#### CONFERENCE PROGRAM

### Monday, April 23

- 09:30–10:30 REGISTRATION
- 10:30–11:15 **Pertti Mattila.** Projections, Hausdorff dimension, and very little about analytic capacity.

#### COFFEE BREAK

- 11:40–12:25 Arno Kuijlaars. The two-periodic Aztec diamond and matrix valued orthogonal polynomials.
- 12:35–13:20 Alexander Logunov. 0.01% improvement of the Liouville property for discrete harmonic functions on  $\mathbb{Z}^2$ .

#### LUNCH

Section A

- 15:00–15:25 Konstantin Dyakonov. Interpolating by functions from star-invariant subspaces.
- 15:30–15:55 Xavier Massaneda. Equidistribution and  $\beta$ -ensembles.

Section B

- 15:00–15:25 Alexandre Sukhov. Analytic geometry of Levi-flat singularities.
- 15:30–15:55 **Nikolai Kruzhilin.** Proper maps of Reinhardt domains. COFFEE BREAK

Section A

- 16:25–16:50 **Stéphane Charpentier.** Small Bergman–Orlicz and Hardy–Orlicz spaces.
- 16:55–17:20 Avner Kiro. Uniqueness theorems in non-isotropic Carleman classes.
- 17:25–17:50 Alexander Pushnitski. Weighted model spaces and Schmidt subspaces of Hankel operators.

Section B

- 16:25–16:50 Vesna Todorčević. Subharmonic behavior and quasiconformal mappings.
- 16:55–17:20 Nicola Arcozzi. Carleson measures for the Dirichlet spaces on the bidisc.

18:00

#### WELCOME PARTY

at The Euler International Mathematical Institute

### Tuesday, April 24

- 09:30–10:15 Armen Sergeev. Seiberg–Witten equations as a complex version of Ginzburg–Landau Equations.
- 10:25–11:10 Josip Globevnik. On complete complex hypersurfaces in  $\mathbb{C}^N$ .

#### COFFEE BREAK

- 11:40–12:25 Alexander Aptekarev. Asymptotics of Hermite–Padé approximants of multi-valued analytic functions.
- 12:35–13:20 Frank Kutzschebauch. Factorization of holomorphic matrices in elementary factors.

#### LUNCH

Section A

- 15:00–15:25 Maksim Mazalov. An L<sup>1</sup>-estimate for Calderón commutators and bianalytic capacities.
- 15:30–15:55 **Petr Borodin.** Density of semigroups in complex Banach spaces.

Section B

- 15:00–15:25 **Rachid Zarouf.** A duality approach to interpolation theory.
- 15:30–15:55 Konstantin Isaev. Representing systems of exponentials in invariant hull and invariant kernel of normed spaces of analytic functions.
- 16:00–17:00 POSTER SESSION / COFFEE

Section A

- 17:05–17:30 Evgenii Dubtsov. Approximation by proper holomorphic maps and tropical power series.
- 17:35–18:00 Faizo Shamoyan. Fourier transform and quasi-analytic classes of functions of bounded type on tubular domain.

Section B

- 17:05–17:30 **Philippe Jaming.** Spectral inequality for Hermite functions and null-controllability of hypoelliptic quadratic equations from thick sets.
- 17:35–18:00 Ilgiz Kayumov. Coefficients problems for Bloch functions.

#### Wednesday, April 25

- 09:30–10:15 **Eero Saksman.** On multiplicative chaos measures as statistical limits.
- 10:25–11:10 Stefanie Petermichl. Factorisation of Hardy spaces.

### COFFEE BREAK

- 11:40–12:25 Alexei Poltoratski. Truncated Toeplitz operators in inverse spectral problems.
- 12:35–13:20 **Stanislav Smirnov.** Percolation crossings and complex analysis.

## LUNCH

18:00

#### CONCERT

**Baroque Ensemble "The Soloists of Catherine the Great"** at St. Petersburg Department of Steklov Math. Institute, Fontanka 27

### Thursday, April 26

10:00–10:45 **Zbigniew Błocki.** Estimates for the Bergman kernel and logarithmic capacity.

#### COFFEE BREAK

- 11:15–12:00 Alexandru Aleman. Hilbert spaces of analytic functions with a contractive backward shift.
- 12:10–12:55 Gady Kozma. On the Cantor uniqueness theorem.

#### LUNCH

- Section A
- 14:45–15:10 **Pascal Thomas.** The Corona Property in Nevanlinna quotient algebras.
- 15:15–15:40 Shahaf Nitzan. Balian–Low type theorems in finite dimensions.
- 15:45–16:10 Alexander Ulanovskii. Discrete translates in function spaces.

Section B

- 14:45–15:10 Vladimir Dubinin. Sharp distortion theorems for meromorphic functions having some connected lemniscates.
- 15:15–15:40 Il'dar Musin. On the Fock type space and its dual.
- 15:45–16:10 **Pavel Mozolyako.** Logarithmic potential estimates on the bi-tree.

#### COFFEE BREAK

Section A

- 16:40–17:05 Roman Bessonov. A spectral Szegö theorem on the real line.
- 17:10–17:35 Yury Belov. The Newman–Shapiro problem and spectral synthesis in the Fock space.
- 17:40–18:05 **Bulat Khabibullin.** On the distributions of zero sets of holomorphic functions.

Section B

- 16:40–17:05 **Petr Paramonov.** The C<sup>1</sup>-approximation on plane compact sets by solutions of elliptic equations of second order.
- 17:10–17:35 Alexander Komlov. Hermite–Padé approximants for meromorphic functions on a compact Riemann surface.
- 17:40–18:05 **Dmitriy Stolyarov.** Generalized Volterra operators on the space of bounded analytic functions.

## 19:15 CONFERENCE DINNER at the Restaurant "Allegro". Griboedov Canal Quay. 55

#### Friday, April 27

- 09:30–10:15 Franc Forstnerič. Nonlinear holomorphic approximation theory.
- 10:25–11:10 Nikolai Nikolski. Bohr's transform vs Kozlov's problem.

#### COFFEE BREAK

- 11:40–12:25 **Evgeni Chirka.** Potentials on a compact Riemann surface.
- 12:35–13:20 Kari Astala. Liquid domains, frozen boundaries and the Beltrami equation.

#### LUNCH

15:00-15:45 Håkan Hedenmalm. Planar orthogonal polynomials and boundary universality in the random normal matrix model.

#### COFFEE BREAK

- 16:10–16:55 Alexander Bufetov. Determinantal point processes and completeness of reproducing kernels.
- 17:05–17:50 Mikhail Sodin. Translation-invariant probability measures on entire functions.

## ABSTRACTS

**Alexandru Aleman.** *Hilbert spaces of analytic functions with a contractive backward shift.* 

The talk is based on joint work with B. Malman, and concerns the structure of Hilbert spaces of analytic functions in the unit disc whose reproducing kernel is normalized at the origin and with the property that the backward shift  $f \mapsto \frac{f-f(0)}{z}$  is contractive. In this generality, the main result asserts that the intersection of such a space with the disc algebra is dense in the space. The approach is based on a variant of the Nagy–Foiaş model for the backward shift. I also intend to present some consequences of this result and if the time permits, I will discuss some special cases, where the structure is more transparent.

# **Alexander Aptekarev.** Asymptotics of Hermite–Padé approximants of multi-valued analytic functions.

We consider a vector of power series

$$\mathbf{f}(z) := \left\{ f_j(z) := \sum_{k=0}^{\infty} \frac{f_{j,k}}{z^{k+1}} \right\}_{j=1}^p$$

which have an analytic continuation along a path in the complex plane that does not intersect a finite set of branch points A:

$$f_j \in \mathcal{A}_A, \quad \mathcal{A}_A := \mathcal{A}(\overline{\mathbb{C}} \setminus A), \quad \#A < \infty.$$

In the series of papers (1978–1984) J. Nuttall put forward a conjecture on the asymptotics of the Hermite–Padé approximants of the vector **f**. The main ingredient in this conjecture is an algebraic Riemann surface  $\Re$  – a (p + 1)-sheeted covering manifold over  $\mathbb{C}$ . In terms of the standard functions on  $\Re$  the conjecture describes the asymptotics, the domains of convergence of approximants, and the limiting sets of the zero distribution of the Hermite–Padé polynomials. For p = 1 the Nuttall conjecture states that the diagonal Padé approximants of a function  $f \in \mathcal{A}_A$  converge in the logarithmic capacity in the "maximal" domain  $\Omega$  of the meromorphic (single-valued) continuation of f, i.e. the boundary of  $\Omega$  is the cut of minimal capacity among cuts making f single-valued. This conjecture was proved in 1985 by H. Stahl (even in the more general case: capA = 0).

In our talk we discuss motivations, problems and the current progress in the proof of the general Nuttall conjecture (p > 1).

The work is supported by RSF project 14-21-00025.

**Nicola Arcozzi.** Carleson measures for the Dirichlet spaces on the bidisc.

We consider the problem of characterizing the Carleson measures for the Dirichlet space on the bidisc and reduce it to a problem concerning a bilinear Hardy operator on the direct product of two trees, which can be solved. Applications to the characterization of the multiplier algebra and exceptional sets are given, and some problems are discussed. Work in collaboration with Pavel Mozolyako, Karl-Mikael Perfekt, and Giulia Sarfatti.

# Kari Astala. Liquid domains, frozen boundaries and the Beltrami equation.

Limit configurations of random structures, in two dimensions in particular, often posses some conformal invariance properties, giving way to methods of geometric analysis. Among the fascinating questions here are the limit configurations of random tilings and the boundaries between their ordered and disordered, or liquid, limit regions.

The liquid domains carry a natural complex structure, which can be described by a quasilinear Beltrami equation with very specific properties. In this talk, based on joint work with E. Duse, I. Prause and X. Zhong, I show how this approach leads to understanding and classifying the geometry of the frozen boundaries for different random tilings and other dimer models.

**Yury Belov.** The Newman–Shapiro problem and spectral synthesis in the Fock space.

In 1966 D. Newman and H. Shapiro posed the following question. Let G be a function from the Fock space  $\mathcal{F}$  such that  $e^{wz}G \in \mathcal{F}$  for any  $w \in \mathbb{C}$ . Is it true that

$$\operatorname{Clos} \left\{ FG : F, FG \in \mathcal{F} \right\} = \overline{\operatorname{Span}} \left\{ e^{wz}G : w \in \mathbb{C} \right\}?$$

Recently we answered this question in the negative. On the other hand, we are able to show that if G satisfies some regularity conditions, then the answer is positive. These results are closely connected to some spectral synthesis problems in the Fock space. The talk is based on a joint work with A. Borichev.

#### Roman Bessonov. A spectral Szegö theorem on the real line.

One version of the classical Szegö theorem describes probability measures on the unit circle with finite entropy integral in terms of their recurrence (or Verblunski) coefficients. I will discuss a version of this result for even measures supported on the real line. According to Krein–de Branges inverse spectral theory, every nonzero Poisson integrable measure on the real line corresponds to a unique canonical Hamiltonian system on the positive half-axis whose Hamiltonian has unit trace almost everywhere on the half-axis. The main subject of the talk is to give a characterization of the Hamiltonians arising from even measures on the real line with finite entropy integral. No background in canonical Hamiltonian systems is assumed. This is a joint work with Serguei Denissov (University of Wisconsin-Madison). **Zbigniew Błocki.** Estimates for the Bergman kernel and logarithmic capacity.

We will present estimates of the Bergman kernel in terms of logarithmic capacity. The central result is the optimal upper bound which answered a question posed by Suita in the 70's. Various related results will be given, including several dimensional counterparts as well as lower bounds for the Bergman kernel.

#### Petr Borodin. Density of semigroups in complex Banach spaces.

We present several results concerning the density of additive semigroups in complex function spaces. The main example of such a semigroup is the set of sums of shifts of a single function (for example, the semigroup of logarithmic derivatives of polynomials).

# **Alexander Bufetov.** *Determinantal point processes and completeness of reproducing kernels.*

Consider a Gaussian Analytic Function on the disk. In joint work with Yanqi Qiu and Alexander Shamov, we show that its zero set is a uniqueness set for the Bergman space on the disk: in other words, almost surely, there does not exist a nonzero square-integrable holomorphic function with these zeros. The distribution of our random subset is invariant under the group of isometries of the Lobachevsky plane; the action of every hyperbolic or parabolic isometry is mixing. It follows, in particular, that our set is neither sampling nor interpolating in the sense of Seip. Nonetheless, in a sequel paper, joint with Yanqi Qiu, we give an explicit procedure to recover a Bergman function from its values on our set.

By the Peres and Virag Theorem, zeros of a Gaussian Analytic Function on the disk are a determinantal point process governed by the Bergman kernel, and we prove, for general determinantal point processes, that reproducing kernels sampled along a trajectory form a complete system in the ambient Hilbert space. The key step in our argument is that the determinantal property is preserved under conditioning.

**Stéphane Charpentier.** Small Bergman–Orlicz and Hardy–Orlicz spaces.

For a convex increasing function  $\psi : \mathbb{R}_+ \to \mathbb{R}_+$ , the Bergman–Orlicz spaces  $A^{\psi}$  and the Hardy–Orlicz spaces  $H^{\psi}$  on the unit ball  $\mathbb{B}_N$  of  $\mathbb{C}^N$  are natural generalizations of the classical Bergman spaces  $A^p$  and Hardy spaces  $H^p$ . We will be first interested in characterizing the spaces  $A^{\psi}$  and  $H^{\psi}$  on which every composition operator is bounded. We will give a necessary and sufficient condition on the function  $\psi$ , in terms of its growth to  $\infty$ . For  $A^{\psi}$ , it will surprisingly appear that those spaces can also be characterized by the growth at the boundary of  $\mathbb{B}_N$  of their elements. We will also see that the boundedness of any operator on these spaces is equivalent to an *a priori* stronger property which, in the setting of  $A^2$ , corresponds to being Hilbert–Schmidt for composition operators. If we have time, we will also say some words about the compactness of composition operators.

#### Evgeni Chirka. Potentials on a compact Riemann surface.

In the talk we will discuss the following topics: Poisson equation, space of delta-subharmonic functions, bipolar Green function, representing kernels for the solutions of the Poisson equation.

# **Vladimir Dubinin.** Sharp distortion theorems for meromorphic functions having some connected lemniscates.

In the talk we discuss the impact of the connectivity of some lemniscates of a meromorphic function f on the distortion of the mapping f. We consider some distortion theorems for polynomials (see, for example, [1]), rational functions [2], and inequalities for the various classes of the meromorphic circumferentially mean p-valent functions [3]. The estimates we obtain are sharp. The proofs of the theorems go by an application of a certain modification of the symmetrization method [4], for which the result of the symmetrization lies on the Riemann surface of the function inverse to a Chebyshev polynomial of the first kind.

Bibliography

1. V. N. Dubinin, On one extremal problem for complex polynomials with constraints on critical values, Siberian Math. J. 55 (2014), 1, 63–71.

2. V. N. Dubinin, An extremal problem for the derivative of a rational function, Math. Notes 100 (2016), 5, 714–719.

3. V. N. Dubinin, Distortion theorems for circumferentially mean *p*-valent functions, J. Math. Sci. (N.Y.) 217 (2016), 1, 28–36.

4. V. N. Dubinin, Circular symmetrization of condensers on Riemann surfaces, Sb. Math. 206 (2015), 1, 61–86.

# **Evgenii Dubtsov.** Approximation by proper holomorphic maps and tropical power series.

Let  $\Omega$  denote the complex plane  $\mathbb{C}$  or the unit disk  $\mathbb{D}$  of  $\mathbb{C}$ . Let w be an unbounded radial weight on  $\Omega$ . We study the following approximation problem: for a sufficiently large n, find a proper holomorphic map f:  $\Omega \to \mathbb{C}^n$  such that |f| is equivalent to w. First, we give characterizations of those w for which the problem is solvable. In particular, for  $\Omega = \mathbb{C}$ , a constructive description is obtained in terms of the tropical power series. Second, motivated by a question of J. Globevnik, we show that a proper holomorphic immersion of  $\mathbb{D}$  into  $\mathbb{C}^2$  may have arbitrary growth. Also, we consider the following extensions to several complex variables:  $\Omega = \mathbb{C}^m$  or  $\Omega$  is the unit ball of  $\mathbb{C}^m$ ,  $m \geq 2$ .

This is joint work with E. Abakumov.

Konstantin Dyakonov. Interpolating by functions from star-invariant subspaces.

Given an interpolating Blaschke product, we discuss a natural interpolation problem for functions from the associated star-invariant subspace in  $H^p$ , where p = 1 or  $p = \infty$ .

#### Franc Forstnerič. Nonlinear holomorphic approximation theory.

In this talk I will discuss recent developments concerning Runge, Mergelyan, Arakelyan and Carleman type approximation of holomorphic maps from Stein manifolds to complex manifolds.

### **Josip Globevnik.** On complete complex hypersurfaces in $\mathbb{C}^N$ .

In 1977 Paul Yang posed the problem about the existence of bounded immersed complete complex submanifolds in  $\mathbb{C}^N$ . In the talk I will speak about the solutions for embedded complex hypersurfaces and related results obtained in recent years by A. Alarcon, F. J. Lopez and the speaker.

Håkan Hedenmalm. Planar orthogonal polynomials and boundary universality in the random normal matrix model.

Motivated by recent interest in the random normal matrix model (RNM), which models the random process of the eigenvalues, the correlation kernel is of fundamental interest. It is essentially the reproducing kernel for the space of polynomials of degree less than a given parameter n, and as such it may be written as a finite sum over the corresponding orthogonal polynomials. The orthogonal polynomials have been of focal interest in many situations, such as on the real line, the unit circle, and so on. In such one-dimensional problems the orthogonal polynomials exhibit strong rigidity, as the zeros are forced to be on the spectral intervals. The modern methods involve the connection with Riemann-Hilbert problems. Here, we look at an

unrestricted situation, where the eigenvalues are generally complex numbers, and very little have been known regarding the orthogonal polynomials. The weights are of the exponentially varying kind, which is natural from the complex-geometric point of view (Kähler geometry etc.). We obtain a complete asymptotic expansion of the orthogonal polynomials, which allows us to analyze the spectral boundary of the RNM model. Previously, only the work of Carleman (1922) and Suetin were available, whose methods and results do not apply here. The method involves a flow of loops which we call the "orthogonal foliation flow". This is a joint work with A. Wennman.

Konstantin Isaev, Rinad Yulmukhametov. Representing systems of exponentials in invariant hull and invariant kernel of normed spaces of analytic functions.

Let D be a bounded convex domain in the complex plane, let  $\varphi$  be a convex function in D, and let

$$H(D,\varphi) = \{f \in H(D) : \sup_{z \in D} |f(z)|e^{-\varphi(z)} < \infty\}$$

be the corresponding uniformly weighted space of analytic functions. For this space we construct a special inductive limit  $\mathcal{H}_i(D,\varphi)$  of normed spaces and a special projective limit  $\mathcal{H}_p(D,\varphi)$  of normed spaces. It is proved that  $\mathcal{H}_i(D,\varphi)$  is the minimal locally convex space containing  $H(D,\varphi)$  and invariant with respect to the operator of differentiation, and  $\mathcal{H}_p(D,\varphi)$  is the maximal locally convex space contained in  $H(D,\varphi)$  and invariant with respect to the operator of differentiation. We construct representing systems of exponentials in the projective limit  $\mathcal{H}_p(D,\varphi)$  and in the inductive limit  $\mathcal{H}_i(D,\varphi)$ , and estimate the superfluity of these systems. **Philippe Jaming.** Spectral inequality for Hermite functions and null-controllability of hypoelliptic quadratic equations from thick sets.

Some recent works have showed that the heat equation on the whole Euclidean space is null-controllable in any positive time if and only if the control set is a thick set. This necessary and sufficient condition for null-controllability is linked to some uncertainty principles as the Logvinenko–Sereda theorem which gives limitations on the simultaneous concentration of a function and its Fourier transform. In the present work, we prove new uncertainty principles of the Logvinenko–Sereda type for finite combinations of Hermite functions with an explicit control of the constant. This spectral inequality allows to derive the null-controllability in any positive time from thick control regions for parabolic equations associated with a general class of hypoelliptic nonselfadjoint quadratic differential operators. Joint work with Karine Beauchard and Karel Pravda-Starov (Rennes).

#### Ilgiz Kayumov. Coefficients problems for Bloch functions.

I am going to talk about various coefficients problems for Bloch functions and their application to the Makarov law of the iterated logarithm.

# **Bulat Khabibullin.** On the distributions of zero sets of holomorphic functions.

Let M be a subharmonic function with Riesz measure  $\nu_M$  in a domain  $D \subset \mathbb{C}^n$ , and f be a nonzero holomorphic function in D that vanishes on a subset  $Z \subset D$ , and  $|f| \leq \exp M$  on D. Then the restrictions on the growth of the Riesz measure  $\nu_M$  of the function M near the boundary of the domain D imply certain restrictions on the (2n - 2)-Hausdorff measure of the set Z. A quantitative form of this phenomenon is given in a subharmonic framework.

#### Avner Kiro. Uniqueness theorems in non-isotropic Carleman classes.

I will consider non-isotropic Carleman classes consisting of smooth functions in the interval [0, 1]. In the talk, I will describe necessary and sufficient conditions which guarantee that any function in such a class is uniquely determined by its Taylor coefficients at the origin. If time permits, I will also discuss the motivation for defining such classes.

## **Alexander Komlov.** *Hermite–Padé approximants for meromorphic functions on a compact Riemann surface.*

In the talk we consider the problem of reconstructing the values of a multivalued algebraic function from an initial germ with the help of the Hermite–Padé polynomials of the first kind.

#### Gady Kozma. On the Cantor uniqueness theorem.

We construct a non-trivial trigonometric series converging to zero everywhere on a subsequence, answering a long-standing question. Contrariwise, we show that such a subsequence must be quite sparse. En route we give a new proof of Cantor's classic uniqueness theorem. Joint work with A. Olevskii.

#### Nikolai Kruzhilin. Proper maps of Reinhardt domains.

Holomorphic maps of bounded Reinhardt domains in complex linear spaces are discussed. While a complete description of the holomorphic automorphism group of such a domain can be given in terms of its logarithmic diagram in any dimension, their proper holomorphic maps are well understood only in small dimensions. For instance, the pairs of 2-dimensional Reinhardt domains connected by proper holomorphic maps can be distinguished and maps between them can be fully described. Moreover, the classes of 2-dimensional Reinhardt domains admitting proper holomorphic maps of multiplicity > 1 onto any twodimensional complex manifolds can be characterized. **Arno Kuijlaars.** The two-periodic Aztec diamond and matrix valued orthogonal polynomials.

Uniform domino tilings of the Aztec diamond have the arctic circle phenomenon: near the corners the pattern is fixed and only one type of domino appears, while in the middle there is disorder and all types appear. The transition is sharp with fluctuations described by the Tracy–Widom distributions.

In the two-periodic Aztec diamond the dominos have a two-periodic weighting and this creates a new phase in the large size limit, where correlations decay at an exponential rate. In recent work with Maurice Duits (KTH Stockholm) we analyze this model with the help of matrix valued orthogonal polynomials. We obtain a remarkably simple double contour integral formula for the correlation kernel that we can analyze in the limit to recover the three phases of the model and the fluctuations near the transition curves.

**Frank Kutzschebauch.** Factorization of holomorphic matrices in elementary factors.

It is standard material in a Linear Algebra course that the group  $\operatorname{SL}_m(\mathbb{C})$  is generated by elementary matrices  $E + \alpha e_{ij}$   $i \neq j$ , i.e., matrices with 1's on the diagonal and such that all entries outside the diagonal are zero, except one entry. The same question for matrices in  $\operatorname{SL}_m(R)$ where R is a commutative ring instead of the field  $\mathbb{C}$  is much more delicate; interesting is the case when R is the ring of complex valued functions (continuous, smooth, algebraic or holomorphic) from a space X. For  $m \geq 3$  (and any n) it is a deep result of Suslin that any matrix in  $\operatorname{SL}_m(\mathbb{C}[\mathbb{C}^n])$  decomposes as a finite product of unipotent (and equivalently elementary) matrices. In the case of continuous complex valued functions on a topological space X the problem was studied and solved by Thurston and Vaserstein. For rings of holomorphic functions on Stein spaces, in particular on  $\mathbb{C}^n$ , this problem was explicitly posed as the *Vaserstein problem* by Gromov in the 1980's. In the talk we explain a complete solution to Gromov's Vaserstein Problem from a joint work with B. Ivarsson. The proof uses a very advanced version of the Oka principle proposed by Gromov and proved in recent years by Forstnerič: An elliptic stratified submersion over a Stein space admits a holomorphic section if and only if it admits a continuous section. Also very recent results on the Symplectic Group are presented.

## **Alexander Logunov.** 0.01% improvement of the Liouville property for discrete harmonic functions on $\mathbb{Z}^2$ .

We prove that if u is a discrete harmonic function on the lattice  $\mathbb{Z}^2$ , and |u| < 1 on 99.99% of  $\mathbb{Z}^2$ , then u is a constant function. Based on a joint work with L. Buhovsky, E. Malinnikova and M. Sodin.

## **Xavier Massaneda.** Equidistribution and $\beta$ -ensembles.

We give the precise rate – in terms of the Kantorovich–Wasserstein distance – at which the empirical measure associated to a  $\beta$ -ensemble converges to its limiting measure. In our setting the  $\beta$ -ensemble is a random point process on the sphere distributed according to the  $\beta$ -power of a determinant of holomorphic polynomials. Joint work with Tom Carrol, Jordi Marzo and Joaquim Ortega-Cerdà.

# **Pertti Mattila.** Projections, Hausdorff dimension, and very little about analytic capacity.

I discuss some recent developments around the question: how do orthogonal projections affect the Hausdorff dimension. I shall also mention a relation to analytic capacity in terms of an old problem of Vitushkin, and a recent partial result of Chang and Tolsa on it. **Maksim Mazalov.** An  $L^1$ -estimate for Calderón commutators and bianalytic capacities.

Bianalytic capacities appear naturally in constructions of uniform approximation of continuous functions by bianalytic functions. Some properties of these capacities are extraordinary; in particular, there is no semiadditivity.

We obtain some estimates for Calderón commutators on Lipschitz curves and apply them to bianalytic capacities.

#### Pavel Mozolyako. Logarithmic potential estimates on the bi-tree.

A bi-tree  $T^2$  is a Cartesian product of two identical dyadic trees T. We will introduce the basics of (bi-logarithmic) potential theory on  $T^2$ and discuss several classical inequalities in this setting, in particular, the Maximum Principle and the strong capacitary inequality.

This talk is based on joint work with N. Arcozzi, K.-M. Perfekt, and G. Sarfatti.

#### Il'dar Musin. On the Fock type space and its dual.

A weighted Hilbert space of entire functions of several variables will be considered in the talk. The weight function is a convex function depending on modules of variables and growing at infinity faster than any linear function. We discuss the problem of description of the strong dual to this space in terms of the Laplace transformation of functionals.

#### Nikolai Nikolski. Bohr's transform vs Kozlov's problem.

We discuss V. Ya. Kozlov's problem on the completeness of dilated systems f(nx), n = 1, 2, ..., on the space  $L^2(0, 1)$  for an odd 2periodic function f coinciding on (0, 1) with the indicator function of the interval (0, a),  $0 < a \leq 1$ . We prove all Kozlov's claims on completeness/incompleteness properties of the systems in question (stated without proofs in 1948–1950), mostly for certain rationals a = p/q, and speculate on related geometric properties of these dilated systems.

#### Shahaf Nitzan. Balian-Low type theorems in finite dimensions.

The classical Balian–Low theorem states that if both a function and its Fourier transform decay too fast, then the Gabor system generated by this function (i.e. the system obtained from this function by taking integer translations and integer modulations) cannot be an orthonormal basis or a Riesz basis.

Though it provides for an excellent "thumbs rule" in time-frequency analysis, the Balian–Low theorem is not adaptable to many applications. This is due to the fact that in realistic situations information about a signal is given by a finite dimensional vector rather then by a function over the real line. In this work we obtain an analog of the Balian–Low theorem in the finite dimensional setting, as well as analogs to some of its extensions. Moreover, we will note that the classical Balian–Low theorem, and its extensions, can be derived from these finite dimensional analogs. The talk is based on joint work with Jan-Fredrik Olsen.

**Petr Paramonov.** The  $C^1$ -approximation on plane compact sets by solutions of elliptic equations of second order.

Criteria are obtained for the  $C^1$ -Whitney-approximability of functions on plane compact sets by solutions of elliptic equations of second order with constant complex coefficients. These results, analogues to Vitushkin's well-known criteria for uniform rational approximation, are obtained both for individual functions and, as corollaries, for classes of functions in terms of related capacities, which are comparable to classic analytic capacity (that is subadditive and has a geometric measure characterization).

#### Stefanie Petermichl. Factorisation of Hardy spaces.

The classical inner-outer factorisation provides us with a factorisation of the Hardy space  $H^1$  into a product of  $H^2$ 's. We discuss the simple duality argument by which a similar factorisation in various product spaces is obtained from certain two-sided commutator estimates of Hilbert or Riesz transforms with multiplication by symbol functions belonging to spaces of bounded mean oscillation. Finally we discuss the two-sided commutator estimates themselves. While the proofs use harmonic analysis, their inspiration stems from Hankel and Toeplitz operators. The investigation started among others with Ferguson– Sadosky in 1999 with the product Hilbert transform case (Toeplitz form), the first corner stone in this direction was the case of two Hilbert transforms on the bi-disk by Ferguson–Lacey in 2000 (Hankel forms). The extension of the latter to Riesz transforms is from 2009. In this talk we will give the complete solution for all arising commutators and BMO spaces (2015–2017).

# **Alexei Poltoratski.** Truncated Toeplitz operators in inverse spectral problems.

An extension of Gelfand–Levitan theory, using de Branges spaces of entire functions and truncated Toeplitz operators, allows one to find explicit solutions to various inverse spectral problems for Krein's canonical systems of differential equations. In my talk I will present these methods along with several new examples of solutions to inverse spectral problems. The talk is based on joint work with N. Makarov.

# **Alexander Pushnitski.** Weighted model spaces and Schmidt subspaces of Hankel operators.

I will describe the structure of Schmidt subspaces of Hankel operators. I will explain that such subspaces can be identified with a class of weighted model spaces in the Hardy space. This can be regarded as an extension of the classical Adamyan–Arov–Krein theory. The talk is based on recent joint results with Patrick Gérard (Paris-Orsay).

#### Eero Saksman. On multiplicative chaos measures as statistical limits.

We consider several situations where multiplicative chaos appears as the limit statistics. Depending how time allows, we consider models arising from random matrix theory, statistical physics, and probability. Talk is based on joint work with Janne Junnila (University of Helsinki), Miika Nikula (Aalto University), and Christian Webb (Aalto University).

# **Armen Sergeev.** Seiberg–Witten equations as a complex version of Ginzburg–Landau Equations.

The Seiberg–Witten equations, as well as the Yang–Mills equations, are the limiting cases of the supersymmetric Yang–Mills theory. But, opposite to Yang–Mills equations, the Seiberg–Witten equations are not under the change of scale. So in order to draw useful information from them we should take the limit  $\lambda \to +\infty$  where  $\lambda$  is the scale parameter.

If we consider this limit for Seiberg–Witten equations on a 4-dimensional symplectic manifold, a solution of these equations will concentrate in a neighborhood of some pseudoholomorphic curve (more precisely, a pseudoholomorphic divisor) while the equations reduce to families of static Ginzburg–Landau equations defined in the normal planes to the limiting pseudoholomorphic curve. Such a limit is called adiabatic as well as the reduced Seiberg–Witten equations. Solutions of these equations may be considered as families of static solutions of Ginzburg–Landau equations in the complex plane with a complex parameter z running along the pseudoholomorphic curve. This parameter plays the role of complex time while the reduced Seiberg–Witten equations have the form of a nonlinear  $\bar{\partial}$ -equation with respect to z.

It turns out that this construction has a non-trivial (2+1)-dimensional analogue. Namely, if we consider in the Ginzburg–Landau equations the "slow-time" limit then these equations will reduce to the adiabatic equations with solutions given by the geodesics on the moduli space of static Ginzburg–Landau solutions (called otherwise vortices) in the metric generated by the kinetic energy functional.

Thus we may consider the adiabatic limit in Seiberg–Witten equations as a (2 + 2)-dimensional analogue of the adiabatic limit for Ginzburg– Landau equations. Solutions of adiabatic Seiberg–Witten equations can be treated as complex analogues of adiabatic geodesics in Ginzburg– Landau case while the nonlinear  $\bar{\partial}$ -equation may be considered as a complex analogue of the Euler equation for these geodesics.

**Faizo Shamoyan.** Fourier transform and quasi-analytic classes of functions of bounded type on tubular domain.

A condition for a function of bounded type to belong to the Hardy class  $H^1$  in terms of the Fourier transform of the boundary values of this function on  $\mathbb{R}^n$  is obtained. Applications of this result to the Hardy classes and to the quasi-analytic classes of functions are given.

#### Stanislav Smirnov. Percolation crossings and complex analysis.

We will present a much shorter and more conceptual proof of the Cardy–Carleson formula (for the scaling limit of the triangular lattice critical percolation crossing probabilities).

This is joint work with Mikhail Khristoforov.

## **Mikhail Sodin.** Translation-invariant probability measures on entire functions.

I shall speak about a somewhat unexpected object: the probability measures on the space of entire functions (of one complex variable) which are (a) invariant with respect to the action of the complex plane by translations, and (b) do not charge the constant functions. The existence (and even an abundance) of such measures was discovered by Benjy Weiss several years ago.

I plan to present recent results from the joint work with Lev Buhovsky, Adi Glucksam, Alexander Logunov, arXiv:1703.08101, and – if time permits – will discuss several open questions.

**Dmitriy Stolyarov.** Generalized Volterra operators on the space of bounded analytic functions.

Let g be an analytic function on the unit disk  $\mathbb{D}$ . Consider the operator

$$I_g[f](z) = \int_0^z f(\zeta)g'(\zeta) \, d\zeta, \quad f \text{ is analytic in } \mathbb{D}.$$

It was conjectured that  $I_g$  acts on  $H^{\infty}$  if and only if the function g has bounded radial variation:

$$\sup_{\theta \in [0,2\pi)} \int_{0}^{1} |g'(re^{i\theta})| \, dr < \infty.$$

We will prove the conjecture in the case of univalent g and disprove it in its full generality. This is a joint work with Wayne Smith and Alexander Volberg.

### Alexandre Sukhov. Analytic geometry of Levi-flat singularities.

We discuss some recent progress in local theory of analytic Levi-flat hypersurfaces in presence of singularities.

**Pascal Thomas.** The Corona Property in Nevanlinna quotient algebras.

Gorkin–Mortini–Nikolskii introduced the Weak Embedding Property (WEP) to characterize the absence of corona and norm-controlled inversions for quotients of the algebra of bounded analytic functions.

We define an analogue of the WEP for quotients of the Nevanlinna Class. This only depends on the zeros of the function by which we take the quotient. We prove that in this case the WEP holds if and only if the zeros are a finite union of Nevanlinna interpolating sequences, in contrast with the situation for bounded functions. This is a joint work with Xavier Massaneda and Artur Nicolau.

**Vesna Todorčević.** Subharmonic behavior and quasiconformal mappings.

This talk is based on a joint work with Pekka Koskela. We prove that the composition of a quasi-nearly subharmonic function and a quasiregular mapping of bounded multiplicity is quasi-nearly subharmonic. Also, we prove that if  $u \circ f$  is quasi-nearly subharmonic for all quasi-nearly subharmonic u and f satisfying some additional conditions, then f is quasiconformal. Similar results are further established for the class of regularly oscillating functions.

## Mikhail Tyaglov. Root location of polynomials with totally nonnegative Hurwitz matrix.

For a given real polynomial

$$p(z) = a_0 z^n + a_1 z^{n-1} + \dots + a_n, \qquad a_0 > 0,$$

the  $n \times n$  matrix  $H_n(p) = (a_{2j-i})$  is called finite Hurwitz matrix, and the matrix  $\mathcal{H}_{\infty}(p) = (a_{2j-i})_{i,j \in \mathbb{Z}}$  is the *infinite* Hurwitz matrix.

It is known that the total positivity of the matrix  $\mathcal{H}_{\infty}(p)$  is equivalent to stability of the polynomial p(z) (roots in the open left half-plane), while the totally nonnegativity of the finite Hurwitz matrix  $H_n(p)$  does not imply stability of p(z). In this talk, we completely describe root location of the polynomial p(z) whose finite Hurwitz matrix  $H_n(p)$  is totally nonnegative.

#### Alexander Ulanovskii. Discrete translates in function spaces.

We consider Banach function spaces X with the property that the space of Schwartz functions is continuously and densely embedded in X. We construct a Schwartz function g such that for every exponentially small perturbation of integers  $\Lambda$ , the set of translates  $\{g(t - \lambda) : \lambda \in \Lambda\}$ spans every X whose norm is "weaker" than  $L^1$  (on bounded functions). This talk is based on a joint work with Alexander Olevskii.

#### **Rachid Zarouf.** A duality approach to interpolation theory.

We introduce a "dual-space approach" to mixed Nevanlinna–Pick/ Carathéodory–Schur interpolation in Banach spaces X of holomorphic functions on the disk  $\mathbb{D}$ . Given a holomorphic Banach space, a sequence  $\{\lambda_i\}_{i=1}^n$  on  $\mathbb{D}$  and a sequence  $\{w_i\}_{i=1}^n$  in  $\mathbb{C}$ , the task is to minimise the norm of  $f \in X$  under the constraint  $f(\lambda_i) = w_i$ . Our approach can be viewed as complementary to the well-known Commutant lifting approach to interpolation theory of D. Sarason and B. Nagy–C.Foiaş. In view of applications, we are particularly interested in the case that  $X = W \subsetneq H^{\infty}$  is the Wiener algebra of absolutely convergent Taylor series

$$W := \Big\{ f = \sum_{j \ge 0} \hat{f}(j) z^j \in \mathcal{H}ol(\mathbb{D}) : \|f\|_W = \sum_{j \ge 0} |\hat{f}(j)| < \infty \Big\}.$$

By von Neumann's inequality, Hilbert space contractions admit an  $H^{\infty}$  functional calculus. As a consequence, Sarason's  $H^{\infty}$ -interpolation theory has contributed significant insight to the study of contractions of Hilbert space. Similarly, *Banach space contractions* are related to

a Wiener algebra functional calculus. Our interest in W comes from developing an analogous theory for contractions on Banach space. The latter applies to exhibit a family of explicit counterexamples that refutes Schäffer's conjecture on norms of matrix inverses. This talk is based on a joint work with Oleg Szehr from the University of Vienna.

## POSTERS

**Astamur Bagapsh.** Boundary behavior and mapping properties of solutions of second order elliptic systems.

We extend a classical result about weighted averages of harmonic functions to the class of solutions of second order strongly elliptic systems with constant coefficients in disks in the complex plane. Let us recall that the non-tangential cluster set of the (harmonic) Poisson integral with a given piecewise continuous boundary function f at every point  $\zeta$  in the unit circle is the segment joining the left- and right-hand side limits of f at  $\zeta$  being taken along the unit circle. Using the recently obtained Poisson-type integral representation formula for solutions of second order strongly elliptic systems of PDE with constant coefficients, we establish an analogous result about weighted averages for solutions of the above-mentioned systems. Mappings of the unit disk by solutions of such systems with piecewise constant boundary data illustrate the nature of the obtained results.

# **Ivan Bochkov.** *Polynomial birth-death processes and the second conjecture of Valent.*

The conjecture of Valent about the type of Jacobi matrices with polynomially growing weights is proved.

**Alexei Bogolyubskii.** On domains of divergence of ray sequences of Frobenius–Padé approximants.

The asymptotics of the Frobenius–Padé approximations of Markov functions is described in terms of the vector equilibrium potential with the Nikishin interaction matrix. For all non-diagonal ray sequences, the pushing effect of the equilibrium measure occurs when the measure equals zero on a part of the segment. Using suitable uniformization, we find the explicit form of the equilibrium measure in terms of an algebraic function. We define the pushing point and the boundary of the domain of convergence of the approximations under consideration.

# **Ekaterina Borovik.** *Quadrature domains and approximative quadrature formulae for holomorphic functions.*

We plan to present a new approach for construction of approximative quadrature formulae for holomorphic functions in Jordan domains in the complex plane. The principal steps of this approach are as follows: first we approximate a given Jordan domain G by a quadrature domain Qin such a way that the boundaries of G and Q are arbitrarily close in the sense of the Hausdorff metric. The desired domain Q is the image of the unit disk  $\mathbb{D}$  under some univalent in  $\mathbb{D}$  polynomial g. Recall that the Schwarz function S of the domain Q has the form  $S(z) = g(1/g^{-1}(z))$ . Then the poles of S in Q (recall that S has no other singularities in Q) will be the nodes of the quadrature identity we construct. Finally, in order to find (numerically) the poles of S in Q and the respective residues we use one special construction based on Padé diagonal approximations.

## **Alexander Dyachenko.** *Rigidity of the Hamburger and Stieltjes moment sequences.*

We determine conditions on a sequence of Hamburger or Stieltjes moments, under which the change of at most a finite number of its entries produces another sequence of the same type. It turns out that a moment sequence allows all small enough perturbations of this kind precisely when it is indeterminate. In turn, determinate moment sequences with a finite index of determinacy only survive under certain perturbations of this kind.

#### Stavros Evdoridis. Bohr's inequality for harmonic mappings.

We state improved versions of Bohr's inequality for sense-preserving harmonic mappings with bounded analytic part, defined in the unit disc. The results are obtained along the lines of an earlier work of Kayumov and Ponnusamy and they are sharp.

# **Emanuel Guariglia.** Euler products, Dirichlet Series and Riemann zeta fractional derivative.

The theorem on Euler products allows to better understand the link between Riemann zeta function, critical strip and the distribution of prime numbers. The  $\alpha$ -order fractional derivative of the Riemann function is a nonmultiplicative Dirichlet series, hence the aforementioned theorem cannot be applied in order to derive the link with prime numbers. Furthermore, the introduction of the strip  $(\alpha, 1 + \alpha)$  as fractional counterpart of the critical strip raises more delicate problems, such as the fractional generalization of both the Riemann hypothesis and the critical line Re s = 1/2. The principal candidate for this role is the line Re  $s = 1/2 + \alpha$ . Based on these considerations, analytic properties of the strip  $(\alpha, 1 + \alpha)$  are derived together with the link with prime numbers.

## **Olesya Krivosheeva.** Singular points of sum of exponential monomials series.

Let  $\Lambda = \{\lambda_k, n_k\}_{k=1}^{\infty}$  be a divisor in the complex plane such that  $|\lambda_k| \to \infty, k \to \infty$ . The series of exponential monomials constructed according to the sequence  $\Lambda$  are considered. Sufficient conditions are obtained for the existence of a sum of a series of exponential monomials that has no singular points on a given arc of a given convergence domain of this series. These conditions are formulated in terms of the relationship between the minimum density of the sequence  $\Lambda$  and the arc length of the convergence domain of the series of exponential monomials.

Maria Lapik, Dmitry Tulyakov. On expanding neighborhoods of local universality for Gaussian Unitary Ensembles.

We will consider classical universality theorem for Christoffel– Darboux kernels of Hermite Polynomials for growing neighborhoods. We will show that uniform convergence of Christoffel–Darboux kernels in local variables  $\tilde{x}, \tilde{y}$  in a neighborhood of  $x^* \in (-\sqrt{2}, \sqrt{2})$  to sinuskernel will take place in expanding intervals.

For  $x^* = 0$  and  $m(n) = o(n^{\frac{2}{3}})$  the Christoffel–Darboux kernel converges when  $\tilde{x}, \tilde{y} \in [-m(n), m(n)]$ . For  $x^* \in (-\sqrt{2}, \sqrt{2}) \setminus \{0\}$  the Christoffel–Darboux kernel converges when  $m(n) = o(n^{\frac{1}{2}})$ .

If  $x^*$  tends to zero like  $O(n^{-\frac{1}{3}})$ , then  $m(n) = o\left(\left(\frac{n}{x^*}\right)^{\frac{1}{2}}\right)$ . If  $x^*$  tends to zero faster, then  $m(n) = o(n^{\frac{2}{3}})$ .

**Andrei Lishanskii.** On hypercyclic rank one perturbations of unitary operators.

Recently, S. Grivaux showed that there exists a rank one perturbation of a unitary operator in a Hilbert space which is hypercyclic. Another construction was suggested later by Baranov and Lishanskii. Using a functional model for rank one perturbations of singular unitary operators, we give yet another construction of hypercyclic rank one perturbation of a unitary operator. In particular, we show that any Carleson set on the circle can be the spectrum of a perturbed (hypercyclic) operator. This is a joint work with Anton Baranov and Vladimir Kapustin.

## **Vladimir Lysov.** On Hermite–Padé approximants for the product of two logarithms.

The Hermite–Padé approximants for systems of functions, containing  $\ln(1+1/z)\ln(1-1/z)$  are considered. Two constructions are considered, for which it is possible to find an explicit form of Hermite–Padé

approximants. Their asymptotic behavior is studied and convergence is proved. The number-theoretic application related to Diophantine approximations for products of logarithms is obtained.

**Khudoyor Mamayusupov.** An application of parabolic surgery to the family of Newton maps of polynomials.

We apply Haïssinsky parabolic surgery to the family of Newton maps of polynomials to obtain the family of Newton maps for the entire functions of the form  $p(z) \exp(q(z))$ , where p and q are polynomials.

**Igor Popov.** Functional model and completeness of resonance states for quantum graphs.

Scattering problem for quantum graph is considered. Resonances and resonance state are described. Completeness of resonance states is studied. Quantum graphs of different geometries with the Schrödinger and Dirac operators at the edges and hybrid manifolds including parts of different dimensions are considered. Functional model approach is used for the proofs. It is a joint work with A. I. Popov, I. V. Blinova, A. G. Belolipetskaya.

#### Anastasia Ulitskaya, Oleg Vinogradov. Zeros of the Zak

transform of totally positive functions and their averages.

Let  $\alpha > 0$  and let  $g \in L_1(\mathbb{R})$  be a continuous function, whose Fourier transform is

$$\widehat{g}(\omega) = C e^{-\gamma \omega^2} e^{-2\pi i \delta \omega} \left( \prod_{\nu=1}^{\infty} \frac{e^{2\pi i \delta_{\nu} \omega}}{1 + 2\pi i \delta_{\nu} \omega} \right) \left( \prod_{j=1}^{m} \frac{e^{\lambda_j - 2\pi i \alpha \omega} - 1}{\lambda_j - 2\pi i \alpha \omega} \right),$$

where  $C > 0, \gamma \ge 0, \delta, \delta_{\nu}, \lambda_j \in \mathbb{R}, \sum_{\nu=1}^{\infty} \delta_{\nu}^2 < \infty, m \in \mathbb{Z}_+$ . We prove that its Zak transform  $Z_{\alpha}g(x,\omega) = \sum_{k\in\mathbb{Z}} g(x+\alpha k)e^{-2\pi i k\alpha \omega}$  has only one zero  $(x^*, \frac{1}{2\alpha})$  in the fundamental domain  $[0, \alpha) \times [0, \frac{1}{\alpha})$ . In particular, the result is valid for totally positive functions. Earlier it was known for such functions without the factor  $e^{-\gamma\omega^2}$ . We also establish that this zero is simple with respect to each variable and give applications to Gabor analysis.

## Sergey Zakharov. Differentiation in small de Branges spaces.

We use recent results of Yu. Belov, T. Mengestie and K. Seip about boundedness of discrete Hilbert transform to give a criterion for boundedness of differentiation in de Branges spaces of small growth in terms of their spectral data.