Function theory and dynamics of point processes

June 1-3, 2017

Yacin Ameur: The two-dimensional Coulomb plasma

Alexey Bufetov: Hall-Littlewood processes and stochastic vertex models

I will discuss recently found connections between vertex models (such as the six-vertex/square ice model) and theory of symmetric functions (in particular, Hall-Littlewood functions). There are two types of applications. On a probabilistic side, these connections allow to analyze the asymptotic behavior of certain vertex models (as well as their degenerations like the asymptotic simple exclusion process) with the use of explicit formulas for their observables. On a combinatorial side, we obtain a natural generalization of the Robinson-Shensted-Knuth algorithm.

Based on joint works with A. Borodin, K. Matveev, M. Wheeler.

Andrey Dymov: A functional limit theorem for the sine-process

It is well-known that a large class of determinantal processes including the sine-process satisfies the Central Limit Theorem. For many dynamical systems satisfying the CLT the Donsker Invariance Principle also takes place. The latter states that, in some appropriate sense, trajectories of the system can be approximated by trajectories of the Brownian motion. I will present results of my joint work with A. Bufetov, where we prove a functional limit theorem for the sine-process, which turns out to be very different from the Donsker Invariance Principle. We show that the anti-derivative of our process can be approximated by the sum of a linear Gaussian process and small independent Gaussian fluctuations whose covariance matrix we compute explicitly.

Subhro Ghosh: Large holes in particle systems : forbidden regions, large deviations and potential theory

In particles systems, a "hole" of size R is defined to be a ball of radius R that is devoid of particles. The study of how the probability of having such a hoe decays to 0 (as R -> infty) is an important and well-studied question in particle systems.

In this talk, we ask what causes a large hole to appear ? In other words, conditioned on having a large hole, how does the configuration of particles outside the hole look like ? Surprisingly, very little is understood about this question, except in the very special case of Gaussian random matrices, where there is an accumulation of particles at the edge of the hole, and equilibrium intensity beyond.

We study this question in the context of zeros of Gaussian random polynomials, and provide a complete description of the intensity profile of the outside particles. A remarkable feature that we find is the appearance of a curious "forbidden region" between the accumulation at the edge of the hole and the equilibrium intensity far beyond. This is in stark contrast to the case of Gaussian random matrices, and seems to be novel even in the wider setting of statistical physics models. Our methods connect to large deviation principles for random polynomials, potential theory and constrained optimization on measure spaces. These ideas can also be applied to other problems, including Jancovici-Lebowitz-Manificat laws for Coulomb systems at general temperatures, and understanding over and under-crowding phenomena for Gaussian zeros. Based on joint works with Alon Nishry.

Haakan Hedenmalm: Bloch functions, asymptotic variance, and geometric zero packing

Andrey Malyutin: The absolute boundary of finitely generated groups.

A.M.Vershik introduced the notion of absolute boundary (also called the ŚabsoluteŠ) for finitely generated groups. The absolute boundary of a group is a topological space that can be regarded as the boundary at infinity (DynkinŠs exit-boundary) of the so-called dynamical graph over the Cayley graph of the group. The absolute boundary contains, in a sense, the Poisson-Furstenberg boundary of the group and is contained in the Martin boundary of the dynamical graph. A part of the absolute boundary can be identified with the set of all minimal positive eigenfunctions of the Laplacian determined by the simple random walk on the group. The absolute boundary of an abelian group is homeomorphic to a closed ball of certain dimension. (The fact that the absolute boundary of the infinite cyclic group is an interval is a reformulation of de Finetti's theorem.) The absolute boundary of the free non-abelian group is homeomorphic to the direct product of the Cantor set by an interval. We will discuss basic notions, examples, and resent developments in the theory of absolute boundary.
The talk is based on joint work with A.M.Vershik.

Alexey Naumov: On the local laws for non-nermitian random matrices

We prove the local circular law for non-Hermitian random matrices and at its generalisation to the products of independent non-Hermitian matrices under weak moment conditions. Our results generalise recent results of P. Bourgade, H.-T. Yau, J. Yin and Y. Nemish. We apply Stein's method and some new ideas which help to simplify the proof of the local laws. The talk will be based on joint results with F. Goetze and A. Tikhomirov.

Nikolai Nessonov: The actions of the full symmetric group on von Neumann algebras

Let M be separable w^*-algebra, and let $\operatorname{Aut} M$ be the automorphism group of M. Denote by $\overline{S}_\infty$ the infinite full symmetric group. Suppose that $\alpha : \overline{S}_\infty \mapsto \operatorname{Aut} M$ is the action of $\overline{S}_\infty$ on M. We will prove that there exists normal α-invariant weight on M.

Tomoyuki Shirai: Determinantal point processes associated with extended kernels and uniform spanning trees on series-parallel graphs

For the existence of determinantal point processes(DPPs), there is a well-known useful condition for self-adjoint kernels, but there is no such a condition for non-self-adjoint kernels. On the other hand, several models such as non-colliding diffusions (random walks), extended sine/Bessel/Airy processes etc. can be described by DPPs associated with extended non-self-adjoint kernels. In this talk, we will discuss its framework and another example related to uniform spanning trees.

Nick Simm: Subcritical multiplicative chaos for regularized counting statistics in the CUE

Abstract: The CUE (Circular Unitary Ensemble) is the group of all $N \times N$ unitary matrices U_N equipped with the uniform (Haar) measure. A suitable counting statistic (or 'height function') can be constructed by counting the number of eigenvalues of U_N in some arc of the unit circle, which we regularize at an N-dependent scale. We show that the exponential of the latter statistic (suitably centered and normalized) converges to a Gaussian multiplicative chaos measure in the entire sub-critical phase as $N \to \infty$. We also compute moments of an associated partition function, viewing the height function as a random Hamiltonian (energy), and prove that all positive integer moments of this partition function converge to Selberg integrals as $N \to \infty$.
This is joint work with Gaultier Lambert and Dmitry Ostrovsky.
(https://arxiv.org/abs/1612.02367)

Yanqi Qiu: Determinantal point processes governed by Bergman kernels

Two important examples of the determinantal point processes governed by Bergman type kernels are the Ginibre point process from random matrix theory and the set of zeros of the Gaussian Analytic Functions on the unit disk. The main objets of this talk are such class of determinantal point processes in greater generality. We are going to discuss the quasi-invariance of these point processes under certain natural group action of the group of compactly supported diffeomorphisms of the phase space. This talk is based partly on the joint works with Alexander I. Bufetov, also with Alexander I. Bufetov, Alexander Shamov and partly on a more recent joint work with Alexander I. Bufetov and Shilei Fan.

Anatoly Vershik: The coding of Bernoulli schemes with Young tableaux and Schutzenberger's Jeu de taquin

Dmitry Zaporozhets: Correlations between conjugate algebraic numbers: a random polynomial approach

We describe the asymptotic distribution of all algebraic numbers of fixed degree and large height in terms of correlation functions of a point process of zeros of some random polynomial.

Based on a joint work with Friedrich Goetze and Denis Koleda:
https://arxiv.org/abs/1703.02289