tree $\mathbb{T}$. We shall consider the point process $N_n = \sum_{|\boldsymbol{v}|=n} \delta_{b_n^{-1} S_{\boldsymbol{v}}}$ for an appropriate scaling sequence $\{b_n\}$ obtained from the limiting measure λ. The point process N_n belongs to the space $\mathcal{M}$ of Radon point measures on $[-\infty, \infty] \setminus \{\mathbf{0}\}$. We shall show that N_n converges weakly to a Cox cluster process in $\mathcal{M}$ endowed with the topology of vague convergence. The limit is a strictly α-stable point process. The limit can also be viewed as a randomly scaled, scale decorated Poisson point process, as conjectured by Brunet and Derrida (2011). As a corollary, we shall show that the rightmost position at the generation n, given by $M_n = \max_{|\boldsymbol{v}|=n} S_{\boldsymbol{v}}$, scales like b_n and converges weakly to a mixture of Frechet distribution.

This is a joint work with A. Bhattacharya, Z. Palmowski and P. Roy.

Extreme Singular Values of Random Matrices

Naumov A. A.

Moscow State University and IITP RAS

E-mail: anaumov@cs.msu.su

In my talk I will discuss the bounds for the smallest and largest singular values of different random matrix ensembles. These bounds play a crucial role in many limit theorems in Random matrix theory. Some applications to numerical mathematics and data analysis will be given as well.

The talk will be based on joint results with F. Götze and A. Tikhomirov.

L_2-Small Ball Asymptotics for Some Gaussian Processes with Respect to a Singular Self-Similar Measure

Alexander I. Nazarov[1]
St.Petersburg Dept of Steklov Math Institute
and St.Petersburg State University, Russia
E-mail: al.il.nazarov@gmail.com

We consider the problem of small ball asymptotics for Gaussian processes in L_2-norm with respect to a singular self-similar measure. Namely, let $X(t)$, $0 \le t \le 1$, be a zero mean Gaussian process. Suppose also that its covariance $G(s, t) = \mathbb{E}X(s)X(t)$, $s, t \in [0, 1]$, is the Green function of the ordinary differential operator $\mathcal{L}$ (such processes are called the Green Gaussian processes). We find the asymptotics of

$$\log \mathbb{P}\left\{ \int_0^1 X^2(t) d\mu < \varepsilon^2 \right\}, \ \varepsilon \to 0,$$

where μ is a Cantor-type self-similar measure or a discrete degenerate self-similar measure.

Note that a number of classical Gaussian processes such as Wiener process, Brownian Bridge, Ornstein-Uhlenbeck process, and their integrated counterparts belong to the class of the Green processes.

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Exponentiality Tests Based on a Special Property of Exponential Law

Ya. Yu. Nikitin and K. Yu. Volkova
Saint-Petersburg State University, Russia
E-mail: yanikit47@mail.ru

The problem of exponentiality testing is as follows. Let $X_1, \ldots, X_n$ be i.i.d. observations with a density f. We wish to test the composite null-hypothesis H_0: $f(x) = \lambda e^{-\lambda x}$, $x \geq 0$, where $\lambda > 0$ is an unknown scale parameter, against the alternative H_1: f is a density of a non-exponential law. There exist numerous tests of exponentiality based on various ideas.

Recently Noughabi and Arghami (2011) used the following "characterization" of exponential law: *Let X_1, X_2 be two i.i.d. rv's with continuous df F. Then $Y = X_1/X_2 \simeq F_{(2,2)}$ iff F is exponential. Here $F_{(2,2)}$ is the df of Fisher's (2,2)-distribution.*

We construct two sensitive and efficient U-empirical test statistics based on this property.This is supported by their high local Bahadur efficiency and considerable power under common alternatives. Our tests are inconsistent against *certain* special alternatives. Hence such tests are more convenient for rejection of H_0 than for acceptance of it.

As an application we apply our tests to the lengths of rule for Western Roman Emperors from Augustus to Theodosius I (27 B.C. - 395 A.D.) In the papers by Khmaladze and his coauthors (2006 - 2008) they came to the surprising agreement of data with the exponential distribution. We conclude, on the contrary, that our tests strongly reject the exponentiality of this data.

Our research was supported by the RFBR grant 16-01-00258.

On the Strong Law of Large Numbers for Some Self-Normalized Processes and Its Applications

E. Palamarchuk

Central Economics and Mathematics Institute of Russian Academy of Sciences;

National Research University Higher School of Economics

E-mail: e.palamarchuck@gmail.com

The strong law of large numbers (SSLN), defined for a pair of stochastic processes, means that their ratio tends to zero almost surely as time tends to infinity. If we apply some functional of numerator as a denominator, then the resulting process is called self-normalized. The integrated squared process (IS), widely used in a likelihood estimation, shows a typical self-normalization option. We consider stochastic processes of Ornstein-Uhlenbeck (OU) type. In this case the total variance (TV) of cumulative disturbances, being a non-random function, represents an important index of long-term behavior. Our first task consists in determining whether it would be possible to move from deterministic TV normalization to self-normalization by the IS. Under appropriate conditions on process parameters, the answer is positive. Secondly, we are to provide various examples of numerators in a way that the respective pairs satisfy the SSLN. For instance, polynomials and Riemann integrals of solutions to OU equations have been shown to meet the requirements. Further applications include Heston volatility model and populations growth dynamics.

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Manna Type Probabilistic Sandpile Model and Polynomial Identities.

Fedor Petrov

PDMI RAS

E-mail: fedor@pdmi.ras.ru

Consider the following finitary analogue of S. S. Manna sandpile model. There are n coins distributed somehow among the vertices of a regular $(n + 1)$-gon, which are enumerated by $0, 1, \ldots, n$. Choose any vertex v containing at least 2 coins and rub it: take 1 coin from the vertex v and put either to left or right neighbor of v with equal probability. Proceed until there remains unique empty vertex. This happens with probability 1, and so we have a distribution on the set of possible remained vertices. It may be expressed via symmetrization operator of rational function. Namely, if initially we have c_i coins at vertex $i = 0, 1, \ldots, n$, then the probability prob $(c_0, c_1, \ldots, c_n)$ that vertex n remains empty equals 1.

$$\frac{1}{n!}\mathrm{Sym}\,\frac{x_0^{c_0}\left(x_0+x_1\right)^{c_1}\ldots\left(x_0+\cdots+x_n\right)^{c_n}}{\left(x_0-x_1\right)\left(x_1-x_2\right)\ldots\left(x_{n-1}-x_n\right)},$$

where Sym denotes a summation over all $(n + 1)!$ permutations of variables $x_0, \ldots, x_n$. This formula leads to a probabilistic proof of a rational function identity previously derived by A. Postnikov by studying volumes of specific convex polytopes.

Small Ball Asymptotics for Detrended Green Gaussian Processes of Arbitrary Order

Y.P. Petrova,

St. Petersburg State University, Russia

E-mail: delafer21@gmail.com

We establish sharp L_2-small ball asymptotics for $(n+1)$-th order detrended Green Gaussian processes

$$X_n(t) := X(t) - \sum_{i=0}^{n-1} a_i t^i$$

under the conditions

$$\int_0^1 t_i X_n(t)dt = 0, \;\; i = 0 \ldots n-1.$$

Here $X(t)$ is a Gaussian process with zero mean and covariance function being the Green function for a boundary value problem:

$$Lu := (-1)^p u^{(2p)} = \lambda u + \text{some boundary conditions.}$$

The case $n = 1$ (centered process) is highly investigated (see, e.g. [1]). The case $p = 1$ was considered in [2]. We deal with arbitrary $n, p \in \mathbb{N}$ under the assumption $n > 2p$. The problem is reduced to the following eigenvalue problem:

$$(-1)^p y^{(2p)}(t) = \mu y^{(2n-2p)}(t)$$

$$y^{(j)}(0) = y^{(j)}(1) = 0, \;\; j = 0 \ldots n-1. \tag{1}$$

Note that the smallest eigenvalue of the problem (1) gives the sharp constant in the embedding theorem $\overset{\circ}{W}{}_2^n(0,1) \hookrightarrow \overset{\circ}{W}{}_2^{n-p}(0,1)$.

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Symmetric α-Stable Distributions for Noninteger $\alpha > 2$ and Associated Stochastic Processes

Platonova M. V.

Chebyshev Laboratory, St. Petersburg State University,

14th Line V.O., 29B, Saint Petersburg 199178 Russia

E-mail: mariyaplat@gmail.com

We construct analogues of symmetric α-stable distributions for noninteger indices $\alpha > 2$ and investigate their links to solutions of the Cauchy problem for some evolution equations.

Inference for Change Point Problems for Fractional Diffusion Processes

Bhagavatula Lakshmi Surya Prakasa Rao (aka) B.L.S. Prakasa Rao
CR Rao Advanced Institute of Mathematics, Statistics and Computer Science,
Hyderabad 500046, India
E-mail address: blsprao@gmail.com

There are some time series which exhibit long-range dependence as noticed by Hurst in his investigations of river water levels along Nile river. Long-range dependence is connected with the concept of self-similarity in that increments of a self-similar process with stationary increments exhibit long-range dependence under some conditions. Fractional Brownian motion is an example of such a process. We discuss statistical inference for stochastic processes modelled by stochastic differential equations driven by a fractional Brownian motion. These processes are termed as fractional diffusion processes. Since fractional Brownian motion is not a semimartingale, it is not possible to extend the notion of a stochastic integral with respect to a fractional Brownian motion following the ideas of Ito integration. There are other methods of extending integration with respect to a fractional Brownian motion. Suppose a complete path of a fractional diffusion process is observed over a finite time interval. We will present some results on inference for change point problems for such processes.

On Data Obfuscation

Bimal Roy

Indian Statistical Institute, Kolkata, India

E-mail: bimal@isical.ac.in

Privacy protection and data security have received lots of attention due to increasing need to protect sensitive information like medical data, credit card data etc. There are various ways to protect data; here we are interested in ways that may retain its statistical uses to some extent. One such way is to mask the data with additive or multiplicative noise and to get back certain desired parameters of the original distribution from the knowledge of the noise distribution & masked data. In this presentation, we discuss the estimation of any desired quantile when masked with additive noise. We also propose a method to choose appropriate parameters of the noise distribution & discuss advantages of this method over some existing methods.

Small Deviation Probabilities of Weighted Sums under Minimal Moment Assumptions

Rozovsky L. V.

Saint - Petersburg Chemical-Pharmaceutical Academy, Department of Mathematics

E-mail: L-Rozovsky@mail.ru

Let $S = \sum_{j\geq 1}\lambda(j)X_j$, where $\{X_i\}$ g are independent copies of a positive random variable X with a distribution function $V(x)$, decreasing at zero as a power of x, and let $\lambda(\cdot)$ be a bounded positive non-increasing function defined on the interval $[1,\infty]$.

Assume that $\limsup_{n\to\infty}\sum_{l\geq 1}\mathbf{E}\min\left(1, \lambda(ln)/\lambda(n)X\right) < \infty$.

The latter condition implies that S converges almost surely, and coincides with this *necessary* assumption for a wide class of sequences $\lambda(n)$ regular enough such as $n^{-\delta}$ or $e^{-n^{\delta}}$ with $\delta > 0$, say.

Our main purpose is to prove that under conditions above

$$\mathbf{P}(S < r) \sim \frac{\exp\left(L(u) + ur\right)}{\sqrt{2\pi u^2 L''(u)}},\ r \to 0,$$

where $u = u(r)$ is a solution of $L'(u) + r = 0$, and $L(u) = \sum_{n\geq 1}\log \mathbf{E}e^{-u\lambda(n)X}$.

Blind Deconvolution Using Natural Image Priors

D. Sarkar

Indian Statistical Institute, Statistics and Mathematics Unit, Delhi, India

E-mail: deepayan@isid.ac.in

Blurring of photographic images due to camera shake is quite common, and recovering the underlying image from such photographs is an interesting inference problem. Ignoring rotations, the blurring process can be modeled as a convolution of the underlying image and a “blur kernel” or “point spread function”, and the problem is thus referred to as “deconvolution”. The problem is well-studied when the blur kernel is known. However, non-blind deconvolution, when the blur kernel is unknown, is more difficult. Considerable progress in this problem has been made during the last decade by making `natural' assumptions about the unknown image in the form of a prior. In this talk, we will give an overview of the problem, summarize the current approaches to solve it, and describe a generalization of the commonly used prior family that appears to give better results. This is joint work with Kaustav Nandy.

Highly Robust and Efficient *M*-Estimates of Scale

Georgy L. Shevlyakov

Peter the Great St. Petersburg Polytechnic University

E-mail: Georgy.Shevlyakov@phmf.spbstu.ru

The problem of estimation of a scale parameter is one of basic in statistical analysis. In present, the most robust and efficient estimate of scale is given by the Q_n-estimate defined as the first quartile of the distance between observations: $Q_n = c\{|x_i - x_j|\}_{(k)}$, where c is a constant providing consistency, $k = C_h^2$, $h = [n/2]+1$. The Q_n-estimate has the highest breakdown point $\varepsilon^*= 0.5$ possible and high efficiency 82% at the Gaussian (Rousseeuw and Croux, 1993). Its drawback is the high computational complexity: generally, it requires $O(n \log n)$ of computational time.

On the contrary, low-complexity Huber's robust M-estimates $\widehat{S}$ of scale defined by $\sum \chi(x_i/\widehat{S}) = 0$, where $\chi(x)$ is a score function, have a potential for enhancing their efficiency. The goal of this work is to construct a computationally fast and highly robust approximation to the Q_n-estimate adapted to data distributions of a general shape. In this case, the following result holds.

Theorem *Under conditions of regularity imposed on distribution densities f, the scores $\chi_\alpha(x)$ of M-estimates $\widehat{S}$ of scale approximating the Q_n-estimate are given by*

$$\chi_\alpha(x) = c_\alpha - 2f(x) - \alpha^2 f''(x)/3, \tag{1}$$

where c_α is a constant that provides consistency and α is a tuning parameter.

The particular cases of interest are: at the Gaussian, the efficiency of the M-estimate (1) is 81%, just only 1% less than that of the Q_n-estimate; at the Cauchy, it is the maximum likelihood estimate. These asymptotic results are confirmed by Monte Carlo experiment on finite samples.

Convergence Rate Estimates in the Global CLT

Irina Shevtsova

Moscow State University, Faculty of Computational Mathematics and Cybernetics;

Institute of Informatics Problems of the Federal Research Center "Computer Science and Control" of RAS,

E-mail: ishevtsova@cs.msu.ru

We consider the sums of independent random variables with finite absolute moments of orders $2+\delta$, where $0<\delta\le 1$, and construct moment-type estimates for the mean metrics between distributions of properly normalized sums and the corresponding limit normal law as the number of summands tends to infinity. Moreover, we also consider random sums, where the number of random summands follows the binomial or the Poisson distributions. A new exact recentering inequality is proved for the third-order absolute moments in the form $\mathsf{E}|X-a|^3 \le K(a)\mathsf{E}|X|^3$ with an explicitly given function $K(a) \le \frac{17+7\sqrt{7}}{27} < 1.32$, where $\mathsf{E}X = a \in [-1,1]$, $\mathsf{E}X^2 = 1$, which (a) allows to compare the obtained bounds of the accuracy of the normal approximation with the known results and (b) improves the earlier results of (Pinelis, 2011) and (Nefedova & Shevtsova, 2012).

Analytic Diffusion Processes: Definition, Properties, Limit Theorems.

N.V. Smorodina

PDMI RAS;

St.-Petersburg State University, St.-Petersburg, Russia

We introduce a concept of an analytic diffusion process. We define such a process as a limit of a sequence of random walks but we understand this limit not in the sense of convergence of measures but in the sense of convergence of generalized functions. In terms of analytic diffusion processes we construct a probabilistic approximation of the Cauchy problem solutions for Schrödinger type evolution equations having in the right hand side elliptic operators with variable coefficients.

Estimation of the kernel function of stable moving average random fields

E. Spodarev

Universität Ulm, Institut für Stochastik, Germany

E-mail: evgeny.spodarev@uni-ulm.de

This is joint work with J. Kampf, L. Palyanitsa, A. Stelmakh and G. Shevchenko.

We consider the problem of estimation of a uniformly continuous symmetric kernel $f : \mathbb{R}^d \to \mathbb{R}$ from observations of the stationary random function

$$X(t) = \int_{\mathbb{R}^d} f(t - s)\Lambda(ds),$$

where Λ is a $S\alpha S$ random measure with independent increments and Lebesgue control measure. This class of stochastic processes includes, e.g., stable CARMA processes which are popular in econometric and financial applications.

We use the smoothed version of an empirical normalized periodogram

$$I_{n,X}(\lambda) = \frac{\left|\sum_{j=1}^{n} X\left(t_{j,n}\right) e^{it_{j,n}\lambda}\right|^2}{\sum_{j=1}^{n} X\left(t_{j,n}\right)^2}$$

of X to (non-parametrically) estimate f from observations $\{X(t_{j,n})\}$ on a high frequency grid of points $\{t_{j,n}\}$. Weak consistency of the estimator as $n \to \infty$ is shown. We conclude with a simulation study of the performance of the estimates.

On the Consistency of Directional Exponentiality Tests against Alternatives with Monotone Failure Rate.

A.V. Tchirina

Saint Petersburg Electrotechnical University;

National Research University Higher School of Economics

E-mail: annatchirina@yandex.ru, achirina@hse.ru

Let X_i, $i = 1, \ldots, n$ be i.i.d. non-negative random variables. We study scale-free tests designed to verify the composite null hypothesis H_0: X_i's have exponential distribution with a density $f(x) = \lambda \exp(-\lambda x)$ for some positive λ, against an alternative composite hypothesis H_0: the distribution of X_i's is not exponential.

If one restricts the set alternatives to some one-parametric family of the type

$$f(x, \theta) = \exp(-x) + \theta h(x) + o(\theta), \theta \to 0,$$

one can use a test equivalent to the likelihood ratio test or the score test. We study signed one-sided tests having asymptotically normal distribution. Such tests perform well against this family of alternatives, but their consistency against other alternatives is not guaranteed.

Among the examples of such tests we can mention such well-known tests as Gini, Moran, Cox and Oakes, Epps and Pulley, Greenwood and some other tests.

The aim of this talk is to show that as soon as alternatives belong to the family of distributions with monotone (increasing or decreasing) failure rate (IFR od DFR) for any small positive θ, the corresponding exponentiality test is also consistent against ANY alternative from IFR or DFR.

Local Laws for Spectrum of Random Matrices

Tikhomirov A. N.

Komi Science Center, Ural Division of RAS;

Syktyvkar State University

E-mail: tikhomirov@dm.komisc.ru

We explain some recent results obtained with F. Götze, A. Naumov and D. Timushev. Following Yau, Erdös et al. we say that the local semicircle law for symmetric matrices or Marchenko-Pastur's law for sample covariance matrices hold if the distance between Stieltjes transforms $m_n(z)$ of the empirical spectral distribution (ESD) function and $s(z)$ of the semicircle law (Marchenko-Pastur's law) is of order $(nv)^{-1}$ (up to the logarithmic factor), where n denotes the order of matrix and $z = u + iv,\ v > 0$. We show that the local semicircle law (or Marchenko-Pastur's law) holds under assumption that $4 + \eta$, $\eta > 0$; moments of matrix entries are finite. Applying these results we prove the optimal bounds for the rate of convergence of the expected ESD of symmetric or covariance matrices to the corresponding limit distribution. We also get the optimal bounds in the problem of delocalization of eigenvectors and optimal bounds (up to the logarithmic factor with the best known power) in the problem of rigidity of eigenvalues of symmetric and sample covariance matrices.

How Many Families Survive for a Long Time

Vatutin V.A., Dyakonova E.E.

(Steklov Mathematical Institute, Moscow)

E-mail: vatutin@mi.ras.ru, elena@mi.ras.ru

Let $\{Z_k,\ k = 0, 1, 2, ...\}$ be a critical branching process in random environment and let $Z_{p,n}$ be the number of particles at time $p < n$ in the process having a positive offspring number at time n. We show that if the associated random walk of the branching process belongs to the domain of attraction of a stable law with parameter $\alpha \in (0, 2]$ then there exists a sequence $\{ c_p^{-1},\ p = 1, 2, ...\}$ such that the conditional law

$$\mathcal{L}(\{ c_p^{-1} \log Z_{pu},\ 0 \leq u < \infty \} \,|Z_n > 0)$$

weakly converges, as $n \gg p \to \infty$ to the law of an α-stable Levy process conditioned to stay nonnegative on the semi-axis $[0, \infty)$.

Basing on this result we prove a conditional functional limit theorem for the properly scaled process $\{\log Z_{pu,n},\ 0 \leq u < \infty\}$ given $Z_n > 0$ and $n \gg p \to \infty$.

This work is supported by the RSF under a grant 14-50-00005.

Highest Zero-One Laws and Theory of Filtrations

Anatolii M. Vershik

St. Petersburg Dept. of the Mathematical Institute of Russian Academy of Sciences;

Mathematical Department of St. Petersburg University;

Moscow Institute of the Problems of Transmission Information

E-mail: avershik@gmail.com

The classical Kolmogorov's result (1933) about sequences of independent random variables asserts that the infinite intersection of the decreasing filtration of the sigma-fields generated by the complement to finite number of variables (so called Bernoulli filtration) is trivial sigmafield. There are many generalizations of Kolmogorov "0-1 Law". Can we say more about structure of infinite filtrations "at infinity" with 0-1 law?

Rigorous posing of the problem is the following: how to distinguish two filtrations $\{A_n\}_{n>0}$ and $\{B_n\}_{n>0}$ if they have isomorphic finite fragments of length n are for all n and 0-1 law? And how to formulate the conditions on the filtrations, which guarantee that all finite invariants (plus $0-1$-law) defined filtration uniquely? We called it as standard or finitely determined filtrations? The Bernoulli filtration is a nontrivial example of finitely determined filtrations. We describe all class of such filtrations; they could be considered as a new natural generalization of Bernoulli filtration. The corresponding sequences of random variables we call as "virtually independent sequences".

Most interesting questions are concerned to the case when filtration is not finitely determined and have some nontrivial invariants on infinity in spite of having 0-1. This is what I called "HIGHEST ZERO-ONE LAWS". Secondary entropy of the process is one of such invariants. There are several connections of this conception with ergodic theory, theory of Markov processes, theory of invariant measures, martingales and random walk.

Large Deviations for the Perimeter of Convex Hulls of Planar Random Walks

Vysotsky V.V.

Arizona State University, Imperial College London,

and St. Petersburg Division of Steklov Institute RAS,

E-mail: vysotsky@asu.edu, v.vysotskiy@imperial.ac.uk

Consider the perimeter L_n of the convex hull of the first n steps of a random walk on a plane. We study large deviations probabilities of the perimeter under the assumption that the Laplace transform of the increments X_k of the walk is finite on $\mathbb{R}^2$. The two cases $\mathbb{P}(L_n \leq 2xn)$ for $x < a$ and $\mathbb{P}(L_n \geq 2xn)$ for $x > a$, where $a = \|\mathbb{E}X_1\|$, are very different. In the former case, we easily find the logarithmic asymptotics and describe the shape of trajectories that result in such behaviour. The later case turns out to be much harder - we are only able to prove certain bounds, which yield the logarithmic asymptotics for several types of random walks, including the ones with either shifted or linearly transformed rotationally invariant distribution of their increments.

This is a joint work with A. Akopyan (IST Austria).

Random Integer Partitions with Restriction on Part Differences

Yuri Yakubovich
Saint Petersburg State University
E-mail: yuyakub@gmail.com

A positive integer n partition is its representation as a sum of positive integers (parts): $n = n_1 + \ldots + n_k$, $n_1 \geq \ldots \geq n_k \geq 1$. It is well known that in the $n \to \infty$ limit the uniformly chosen random partition of n after rescaling converges in probability to a deterministic object called the "limit shape": roughly speaking, it means that $\frac{1}{\sqrt{n}} n_{\lfloor x\sqrt{n} \rfloor} \to y$, for $x, y > 0$ satisfying $e^{-cx} + e^{-cy} = 1$, $c = \frac{\pi}{\sqrt{6}}$ (Vershik's curve). About 10 years ago A. Comtet with collaborators used physical argumentation to extend this result to partitions satisfying additional restriction $n_j - n_{j+1} \geq q$, $j = 1, \ldots, k-1$, and pointed out links to several physical models. These links are of particular interest when one performs a trick and treats the integer parameter q as real.

We provide a combinatorial model that justifies this trick by introducing restrictions $n_j - n_{j+1} \geq q_{k-j}$, $j = 1, \ldots, k-1$, for a given sequence $\mathsf{q} = (q_i)$ of nonnegative integers. We show that if q satisfies the condition $q_1 + \ldots + q_k = q \cdot k + O(k^\beta)$ as $k \to \infty$, for some $q \geq 0$ and $\beta \in [0, 1)$, then the limit shape result holds and the limit shape depends only on q, recovering formulas found by A. Comtet et. al. We also note that the conditions are satisfied by a class of random sequences q independent of the random partition, providing a model for "random partitions in a random environment".

This is joint work with Leonid Bogachev.

Measurable Linear and Multilinear Mappings.

Yurova E. V.

Lomonosov Moscow State University

Tel.: 89163077409, E-mail: yurova-k@rambler.ru

Let X and Y are two separable Fréchet spaces, μ is a Radon probability measure on X, then there are two natural definitions of linear measurable operator $A : X \to Y$:

(i) there is a sequence of continuous linear operators $A_n : X \to Y$ converging to A almost everywhere,

(ii) there exist a separable reflexive Banach space E of full measure that is compactly embedded into X and a continuous linear operator $\widetilde{A} : E \to Y$ that coincides with A almost everywhere.

Theorem 1. Property (i) implies property (ii). If Y is a Banach space with a Schauder basis then they are equivalent.

The question naturally arises whether these properties or their analogs hold for measurable multilinear mappings $B(x_1, \ldots, x_n)$: $X \times \ldots \times X \to Y$ (X is equipped with centered Gaussian measure).

There are two natural analogs of property (ii):

(a) the existence of a full measure separable reflexive Banach space L compactly embedded into X and a continuous multilinear mapping $\widetilde{B}(x_1, \ldots, x_n)$: $L \times \ldots \times L \to Y$ such that $\widetilde{B}(x_1, \ldots, x_n) = B(x_1, \ldots, x_n)$ for almost all $(x_1, \ldots, x_n) \in L \times \ldots \times L$;

(b) the existence of a full measure separable reflexive Banach space E compactly embedded into $X \times \ldots \times X$ and a continuous multilinear mapping $\widetilde{B}(x_1, \ldots, x_n)$: $E \to Y$ such that $\widetilde{B}(x_1, \ldots, x_n) = B(x_1, \ldots, x_n)$ for almost all $(x_1, \ldots, x_n) \in E$.

Theorem 2. If E is a full measure separable reflexive Banach space continuously embedded into X^n, then there is a full measure separable reflexive Banach space L continuously embedded into X such that $L^n \subset E$ and the embedding is compact.

Therefore, properties (a) and (b) are equivalent. But it is shown that even for $n = 2$ they are not equivalent to the natural analogs of property (i).

Arak Inequalities for Concentration Functions and the Littlewood-Offord Problem

Zaitsev A.Yu.

St. Petersburg Department of Steklov Mathematical Institute

E-mail: zaitsev@pdmi.ras.ru

We discuss the behavior of concentration functions of weighted sums of independent random variables with respect to the arithmetic structure of coefficients. Recently, Tao and Vu [4] and Nguyen and Vu [3] formulated a so-called Inverse Principle in the Littlewood–Offord problem. We discuss the relations between this Inverse Principle and a similar principle formulated for sums of arbitrarily distributed independent random variables formulated by T. Arak in the 1980's (see [1] and [2]).

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