Control Theory, Integral Geometry, Inverse Problems

12-18 June 2017

Euler International Mathematical Institute, St. Petersburg, Russia

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12 June, Monday

9:00–9:55 Registration
9:55–10:00 Opening ceremony

Morning session, ch. J.Boman

10:00–10:40 *Yu.Egorov.* On the optimal form of nano-rods.
Cofee break
11:00 – 11:40 *B.Vainberg.* D-bar method for inverse two-dimensional problems and its applications.
11:45–12:25 *H.Kang.* The spectral theory of the Neumann-Poincare operator and plasmon resonance.
Lunch

Evening session, ch. V.Sharafutdinov

14:15–14:55 *A.El Badia.* Some recent results on some inverse source problems: identification and stability.
Coffee break
15:15–15:55 *M.I.Ismailov.* An externally controlled problem for diffusion equation with nonlocal Wentzell boundary condition.

13 June, Tuesday

Morning session, ch. Yu.Egorov

10:00–10:40 *A.A.Shkalikov.* Recovering of potentials in the scale of Sobolev spaces.
Cofee break
11:00 – 11:40 *M.Eller.* Stabilization of Maxwell's equation.
11:45–12:25 *Rakesh.* The exterior Goursat problem for the wave equation.
Lunch

Evening session, ch. G.Nakamura

14:15–14:55 *A.V.Fursikov.* Parabolic equation of normal type connected with 3D Helmholtz system and its nonlocal stabilization.
Coffee break
15:15–15:55 *S.Avdonin.* Control and Inverse Problems for Differential Equations on Graphs.

14 June, Wednesday

Morning session, ch. H.Kang

10:00–10:40 *M.Di Cristo.* A stability result for quantitative photoacoustic tomography.
Coffee break
11:00 – 11:40 *V.Serov.* Inverse scattering problems for perturbation of the bi-harmonic operator.
11:45–12:25 *M.Salo.* The attenuated geodesic X-ray transform.

Lunch, boat trip

15 June, Thursday

Morning session, ch. A.V.Fursikov

10:00–10:40 *J.Boman*. Stability estimates in tensor tomography.
Coffee break
11:00 – 11:40 *A.Denisiuk*. Inversion in the cone-beam vector tomography with two sources.
11:45–12:25 *T.Quinto*. Artifacts in Arbitrary Limited Data Tomography Problems.
Lunch

Evening session, ch. R.Novikov

14:15–14:55 *T.Hohage*. Variational Source Conditions and Stability Estimates.
Coffee break
15:15–15:55 *A.Jollivet*. Steklov zeta-invariants and a compactness theorem for isospectral families of planar domains.
BANQUET!

16 June, Friday

Morning session, ch. M.Eller

10:00–10:40 *G.Nakamura*. Inverse boundary value problem for identifying elasticity tensor by boundary measurements.
Coffee break
11:00 – 11:40 *R.Novikov*. Inverse scattering without phase information.
11:45–12:25 *V.Sharafutdinov*. The Reshetnyak formula and Natterer stability estimates in tensor tomography.
Lunch

Evening session, ch. V.Agoshkov

14:15–14:55 *A.V.Baev*. Solution of an Inverse Scattering Problem for the Acoustic Wave Equation in Three-Dimensional Layered Media.
Coffee break
15:15–15:55 *A.S.Demidov, M.A.Galchenkova*. Inverse magnetoencephalography problem and its flat approximation.

17 June, Saturday

Morning session, ch. Rakesh

10:00–10:40 *V.Agoshkov*. Statement and study some inverse problems in modeling of hydrophysical fields in water areas with “liquid” boundaries.
Coffee break
11:00 – 11:40 *P.Kurasov*. Inverse problems for graphs.
11:45–12:25 *Ye.M.Assylbekov*. Inversion formulas and range characterizations for the attenuated geodesic ray transform.

Lunch

Evening session, ch. P.Kurasov

14:15–14:55 *L.Pestov*. On the boundary rigidity problem for simple manifolds.
Coffee break
15:15–15:55 *S.A.Simonov*. Wave models of Sturm-Liouville operators.

18 June, Sunday

11:00–15:00 Round table and discussions.
Closing ceremony

Statement and study some inverse problems in modeling of hydrophysical fields in water areas with "liquid" boundaries
Valeriy Agoshkov

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There are different approaches for modeling boundary conditions describing hydrophysical fields in water areas with "liquid" boundaries. Variational data assimilation may also be considered as one of such approaches. Development of computer equipment, together with an increase in the quantity and quality of data from the satellites and other monitoring tools proves that the development of this particular approach is perspective. The range of connected the problems is wide - different recording forms of boundary conditions, observational data assimilation procedures and used models of hydrodynamics are possible.

In this work some inverse problems and corresponding variational data assimilation ones, connected with mathematical modeling of hydrophysical fields in water areas (seas and oceans) with "liquid" ("open") boundaries, are formulated and studied. Note that the surface of water area (which can also be considered as a "liquid" boundary) is not included in the set of "liquid" boundaries, in this case "liquid" boundaries are borders between the areas "water-water". In the work, mathematical model of hydrothermodynamics in the water areas with "liquid" ("open") part of the boundary, a generalized statement of the problem and the splitting method for time approximation are formulated. Also the problem of variational data assimilation and iterative algorithm for solving inverse problems mentioned above are formulated. The work is based on [1].

The work was partly supported by the Russian Science Foundation (project 14–11–00609, the general formulation of the inverse problems) and by the Russian Foundation for Basic Research (project 16–01–00548, the formulation of the problem and its study).

References

- [1] V.I. Agoshkov, Methods for solving inverse problems and variational data assimilation problems of observations in the problems of the large-scale dynamics of the oceans and seas, Institute of Numerical Mathematics, RAS, Moscow, 2016 (in Russian).

Inversion formulas and range characterizations for the attenuated geodesic ray transform
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We present two range characterizations for the attenuated geodesic X-ray transform defined on pairs of functions and one-forms on simple surfaces. Such characterizations are based on first isolating the range over sums of functions and one-forms, then separating each sub-range in two ways, first by implicit conditions, second by deriving new inversion formulas for sums of functions and one-forms. This is a joint work with François Monard and Gunther Uhlmann.

Control and Inverse Problems for Differential Equations on Graphs

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Quantum graphs are metric graphs with differential equations defined on the edges. Recent interest in control and inverse problems for quantum graphs is motivated by applications to important problems of classical and quantum physics, chemistry, biology, and engineering.

In this talk we describe some new controllability and identifiability results for partial differential equations on compact graphs. In particular, we consider graph-like networks of inhomogeneous strings with masses attached at the interior vertices. We show that the wave transmitted through a mass is more regular than the incoming wave. Therefore, the regularity of the solution to the initial boundary value problem on an edge depends on the combinatorial distance of this edge from the source, that makes control and inverse problems for such systems more difficult.

We prove the exact controllability of the systems with the optimal number of controls and propose an algorithm recovering the unknown densities of the strings, lengths of the edges, attached masses, and the topology of the graph. The proofs are based on the boundary control and leaf peeling methods [1, 2].

[1] S. Avdonin and P. Kurasov, "Inverse problems for quantum trees", *Inverse Problems and Imaging*, **2** (2008), no. 1, 1–21.
[2] S. Avdonin, G. Leugering and V. Mikhaylov, "On an inverse problem for tree-like networks of elastic strings", *Zeit. Angew. Math. Mech.*, **90** (2010), no. 2, 136–150.

Some recent results on some inverse source problems: Identification and stability

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This talk deals with some inverse source problems. We focus particularly on the identification and stability issues.

Solution of an Inverse Scattering Problem for the Acoustic Wave Equation in Three-Dimensional Layered Media
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In this work, we consider a three-dimensional medium characterized by two material parameters: density and the speed of sound. It might be possible to consider another pair of functions, namely, density and acoustic impedance. It is shown that, along with the density, the other two characteristics are recovered from precise scattering data in a coordinate system fixed to the observer. From a practical point of view, this makes it possible to interpret the medium as a substance. This result can be useful, for example, in the interpretation of geophysical exploration data.

Stability estimates in tensor tomography
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This is joint work with **Vladimir Sharafutdinov**
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For a compactly supported vector field $f = (f_1, \dots, f_n)$ in $\mathbf{R}^n$ the ray transform If is defined by

$$If(x,\theta) = \sum_{j=1}^{n} \int_{\mathbf{R}} f_j(x+t\theta)\theta_j dt, \quad (x,\theta) \in \mathbf{R}^{2n},$$

and for a symmetric 2-tensor field $(f_{jk})_{j,k=1}^{n}$ one defines

$$If(x,\theta) = \sum_{j,k=1}^{n} \int_{\mathbf{R}} f_{jk}(x+t\theta)\theta_j\theta_k dt, \quad (x,\theta) \in \mathbf{R}^{2n}.$$

Since $If = 0$ if f is the gradient of a scalar function, one can at most recover the so-called solenoidal part ${}^s\!f$ of f from If. An analogous fact is true for second and higher rank tensor fields. Similarly, if the tensor field f is only defined in a bounded convex subset $\Omega \subset \mathbf{R}^n$, there is a natural definition of the solenoidal part ${}^s f$ of f relative to Ω. For tensor fields of arbitrary rank we give estimates for the norm of ${}^s f$ of the type

$$\|{}^s f\| \leq C\|If\|_{1/2},$$

where $\|\cdot\|$ is the L^2-norm and $\|\cdot\|_{1/2}$ is a Sobolev norm of order 1/2. The proof is based on a comparison of the Dirichlet integrals for the exterior and interior Dirichlet problems and a generalization of the Korn inequality to symmetric tensor fields of arbitrary rank. Weaker estimates for $\|{}^s f\|$ were given by L. Pestov and V. Sharafutdinov in 1988 for the more general case of the geodesic ray transform with respect to a Riemannian metric under a certain curvature condition on the metric.

A stability result for quantitative photoacoustic tomography

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We treat the stability issue for the three dimensional inverse imaging modality called Quantitative Photoacoustic Tomography. We provide universal choices of the illuminations which enable to recover, in a Hölder stable fashion, the diffusion and absorption coefficients from the interior pressure data. With such choices of illuminations we do not need the non-degeneracy conditions commonly used in previous studies, which are difficult to be verified a-priori.

The inverse Magnetoencephalography problem and its flat approximation

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Contrary to the prevailing opinion about the incorrectness of the inverse MEEG-problem, we prove that its uniqueness solution in the framework of the electrodynamic system of Maxwell equations [1]. The solution of this problem is the distribution of $\mathbf{y} \mapsto \mathbf{q}(\mathbf{y})$ current dipoles of brain neurons that occupies the region $Y \subset \mathbb{R}^3$. It is uniquely determined by the non-invasive measurements of the electric and magnetic fields induced by the current dipoles of neurons on the patient's head. The solution can be represented in the form $\mathbf{q} = \mathbf{q}^* + \rho\delta\Big|_{\partial Y}$, where $\mathbf{q}^*$ is the usual function defined in Y, and $\rho\delta\Big|_{\partial Y}$ is a δ-function on the boundary of the domain Y with a certain density ρ. However, in cases where the conductivity is assumed to be everywhere the same (in the brain, skull, ambient air) and, in addition, it is not possible (or impossible) to record *in time* the electric and magnetic inductions, it is impossible to completely find $\mathbf{q}$. Nevertheless, it is still possible to obtain partial information about the distribution of $\mathbf{q} : Y \ni \mathbf{y} \mapsto \mathbf{q}(\mathbf{y})$. This question is considered in detail in a flat model situation.

References

[1] A.S. Demidov (2017) Unique solvability of the inverse MEEG-problem (to appear)

[2] T.A. Stroganova et al. (2011) EEG alpha activity in the human brain during perception of an illusory kanizsa square, *Neuroscience and Behavioral Physiology*, V. 41 (2), 130-139.

[3] M. Hämäläinen et al. (1993) Magnetoencephalography – theory, instrumentation, and applications to noninvasive studies of the working human brain, *Reviews of Modern Physics*, V. 65, No 2, 413-497.

[4] A.S. Demidov (1973) Elliptic pseudodifferential boundary value problems with a small parameter in the coefficient of the leading operator, *Math. USSR-Sb.*, 20:3, 439-463.

[5] A.S. Demidov (1975) Asymptotics of the solution of the boundary value problem for elliptic pseudo-differential equations with a small parameter with the highest operator, *Trudy Moskov. Math. obshchestva*, 32, Moscow University Press, M. 119+-146 (In Russian).

[6] A.S. Demidov, M.A. Galchenkova, A.S. Kochurov (2015) On inverse problem magnetoence-phalography, *Quasilinear equations, inverse problems and their applications.* Moscow, 30.11.2015–02.12.2015, Conference handbook and proceedings, p. 22.

Inversion in the cone-beam vector tomography with two sources

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Consider the problem of reconstruction of solenoidal part of a vector field from its X-ray transform, known for a family of rays, coming out from two given curves in $\mathbb{R}^3$. A new exact inversion formula and a filtered backprojection reconstruction algorithm will be presented.

On the optimal form of nano-rods
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We consider an inverse spectral problem for a differential operator of fourth order: to find the potential $Q \in C(0,1)$, such that $Q(x) > 0$,

$$\int_0^1 Q(x)^\alpha dx = 1, \quad 0 < \alpha \leq 1,$$

and the first eigenvalue l of the problem:

$$(Qy'')'' - ky'''' + ly'' = 0, \quad y(0) = y'(0) = y(1) = y'(1) = 0$$

is maximal. Here $k > 0$ is a given constant.

Such the problem arises when one is looking for the optimal form of elastic rods using the quantum mechanics laws.

Stabilization of Maxwell's equation
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We will discuss the stabilizability of the dynamic Maxwell equations by means of a conductivity term. Various results will be given, depending on the set in which the conductivity is known to be strictly positive. The initial configuration of the magnetic field will be assumed to be divergence free which is in agreement with most physical models. For some of the results the electric field needs to be divergence free as well.

PARABOLIC EQUATION OF NORMAL TYPE CONNECTED WITH 3D HELMHOLTZ SYSTEM AND ITS NONLOCAL STABILIZATION
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The talk will be devoted to the normal parabolic equation (NPE) connected with 3D Helmholtz system whose nonlinear term B(v) is orthogonal projection of nonlinear term for

Helmholtz system on the ray generated by vector v. Interest to NPE arised in connection with attempts to find approaches to solve problem on non local existence of smooth solution for 3D Navier-Stokes equations.

As it became clear now the studies of NPE has been opened the way to construct the method of nonlocal stabilization by feedback control for 3D Helmholtz as well as for 3D Navier-Stokes equations.

First we describe the structure of dynamical flow corresponding to this NPE (see [1]). After, the non local stabilization problem for NPE by starting control supported on arbitrary fixed subdomain will be formulated. The main steps of solution to this problem will be discussed (see [2]). At last how to apply this result for solution of nonlocal stabilization problem with impulse control for 3D Helmholtz system will be explained.

Literature

[1] A.V.Fursikov. "On the Normal-type Parabolic System Corresponding to the three-dimensional Helmholtz System".- Advances in Mathematical Analysis of PDEs. AMS Transl.Series 2, v.232 (2014), 99-118.

[2] A.V.Fursikov, L.S.Shatina. "Nonlocal stabilization of the normal equation connected with Helmholtz system by starting control."-ArXiv: 1609.08679v2[math.OC] 26 Feb. 2017, p.1-55

Variational Source Conditions and Stability Estimates

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Over the last years Variational Source Conditions (VSCs), i.e. source conditions in the form of variational inequalities, have become a standard assumption for the analysis of variational regularization methods. Compared to classical spectral source conditions they have a number of advantages: They can be used for general non-quadratic penalty and data fidelity terms, lead to simpler proofs, are often not only sufficient, but even necessary for certain convergence rates, and they do not involve the derivative of the forward operator (and hence do not require restrictive assumptions such as a tangential cone condition). However, until recently only very few sufficient conditions for VSCs for specific inverse problems had been known.

To overcome this drawback, we describe a general strategy for the verification of VSCs, by making a connection to the well-studied field of conditional stability estimates. More precisely we show that a VSC follows from two assumptions: One of them describes the smoothness of the solution, whereas the other one is a special type of stability estimate and hence describes the degree of ill-posedness. VSCs always imply stability estimates, but the reverse implication is not obvious. Typically extra work is required for specific inverse problems to verify the second assumption by extending known proofs of stability estimates.

For a number of important linear inverse problems the approach above leads to equivalent characterizations of VSCs in terms of Besov spaces and necessary and sufficient conditions for rates of convergence. We also discuss the application of our strategy to nonlinear parameter identification and inverse medium scattering problems where it provides sufficient conditions for VSCs in terms of standard function spaces.

References

[1] T. Hohage and F. Weidling. Verification of a variational source condition for acoustic inverse medium scattering problems. *Inverse Problems*, 31(7):075006, 14, 2015.

[2] T. Hohage and F. Weidling. Characterizations of variational source conditions, converse results, and maxisets of spectral regularization methods. *SIAM J. Numer. Anal.*, 55(2):598–620, 2017.

[3] F. Weidling and T. Hohage. Variational source conditions and stability estimates for inverse electromagnetic medium scattering problems. *Inverse Problems and Imaging*, 11(1):203–220, 2017.

An externally controlled problem for diffusion equation with nonlocal Wentzell boundary condition

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We consider the heat equation

$$u_t = u_{xx} + F(x,t,u),\ (x,t) \in \Omega_T \tag{1}$$

with the initial condition

$$u(x,0) = \varphi(x),\ x \in [0,1], \tag{2}$$

Dirichlet boundary condition

$$u(0,t) = 0,\ t \in [0,T], \tag{3}$$

and nonlocal Wentzell-Neumann boundary condition

$$u_x(0,t) + \alpha u_{xx}(1,t) = 0,\ t \in [0,T] \tag{4}$$

for $\alpha > 0$, where $\Omega_T = \{(x,t) : 0 < x < 1,\ 0 < t \leq T\}$ with fixed $T > 0$. The function φ is given functions in $0 \leq x \leq 1$.

When the source term $F(x,t,u)$, $(x,t) \in \Omega_T$ is also given, the problem of finding $u(x,t)$ from using equation (1), initial condition (2) and boundary conditions (3) and (4) is termed as the direct (or forward) problem.

When the function $F(x,t,u)$, $(x,t) \in \Omega_T$ is unknown, the inverse problem is formulated as a problem of finding a pair of functions $\{r(t),\ u(x,t)\}$ which satisfy the equation (1), initial condition (2), boundary conditions (3) and (4) and overdetermination condition

$$\int_0^1 u(x,t)dx = E(t),\ t \in [0,T], \tag{5}$$

where $E(t)$ is a given function whilst the prescription of the total energy, or mass. The following inverse problems will be studied:

First inverse problem (IP1): In the case $F(x,t,u) = p(t)u + f(x,t)$, the inverse problem is formulated as the problem of finding the pair $\{u(x,t), p(t)\}$ from (1)-(5).

Second inverse problem (IP2): Let $F(x,t,u) = r(t)f(x,t)$, the inverse problem is formulated as the problem of finding the pair $\{u(x,t), r(t)\}$ from (1)-(5).

In contrast to direct problems, the inverse parabolic problems with dynamic (and the related general Wentzell) boundary conditions are scarce ([1 - 3]) and needs additional consideration.

Under some regularity, consistency and orthogonality conditions on initial data and source term, the existence, uniqueness and stability of the classical solution of IP1 and IP2 are shown by using the method of expansion in terms of eigenfunctions of auxiliary spectral problem:

$$\begin{cases} -y''(x) = \lambda y(x),\ 0 \le x \le 1, \\ y(0) = 0,\ y'(0) - \alpha\lambda y(1) = 0. \end{cases}$$

This problem is considered with $\alpha \neq \frac{1}{x_i \sin x_i}$ where x_i are the roots of the equation $sinx + xcosx = 0$ on $(0, +\infty)$. It is known from [4] that this problem has at most infinitely many complex eigenvalues and their numbers depend on α and the system of eigenfunctions with one deleted is a Riesz basis in $L_2[0,1]$.

Such type of inverse problems for the model of microwave heating gives an idea of how total energy content might be externally controlled. However, the dielectric constant of the target material varies in space and time, resulting in spatially heterogeneous conversion of electromagnetic energy to heat. This can correspond to source term $F = r(t)f(x,t)$, where $r(t)$ is proportional to power of external energy source and $f(x,t)$ is local conversion rate of microwave energy. In ths way, the external energy is supplied to a target at a controlled level by the microwave generating equipment.

References

[1] Kerimov, N. B., Ismailov, M. I., *Direct and inverse problems for the heat equation with a dynamic-type boundary condition*. IMA J. Appl. Math. 80 (5) (2015) 1519–1533.

[2] Hazanee A., Lesnic D., Ismailov M.I., Kerimov N.B. *An inverse time-dependent source problem for the heat equation with a non-classical buondary conditions*, Applied Mathematical Modelling, 2015, v. 39, p. 6258-6272.

[3] Slodicka, M., *A., parabolic inverse source problem with a dynamical boundary condition.* Appl. Math. Comput. 256 (2015), 529–539.

[4] Marchenkov, D.B., *On the convergence of spectral expansions of functions for a problem with a spectral parameter in the boundary condition.* Differential Equations, 41 (10) (2005), pp 1496–1500.

Steklov zeta-invariants and a compactness theorem for isospectral families of planar domains
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this is joint work with **Vladimir Sharafutdinov**,
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We address the question of determining a bounded smooth and simply connected planar domain Ω from the spectrum of its Dirichlet-to-Neumann operator (Steklov spectrum). We state two analog formulations of this problem. Then by studying the zeta function of the Steklov spectrum and the related zeta invariants [2] we show that isospectral sets are compact in the C^∞ topology up to natural gauge invariance, which generalizes a result in [1]. This is a joint work with Vladimir Sharafutdinov.

References

[1] J. Edward, *Pre-compactness of isospectral sets for the Neumann operator on planar domains*, Commun. in PDE's, **18** no. 7–8 (1993), 1249–1270.

[2] E. Mal'kovich and V. Sharafutdinov, *Zeta-invariants of the Steklov spectrum of a planar domain,* Siberian Math. J., 56 (2015), no. 4, 678–698

The spectral theory of the Neumann-Poincare operator and plasmon resonance
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Abstract: On the surface of dielectric materials with the negative dielectric constant a resonance occurs. This resonance is called the surface plasmon resonance and is underlying physical phenomenon of important imaging modalities such as SERS (surface enhanced Raman spectroscopy). It turns out that the Plasmon resonance is closely related to the spectrum of the Neumann-Poincare (NP) operator defined on the surface. In this talk I will explain the connection of the plasmon resonance and the spectrum of the NP operator and review recent development in the spectral theory of the NP operator. The NP spectrum is also closely related to the anomalous localized resonance which attracts much attention in connection with the invisibility cloaking. I will explain this as well.

Inverse problems for graphs

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This is a joint work with **Jan Boman** and **Rune Suhr** (Stockholm).

We prove a direct analog of the celebrated Ambartzumian theorem for quantum graphs - Schrödinger operators on metric graphs. The theorem is proven is full generality, namely it is shown that if the spectrum of a quantum graph coincides with the spectrum of the second derivative operator on a an interval with Neumann boundary conditions, then the underlying metric graph is the interval and the corresponding potential is identically zero.

Inverse boundary value problem for identifying elasticity tensor by boundary measurements

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The uniqueness of Calderon problem for three dimensional elasticity has not been solved even for isotropic elasticity in the static case. So far there is a partial result by Nakamura-Uhlmann and Eskin-Ralston for isotropic elasticity. In this talk I will consider the case that the elasticity tensor is piecewise constant or piecewise analytic. The anisotropy of elasticity tensor can be allowed in some cases.

Inverse scattering without phase information

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We consider direct and inverse scattering for the Schrödinger equation of quantum mechanics and for the Helmholtz equation of acoustics or electrodynamics. In addition, scattering data without phase information, only, can be measured directly in practice in quantum mechanics and in some other cases. In particular, in quantum mechanics this limitation is related with the probabilistic interpretation of the wavefunction, proposed originally by M. Born in 1926.

In this connection we report on non-uniqueness, uniqueness and reconstruction results for inverse scattering without phase information. We are motivated by recent and very essential progress in this domain. For more information we refer to [1], [3], [4] and references therein.

References

- [1] A.D. Agaltsov, R.G. Novikov, Error estimates for phaseless inverse scattering in the Born approximation at high energies, arXiv:1604.06555v2
- [2] A.D. Agaltsov, T. Hohage, R.G. Novikov, An iterative approach to monochromatic phaseless inverse scattering, in preparation.
- [3] M.V. Klibanov, V.G. Romanov, Reconstruction procedures for two inverse scattering problems without the phase information, SIAM J. Appl. Math. 76 (2016), no. 1, 178-196.
- [4] R.G. Novikov, Inverse scattering without phase information, Séminaire Laurent Schwartz - EDP et applications (2014-2015), Exp. No16, 13p., doi: 10.5802/slsedp.74

On the boundary rigidity problem for simple manifolds

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Let (M, g) be a smooth compact riemannian manifold, $\dim M \geq 2$, with smooth boundary ∂M, and d_g is the boundary distance function. We assume that (M, g) is simple , i.e. the boundary ∂M is strictly convex and exponential map $\exp_x : T_x \to M$ is a diffeomorphism for each point $x \in M$. R. Michel conjectured in [1] that simple manifolds are boundary distance rigid: that is d_g determines g up to an isometry which is the identity on the boundary.

This is known for simple subspaces of Euclidean space (see [2]), simple subspaces of an open hemisphere in two dimensions ([1]), simple subspaces of constant negative curvature ([3]), riemannian manifolds with metrics close to flat one ([5]), two dimensional simple manifolds ([6]). Recently, boundary rigidity were obtained in ([7] when there is a ctrictly convex function on (M, g).

In this talk we discuss Michel's conjecture for any dimensions using Jacobee fields.

Acknowledgements. *This work was supported by the Russian Science Foundation under grant 16-11-10027.*

References

[1] R.Michel, *Sur la rigite impose par la longueur des geodesiques.* Invent. Math. 65 (1981), no. 1, 71-84.

[2] M. Gromov, *Filling Riemannian manifolds.* J. Differential Geom. 33 (1991), 445-464.

[3] G.Besson, G. Courtois, and S. Gallot, *Entropies et rigidiies des espaces localement symetriques de courbure strictement negative.* Geom. Func. Anal. 5 (1995), 731-799. 1-41.

[4] C. Croke, *Rigidity for surface of nonpositive curvature.* Cooment. Math.Helv. 65 (1990), 150-169.

[5] A. D. Burago and S. Ivanov, *Boundary rigidity and filling volume minimality of metrics close to a flat one*, manuscript, 2005.

[6] L. Pestov, G. Uhlmann, *Two dimensional compact sipmle Riemannian manifolds are boundary distance rigid.* Annals of Math. 161 (2005), 1093-1110.

[7] P.Stefanov, G. Uhlmann, and A. Vasy, *Local and global boundaty rigidityand the geodesic X-ray transfrom in the normal guage*, preprint, 2017.

Artifacts in Arbitrary Limited Data Tomography Problems

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In this talk, we will describe work of the speaker along with Leise Borg, Jürgen Frikel, and Jakob Jørgensen characterizing how artifacts appear in limited data X-ray tomography with arbitrary data sets both when the boundary of the data set is smooth and when it is not smooth. We also provide estimates of the strength of the added artifacts in some cases, and we illustrate our results using standard and non-standard limited data tomography problems with real and simulated data.

We put our results in a general mathematical framework that can be used on a range of limited data problems in which the forward and reconstruction operators are Fourier integral operators. We outline the proof, which is based on microlocal analysis.

This work is motivated by an unusual synchrotron CT data set that has artifacts not seen in standard limited data problems.

The exterior Goursat problem for the wave equation

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If $u(x,t)$ is the solution of an IVP for the wave equation in $(x,t) \in \mathbb{R}^n \times \mathbb{R}$ then we consider the question of recovering the initial data from the trace of $u(x,t)$ on the characteristic cone $|t| = |x|$. We show that the map from the initial data to the trace on the characteristic cone is an isometry, we construct the inverse of this map, and we characterize the range. What is significant about this problem is that we are attempting to solve the characteristic BVP for the wave equation outside the characteristic cone.

The attenuated geodesic X-ray transform

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The standard X-ray transform, where one integrates functions over straight lines, is a well-studied object and forms the basis of medical imaging techniques such as CT and PET. This transform has useful generalizations involving other families of curves, weight factors, and integration of tensor fields. These more general transforms come up in seismic imaging (travel time tomography), in medical imaging (SPECT and ultrasound), and in the mathematical analysis of other inverse problems.

In this talk we discuss certain recent results related to attenuated geodesic ray transforms, including the case where the attenuation is given by a connection or Higgs field on a vector bundle. The talk is based on joint works with C. Guillarmou (Orsay), G. Paternain (Cambridge), G. Uhlmann (Washington), and H. Zhou (Cambridge).

Inverse scattering problems for perturbation of the bi-harmonic operator
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Some inverse scattering problems for operator of order 4 which is the perturbation of the bi-harmonic operator are considered. The classical scattering theory is developed. The uniqueness of these inverse problems is proved. The method of Born approximation and an analogue of Saito's formula are justified.

We deal with the operator of order 4 in the form

$$L_4u(x) := \Delta^2 u(x) + 2i\vec{W}(x)\nabla u(x) + i(\nabla\vec{W})u(x) + V(x)u(x), \quad x \in R^3,$$

with real-valued functions $\vec{W}$ and V. The first result is the analogue of Saito's formula for 4th order operator.
If $\vec{W} \in W^1_{p,\delta}(R^3)$ and $V \in L^p_\delta(R^3)$ where $3 < p \leq \infty$ and $\delta > 3 - \frac{3}{p}$, then the limit

$$\lim_{k\to+\infty} k^4 \int\limits_{S^2\times S^2} e^{-ik(\theta-\theta',x)} A(k,\theta,\theta')\, d\theta\, d\theta' = 8\pi^2 \int\limits_{R^3} \frac{V(y)}{|x-y|^2}\, dy$$

holds uniformly in $x \in R^3$.

An important consequence of Saito's formula is the uniqueness result for the inverse scattering problem with full data.

As a different data for the reconstruction of unknown potential $V(x)$ we consider the kernel $G_p(x,y,k)$ of the integral operator $(L_4 - k^4 - i0)^{-1}$. The knowledge of the function $G_p(x,y,k)$ for large values of x, y, k allow us to calculate at every point ξ the Fourier transform of V by the formula

$$F(V)(\xi) = \lim_{x,y\to\infty, k\to+\infty} 64\pi^2 k^4 |x||y| e^{-ik(|x|+|y|)} \left(G_k^+(|x-y|) - G_p(x,y,k)\right),$$

where $\xi = -k\left(\frac{x}{|x|} + \frac{y}{|y|}\right)$ and fixed.

Our next interest concerns to the particular case $\theta' = -\theta$. This case leads to the backscattering Born approximation.

The inverse backscattering Born approximation $V_B^b(x)$ in the operator L_4 is defined as

$$V_B^b(x) = \frac{1}{4(2\pi)^3} \int\limits_0^\infty k^4\, dk \int\limits_{S^2} e^{-ik(x,\theta)} A\left(\frac{k}{2}, \theta, -\theta\right) d\theta.$$

We obtain that

$$V_B^b(x) = V(x) + V_1(x) + V_{rest}(x),$$

where the quadratic form $V_1(x)$ is equal to

$$V_1(x) = -\frac{1}{(2\pi)^3} F^{-1}_{\xi\to x}\left(\int\limits_{R^3} \frac{F(\overline{\tilde{V}})(\xi-\eta)F(\tilde{V})(\eta)}{\xi^2(\eta^2-(\eta,\xi)-i0)(\eta^2-(\eta,\xi)+\frac{\xi^2}{2})}\,d\eta\right)+$$

$$+\frac{1}{4(2\pi)^3} F^{-1}_{\xi\to x}\left(\int\limits_{R^3} \frac{\xi F(\vec{W})(\xi-\eta)(\xi+\eta)F(\vec{W})(\eta)}{\xi^2(\eta^2-(\eta,\xi)-i0)(\eta^2-(\eta,\xi)+\frac{\xi^2}{2})}\,d\eta\right).$$

It turns out that V_1 is continuous and V_{rest} belongs to the Sobolev space $H^t(R^3)$ with any $t<\frac{3}{2}$. Thus, using the Born approximation we can reconstruct all local singularities from $L^p(R^3)$ of $V(x)$ for any $3<p<\infty$.

The Reshetnyak formula and Natterer stability estimates in tensor tomography
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The Reshetnyak formula states that the Radon transform R is an isometry between $L^2(\mathbb{R}^n)$ and a Hilbert space $H^{(n-1)/2}_{(n-1)/2,e}(\mathbb{S}^{n-1}\times\mathbb{R})$ of even functions on $\mathbb{S}^{n-1}\times\mathbb{R}$ furnished with some special norm. We generalize this result to Sobolev spaces: R is an isometry between $H^s(\mathbb{R}^n)$ and $H^{s+(n-1)/2}_{(n-1)/2,e}(\mathbb{S}^{n-1}\times\mathbb{R})$ for every real s. Moreover, using Riesz potentials, we define some new Hilbert spaces $H^s_t(\mathbb{R}^n)$ $(t>-n/2)$ and prove that R is an isometry between $H^s_t(\mathbb{R}^n)$ $H^{s+(n-1)/2}_{t+(n-1)/2,e}(\mathbb{S}^{n-1}\times\mathbb{R})$. The generalized Reshetnyak formula is closely related to the Natterer stability estimates: $c\|f\|_{H^s(\mathbb{R}^n)}\le\|Rf\|_{H^{s+(n-1)/2}(\mathbb{S}^{n-1}\times\mathbb{R})}\le C\|f\|_{H^s(\mathbb{R}^n)}$ for functions f supported in a fixed ball. Then we obtain analogies of these results for the X-ray transform of symmetric tensor fields.

Recovering of potentials in the scale of Sobolev spaces
from finite set of spectral data
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this is joint work with **A.M.Savchuk.**

We consider classical inverse problems associated with the Sturm-Liouvolle equation

$$Ly=-y''+q(x)y=\lambda y$$

on the finite interval $[0,1]$. Assume that a sequence $\{s_k\}_{k=1}^{\infty}$ corresponds to the spectral data associated with the recovering of the potential q by two spectra (Borg problem) or by the spectral function of the operator L. Let $\{s_k\}_{k=1}^{N}$ be a finite set of this sequence and q_N be the N-approximation for q, i.e. the potential which corresponds to the the sequence $\{s_1, s_2, \dots s_N, 0.0. \dots\}$. It turns out that the N-approximation q_N can be effectively constructed. One of the main results of the talk is the following:

Assume that $q \in W_2^{\theta}[0,1]$ *with some* $\theta > -1$, *where* W_2^{θ} *is the Sobolev space, and* $\|q\|_{\theta} \leqslant R$. *Then*

$$\|q - q_N\|_{\tau} \leqslant C N^{-(\theta-\tau)} \quad \text{for any } -1 \leqslant \tau < \theta,$$

with the constant C depending only on R, τ and θ but not depending on N and q.

To prove this result we need to establish some new results in the inverse spectral theory for the Sturm-Liouville equation and sone new results in the nonlinear interpolation theory having independent interst. These ones will be discussed in the talk.

Wave models of Sturm-Liouville operators
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For a semi-bounded symmetric operator the notion of the wave spectrum was introduced in [1]. The wave spectrum is a topological space determined by the operator in a canonical way. The definition uses a dynamical system associated with the operator: the wave spectrum is constructed from its reachable sets. We illustrate this notion by two examples: we give a description of the wave spectrum of two symmetric Sturm-Liouville operators defined by the differential expression $-\frac{d^2}{dx^2} + q$ considered on the half-line $(0, +\infty)$ and on the interval $(0, l)$, $0 < l < +\infty$. They act in the spaces $L_2(0, +\infty)$ and $L_2(0, l)$ and have defect indices $(1, 1)$ and $(2, 2)$, respectively. We construct functional wave models of these operators. In these models the elements of the original spaces are realized as functions on the wave spectra. These constructions can be used for solving inverse problems.

References

[1] M.I. Belishev, A unitary invariant of a semi-bounded operator, *J. Operator Theory*, **69**(2), 299–326 (2013), arXiv: 1208.3084.

[2] M.I. Belishev, S.A. Simonov, Wave model of the Sturm-Liouville operator on the half-line, *To appear in St. Petersburg Math. J.*, arXiv: 1703.00176.

D-bar method for inverse two-dimensional problems and its applications
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this is joint work with **Evgeny Lakshtanov**

We will discuss an inverse problem for the two-dimensional Schrodinger and Dirac equations with L^p-potentials. Using the $\overline{\partial}$-method, the potential is recovered from the Dirichlet-to-Neumann map on the boundary of a domain containing the support of the potential. We do not assume that the potential is small or that the Faddeev scattering problem does not have exceptional points. The results are applied to solve the inverse conductivity problem with minimal assumptions on smoothness of the conductivity and to construct solutions of non-linear equations such as the Davey-Stewartson II equation.

Let us focus only on the inverse problem for the Schrodinger equation:

$$(\Delta + E + v)u(x) = 0, \quad x \in \mathcal{O} \subset \mathbb{R}^2, \tag{1}$$

with a fixed positive value of energy E. Let $z = x_1 + ix_2$ and $\lambda \in \mathbb{C}' = \mathbb{C}\backslash(\{0\} \bigcup \{|\lambda| = 1\}$. The Faddeev scattering solutions $\psi(\lambda, z, E)$ are defined as solutions of the problem

$$(\Delta + E + v)\psi = 0, \quad x \in \mathbb{R}^2, \quad \psi e^{-i\frac{\sqrt{E}}{2}(\lambda\bar{z}+z/\lambda)} \to 1, \quad |z| \to \infty. \tag{2}$$

The unique solvability of the corresponding integral (Lippmann-Schwinger) equation may be violated for certain $\lambda \in \mathbb{C}'$. These points are called exceptional.

The scattering data $h(\varsigma, \lambda)$ of the Faddeev scattering problem is defined by the following formula involving the Cauchy data of $\psi(\lambda, z)$ on $\partial\mathcal{O}$:

$$h(\varsigma, \lambda) = \frac{1}{(2\pi)^2} \int_{\partial\mathcal{O}} [\psi(z, \lambda) \frac{\partial}{\partial\nu} e^{-i\frac{\sqrt{E}}{2}(\varsigma\bar{z}+z/\varsigma)} - e^{-i\frac{\sqrt{E}}{2}(\varsigma\bar{z}+z/\varsigma)} \frac{\partial}{\partial\nu}\psi(z, \lambda)] dl_z, \quad \varsigma, \lambda \in \mathbb{C}', \tag{3}$$

where ν is the outer unit normal to $\partial\mathcal{O}$.

The Dirichlet-to-Neumann map Λ_v immediately determines the Cauchy data of ψ:

$$\psi(z, \lambda) = (I + S_\lambda(\Lambda_v - \Lambda_0))^{-1} e^{i\frac{\sqrt{E}}{2}(\lambda\bar{z}+z/\lambda)}, \quad z \in \partial\mathcal{O}, \tag{4}$$

where S_λ is the single layer operator corresponding to Faddeev's Green function. Thus the scattering data (3) can be found through Λ_v. We will show that ψ can be recovered if h is known. Then the potential v can be found, for example from (2): $v(z) = -\frac{(\Delta+E)\psi}{\psi}$.

Function ψ could be determined from the following $\overline{\partial}$-equation:

$$\frac{\partial}{\partial\bar{\lambda}}\psi(z, \lambda) = r(\lambda)\psi\left(z, -\frac{1}{\bar{\lambda}}\right), \quad |\lambda| \neq 0, 1, \tag{5}$$

complemented by specific asymptotic behavior at infinity. Here r is defined by h:

$$r(\lambda) = \frac{\operatorname{sgn}(|\lambda|^2 - 1)\pi}{\overline{\lambda}} h(-\frac{1}{\overline{\lambda}}, \lambda).$$

This approach requires the existence of ψ and the validity of (5) for all the values of λ. So, until recently, the $\overline{\partial}$-method was restricted by the assumption of the absence of exceptional points. We will show how ψ can be found in the presence of exceptional points (or curves). A prototype of this approach for $E < 0$ was introduced by R. Novikov. More details and applications can be found in [1]-[4].

References

[1] Lakshtanov, E., Vainberg, B., (2016), On reconstruction of complex-valued once differentiable conductivities, J. Spectr. Theory 6, 4, 881-902, arXiv:1511.08780

[2] Lakshtanov E., Novikov R., Vainberg B. (2016), A global Riemann-Hilbert problem for two-dimensional inverse scattering at fixed energy, Rend. Istit. Mat. Univ. Trieste, 48, 1-27, arXiv:1509.06495

[3] Lakshtanov, E., Vainberg, B., (2016), Solution of the initial value problem for the focusing Davey-Stewartson II system, arXiv:1604.01182.

[4] Lakshtanov, E., Vainberg, B., (2016), Recovery of L^p-potential in the plane, Journal of Inverse and Ill-Posed Problems, accepted, arXiv:1608.05807