"Bernoulli and Plancherel measures, RSK and Schutzenberger dynamics"

ANATOLII M. VERSHIK (ST. PETERSBURG DEPT. OF THE MATHEMATICAL INSTITUTE OF RUSSIAN ACADEMY OF SCIENCES, MATHEMATICAL DEPARTMENT OF ST.PETERSBURG UNIVERSITY, MOSCOW INSTITUTE OF THE PROBLEMS OF TRANSMISSION INFORMATION)

August 6, 2017

CONTENT

1.Regular Enumerations of tableaux of the set of points of locally finite posets, and central probability measures.
Example:$D = \mathbb{Z}_+^2$, Hasse diagram is the Young graph and central measures.

CONTENT

1.Regular Enumerations of tableaux of the set of points of locally finite posets, and central probability measures.
Example: $D = \mathbb{Z}_+^2$, Hasse diagram is the Young graph and central measures.
2. Theorem (VK 1987) Infinite RSK as homomorphism of Bernoulli measure onto Plancherel measures. Several proofs.

CONTENT

1.Regular Enumerations of tableaux of the set of points of locally finite posets, and central probability measures.
Example:$D = \mathbb{Z}_+^2$, Hasse diagram is the Young graph and central measures.
2. Theorem (VK 1987) Infinite RSK as homomorphism of Bernoulli measure onto Plancherel measures. Several proofs.
3.M.Schitzenberger "Jeu de Taquin". Main lemma. and infinite shift "Sch". Invarinat measures. Theorem (Sniady-Romic - 2014) about isomorphism.

$$(X, \mu, S) \approx (\mathbb{T}(\mathbb{Y}), Pl, Sch).$$

Comments.

CONTENT

1.Regular Enumerations of tableaux of the set of points of locally finite posets, and central probability measures.
Example:$D = \mathbb{Z}_+^2$, Hasse diagram is the Young graph and central measures.
2. Theorem (VK 1987) Infinite RSK as homomorphism of Bernoulli measure onto Plancherel measures. Several proofs.
3.M.Schitzenberger "Jeu de Taquin". Main lemma. and infinite shift "Sch". Invarinat measures. Theorem (Sniady-Romic - 2014) about isomorphism.

$$(X, \mu, S) \approx (\mathbb{T}(\mathbb{Y}), Pl, Sch).$$

Comments.
4.Theorem about "Nerve of tableau". Connection with theorem about limit shape (S-R). Tree of the nerves. Remark about isomorphism and connection with theory of filtrations.

CONTENT

1.Regular Enumerations of tableaux of the set of points of locally finite posets, and central probability measures.
Example:$D = \mathbb{Z}_+^2$, Hasse diagram is the Young graph and central measures.
2. Theorem (VK 1987) Infinite RSK as homomorphism of Bernoulli measure onto Plancherel measures. Several proofs.
3.M.Schitzenberger "Jeu de Taquin". Main lemma. and infinite shift "Sch". Invarinat measures. Theorem (Sniady-Romic - 2014) about isomorphism.

$$(X, \mu, S) \approx (\mathbb{T}(\mathbb{Y}), Pl, Sch).$$

Comments.
4.Theorem about "Nerve of tableau". Connection with theorem about limit shape (S-R). Tree of the nerves. Remark about isomorphism and connection with theory of filtrations.
5. Possible generalization for higher dimension in combinatorics. How to extend these results on the general lattices $\mathbb{Z}_+^d$? Does thee exist the generalization of RSK?

EXAMPLE

EXAMPLE

Evolution of the simplest numeration (Dynamics on Young tableaux).

EXAMPLE

Evolution of the simplest numeration (Dynamics on Young tableaux).

"Diagonal numeration of the quadrant:

EXAMPLE

Evolution of the simplest numeration (Dynamics on Young tableaux).

"Diagonal numeration of the quadrant:

$$T = \begin{pmatrix} 1 & 2 & 4 & 7 & 11 \\ 3 & 5 & 8 & 12 & 17 \\ 6 & 9 & 13 & 18 & 24 \\ 10 & 14 & 19 & 25 & 32 \\ 15 & 20 & 26 & 33 & 41 \end{pmatrix}$$

EXAMPLE

Evolution of the simplest numeration (Dynamics on Young tableaux).

"Diagonal numeration of the quadrant:

$$T = \begin{pmatrix} 1 & 2 & 4 & 7 & 11 \\ 3 & 5 & 8 & 12 & 17 \\ 6 & 9 & 13 & 18 & 24 \\ 10 & 14 & 19 & 25 & 32 \\ 15 & 20 & 26 & 33 & 41 \end{pmatrix}$$

$$Sch \cdot T = \begin{pmatrix} 1 & 3 & 6 & 10 & 15 \\ 2 & 4 & 7 & 11 & 16 \\ 5 & 8 & 12 & 17 & 23 \\ 9 & 13 & 18 & 24 & 31 \\ 14 & 19 & 25 & 32 & 40 \end{pmatrix}$$

EXAMPLE

Evolution of the simplest numeration (Dynamics on Young tableaux).

”Diagonal numeration of the quadrant:

$$T = \begin{pmatrix} 1 & 2 & 4 & 7 & 11 \\ 3 & 5 & 8 & 12 & 17 \\ 6 & 9 & 13 & 18 & 24 \\ 10 & 14 & 19 & 25 & 32 \\ 15 & 20 & 26 & 33 & 41 \end{pmatrix}$$

$$Sch \cdot T = \begin{pmatrix} 1 & 3 & 6 & 10 & 15 \\ 2 & 4 & 7 & 11 & 16 \\ 5 & 8 & 12 & 17 & 23 \\ 9 & 13 & 18 & 24 & 31 \\ 14 & 19 & 25 & 32 & 40 \end{pmatrix}$$

$$Sch^2 \cdot T = \begin{pmatrix} 1 & 2 & 5 & 9 & 14 \\ 3 & 6 & 10 & 15 & 21 \\ 4 & 7 & 11 & 16 & 22 \\ 8 & 12 & 17 & 23 & 30 \end{pmatrix}$$

Continuation

Continuation

$$Sch^{14} \cdot T = \begin{pmatrix} 1 & 2 & 8 & 15 & 23 \\ 3 & 9 & 16 & 24 & 33 \\ 4 & 10 & 17 & 25 & 34 \\ 5 & 11 & 18 & 26 & 35 \\ 6 & 12 & 19 & 27 & 36 \end{pmatrix}$$

Continuation

$$Sch^{14} \cdot T = \begin{pmatrix} 1 & 2 & 8 & 15 & 23 \\ 3 & 9 & 16 & 24 & 33 \\ 4 & 10 & 17 & 25 & 34 \\ 5 & 11 & 18 & 26 & 35 \\ 6 & 12 & 19 & 27 & 36 \end{pmatrix}$$

$$Sch^{15} \cdot T = \begin{pmatrix} 1 & 7 & 14 & 22 & 31 \\ 2 & 8 & 15 & 23 & 32 \\ 3 & 9 & 16 & 24 & 33 \\ 4 & 10 & 17 & 25 & 34 \\ 5 & 11 & 18 & 26 & 35 \end{pmatrix}$$

Continuation

$$Sch^{14} \cdot T = \begin{pmatrix} 1 & 2 & 8 & 15 & 23 \\ 3 & 9 & 16 & 24 & 33 \\ 4 & 10 & 17 & 25 & 34 \\ 5 & 11 & 18 & 26 & 35 \\ 6 & 12 & 19 & 27 & 36 \end{pmatrix}$$

$$Sch^{15} \cdot T = \begin{pmatrix} 1 & 7 & 14 & 22 & 31 \\ 2 & 8 & 15 & 23 & 32 \\ 3 & 9 & 16 & 24 & 33 \\ 4 & 10 & 17 & 25 & 34 \\ 5 & 11 & 18 & 26 & 35 \end{pmatrix}$$

At the last tableau the 6-th row is:
6 12 19 27 36 46
and 6-th column is: 41 42 43 44 45.
So, the fragment 66 has exactly vertical numeration. Out of fragment the diagonal lines took place.