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Abstracts

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ON WORD AND DIVISIBILITY PROBLEMS FOR ONE RELATOR SEMIGROUPS

SERGEI I. ADIAN

For semigroups presented by a single left irreducible defining relation we describe an algorithm that computes the shortest proof of divisibility of a given word by a given letter of the semigroup alphabet. It is guaranteed that the algorithm terminates each time the given word is in fact divisible by the given letter in that semigroup. The problem of termination of this algorithm is of crucial importance for applications. In particular, it yields a complete solution to the well-known word problem for one relator semigroups.

I have described this algorithm in 1976 while trying hard to find a positive solution of the now famous but still open problem of decidability of the word problem for one relator semigroups. It was published in volume 15 of *Algebra and Logic* in a paper dedicated to the memory of Mikhail Ivanovich Kargapolov after his untimely death [1]. Shortly before that, during an algebraic conference in Novosibirsk, he announced a contest for the young participants to solve this problem. In his book *Algorithms and Recursive Functions* [3], A. I. Maltsev mentioned that this problem was "nearly solved" by S. I. Adian, however by now it is clear that the problem is notoriously difficult and, quoting P. S. Novikov, "contains something transcendental". Its solution is a task for the future generations of mathematicians.

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REBUILDING EPISTEMIC LOGIC

SERGEI ARTEMOV

All three foundational pillars on which Epistemic Logic rests: modal language, Kripke-style semantics, and proof theory need rebuilding and modernization.

1. Modal language alone does not support such central topics in Epistemology as "knowledge vs. justified true belief" discussion, (cf. [7, 5, 6], etc.) due to a lack of justification objects in epistemic logic. Situation is gradually improving with the introduction of Justification Logic ([1, 2]) but there is still a long way to go. We provide examples of situations in which Justification Logic methods offer a superior epistemic analysis. In a more general setting, it is the Justification Logic framework which has introduced much needed hyperintensionality into Epistemology [4].

2. Kripke semantics of possible worlds for epistemic logic is based on a hidden assumption of common knowledge of the model, *CKM*, manifested in the condition "if a sentence is valid at all possible states, then it is known". In social scenarios, however, agents may possess asymmetric knowledge of the situation and *CKM* as a uniform assumption should be resisted. What we need here is a new theory of epistemic modeling in a general setting without assuming common knowledge of the model. We introduce epistemic models which do not rely on *CKM* [3]. Conceptually, such general epistemic models can be viewed as *observable fragments* of comprehensive Kripke models.

3. A well-principled notion of *epistemic theory* as an axiomatic description of a given scenario incorporated into the possible worlds environment is conspicuously absent. Moreover, given an informal verbal description of a situation, a typical epistemic user cherry-picks a "natural model" and simple-mindedly regards it as a formalization of the original description, i.e. uses a model in lieu of a theory and ignores the fact that there might be different "natural models" of the same description. In this respect, a systematic confusion of a theory and a model in Epistemic Logic resembles the pre-Gödelian state of mathematical logic, without a clear distinction between theories and models. We describe a framework of *hypertheories* for epistemic reasoning with partial information. Remarkably, natural semantic counterparts of hypertheories are epistemic models from (2), not Kripke models.

Together with epistemic models, hypertheories provide a new and balanced syntactic/semantic foundation for epistemic reasoning.

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THE MECHANIZATION OF MATHEMATICS

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The phrase “formal methods” is used to describe a body of methods in computer science for specifying, developing, and verifying complex hardware and software systems. The word “formal” indicates the use of formal languages to write assertions, define objects, and specify constraints. It also indicates the use of formal semantics, that is, accounts of the meaning of a syntactic expression, which can be used to specify the desired behavior of a system or the properties of an object sought. Finally, the word “formal” suggests the use of formal rules of inference, which can be used to verify claims or guide a search.

Such methods hold great promise for mathematical discovery and verification of mathematics as well. In this talk, I will survey some applications, including verifying mathematical proofs, verifying the correctness of mathematical computation, searching for mathematical objects, and storing and communicating mathematical results.

Interactive theorem proving involves the use of computational proof assistants to construct formal proofs of mathematical claims, using the axioms and rules of a formal foundation that is implemented by the system. The user of such an assistant generally has a proof in mind and works interactively with the system to transform it into a formal derivation. Proofs are presented to the system using a specialized proof language, much like a programming language. I will discuss the current state of the field, and some recent milestone formalizations.

One place for formal verification is especially useful is in the case of mathematical proofs that rely on substantial uses of computation, where the associated code is subtle and susceptible to error. I will discuss various strategies that are employed to make such computational results more reliable.

The use of formal search methods to establish theorems of core mathematics is less common, but nonetheless I will discuss a few notable successes to date, as well as prospects for the future.

Finally, I will briefly discuss projects like the *Formal Abstracts* project, which aim to provide digital infrastructure to support mathematical activity.

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THE CONCEPT OF PROOF

MATTHIAS BAAZ

The concept of proof is one of the most fundamental building blocks of mathematics. The Hilbertian revolution at the beginning of the 20th century is based on an atomic notion of proof which is the foundation of the axiomatic method:

"A proof is a finite sequence of formulas $A_1, \ldots, A_n$ such that each A_i is instance of an axiom or follows by direct application of a rule from $A_{i_1}, \ldots, A_{i_k}$ with all $i_j < i$".

No scientific revolution is however total, but there is a trend to disregard all alternatives to the successful method. In this lecture we discuss more global notions of proof, where subproofs are not necessarily proofs themselves. Examples are among others:

1. protoproofs in the sense of Euler's famous solution to the Basel problem, which uses analogical reasoning and where additional external justifications are necessary;
2. circular notions of proof, where the concept of proof itself incooperates induction. The most significant example is Pierre de Fermat's Methode de Descente, for a modern setting cf. [1];
3. sound proofs based on locally unsound rules cf. [2];
4. proofs based on abstract proof descriptions prominent e.g. in Bourbaki, where only the choice of a suitable result makes a verification possible cf. [3].

We discuss the benefits of these alternative concepts and the possibility that innovative concepts of proof adapted to the problems in question might lead to strong mathematical results and constitute a novel area of Proof Theory.

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ON THE COMPUTATIONAL CONTENT OF THEOREMS

VASCO BRATTKA

To analyze the computational content of theorems is a research topic at least since Turing's seminal work on computable numbers in which he started the investigation of computable versions of theorems in analysis. In the sequel this topic was taken up by many other researchers such as Specker, Lacombe, Shore and Nerode, Pour-El and Richards [2], and Weihrauch [4]. A related but formally different approach has been started by Friedman and Simpson [3] who have characterized axioms that are sufficient and often necessary to prove certain theorems in second-order arithmetic. In recent years the interaction between these two research trends has been intensified and overlaps in what is called Weihrauch complexity. Weihrauch complexity is a computability theoretic approach to the classification of the computational content of theorems that yields results that can be seen as a uniform and resource sensitive version of reverse mathematics. The benefit of this theory is that it yields fine grained computational results that answer typical questions from the computable analysis perspective, while being compatible with reverse mathematics. Sometimes results can be imported from reverse mathematics and computable analysis, but often completely new methods and techniques are required. We will present a survey on this approach that is based on a recent survey article [1] on this topic.

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ENCOUNTERS WITH INFINITY

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Questions about the ontological status of the objects about which mathematicians reason have been with us since ancient times. In my talk I will emphasize the role of mathematical practice in expanding the realm of mathematical discourse. I will begin with the example of Gödel's struggles with the philosophical consequences of his two main discoveries: the inevitability of undecidability and the consistency of the continuum hypothesis.

I will then present a number of revealing examples from the history of mathematics. The solution in terms of radicals of cubic equations seemed to force practitioners to work with square roots of negative numbers although these were thought to be impossible. Torricelli considered the region bounded above by a rectangular hyperbola, below by one of its asymptotes, and to the left by a perpendicular from the hyperbola to that asymptote. He was able to show that while that region is of infinite extent and has an infinite area, the solid formed by revolving it about the asymptote has a finite volume. This provided a shock to the world of 17th century mathematics, contradicting what Aristotle had taught about infinity.

Leibniz's infinitesimal calculus yielded useful answers although reasoning with his infinitesimals seemed to lead to contradictions. By assuming that the relationships between the zeros and the coefficients of a polynomial would hold as well for an infinite power series, Euler was able to obtain the sum of the series $\sum_{n=1}^{\infty} 1/n^2$. In solving a partial differential equation for heat conduction, Fourier used trigonometric series with a quite unjustified expansive freedom. This led Dirichlet to the modern notion of a function as an arbitrary mapping.

Cantor's investigation of uniqueness theorems for trigonometric series led him to develop his transfinite ordinal numbers. Contemporary set theorists were able to resolve hitherto intractable problems concerning the hierarchy of projective sets by invoking the determinacy of projective sets as a new axiom. More recently, assumptions about the hierarchy of large cardinals was used to prove this axiom.

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THE COMPUTABILITY VIA DEFINABILITY IN SEMANTIC MODELING

SERGEI S. GONCHAROV*

This is joint work with D. I. Sviridenko and A. Nechesov.

The construction of computability on abstract structures was founded in the theory of semantic programming in [1–6]. We will discuss some problems in this approach connected with computability and definability. The main idea of this construction was created on the base of restricted quantifiers. In [1–4] construction of a programming language of logical type was proposed for creating the programming systems that provide control of complex systems in which control under different conditions depends on the type of the input data represented by formalisms of logical type on the basis of logical structures. For constructing an enrichment of the language with restricted quantifiers, we extend the construction of conditional terms. We show that the so-obtained extension of the language of formulas with restricted quantifiers over structures with hereditary finite lists is a conservative enrichment. For constructing some computability theory over abstract structures, in [6–7], Yu. L. Ershov considered a superstructure of hereditarily finite sets. From the problems in Computer Science the superstructure of hereditarily finite lists was constructed in [3], and the computability theory was developed in terms of Σ-definability in this superstructure. From the standpoint of constructing a programming language, such an approach seems more natural for accompanying logical programs since for a specific implementation of a language of logical type on sets, we must externally define the sequence of an efficient exhaustion of their elements. In choosing a list of elements, the order is already contained in the model, and we have a definition in the model of operations that explicitly define the work with the list items. However, from the viewpoint of the construction of programs, taking into account the complexity of their implementation, it is preferable to consider their constructions based on the Δ_0-construction while retaining sufficiently broad logical means of definitions, and on the other hand, ensuring more imperative constructions in the required estimates of performance complexity.

In this talk, we consider the questions of definability on the basis of the Δ_0-formulas whose verification of truth has bounded complexity

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with respect to the basic terms and relations in the basic model, as well as the implementation of the list operations in the superstructure.

We will define some new construction for new extension of the notion of term be conditional terms, bounded recursion and some bounded Δ_0-definable function in such way that each term in this definition will give computable function are polynomial if basic function and relations in our superstructure have polynomial complexity and for each Δ_0^*-formulas in with these terms we have polynomial algorithm to verify their truth [8].

The next theorem about this construction gives us these possibilities.

Theorem (about conservatism)**.** *There exists some algorithm for construction by Δ_0^*-formula φ Δ_0-formulas ψ without non-standard terms such that*

$$HW\left(\mathfrak{M}\right) \models (\forall \overline{v})(\varphi\left(\overline{v}\right) \Leftrightarrow \psi\left(\overline{v}\right)).$$

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LOGIC IN COMPUTER SCIENCE, COMPUTER ENGINEERING AND MATHEMATICS

YURI GUREVICH

In software industry, engineers do formal logic day in and day out, even though they may not and usually do not realize that. As a rule, they have not studied logic. Instead, they studied calculus which they use rarely, if ever.

We illustrate why logic is so relevant to computer science and to computer industry and why it is so hard for software engineers to pick it up.

At the end we discuss the uses of formal logic in mathematics and the prospects of logic in mathematics departments.

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IN PURSUIT OF A MEDIEVAL MODEL THEORY

WILFRID HODGES

Logicians have always worked with some notion of logical consequence. Usually their notion of logical consequence belongs to one of the following two types. We say that θ is a logical consequence of the premises Φ if:

> (Proof-theoretic) *There is* a pattern of inference steps (a 'derivation') that leads from Φ to θ.
>
> (Model-theoretic) *Every* interpretation making Φ true (i.e. every model of Φ) also makes θ true.

The difference between 'There is' and 'Every' implies that these two notions of consequence will be used in very different ways.

The proof-theoretic notion goes back to Aristotle. The model-theoretic notion was introduced as a basic notion of model theory in papers of Abraham Robinson and Tarski in 1949–1954, after Tarski had called the attention of philosophers to this notion in a paper of 1936. In fact truth-tables (Wittgenstein and Post around 1920, Peirce a little earlier) had already introduced a propositional version of model-theoretic consequence. For countable first-order logic, the agreement between proof-theoretic and model-theoretic notions of consequence was stated and proved in Gödel's doctoral dissertation in 1929. In these ways the notion began to be used in Western logic in the 20th century, over two thousand years after Aristotle had first introduced logic. Why so late?

During the last six months it came to light that in the mid 12th century Abū al-Barakāt al-Baghdādī, a Jew based in Baghdad, had a system of syllogistic logic up and running, in which he used only the model-theoretic notion of consequence. Barakāt was already known as a perceptive philosopher and physicist—he was the first to state that bodies fall with constant acceleration. But it was not realised that he broke the mould in logic too. The present talk will be to some extent a preliminary report on this discovery.

In brief, Barakāt showed how we can deduce the conclusion of a productive syllogism (i.e. one that has a conclusion) by listing representatives of all possible models of the premises, and looking to see what propositions are true in all these models. He also invented a notation to accompany these calculations. His notation is interesting as the earliest known system of logical diagrams for proving consequences, anticipating

Leibniz by 500 years. Unlike the diagrams of Leibniz, Euler and Venn, Barakāt's diagrams represent models, not propositions.

About a hundred years before Barakāt, Ibn Sīnā (= Avicenna) started to develop model-theoretical consequence for proving non-entailments—similar to Hilbert's *Grundlagen der Geometrie*, though done entirely within logic. Unlike Barakāt, Ibn Sīnā made significant mistakes. But his models are more concrete than those of Barakāt, and there are clear signs that he took them to consist of a set on which relations are defined, just as in today's model theory.

In keeping with the interest in 'perspectives' at this meeting, we discuss how this sudden appearance of model theory in the 11th and 12th centuries, and its equally sudden disappearance after the death of Barakāt, make sense within the history of logic as a whole. For example, what logical tools needed to be developed to sustain model theory, and why was there an incentive to build these tools in the 20th century but not in the 13th? We note that Barakāt's ideas were in a sense already implicit in Aristotle's work in the 4th century BC. This raises further questions: can we trace a development from Aristotle to Barakāt? (The 6th century logician Paul the Persian is a likely intermediary.) Why was there no development along these lines for maybe a thousand years after Aristotle himself?

Saloua Chatti, Amirouche Moktefi, Seyed Mousavian, Lukas Muehlethaler, Moshe Pavlov, Richard Sorabji and Robert Wisnovsky have all provided valuable input into this work, for which I thank them.

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HYPE: A SYSTEM OF HYPERINTENSIONAL LOGIC (WITH AN APPLICATION TO SEMANTIC PARADOXES)

HANNES LEITGEB

This lecture introduces, studies, and applies a new system of logic which is called 'HYPE'. In HYPE, formulas are evaluated at states that may exhibit truth value gaps (partiality) and truth value gluts (overdeterminedness). Simple and natural semantic rules for negation and the conditional operator are formulated based on an incompatibility relation and a fusion operation on states. The semantics is worked out in formal and philosophical detail, and a sound and complete axiomatization is provided both for the propositional and the predicate logic of the system. The propositional logic of HYPE can be shown to contain first-degree entailment, to have the Finite Model Property, to be decidable, to have the Disjunction Property, and to extend intuitionistic propositional logic conservatively when intuitionistic negation is defined appropriately by HYPE's logical connectives. Furthermore, HYPE's first-order logic is a conservative extension of intuitionistic logic with the Constant Domain Axiom, when intuitionistic negation is again defined appropriately. The system allows for simple model constructions and intuitive Euler-Venn-like diagrams, and its logical structure matches structures well-known from ordinary mathematics, such as from optimization theory, combinatorics, and graph theory. HYPE may also be used as a general logical framework in which different systems of logic can be studied, compared, and combined. In particular, HYPE is found to relate in interesting ways to classical logic and various systems of relevance and paraconsistent logic, many-valued logic, and truthmaker semantics. On the philosophical side, if used as a logic for theories of type-free truth, HYPE is shown to address semantic paradoxes such as the Liar Paradox by extending non-classical fixed-point interpretations of truth by a conditional as well-behaved as that of intuitionistic logic. Finally, HYPE may be used as a background system for modal operators that create hyperintensional contexts, though the details of this application need to be left to follow-up work.

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UNIVERSALLY BAIRE SETS AND BOREL CANONIZATION

MENACHEM MAGIDOR

Using the axiom of choice, one can construct sets of reals which are quite pathological: e.g. non-measurable, not having the Baire property etc. But the guiding principle of descriptive set theory is that if the set is "nicely" definable then it is not pathological. An possible definition of the maximal family of "nice" sets of reals (or of any Polish space) is the family of Universally Baire sets, introduced in [4].

In this talk we shall survey the definition and some of the basic "niceness" properties of the family of universally Baire subsets of a Polish space. Some of these properties depends on the assumptions of strong axioms of infinity. ("The existence of large cardinals".)

As an example of the regularity properties of universally Baire sets, we shall discuss the problem of Borel canonization. This problem was introduced by Kanovei, Sabok and Zapletal ([5]). In the original setting we are given an analytic equivalence relation E and an ideal I on the reals. The problem is to find a Borel set B which is not in the ideal such that E restricted to B is Borel. In this generality the answer is "NO", but if we put some "nicety" conditions on I and the equivalence relation E one can get a positive answer, assuming some large cardinals. (These results are due to W. Chan and O. Drucker, independently: [3] and [1].)

In the talk we shall survey some possible generalizations of these results. For instance when we assume that the relation E is universally Baire. (Some of the results are joint results with W. Chan [2].)

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THE UNDECIDABILITY OF VARIOUS AFFINE PAPPIAN GEOMETRIES: WRONG PROOFS AND NEW TRUE THEOREMS

JOHANN A. MAKOWSKY

In his Ph.D. thesis [3], W. Rautenberg claimed to have proven that the set of first order consequences of affine incidence geometry is undecidable. Although his proof is cited in many followup papers, his proof is based on wrong reduction to the undecidablity of the first order theory of fields. To the best of our knowledge, we have not found a complete and correct proof in the literature. In this paper we analyze his mistake, give a correct proof, and extend the result to many other axiomatizations of geometry. These include the geometry of Hilbert and Euclidean planes, Wu's geometry [4] and Origami geometry [1]. We also discuss applications to automated theorem proving. An important tool is M. Ziegler's Theorem proving that no finite subset of the theory of real closed fields is decidable, [5], translated in [2].

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ON CONSTRUCTIVE VERSIONS OF INDEPENDENCE-FRIENDLY LOGIC

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This talk presents the recent results of the joint work of the author, S. O. Speranski and I. Yu. Shevchenko, which are partly presented in [9] and partly are in progress.

The independence-friendly first-order logic (IF-FOL) suggested in [4] has generated numerous dependence and independence logics — whose specific operators can be easily defined in terms of so-called *teams* (see [5, 6]). Recall that a *team* is a family of assignments of elements of the domain of a first-order structure to individual variables, or a family of valuations of propositional variables in the set of truth values; usually it is assumed that all members of a team have the same domain. The logic IF-FOL is an extension of first-order logic (FOL) by means of *independent quantifiers* of the form $\exists x \backslash X$ where $\{x\} \cup X$ is a finite set of individual variables. The validity of a formula $\exists x \backslash X\, \varphi$ in a structure $\mathfrak{M}$ on a team T means that the formula φ is valid in $\mathfrak{M}$ on a team $T' = \{s\,(x/f\,(s)) \mid s \in T\}$* where f is a function from T to the domain of $\mathfrak{M}$ such that $f\,(s) = f\,(s')$ whenever $s\,(y) = s'\,(y)$ for all $y \in \mathrm{dom}\,(s) \setminus X$ — in this way, the value of s on x is independent of its values on the variables in X. Equivalently, IF-FOL can be easily interpreted using skolemisations, so as Skolem terms for occurences of $\exists x \backslash X$ do not contain variables from X.

The logic IF-FOL admits a game theoretical interpretation too. To obtain a game theoretical semantics (GTS) for IF-FOL, we have to pass from standard games used to interpret formulas in FOL to games with imperfect information. Hintikka [3, Chapter 6] motivates the game theoretical approach to interpreting IF-FOL as follows:

> The approach presented in this book has a strong spiritual kinship with constructivistic ideas. This kinship can be illustrated in a variety of ways. One of the basic ideas of constructivists like Michael Dummett [1, 2] is that meaning has to be mediated by teachable, learnable, and practicable human activities. This is precisely the job which semantical games do in game-theoretical semantics.

*Here $s\,(x/a)$ denotes the assignment with domain $\mathrm{dom}\,(s) \cup \{x\}$ such that $s\,(x/a)\,(x) = a$, and $s\,(x/a)\,(y) = s\,(y)$ for $y \neq x$.

In fact this statement made by Hintikka motivated us to compare GTS for FOL and IF-FOL with one standard constructive semantics, namely with the modification of realizability semantics suggested by D. Nelson [8]. To be more precise, let $\sigma_{\mathbb{N}}$ and $\mathfrak{N}$ be the signature of Peano arithmetic and its standard model, i.e.

$$\sigma_{\mathbb{N}} \ := \ \{0, \mathsf{s}, +, \times, =\} \quad \text{and} \quad \mathfrak{N} \ := \ \langle \mathbb{N}; 0^{\mathbb{N}}, \mathsf{s}^{\mathbb{N}}, +^{\mathbb{N}}, \times^{\mathbb{N}}, =^{\mathbb{N}} \rangle.$$

For any $e \in \mathbb{N}$, assignment s in $\mathfrak{N}$ and first-order $\sigma_{\mathbb{N}}$-formula ϕ with $FV(\phi) \subseteq \mathrm{dom}(s)$, D. Nelson [8] inductively defines

$$e \, \text{Ⓟ} \, s, \phi \quad \text{and} \quad e \, \text{Ⓝ} \, s, \phi.$$

If $e \, \text{Ⓟ} \, s, \phi$ (respectively $e \, \text{Ⓝ} \, s, \phi$), then the number e is called *a positive* (*negative*) *realization for ϕ under s*. Roughly speaking, each positive (negative) realization of ϕ under s encodes an effective verification (respectively falsification) procedure for ϕ in $\mathfrak{N}$ under s. Negation can be viewed as a kind of switch between verification and falsification procedures in Nelson's semantics, which is similar to how it behaves in GTS, where the players switch their roles when they see $\neg$. This observation explains our choice of constructive semantics for comparing with GTS. On this way we obtain the following results.

i. First, omitting the requirements of constructivity in the definition of realizations, we define, for any pair s, ϕ with $FV(\phi) \subseteq \mathrm{dom}(s)$, two families of set theoretical objects $S^{+}(s, \phi)$ and $S^{-}(s, \phi)$. In GTS for FOL, two strategies of the same player are called equivalent if the sets of histories played according to these strategies coincide. It turns out that there is a natural one-to-one correspondence between elements of $S^{+}(s, \phi)$, where ϕ is implication-free, and winning strategies for Eloise (the initial verifier in GTS) up to the equivalence just defined. Similarly, there is a natural bijection between $S^{+}(s, \phi)$ and the equivalence classes of winning strategies for Abelard (who is the initial falsifier). By distinguishing effective objects in $S^{+}(s, \phi)$ and $S^{-}(s, \phi)$ and codifying them by natural numbers we get back to positive and negative Nelson's realizations for ψ under s. In this sense Nelson's realizability restricted to the implication-free first-order formulas can be viewed as an effective version of GTS for FOL.

ii. Next we propose a realizability interpretation for IF-FOL. More precisely, for any $e \in \mathbb{N}$, team T of assignments in $\mathfrak{N}$ and IF-FOL-$\sigma_{\mathbb{N}}$-formula ϕ with $FV(\phi) \subseteq \mathrm{dom}(T)$, we inductively define

$$e \, \boxed{\mathtt{P}} \, T, \phi \quad \text{and} \quad e \, \boxed{\mathtt{N}} \, T, \phi.$$

We show that the resulting realizability semantics is related to GTS for IF-FOL in exactly the same way as Nelson's restricted realizability to GTS for FOL.

iii. Finally, we show that the team realizability interpretation for IF-FOL appropriately generalises Nelson's restricted realizability interpretation for the implication-free first-order formulas. In fact, we establish that for 'effective' teams and implication-free first-order formulas, team realizations can be identified with computable sequences of Nelson's realizations.

In conclusion we shall discuss another approach to 'effectivizing' IF-FOL, which is based on the notion of effective strategy (defined as a computable function from sequences of actions to actions). A sketch of this approach can be found already in [3, Chapter 6]. We shall also discuss the equivalence of this approach and the one described above (which is based on the possibility of codifying elements of $S^{+}(s, \phi)$ and $S^{-}(s, \phi)$ by natural numbers). Lastly, we shall describe a version of IF-FOL with implication and discuss the possibility of defining a kind of independent implication. Nelson's realizability gives us a hint for such a definition.

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LOGIC AND COMPLEXITY

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The Π_1 reflection principle for a theory T says that if a Π_1 sentence is provable in T, then it is true.* This principle can also be stated as disjointness of two r.e. sets: the set of T-provable Π_1 sentences and the set of false Π_1 sentences. We are interested in this kind of sentences scaled down to nondeterministic polynomial time computations. For a formal system P, Razborov defined the *canonical disjoint NP pair* of the proof system P as the pair of sets

$$\{(\phi, 0^n) \mid \exists\pi,\ \pi \text{ is a } P\text{-proof of } \phi \text{ of length} \leq n\},$$

$$\{(\phi, 0^n) \mid \neg\phi \text{ is satisfiable}\},$$

where ϕ denotes a Boolean formula and 0^n is padding [2]. P can be any formal system in which one can prove Boolean tautologies, e.g. ZFC, but we are mainly interested in well-known propositional proof systems.

There at least three good reasons why we are interested in canonical pairs.

1. A canonical pair, as any disjoint NP pair, presents a *computational* problem of separating the two sets. We believe that the computational complexity of this problem is inherently connected with the logical strength of the system. We do not have quantitative measures of hardness of separation of disjoint pairs, but there is a natural concept of reduction that enables us to compare disjoint NP pairs.
2. Canonical pairs are connected with the property of propositional proof systems called *feasible interpolation.* The latter property enables one to prove lower bounds for some weak proof systems such as Resolution and Cutting-Planes. An important open problem in proof complexity is: for how strong proof systems can one use some form of feasible interpolation?
3. There are several types of finite combinatorial games whose complexity has not been determined; namely, it is not known how hard it is to decide which player has a winning strategy. However, it has been shown that they are reducible to some canonical pairs of bounded depth Frege proof systems [1].

*This principle is equivalent to the consistency of T, but this fact is not important here.

In this lecture we will report on our work on characterization of canonical pairs of bounded depth Frege proof systems. We will first describe games by means of which we characterize the canonical pairs. Then we will present a proof system equivalent to bounded depth Frege systems that we use in our proof. The system is interesting on its own right because of its symmetry and simplicity.

Our games generalize the standard concept of a game where two players play some symbols in the following way. After playing in the usual way, the players play backward and rewrite the symbols played until they arrive at the beginning of the play. According to a given parameter, they may play in the forward direction and backward direction several times before the game ends. What is important is that one can define a natural concept of positional strategy; such a strategy has a short description and one can check if it is a winning strategy in polynomial time. Thus we can define a disjoint NP pair by taking games in which the first player has a positional winning strategy as one NP set and taking similarly defined set for the second player as the other NP set. These pairs are equivalent to the canonical pairs of bounded depth Frege systems.

Our proof system is inspired by the system of Skelley and Thapen [3], but it is more symmetric. The main rules are resolution and dual resolution, and the rules can be applied internally (deep in a given formula).

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MODAL LOGIC MEETS SIMPLICIAL SETS

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Simplicial semantics for modal and superintuitionistic predicate logics was introduced by Dmitry Skvortsov in the early 1990s as a “maximal” Kripke-type semantics. The proofs of basic results on this semantics — soundness and “maximality” theorems (sketchy) and completeness theorem (for a certain class of logics) were given in [1]. The book [2] (Chapter 5) contains a detailed proof of soundness for metaframe semantics, which is a particular case of simplicial semantics*. In this note we state some further results on completeness and incompleteness.

Let us briefly recall the main definitions.

Modal (predicate) formulas are constructed from predicate letters P_k^n (countably many for each arity $n \geq 0$), a countable set of individual variables, classical propositional connectives, quantifiers, and the modal operator $\Box$. Individual constants, function letters and equality are not used.

A *modal predicate logic* is a set of modal predicate formulas containing classical predicate axioms, the axiom of $\mathbf{K}$ and closed under Modus Ponens, Generalization, $\Box$-introduction, and predicate substitutions. $\mathbf{Q\Lambda}$ denotes the smallest predicate logic containing the propositional logic $\mathbf{\Lambda}$.

A *predicate Kripke frame* over a propositional frame $F = (W, R)$ is a pair (F, D), in which $D = (D_u)_{u \in W}$, $D_u \neq \varnothing$, and $D_u \subseteq D_v$ whenever uRv. The notion of validity for Kripke frames is standard and well-known. By soundness theorem, the set of formulas valid in a certain class of frames is always a modal predicate logic. Logics of this form are called *Kripke complete.*

Many modal predicate logics are known to be Kripke incomplete, so other Kripke-type semantics were proposed, in particular, Kripke sheaf semantics, Ghilardi’s functor semantics, metaframe semantics, and simplicial semantics — the strongest of them. It is defined as follows.

Let $I_n = \{1, \dots, n\}$, $I_0 = \varnothing$, and let Σ_{mn} be the set of all maps from I_m to I_n (Σ_{0n} consists of a single map $\varnothing_n$, and $\Sigma_{m0} = \varnothing$ for $m > 0$). Also let $\Sigma = \bigcup\limits_{m \geq 0,\, n > 0} \Sigma_{mn}$. There are specific maps:

$\delta_i^n \in \Sigma_{n-1,n}$ sends $1, \dots, n-1$ respectively to $1, \dots, i-1, i+1, \dots, n$;

*The name ‘simplicial semantics’ was introduced in [2] as an allusion to algebraic topology. ‘Simplicial frames’ from [2] correspond to ‘metaframes’ from [1], and ‘metaframes’ from [2] correspond to ‘Cartesian metaframes’ from [1].

$\sigma^+ \in \Sigma_{m+1,n+1}$ prolongs $\sigma \in \Sigma_{mn}$ with $\sigma^+(m+1) = n+1$.

A *simplicial frame* based on a propositional Kripke frame F is a tuple $\mathbb{F} = (F, \overrightarrow{D}, \overrightarrow{R}, \pi)$, where $\overrightarrow{D} = (D^n)_{n\geq 0}$ is a family of (non-empty) sets, $\overrightarrow{R} = (R^n)_{n\geq 0}$ is a family of relations $R^n \subseteq D^n \times D^n$, with $F = \left(D^0, R^0\right)$; $\pi = (\pi_\sigma)_{\sigma\in\Sigma}$ is a family of maps $\pi_\sigma : D^n \longrightarrow D^m$ for $\sigma \in \Sigma_{mn}$.

If D^n is the Cartesian power of a set $D\,(= D^1)$, and $\pi_\sigma(a_1, \ldots, a_n) = (a_1, \ldots, a_n) \cdot \sigma := (a_{\sigma(1)}, \ldots, a_{\sigma(m)})$ for $n > 0$, $\mathbb{F}$ is called a *metaframe.*

Remark. *Simplicial sets* are mathematical structures closely related to simplicial frames. By definition [3], a simplicial set consists of non-empty sets $(D^n)_{n\geq 1}$ and maps $\pi_\sigma : D^n \longrightarrow D^m$ corresponding to $\sigma : I_m \longrightarrow I_n$ that are monotonic w.r.t. $\leq$. π_σ should also preserve composition and identity as in sound simplicial frames (cf. Theorem 1 below). So a sound simplicial frame (without level 0) can be regarded as a simplicial set with extra maps π_σ corresponding to permutations $I_n \longrightarrow I_n$ and with extra relations R_n.

A *valuation* in a simplicial frame $\mathbb{F}$ is a function ξ sending every predicate letter P^n_k to a subset $\xi(P^n_k) \subseteq D^n$. An *assignment* of length n in $\mathbb{F}$ is a pair $(\mathbf{x}, \mathbf{a})$, where $\mathbf{a} \in D^n$, $\mathbf{x}$ is a list of different variables of length n. For a formula A, an assignment $(\mathbf{x}, \mathbf{a})$ involving all its parameters and a model $M = (\mathbb{F}, \xi)$ the *truth relation* $M, \mathbf{a}/\mathbf{x} \vDash A$ is defined by induction, in particular

- $M, \mathbf{a}/\mathbf{x} \vDash P^m_k(\mathbf{x}\cdot\sigma)$ iff $\pi_\sigma \mathbf{a} \in \xi(P^m_k)$ (for $\sigma \in \Sigma_{mn}$);
- $M, \mathbf{a}/\mathbf{x} \vDash \Box B$ iff $\forall \mathbf{b} \in R^n(\mathbf{a})\ M, \mathbf{b}/\mathbf{x} \vDash B$;
- $M, \mathbf{a}/\mathbf{x} \vDash \exists y B$ iff $\exists \mathbf{c} \in D^{n+1}\left(\pi_{\delta^{n+1}_{n+1}}\mathbf{c} = \mathbf{a}\ \&\ M, \mathbf{c}/\mathbf{x}y \vDash B\right)$, where y does not occur in $\mathbf{x}$;
- $M, \mathbf{a}/\mathbf{x} \vDash \exists x_i B$ iff $M, \pi_{\delta^n_i}\mathbf{a}/(\mathbf{x}\cdot\delta^n_i) \vDash \exists x_i B$.

A formula is called *valid* in a simplicial frame if it is true under every valuation and variable assignment (for its parameters); a formula is *strongly valid* if all its substitution instances are valid.

Theorem 1 ([1])**.** *Let $\mathbb{F} = (F, \overrightarrow{D}, \overrightarrow{R}, \pi)$ be a simplicial frame such that:*

- $\pi_{\varnothing_1}$ *is surjective;*
- *every π_σ for $\sigma \in \Sigma_{mn}$ is a p-morphism from (D^n, R^n) to (D^m, R^m), i.e., $\pi_\sigma(R^n(\mathbf{a})) = R^m(\pi_\sigma(\mathbf{a}))$ for every $\mathbf{a}$;*
- *π preserves composition and sends identity maps to identity maps;*
- *if $\pi_{\delta^{m+1}_{m+1}}(\mathbf{b}) = \pi_\sigma(\mathbf{a})$, $\sigma \in \Sigma_{mn}$, then there exists $\mathbf{c} \in D^{n+1}$ such that $\pi_{\sigma^+}(\mathbf{c}) = \mathbf{b}$, $\pi_{\delta^{n+1}_{n+1}}(\mathbf{c}) = \mathbf{a}$.*

Then the set of formulas strongly valid in $\mathbb{F}$ *is a modal predicate logic.*

A simplicial frame satisfying these conditions is called *sound*. Note that metaframes always satisfy the fourth condition.

A modal logic of some class of sound simplicial frames (respectively, metaframes) is called *simplicially complete* (respectively, *metaframe complete*).

Theorem 2 ([1]). *If* $\mathbf{\Lambda}$ *is a canonical (d-persistent) propositional modal logic, then* $\mathbf{Q\Lambda}$ *is simplicially complete.*

Now consider the propositional modal logics

$$\begin{aligned} \mathbf{D4.1} &:= \mathbf{K} + \Box p \to \Box\Box p + \Diamond\top + \Box\Diamond p \to \Diamond\Box p, \\ \mathbf{S4.1} &:= \mathbf{K} + \Box p \to \Box\Box p + \Box p \to p + \Box\Diamond p \to \Diamond\Box p, \\ \mathbf{SL4} &:= \mathbf{K} + \Box p \leftrightarrow \Diamond p + \Box p \to \Box\Box p. \end{aligned}$$

Theorem 3. *Let* $\mathbf{\Lambda}$ *be a propositional modal logic between* $\mathbf{D4.1}$ *and* $\mathbf{SL4}$. *Then* $\mathbf{Q\Lambda}$ *is metaframe incomplete.*

The crucial formula for the proof is

$$\Box\Diamond\forall x\forall y(\Box\Diamond P(x,y) \to \exists x'\exists y'(P(x',y') \wedge \Diamond P(x,y'))).$$

It is strongly valid in sound metaframes strongly validating $\mathbf{QD4.1}$. However, it is not provable in $\mathbf{QSL4}$, because it can be refuted in a simplicial frame strongly validating $\mathbf{QSL4}$.

Corollary 4. *The logics* $\mathbf{QD4.1}$, $\mathbf{QS4.1}$, $\mathbf{QSL4}$ *are simplicially complete, but metaframe incomplete.*

In fact, completeness follows from Theorem 2 and incompleteness from Theorem 3.

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AN EXTENSION OF A THEOREM OF ZERMELO

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Zermelo [2] proved the following categoricity result for set theory. Suppose M is a set and $\in_1, \in_2$ are two binary relations on M. If both $(M, \in_1)$ and $(M, \in_2)$ satisfy the second order Zermelo–Fraenkel axioms ZFC^2, then $(M, \in_1) \cong (M, \in_2)$. Of course, the same is not true for first order ZFC. However, we show that if first order ZFC is formulated in the extended vocabulary $\{\in_1, \in_2\}$, then Zermelo's result holds even in the first order case. Similarly, Dedekind's categoricity result [1] for second order Peano arithmetic has an extension to a result about first order Peano.

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CONNEXIVE CONDITIONAL LOGIC

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Connexive logics are contra-classical logics. They are neither subsystems nor supersystems of classical logic, and what is characteristic of them is that they validate the so-called Aristotle's Theses and Boethius' Theses:

(AT) $\sim(\sim A \to A)$,

(AT)′ $\sim(A \to \sim A)$,

(BT) $(A \to B) \to \sim(A \to \sim B)$,

(BT)′ $(A \to \sim B) \to \sim(A \to B)$.

There are several ways of defining systems of connexive logic by making use of various semantical constructions and proof-theoretical frameworks, for surveys and some recent contributions see [3, 6, 7].

In this paper, first some propositional conditional logics based on Belnap and Dunn's paraconsistent four-valued logic of first-degree entailment, FDE, are introduced semantically, which are then turned into systems of connexive conditional logic. The general frame semantics for conditional logics (see [1, 4, 5]) is generalized, so that it utilizes a set of permissible extension/anti-extension pairs. Sound and complete tableau calculi for the basic connexive conditional logics are presented. Moreover, an expansion of these systems by a constructive implication is considered, which gives us an opportunity to discuss recent related work by Kapsner and Omori [2], motivated by the combination of indicative and counterfactual conditionals. Tableau calculi for the basic constructive connexive conditional logics are defined and shown to be sound and complete with respect to their semantics. This semantics has to ensure a persistence property with respect to the preorder that is used to interpret the constructive implication.

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HIGHER TYPES OF RECURSION AND LOW LEVELS OF DETERMINACY

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We consider how one may lift Kleene's theory of recursion in finite types ([3]) to more expanded notions of recursion. Kleene himself in [4] and [5] sought to show how his previous definitions using an equational calculus could also be grounded in an equivalent formulation using Turing machines (thus perhaps providing backing for the higher type recursive notions). 'Kleene Recursion' (at type 2) has come down to us as a theory of hyperarithmetic sets of reals.

A notion of a more 'generalized-recursion' can roughly speaking be obtained by replacing Turing machines in Kleene's [4] and [5], by so called *infinite time Turing machines* (ittm's) [2]. The characteristics of such conceptual devices have been investigated in [7] and they can be shown to compute codes for an initial segment of the constructible universe ([1]). This higher type involvement of ittm's is an interesting construction in its own right, and deserves, we believe, further investigation. Here it is shown that there are applications of ittm-theory to classical descriptive set theory. For, we can already give an exact characterisation of complete ittm-semi-decidable sets formed relative to a particular type 2 functional. (This results in a definition similar to one already used by [6].)

This comes through a theorem connected with low level determinacy. For the 'Kleene recursion' above, there were already connections with open determinacy: Player I in an open, or Σ^0_1, game on Cantor or Baire space has a hyperarithmetic, hence 'Kleene recursive', strategy. A listing of the open games won by Player I formed a complete 'Kleene semi-decidable-in-oJ' (for *ordinary jump*) set of integers.

We show the equivalence between the existence of winning strategies for $G_{\delta\sigma}$ (or Σ^0_3) games in Cantor or Baire space, and the existence of functions generalized ittm-recursive in a certain higher type-2 functional eJ' (for *extended jump*). This allows us to lift in a natural fashion the Kleenean results to this level: the list of Σ^0_3 games won by Player I is now a complete generalized semi-decidable-in-eJ set of integers. (See [8].)

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BETWEEN MODEL THEORY AND PHYSICS

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There are several important issues in physics which model theory have potential to help with. First of all, there is the issue of adequate language and formalism, and closely related to this there is a more specific problem of giving rigorous meanings to limits and integrals used by physicists.

I will present a variation of *positive model theory* which addresses these issues and will discuss some progress in defining and calculating oscillating integrals of importance in quantum physics.

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