Geometry, Analysis, Groups

October 1-5, 2018

Euler International Mathematical Institute

Programme

Monday

09:30-10:00 Registration
10:00-11:00 Nevo
11:00-11:30 Coffee break
11:30-12:30 Kato
12:30-14:30 Lunch
14:45-15:45 Petrov
15:45-16:00 Coffee break
16:00-17:00 Vershik
17:00-18:00 Zuk
18:00 Reception

Tuesday

10:00-11:00 Nevo
11:00-11:30 Coffee Break
11:30-12:30 Kato
12:30-14:30 Lunch
14:45-15:45 Kahrobaei
15:45-16:00 Coffee break
16:00-17:00 Carbone
17:00-18:00 Ivanov

Wednesday

10:00-11:00 Nevo
11:00-11:30 Coffee break
11:30-12:30 Kato
12:30-14:30 Lunch
16:00-18:00 Visit to the Hermitage
19:00 Conference Dinner

Thursday

10:00-11:00 Nevo
11:00-11:30 Coffee break
11:30-12:30 Kato

12:30-14:30 Lunch
14:45-15:45 Brothier
15:45-16:00 Coffee break
16:00-17:00 Brieussel
17:00-18:00 Tsujimoto

Friday

10:00-11:00 Nevo
11:00-11:30 Coffee Break
11:30-12:30 Stavrova
12:30-14:30 Lunch

Lectures series:

Amos Nevo (Technion) - “Effective ergodic theorems for algebraic and arithmetic groups, and their applications to lattice point counting problems and Diophantine approximation"

We will begin by discussing aspects of ergodic theory and representation theory of group actions, including a discussion of property T and the spectral gap property.

We will then show that for algebraic and arithmetic groups it is possible to sharpen and extend the spectral estimates arising from property T considerably beyond their usual formulations. We will also discuss how to derive, in certain favorable situations, best possible spectral estimates via representation theory.

In turn, these spectral estimates will be used to derive effective ergodic theorems in actions of the groups under consideration. We will then show how the rate of convergence in the effective ergodic theorems implies effective solutions in a host of natural problems.

These will include non-Euclidean lattice point counting problems, fast equidistribution of lattice orbits on homogenous spaces, exponents of Diophantine approximation on homogeneous spaces (including some best possible ones), and effective solution counts to intrinsic Diophantine inequalities on homogeneous algebraic varieties, as time permits.

Our goal is to give an exposition of the subjects mentioned which is suitable for a diverse audience, and which is fairly explicit and not too technical. Many open problems will be indicated along the way.

Tsuyoshi Kato (Kyoto University) - “Non commutative geometry and gauge theory ”

In my series of lectures, I will introduce extensions of gauge theory
by integrating it with non commutative geometry from the view point of fundamental groups.
I will select some of topics among the following lists.

(1) Twisted Donaldson invariant

Abstract: We define a twisted Donaldson's invariant using the Dirac operator twisted by flat connections when the fundamental group of a four manifold is free abelian. We also verify non triviality of the invariant by presenting some exotic pairs which are obtained from them. Then using Connes-Moscovici's index theorem in non commutative geometry, we introduce the construction of twisted Donaldson's invariant when fundamental group is non-abelian. It leads to a canonical question which we would address, whether the twisted invariant vanishes when a four manifold admits a connected sum decomposition. We also include basic subjects on both Yang-Mills theory and non commutative geometry.

(2) Higher Nahm transform in non commutative geometry

Abstract: Anti-self-dual (ASD) connections over a compact four manifold X attain the critical values on Yang-Mill functional. Nahm transform is a correspondence from a vector bundle with a connection on X to another vector bundle with a connection on the Picard torus. In the case of four torus, it transforms ASD to ASD. In this talk we propose a noncommutative geometric version of Nahm transform, which generalise Connes-Yang-Mills functional via higher Dixmier trace.

(3) Higher degree of the covering monopole map
Abstract: I will introduce a monopole map over universal covering spaces of compact four manifolds. In particular we can formulate higher degree of the covering monopole map when the linearized maps are isomorphic. It induces a homomorphism between K theory of group C^* algebras. As an application we propose an aspherical inequality on compact aspherical four manifolds. This presents a stronger version to 10/8 inequality by Furuta, in the aspherical class of four manifolds. This holds for many cases which include some complex surfaces of general type. Technically the construction of the covering monopole map requires non linear estimates in Sobolev spaces and will motivate L^p analysis on non compact manifolds.

Talks:

Arnaud Brothier (Sydney) - "Unitary representations of the Thompson's group constructed with a category/functor method due to Jones"

Jérémie Brieussel (Montpellier) - "Numerical upper bounds on growth exponents of automata groups"

Lisa Carbone (Rutgers) - "A Lie group analog for the Monster Lie algebra"

Fedor Petrov (Saint Petersburg) - "Asymptotics of traces of paths on graded graphs"

Sergei Ivanov (Saint Petersburg) - "Transfinite invariants of groups and spaces"

Delaram Kahrobaei (New York) - "Algorithmic Problems in Right Angled Groups and Applications"

Anastasia Stavrova (Saint Petersburg) - “Non-split groups graded by root systems”

Satoshi Tsujimoto (Kyoto) - “Dynamics of the box-ball system with random initial conditions via Pitman's transformation”

Anatoly Vershik (Saint Petersburg) - "New results about limit shape for Plancherel measure"

Andrzej Zuk (Paris) - “Ultradiscrete limits of PDE’s”

Abstracts:

Arnaud Brothier (Sydney University)

Title: Unitary representations of the Thompson's group
constructed with a category/functor method due to Jones

Abstract: The Thompson group is the group of homeomorphisms of [0,1] that are piecewise linear with slopes a power of 2 and breakpoints a dyadic rational. It is one of the most studied discrete group which still remains very mysterious.
Motivating in constructing conformal field theories Jones recently discovered a very general process that produces a unitary representation of the Thompson group from very few data such as an isometry between Hilbert spaces or a planar algebra together with a certain element. We will present this construction and provide concrete examples.
This is a joint work with Jones.

Jérémie Brieussel (Université de Montpellier)

Title: Numerical upper bounds on growth exponents of automata groups

Lisa Carbone (Rutgers University)

Title: A Lie group analog for the Monster Lie algebra

Abstract: The Monster Lie algebra m, which admits an action of the Monster finite simple group M, was constructed by Borcherds as part of his program to solve the Conway-Norton conjecture about the representation theory of M. We associate the analog of a Lie group G(m) to the Monster Lie algebra m. We give generators for large free subgroups and we describe relations in G(m).

Fedor Petrov (Saint Petersburg)

Title: Asymptotics of traces of paths on graded graphs

Abstract: Let G be a graded graph with levels $V_0,V_1,\dots$. Fix m and choose a vertex v on the level $V_n, n\geqslant m$.
Consider the uniform measure on the paths from V_0 to the vertex v. Each such a path has a unique vertex on the level V_m, and so the measure ν_v^m on V_m is induced. It is natural to expect that such measures have a limit when vertex v goes to infnity by somehow ``regular''. This general approach to ``Plancherel'' measure on the path space was proposed by Vershik. We justify it for the classical graphs of Young and Schur which correspond to linear and projective representations, respectively, of the symmetric group.

Sergei Ivanov (Saint Petersburg)

Title: Transfinite invariants of groups and spaces

Abstract: Let R be one of the rings Z,Q,Z/p. Bousfield has defined the R-homological localization of a space X as the universal map in the homotopy category X --> X_R that induces an isomorphism on homology H_*(-,R). This notion is closely related to purely algebraic notion of HR-localization of a group, which can be defined as a version an algebraic closure with respect to some class of group equations. HR-localisation of a group is a transfinitely (R-)nilpotent group. We are interested in the length of the transfinite lower central series of HR-localization and call it HR-length of the group. For a free group F we proved that HZ-length(F)> omega+1; HZ/p-length(F)> omega; HQ-length(F)> omega.

We are also interested in HR-length of semidirect products of the cyclic group Z on Z^n, where the action of Z on Z^n is given by some matrix. We proved that under some assumptions the HZ-length of such group can be described on the language of eigenvalues of the matrix. Using this approach we show how to distinguish some fibrations (S^1)^n --> X --> S^1 up to homological equivalence. I will also show a connection of this theory with discrete homology of pro-p-groups and give a sketch of a prove that H_2(F,Z/p) is not trivial, where F is a free pro-p-group. The proof uses a lot of different techniques including Baire category theorem.

Delaram Kahrobaei (CUNY, New York)

Title: Algorithmic Problems in Right-Angled Artin Groups: Complexity and Applications

Abstract. In this talk we consider several classical and novel algorithmic problems for right-angled Artin groups, some of which are closely related to graph theoretic problems, and study their computational complexity. We study these problems with a view towards applications to cryptography. This is joint work with Ramón Flores and Thomas Koberda.

Anastasia Stavrova (Saint Petersburg)

Title: Non-split groups graded by root systems

Abstract. We discuss several results in the spirit of the congruence subgroup problem for a class of groups graded by root systems in the sense of Ershov, Jaikin-Zapirain, and Kassabov. These groups arise from isotropic simple algebraic groups and generalize elementary Chevalley groups and simple Lie groups.

Satoshi Tsujimoto (Kyoto University)

Title: Dynamics of the box-ball system with random initial conditions via Pitman's transformation

Abstract: The box-ball system (BBS), introduced by Takahashi and Satsuma in 1990, is a cellular automaton that exhibits solitonic behaviour. It is related to the Korteweg–de Vries equation, which is used for modelling shallow water waves. In a joint work with David Croydon (Kyoto University), Tsuyoshi Kato (Kyoto University) and Makiko Sasada (University of Tokyo), we explore the dynamics of the BBS started from random initial conditions. In particular, we show that the model can be described using the transformation of a nearest neighbour path encoding of the particle configuration given by `reflection in the past maximum', which was shown by Pitman to connect Brownian motion and a three-dimensional Bessel process. We use this to characterise the set of configurations for which the dynamics are well-defined and reversible for all times. We give simple sufficient conditions for random initial conditions to be invariant in distribution under the BBS dynamics, which we check in several natural examples, and also investigate the ergodicity of the relevant transformation. Furthermore, we analyse various probabilistic properties of the BBS that are commonly studied for interacting particle systems, such as the asymptotic behavior of the integrated current of particles and of a tagged particle.

Anatoly Vershik (Saint Petersburg)

Title: New results about limit shape for Plancherel measure

Andrzej Zuk (Université Paris 7)

Title: Ultradiscrete limits of PDE’s