

Nikolay Bogachev (Moscow Institute of Physics and Technology)

On Reflective Hyperbolic Lattices

Suppose F that is a totally real algebraic number field with the ring of integers A . A finitely generated free A -module with an inner product of signature $(n,1)$ with values in A is said to be a hyperbolic lattice with the ground field F if its inner product is positive definite for all non-identity embeddings of the field F into the field of real numbers.

A hyperbolic lattice is said to be reflective if its automorphism group is up to finite index generated by reflections. It is known (Nikulin, 2007) that there are only finitely many reflective hyperbolic lattices as well as that there are only finitely many their ground fields. However, the problem of their classification (posed by Vinberg in 1967) and the problem of enumeration of their ground fields (posed by Nikulin in the 1980s) are still very far from being completely solved.

In this talk I will describe some new methods (which are modifications of Nikulin's ideas) which enable one to obtain some classification results for reflective hyperbolic lattices of rank 4 with ground fields Q and $Q(\sqrt{2})$, as well as to improve the known upper bounds on degrees of ground fields F over the field Q .

Fabien Clery (Loughborough University)

Vector-valued Siegel modular forms of degree 2 with character.

The link between covariants of binary sextics and Siegel modular forms of degree 2, recently obtained in a joint work with G. van der Geer and C. Faber, is an efficient way for producing Siegel modular forms. Using this link, the structure of some graded rings of vector-valued Siegel modular forms of degree 2 with character will be given. This is a joint work with G. van der Geer and C. Faber.

Gavril Farkas (Humboldt University, Berlin)

Moduli of K3 surfaces via cubic 4-folds

In a celebrated series of papers, Mukai established structure theorems for polarized K3 surfaces of all genera $g < 21$, with the exception of the case $g=14$. Using Hassett's identification between the moduli space of polarized K3 surfaces of genus 14 and the moduli space of special cubic fourfolds of discriminant 26, we establish the rationality of the universal K3 surface of genus 14. I will also discuss similar result in genus 22.

This is joint work with Alessandro Verra.

Evgeny Ferapontov (Loughborough University)

Dispersionless Hirota equations and the genus 3 hyperelliptic divisor

(based on joint work with F. Clery)

Three-dimensional equations of the dispersionless Hirota type have been thoroughly investigated in the mathematical physics literature. Geometrically, such equations correspond to hypersurfaces in the Lagrangian Grassmannian. It has been known that the parameter space of integrable Hirota type equations is 21-dimensional and that the action of the natural equivalence group $Sp(6, \mathbb{R})$ on the

parameter space possesses an open orbit. However the structure of the 'master-equation' corresponding to this orbit remained elusive. Our main observation is that the hypersurface corresponding to the Hirota master-equation coincides with the genus 3 hyperelliptic divisor.

The rich geometry of integrable Hirota type equations sheds new light on local differential geometry of the genus 3 hyperelliptic divisor.

Dima Grigoriev (CNRS, University Lille, France)

Generalized RSK (jointly with A. Berenstein, A. Kirillov, G. Koshevoy)

The combinatorial RSK correspondence is a bijection between integer non-negative matrices and plane partitions. We introduce an algebraic generalization of RSK being a bijection between the cones of a pair of non-archimedean valuations. An ensemble of rational morphisms of tori in a Schubert cell of a Kac-Moody group is constructed which provides a commuting family of RSK correspondences of their induced valuations. As a particular case of this construction one gets the classical combinatorial RSK correspondence.

Also a geometric analog of RSK correspondence is defined for open embeddings of tori into an irreducible variety. It is proved that the tropicalization of the geometric RSK correspondence of the constructed ensemble in a Kac-Moody group (for the longest word in Weyl group) coincides with the algebraic RSK correspondence of this ensemble in the dual group.

Valery Gritsenko (University Lille/IUF/NRU HSE)

Theta-blocks, Borcherds products and applications to geometry

A theta-block is a special quotient of Jacobi thetas and Dedekind eta's. We show how to construct infinite family of holomorphic Borcherds products using theta-blocks. In particular, we get infinite families of Siegel paramodular forms of weight 2 and 3. We discuss some applications to the moduli spaces of polarized abelian and Kummer surfaces and to motivic L-functions of Calabi-Yau 3-folds.

Klaus Hulek (University of Hannover)

Elliptic K3 surfaces - monodromy strata versus lattice polarizations

Bogomolov, Petrov and Tschinkel defined monodromy strata in the moduli space of elliptic K3 surfaces (BPT strata) and they also proved that these strata are rational. Here we compare (some of) these BPT strata to moduli spaces of lattice polarized K3 surfaces. More precisely, we classify all moduli spaces of lattice polarized K3 surfaces which dominate finite to one of the BPT strata. This is closely related to Shimada's classification of connected components of the moduli of elliptic K3 surfaces. This is joint work with Michael L\"onne.

Klaus Hulek (University of Hannover)

K3 surfaces and friends

(Colloquium of the Chebyshev Laboratory of SPbSU)

K3 surfaces are ubiquitous in mathematics. They appear in complex geometry, but also play a role in arithmetic or string theory. In this talk I will discuss K3 surfaces and their higher dimensional analogues, especially irreducible holomorphic symplectic manifolds (IHSM), also known as hyperkahler manifolds (HK). I will also discuss the classification problem for these varieties, in particular the geometry of their moduli spaces.

Jong Hae Keum (KIAS)

The bicanonical map of fake projective planes

Two-dimensional ball quotients are interesting subjects to study. Unfortunately not much is known about their automorphic forms and projective models. Fake projective planes are ball quotients with the minimum possible Betti numbers, i.e. the Betti numbers of the projective plane. We show, for several fake projective planes, that the bicanonical map is an embedding. To see this, we first recall I. Reider's theorem of very ampleness of adjoint linear systems. Then we show that some fake projective planes do not contain curves with small degree on which the bicanonical map may fail to be an embedding. This is joint work with F. Catanese.

Shigeyuki Kondo (University of Nagoya)

1-dimensional families of Enriques surfaces in characteristic 2 covered by the supersingular K3 surface with Artin invariant 1

Bombieri and Mumford classified Enriques surfaces into three classes, namely, singular, classical and supersingular Enriques surfaces. We give two 1-dimensional families of classical and supersingular Enriques surfaces in characteristic 2 covered by the supersingular K3 surface with Artin invariant 1. This is a joint work with Toshiyuki Katsura.

Radu Laza (Stony Brook)

Birational Geometry of the moduli space of K3 surfaces

I will discuss a program, joint with K. O'Grady, to investigate the birational geometry of locally symmetric varieties of K3 type (similar considerations apply to the case of ball quotients).

The motivation for our study is the search for geometric compactifications for the moduli of polarized K3 surfaces. Namely, as a consequence of Torelli theorem, the moduli of polarized K3 surfaces (with canonical singularities) can be identified to a locally symmetric variety D/Γ . As such, there are natural 'arithmetic' compactifications, e.g. the Baily-Borel (BB) compactification. Unfortunately, the BB compactification has obscure geometric meaning. Consequently, it is natural to compare it with more geometric compactifications, such as those given by GIT. I will explain that there is a natural continuous interpolation between BB and GIT compactifications, and that there is a rich geometric and arithmetic structure behind this picture. In particular, I will show that the Borchers-Gritsenko relations provide an explanation to some surprising geometric behavior.

The focus of the talk will be on the quartic K3 case. Some new results on degree 6 K3s (with F. Greer) will be briefly discussed.

Shouhei Ma (Tokyo Tech)

Siegel modular forms and universal abelian variety

We give a correspondence between Siegel modular forms and pluricanonical forms on the universal abelian variety and its rank 1 partial compactification for every arithmetic group for a symplectic lattice of rank $2g > 2$.

We first show that the graded ring of Siegel modular forms of weight divisible by $g+2$ is isomorphic to the canonical ring of the nonsingular locus of the universal abelian (or Kummer) variety. At the partial compactification of the universal family, this extends to an isomorphism between the first ring and the log canonical ring of the partial compactification, which maps the canonical ring to the subring of modular forms with vanishing condition at the rank 1 cusps. When $g > 3$, the partial compactification has canonical singularities.

Vyacheslav Nikulin (MIAN/University of Liverpool)

Degenerations and Picard lattices of K3 surfaces with finite symplectic automorphism groups

The talk will be devoted to my recent results about classification of degenerations and Picard lattices of K3 surfaces with finite symplectic automorphism groups.

Viktor Petrov (SPbSU/POMI)

Symmetric space of type EIII as a Grassmannian for Brown algebra.

It is known that the group of type G_2 can be constructed as the automorphism group of the octave algebra, while its symmetric space of type G turns out to be the variety of the quaternion subalgebras in the octave algebra. Brown has constructed a 56-dimensional non-associative algebra with involution whose automorphism group is of type E_6 . We consider the variety of quaternion subalgebras in this algebra and show that, on the one hand, it is the 32-dimensional symmetric space of type EIII, and, on the other hand, is an open subvariety in the complexified Cayley plane (the latter reflects the fact that EIII is a "Rosenfeld plane" over complexified octaves). The structure of the projective plane and Witt extension theorem for Hermitian forms allow to prove a Skolem-Noether type theorem over any base field (of characteristic not 2 and not 3): two quaternion subalgebras in a twisted form of the Brown algebra are conjugate by an automorphism if and only if they are isomorphic. This gives a description of the orbits of the action of a twisted form of EIII, and, in terms of Galois cohomology, of the fibers of the map $H^1(F, \{ \}^2 D_5) \rightarrow H^1(F, \{ \}^2 E_6)$ (the latter map is surjective up to cubic extensions of the base field).

Nikolay Proskurin (POMI)

Distribution of zeros of L-functions admitting no Euler product expansion. Numerical observations and some conjectures.

Let $L: \mathbb{C} \rightarrow \mathbb{C}$ be a meromorphic function defined by Dirichlet series (convergent in some half-plane) and satisfying a functional equation similar to that for the Riemann zeta function (with arbitrary finite number of gamma-factors involved). Assume, L admits no Euler expansion as a product of local factors. As to

the distribution of zeros of L , we may not expect anything similar to the Riemann hypothesis for the zeta function. We have studied numerically some of such functions L aiming to understand possible distribution of real parts of their zeros. That are the Epstein zeta functions of some quadratic forms, the zeta function of the Leech lattice, the L -function attached to the Kubota-Patterson cubic theta function. For computation, we have used 'functional equations with free parameters' which are known also as 'smoothed functional equations'.

The functions considered have some 'trivial' zeros (located on the real line) and some 'on-line' zeros (located on the critical line). There are some observations concerning other zeros, i.e. the 'off-line' ones. In particular, it is seen that the 'off-line' zeros are concentrated mainly near some vertical lines.

Anastasia Stavrova (Chebyshev Laboratory, St.Petersburg University)

Generalized congruence subgroup problem for isotropic simple algebraic groups

The congruence subgroup problem for semisimple algebraic groups goes back to the 1967 paper of H. Bass, J. Milnor, and J.-P. Serre. In its classical form, it asks whether every finite index normal subgroup of a linear group over an arithmetic ring contains a congruence subgroup. More generally, for any linear group over a commutative ring, one may ask how far an arbitrary normal subgroup is from being a congruence subgroup. We will discuss two recent results on this problem. Let G be a simple algebraic group G of isotropic rank at least 2 over a Noetherian commutative ring R with invertible 2 and 3. The first result concerns centrality of the profinite congruence kernel associated to G . The second one is the following theorem of the speaker and A. V. Stepanov. For any normal subgroup H of $G(R)$ there is a unique ideal I of R such that H lies between $E(R, I)$ and $C(R, I)$, where $E(R, I)$ is the normal closure of the set of elementary unipotents congruent to 1 modulo I in the subgroup generated by all elementary unipotents, and $C(R, I)$ is the set of elements in $G(R)$ central modulo I .

Let G be a simple algebraic group of isotropic rank at least 2 over a commutative ring R . We show that every normal subgroup H of the group of points $G(R)$ lies between the elementary congruence subgroup and the principal congruence subgroup corresponding to a uniquely determined ideal $I=I(H)$ of R . (joint work with Alexei Stepanov).

Xavier Roulleau (Universite d'Aix-Marseille)

Construction of Nikulin configurations on some Kummer surfaces

(Joint work with Alessandra Sarti.)

A Nikulin configuration on a K3 surface is a set C of 16 smooth disjoint rational curves. By the results of Nikulin, any K3 surface X containing a Nikulin configuration is a Kummer surface, which means that there exists an abelian surface A such that X is the minimal resolution of the quotient $A/[-1]$ and the exceptional curves of the resolution $X \rightarrow A/[-1]$ are the 16 curves of the Nikulin configuration C (this is denoted $X = \text{Km}(A)$).

In this talk, starting with a Kummer configuration C on some polarised Kummer surface X , we will construct another Kummer configuration C' on X such that if A and A' denotes the associated Abelian surfaces, although one has: $\text{Km}(A) = X = \text{Km}(A')$, the Abelian surfaces A and A' are not isomorphic (unless X is a Jacobian Kummer surface). The first example of that phenomena was obtained by Gritsenko and

Hulek.

Our construction uses the Torelli Theorem for $K3$. As a by-product we obtain some new knowledge on the automorphisms group of Kummer surfaces, moreover it brings some interesting configurations of rational curves on $K3$'s. If we have enough time we will derive some applications on the construction surfaces of general type, like the Schoen surfaces.

Nils-Peter Skoruppa (University of Siegen)

Theta blocks

The construction of various types of automorphic forms depends on the explicit construction of Jacobi forms. One of the most efficient tools for the latter are theta blocks. After an introduction of theta blocks and their basic properties we report about recent developments and experiments.

Misha Verbitsky (Instituto Nacional de Matematica Pura e Aplicada/NRU HSE)

Contraction loci in hyperkahler manifolds

An MBM curve on a hyperkahler manifold is a rational curve with negative BBF square and minimal possible dimension of its Barlet deformation space. It is known that (up to a possible birational transform) MBM curves survive in all deformations of M which leave its homology class of type $(1,1)$. The MBM locus of an MBM curve is the union of all its deformations in the ambient manifold M . When M is projective, this is a birational contraction locus, and all birational contraction loci are obtained this way (when M is non-projective, a similar result is conjectured). I will prove that all MBM loci in a given deformation class are homeomorphic. This is a joint work with Ekaterina Amerik.

Noriko Yui (Queen's University)

Supercongruences for rigid hypergeometric Calabi–Yau threefolds

We give two proofs to the supercongruences for the fourteen rigid hypergeometric Calabi–Yau threefolds defined over \mathbb{Q} . The existence of such supercongruences was conjectured (based on numerical evidence) by F. Rodriguez-Villegas in 2003. This is a joint work with Ling Long, Fang-Ting Tu and Wadim Zudilin.

Wadim Zudilin (Radboud University)

Special hypergeometric motives and their L-functions

I will talk about reincarnation of an arithmetic phenomenon going back to Ramanujan's work from 1914, in terms of hypergeometric motives--a notion introduced and studied recently by F. Rodriguez-Villegas, D.Roberts, M.Watkins and others. The developed theory allows one to investigate, both rigorously and experimentally, several remarkable features including connections to modular forms, (expectedly) Siegel modular forms and their \mathcal{L} -functions. The talk is joint work in progress with Lassina Dembélé and Alexey Panchishkin.