Conference

August 7–11

August 7, Monday

09:00–09:55 *Registration*

09:55–10:00 *Welcome word*

10:00–10:45 **A. Vershik**
New results about limit shape of Young diagrams and its generalizations

10:45–11:15 *Coffee break*

11:15–12:00 **C. Boutillier**
Elliptic combinatorics and integrable statistical mechanics on isoradial graphs

12:00–12:30 **K. Izyurov**
Scaling limits of critical Ising correlations: convergence, fusion rules, applications to SLE

12:30–14:30 *Lunch*

14:30–15:15 **E. Sklyanin**
Dunkl operators as Lax matrices

15:15–15:45 *Coffee break*

15:45–16:30 **K. Eloranta**
The bounded 19-vertex Model

16:30–17:00 **A. Knizel**
Gap probabilities in tiling models and Painlevé equations

17:30–20:00 *Welcome party*

August 8, Tuesday

10:00–10:45 **A. Klümper**
Thermodynamics, contact and density profiles of the repulsive Gaudin-Yang model

10:45–11:15 *Coffee break*

11:15–12:00 **V. Terras**
Heisenberg spin chains by separation of variables: recent advances

12:00–12:30 **A. Sridhar**
TBA

12:30–14:30 *Lunch*

14:30–15:15 **R. Longo**
Standard subspaces and infinite spin particles

15:15–15:45 *Coffee break*

15:45–16:30 **S. Shlosman**
Plane partitions and their pedestal polynomials

16:30–17:00 **G. Schrader**
Towards a modular functor from quantum higher Teichmuller theory II: cutting and gluing

August 9, Wednesday

10:00–10:45 **N. Gromov**
New construction of eigenstates and separation of variables for $SU(N)$ quantum spin chains

10:45–11:15 *Coffee break*

11:15–12:00 **A. Belavin**
Superstring compactification and Frobenius manifold structures

12:00–12:30 **V. Tarasov**
TBA

12:30–14:30 *Lunch*

14:30–15:15 **G. Olshansky**
The infinite-dimensional q-Beta distribution

15:15–15:45 *Coffee break*

15:45–16:30 **J. Stokman**
Boundary correlation functions

16:30–17:00 **O. Postnova**
Path model for decomposition of the graded tensor powers into Weyl modules

18:00–20:30 *Conference dinner*

August 10, Thursday

10:00–12:30 *Bus trip*

12:30–14:30 *Lunch*

14:30–15:00 **A. Lodkin**
Mathematical explanation of the phyllotaxis

15:00–15:30 **A. Minabutdinov**
Limiting curves for a class ofself-similaradic transformations

15:30–16:00 *Coffee break*

16:00–16:30 **A. Shapiro**
Towards a modular functor from quantum higher Teichmuller theory I: the quantum group

16:30–17:00 **T. Salnikova**
Non-equilibrium thermodynamics in the Poincaré cycles

August 11, Friday

10:00–10:45 **A. Bobenko**
Discrete pluri-Lagrangian systems and discrete pluri-harmonic functions

10:45–11:15 *Coffee break*

11:15–12:00 **V. Gorbunov**
Quantum integrable systems in cohomology

12:00–12:30 **S. Ramassamy**
Miquel dynamics for circle patterns

12:30–14:30 *Lunch*

14:30–15:00 **P. Gavrylenko**
General Painlevé VI tau-function as Fredholm determinant

15:00–16:30 **A. Shchechkin**
Backlund transformation of Painleve tau function from representation theory

15:30–16:00 *Coffee break*

16:00–16:30 **A. Liashyk**
Sum formula for scalar product of off-shell Bethe vectors

Titles and Abstracts of Talks

Alexander Belavin (Landau Institute)

Superstring compactification and Frobenius manifold structures

For finding Effective Lagrangian which appears after the compactification of Superstring theory on a Calabi-Yau (CY) manifold we need to know the so called Special Kahler geometry on the Moduli space of the CY manifold. We present a simple way for computing the Special Kahler metric. This method uses the fact that the Moduli space is a subspace of the Frobenius manifold connected with the given CY manifold. Due to this fact we are able to calculate two special basises of periods of the CY holomorphic 3-form. The knowledge these basises together with the holomorphic metric on the Frobenius manifold makes it possible finding the exact expression for Kahler metric on the Moduli space.

Alexander Bobenko (TU, Berlin)

Discrete pluri-Lagrangian systems and discrete pluri-harmonic functions

Pluri-Lagrangian systems are variational systems with the multi-dimensional consistency property closely related to integrability (or even understood as integrability). We investigate discrete 2-dimensional linear pluri-Lagrangian systems, i.e. those with quadratic Lagrangians. The action is a discrete analogue of the Dirichlet energy, and solutions are called discrete pluri-harmonic functions. We classify linear pluri-Lagrangian systems with Lagrangians depending on diagonals. They are described by generalizations of the star-triangle map. Relation to discrete complex analysis and discrete Riemann surfaces is explained. Examples of more general Lagrangians are also considered.

Cédric Boutillier (Universit Pierre et Marie Curie)

Elliptic combinatorics and integrable statistical mechanics on isoradial graphs

Isoradial graphs are planar graphs embedded in such a way that all bounded faces are inscribed in a circle of radius 1. After a presentation of some properties of these graphs, we will introduce a one-parameter family of massive Laplacians on these graphs, for which conductances and masses are expressed in terms of elliptic functions. We will discuss some integrability properties of these operators, and their consequences for related models in statistical mechanics, namely spanning forests and the Ising model. This is joint work with Batrice de Tilire and Kilian Raschel.

Kari Eloranta (University of Helsinki)

The bounded 19-vertex Model

We investigate the 19-vertex model in a bounded domain on the square lattice. By establishing its elementary actions and the configuration space irreduciblity one can proceed to study the dynamic version of the Model. Through parametrization of the actions one obtains the Ice Model, its dilute relatives and more exotic beasts. With domain wall boundary conditions in a square they seem to generically exhibit non-trivial limit shapes. The action distributions reveal some intriguing entropy geometry.

Pavlo Gavrylenko (Higher School of Economics)

General Painlevé VI tau-function as Fredholm determinant

In this talk I will consider first the toy example of exactly computable Fredholm determinant which gives normalization constant for z-measure. Then I will demonstrate that this determinant has natural matrix generalization which produces the general tau-function of Painlev VI equation. I will also demonstrate that combinatorial structure which arises in the minor expansion of this determinant consists of Nekrasov partition functions and is the natural generalization of z-measure to the case of pair of Young diagrams and extra integer number.

Vasily Gorbunov (University of Aberdeen)

Quantum integrable systems in cohomology

In the talk we will describe a new feature of the classical Schubert calculus which holds for all types of the classical Lie groups. As the main example we will use the type A Grassmanians. The usual definition of the Schubert cycles involves a choice of a parameter, namely a choice of a full flag. Studying the dependence of the construction of the Schubert cycles on these parameters in the equivariant cohomology leads to an interesting solution to the quantum Yang Baxter equation. This connects the Schubert calculus to the theory of quantum integrable systems. We show the above solution is the 'Baxterization' of the natural action of the nil Coxeter algebra of Bernstein-Gelfand-Gelfand-Demazure difference operators in the equivariant cohomology and study the appropriate quantum integrable system.

Nikolay Gromov (Kings College London)

New construction of eigenstates and separation of variables for $SU(N)$ quantum spin chains

We present a new way to construct eigenstates of integrable XXX quantum spin chains with $SU(N)$symmetry which bypasses the nesting procedure. The states are built by repeatedly acting on the vacuum with a single operator $B_{\text{good}}(u)$ evaluated at the Bethe roots. Our proposal serves as a compact alternative to the usual nested algebraic Bethe ansatz, which produces the states with factorial complexity. Furthermore, the roots of this operator give the separated variables of the model, explicitly generalizing Sklyanin's approach to the $SU(N)$ case. We present many tests of the conjecture and prove it in several special cases. We focus on rational spin chains with fundamental representation at each site, but expect that our main results are valid more generally.

Konstantin Izyurov (University of Helsinki)

Scaling limits of critical Ising correlations: convergence, fusion rules, applications to SLE

We prove convergence to conformally covariant scaling limits for a family of observables in the critical 2D Ising model, including spins, energies, disorders and fermions. We also check that the limits satisfy fusion rules as predicted by Conformal Field Theory. I will also explain how to apply these results to deduce convergence of Ising interfaces to SLE(3) variants in a general setting.

Andreas Kluemper (Wuppertal University)

Thermodynamics, contact and density profiles of the repulsive Gaudin-Yang model

We address the problem of computing the thermodynamic properties of the repulsive one-dimensional two-component Fermi gas with contact interaction (Gaudin-Yang model). We derive an exact system of only two non-linear integral equations for the thermodynamics of the homogeneous model. This system allows for an easy and extremely accurate calculation of thermodynamic properties circumventing the difficulties associated with the truncation of the thermodynamic Bethe ansatz system of equations. We present extensive results for the densities, polarization, magnetic susceptibility, specific heat, interaction energy, Tan contact and local correlation function of opposite spins. Our results show that at low and intermediate temperatures the experimentally accessible contact is a non-monotonic function of the coupling strength. As a function of the temperature the contact presents a pronounced local minimum in the Tonks-Girardeau regime which signals an abrupt change of the momentum distribution in a small interval of temperature. The density profiles of the system in the presence of a harmonic trapping potential are computed using the exact solution of the homogeneous model coupled with the local density approximation. At finite temperature the density profile presents a double shell structure (partially polarized centre and fully polarized wings) only when the polarization in the center of the trap is above a critical value.

Alisa Knizel (Columbia University)

Gap probabilities in tiling models and Painlevé equations

We consider tilings of a hexagon by rhombi, viewed as 3D random stepped surfaces with a measure proportional to q^{Volume}. Such model is closely related to q-Hahn orthogonal polynomial ensembles, and we use this connection to obtain results about the local behavior of this model. In terms of the q-Hahn orthogonal polynomial ensemble, our goal is to show that the one-interval gap probability function can be expressed through a solution of the asymmetric q-Painlevé V equation.

Andrew Liashyk

Sum formula for scalar product of off-shell Bethe vectors

In the talk we will describe scalar products of Bethe vectors in the models solvable by the nested algebraic Bethe ansatz and described by $gl(m|n)$ superalgebra. Using coproduct properties of the Bethe vectors we obtain a sum formula for their scalar products. This formula describes the scalar product in terms of a sum over partitions of Bethe parameters. In addition, we show that when the Bethe vectors are on-shell, their norm takes the form of a Gaudin determinant.

Andrey Lodkin (St. Petersburg State University)

Mathematical explanation of the phyllotaxis

The regular spiral arrangement of various parts of biological objects (leaves, branches, florets, seeds, etc.), known as phyllotaxis, could not find an explanation during several centuries (Leonardo, Kepler, Goethe, ...). Some quantitative parameters of the phyllotaxis (the divergence angle being the principal one) show that the organization in question is, in a sense, the same in the large family of living objects, and the divergence angle is mainly close to the golden number. This was a mystery, and numerous explanations of this phenomenon remained "lyrical". Later, similar patterns were discovered in inorganic objects (L. Levitov, S. Douady & Y. Couder, et al). After a series of computer models, it was only in the XXI century that the rigorous explanation of the appearance of the golden number in a simple mathematical model

has been given. The resulting pattern is related to stable fixed points of an operator acting on a finite-dimensional torus and depends on a real parameter. The variation of this parameter leads to an interesting bifurcation diagram (actually, a binary tree) where the limiting object (a kind of the boundary of the tree) is the $SL(2, Z)$-orbit of the golden number on the segment [0,1].

Roberto Longo (University of Rome Tor Vergta)

Standard subspaces and infinite spin particles

The family of Real Hilbert subspaces of a complex Hilbert Hilbert space shows up a surprisingly rich structure. Among various applications, we focus here on the analysis of the possible localisation of infinite spin particles, and its general consequences in QFT.

Alexey Minabutdinov (Higher School of Economics)

Limiting curves for a class ofself-similaradic transformations

We prove the existence of limiting curves resulting from deviations of partial sums in the ergodic theorem in the case of polynomial adic systems (generalized odometers) and cylindrical functions. These systems are related to self-similar graphs. For a general ergodic measure-preserving transformation and a summable function we give a necessary condition for a limiting curve to exist. Our work generalizes results by É. Janvresse, T. de la Rue and Y. Velenik and answers several questions from their work.

Grigori Olshanski (IITP, Skoltech, and NRU HSE, Moscow)

The infinite-dimensional q-Beta distribution

The N-dimensional q-Beta distribution serves as the weight measure for a family of N-variate symmetric orthogonal polynomials introduced by Stokman and Koornwinder (1997). This distribution is a (q, t)-analogue of the measure appearing in Selbergs integral. I will describe a large-N limit transition which leads to an infinite-dimensional version of Stokman-Koornwinders big q-Jacobi orthogonal polynomials and their weight measure. The limit weight measure gives rise to a lattice random point process with infinitely many interacting particles.

Olga Postnova (St. Petersburg State University)

Path model for decomposition of the graded tensor powers into Weyl modules

We study modules of current algebra that appear as Feigin-Loktev graded tensor product of simple Lie algebra representations.We present a model for the graded characters of tensor product modules of simple Lie algebras that is based on the counting of paths on Pascal pyramids. Using this model we propose a method to decompose the graded tensor powers into combination of Weyl modules. This talk is based on joint work with Anton Nazarov.

Sanjay Ramassamy (Ecole normale superieure de Lyon)

Miquel dynamics for circle patterns

A circle pattern with the combinatorics of the square grid is an embedding of the square grid such that every face admits a circumcircle. Using Miquels six circles theorem, we define a discrete-time dynamical system on the space of such circle patterns on a flat torus. I will describe some properties of this dynamics that suggest it may be an integrable system.

Anton Shchechkin (Skoltech, NRU HSE)

Backlund transformation of Painleve tau function from representation theory

In this talk I present results about bilinear relations on conformal blocks which refer to Painleve tau functions and it's Backlund transformations. At first I very briefly talk about Painleve equations and their connection to CFT. Then I present important representation-theoretic tool — decomposition of the Super Virasoro Verma modules into direct sum of tensor product of two Virasoro Verma modules. Finally I use this tool to obtain some relation on conformal blocks and τ functions.

Tatiana Salnikova (Moscow State University)

Non-equilibrium thermodynamics in the Poincar cycles

Henri Poincaré considered a one-dimensional ideal gas uniformly filling an interval. The ideal gas is considered as a system of noninteracting particles. In particular, they cannot collide with each other. Each particle of this medium moves inertially, independently of the other particles, reflecting elastically from the boundaries of this interval. Poincarés basic observation was that, independently of the initial distribution, gas eventually tends to uniform filling of interval. Thus, the ideal gas shows the irreversible behavior. Every particle of the gas approaches arbitrary close to the initial position infinitely many times. However, such individual returnability is not uniform, which results in a diffusion in a reversible and conservative system. Thus, the compatibility of the reversibility and retainability properties with irreversible behavior of a dynamical system was shown.
Lets consider an equilibrium of a one-dimensional ideal gas. A gravitating body approaches the interval from infinity, the gas is allowed to attain a new equilibrium, after which the body recedes back to infinity. After that, the gas tends to fill the interval uniformly. So, the colisionless gas has performed a closed cycle. This cycle is defined by V.V.Kozlov as a Poincaré cycle (similarly to the Carnot cycle). But in contrast to the Carnot cycle, the Poincaré cycle is non-equilibrium and irreversible.
We take the initial velocity distribution density proportional to its square. In particular, Maxwell distribution doesnt satisfy this condition. We add the force field and wait for attaining equilibrium. Then we remove the force field and again wait for attaining equilibrium. Modeling many times these cycles, we obtain the different intermediate types of the density and the asymptotic behavior.

Gus Schrader (University of California at Berkeley)

Towards a modular functor from quantum Teichmuller theory II: cutting and gluing

Fock and Goncharov's modular functor conjecture in quantum higher Teichmuller theory is closely related to another conjecture in the representation theory of the quantum group $U_q(sl_n)$. The latter asserts that a certain category of representations of the quantum group, namely its category of positive representations, is closed under tensor product. We will outline a proof of the latter conjecture and argue that it gives a key step towards the proof of the former one. The proof will be based on the cluster realization of the quantum group presented in the talk by A. Shapiro.

Alexander Shapiro (University of Toronto)

Towards a modular functor from quantum higher Teichmuller theory I: the quantum group

We will discuss a new realization of quantum groups. Namely, we show that the quantum group $U_q(sl_n)$ admits an embedding into a quantum cluster algebra associated to the moduli space of framed PGL_n local systems on a punctured disk with two marked points on its boundary. This embedding allows us to give a geometric realization of the R-matrix as the generator of the mapping class group of a twice punctured disk with two boundary marked points.

Evgeny Sklyanin (University of York)

Dunkl operators as Lax matrices

The striking similarity between Dunkl operators and Lax matrices for the Calogero-Moser models has been apparent for many years. However, only recently the exact nature of the correspondence has been revealed. The key trick is to restrict the Lax operator on the space of vector-valued functions having a special symmetry. Then the Lax operator becomes equivalent to the Dunkl operator, more precisely, Dunkl-Heckman operators that do not commute but are covariant relative to the Weyl group. We apply the same recipe to the relativistic Ruijsenaars-Schneider model and Cherednik operators and derive the quantum Lax operator for the Macdonald polynomials. The results are used then to compute the Lax matrix in the infinite N limit.

Senya Shlosman (Centre de Physique Theorique, CNRS)

Plane partitions and their pedestal polynomials

I will explain the construction of some multivariate polynomials, which generalize the hook polynomial. The following topics will be discussed: 1) the definition; 2) the relation to MacMahon generating functions; 3) other stuff. This talk is based on joint work with Oleg Ogievetsky.

Jasper Stokman (University of Amsterdam)

Boundary correlation functions

I will introduce a boundary version of the system of Knizhnik-Zamolodchikov-Bernard (KZB) equations. Besides Felder's classical dynamical r-matrix, the equations involve new solutions of the associated dynamical reflection equation. I will construct solutions of the boundary KZB equations, called boundary correlation functions, as matrix coefficients of products of vertex operators. This talk is based on joint work with Nicolai Reshetikhin.

Veronique Terras (CNRS)

Heisenberg spin chains by separation of variables: recent advances

We review recent advances concerning the solution of different variants of Heisenberg spin 1/2 chains by separation of variables. We notably discuss the case of the XYZ spin chain with quasi-periodic boundary conditions, and the case of the XXX and XXZ open spin chains with non-diagonal boundary conditions.

Anatoly Vershik (Steklov, St. Petersburg)

New results about limit shape of Young diagrams and its generalizations

Robinson-Schensted-Knuth algorithm, Jeu de taquin—Schutzenberger shift, isomorphism with Bernoulli shift (Vershik-Kerov, Sniady-Romic). New approach to limit shape. Generalization for distributive lattices.