International Conference PDEs and Mathematical Hydrodynamics: in Honor of Vsevolod Alekseevich Solonnikov's 85-th Birthday

July 30 — August 3, 2018
Euler International Mathematical Institute
St. Petersburg, Russia

ABSTRACTS

The international conference “PDEs and Mathematical Hydrodynamics: in Honor of Vsevolod Alekseevich Solonnikov’s 85’th Birthday” will be held on July 30 — August 3, 2018, at the Euler International Mathematical Institute, St.-Petersburg, Pesochnaya nab. 10.

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Axisymmetric flows in the exterior of a cylinder

Ken Abe
Osaka City University, Japan
kabe@math.osaka-cu.ac.jp

We consider the three-dimensional Navier-Stokes equations for axisymmetric initial data. It is known that the Cauchy problem is globally well-posed for large axisymmetric initial data without swirl. However, regularity and uniqueness is unknown in general for solutions with non-trivial swirls. In this talk, we study axisymmetric flows in the exterior of a cylinder subject to the slip boundary condition. We show that unique global-in-time solutions exist for large axisymmetric data in L_3 with finite energy satisfying a decay condition of the swirl component. This talk is based on a joint work with G. Seregin (PDMI/ University of Oxford).

Degenerate parabolic equations

Herbert Amann
Mathematisches Institut, Universitat Zurich, Switzerland
herbert.amann@math.uzh.ch

We describe recent results on the local existence and regularity of degenerate linear and nonlinear parabolic boundary value problems. The equations under consideration can be embedded in the class of parabolic problems on suitable noncompact Riemannian manifolds. This leads to a very general unified theory with a wide range of applications.

Generalized Kelvin-Voigt equations with p-Laplacian and source/absorption terms for nonhomogeneous incompressible fluid

Stanislav Antontsev
CMAF-CIO, University of Lisbon, Portugal, and
Lavrentyev Institute of Hydrodynamics, SB RAS, Novosibirsk, Russia
antontsevsn@mail.ru

This talk is devoted to study the generalized Kelvin-Voigt equations with p-Laplacian and source/absorption terms for incompressible and nonhomogeneous fluid, that is, with a variable density. Existence of solutions and large time behavior properties are considered for the following initial-boundary value problem:

$$\frac{\partial(\rho\,\mathbf{v})}{\partial t} + \mathbf{div}(\rho\,\mathbf{v}\otimes\mathbf{v}) = \nabla\pi + \mathbf{div}\left(\varkappa|\mathbf{D}(\mathbf{v})|^{q-2}\frac{\partial\,\mathbf{D}(\mathbf{v})}{\partial t} + \mu|\mathbf{D}(\mathbf{v})|^{p-2}\mathbf{D}(\mathbf{v})\right) + \\ +\gamma|\mathbf{v}|^{m-2}\mathbf{v} + \rho\,\mathbf{f} \quad \text{in } Q_T,$$

$$\frac{\partial\rho}{\partial t} + \operatorname{div}(\rho\,\mathbf{v}) = 0 \quad \text{and} \quad \operatorname{div}\mathbf{v} = 0 \quad \text{in } Q_T,$$

$$\rho\mathbf{v} = \rho_0\mathbf{v}_0 \quad \text{and} \quad \rho = \rho_0 \quad \text{in } \Omega, \quad \text{when} \;\; t = 0,$$

$$\mathbf{v} = \mathbf{0} \quad \text{on } \Gamma_T$$

where $\mathbf{D}(\mathbf{v}) = 1/2(\nabla\mathbf{v} + \nabla\mathbf{v}^T)$, $\Omega \subset \mathbb{R}^d$, $d \geq 2$. Here the unknowns are the vector field $\mathbf{v}$, the density ρ and the scalar field of pressure π. The coefficients $\varkappa$, μ and γ and the exponents p, q and m are given constants. A fundamentally new point here is the presence of a hyperbolic transport equation for the density ρ and nonlinear terms describing the presence of sources or absorption and the case $p \in (1, \infty)$. Two different cases are considered. In the case $\gamma < 0$ (with an absorption term), we prove that the solution of the associated problem exists globally in time and exponentially and power decay. In the case $\gamma > 0$ (with nonlinear source term), we analyze local existence under suitable assumptions on the exponents p, q, m, on the coefficients μ, $\varkappa, \gamma$, and for certain initial data. The detailed proofs can be found in [1, 2, 3].

Joint work with H. B. de Oliveira, Universidade do Algarve, Portugal and Kh. Khompysh, Farabi Kazakh National University, Kazakhstan.

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How far p is equivalent to v^2 in the regularity theory of the Navier-Stokes equations?

Hugo Beirão da Veiga

Pisa University (Italy) and CMAF-CIO (Portugal)

bveiga@dma.unipi.it

The starting point of these talk is the well known sufficient condition for regularity of weak solutions to the evolution Navier-Stokes equations, sometimes called Ladyzhenskaya-Prodi-Serrin's condition (LPS condition). Roughly speaking, it establishes that solutions $\boldsymbol{v}$ which belong to the functional space $L^r(0, T; L^q(\Omega))$ where $2/r + n/q = 1$ and $q > n$, are regular. On the other hand, a formal equivalence $p \cong |\boldsymbol{v}|^2$ is suggested by the well known equation

$$-\Delta p \ = \ \sum_{i,j=1}^{n} \partial_i \, \partial_j (v_i v_j).$$

In three papers published nearly twenty years ago we have proved some results which support this equivalence. In a recent paper we obtained new results in this direction. Interesting open problems still remain.

Nonlocal approach to asymptotic methods of perturbation theory

Vladimir S. Belonosov

Sobolev Institute of Mathematics, Novosibirsk, Russia

belonosov@academ.org

The fundamental role in the perturbation theory is played by the methods of asymptotic expansions in powers of a small parameter, united by the idea of representing a nonstationary process as a composition of smooth evolutionary changes and small rapid oscillations [1]–[2]. They include the method of normal forms, the averaging method, and some others. These methods lead to autonomous differential equations approximately describing the features of the exact solutions – fixed points, cycles, areas of separatrices, etc. Generally speaking, these features also depend on the parameter. When it is reduced, they can "go" beyond the areas in which the estimates of approximation errors are established, leading to a loss of the efficiency of asymptotic methods.

This problem can be overcome if the corresponding method is modified so that the phase portrait of the averaged equation does not depend on a small parameter, but still reflects the characteristic features of the solutions. In general, this task is quite difficult. However, it is possible to solve it for generalized Mathieu–Hill equations $u''(t) = -Au(t) + \varepsilon F(t,u)$ in finite-dimensional space. Here we have a linear self-adjoint positive operator A, a small parameter ε, a polynomial $F(t,u)$ with respect to the components of the vector u with almost periodic in t coefficients. It is proposed to change the variables by the formula $v = \varepsilon^{\alpha} u$, and then use the Krylov–Bogolyubov averaging method. With a suitable choice of the exponent α and the degree of averaging defined by the structure of the polynomial $F(t,u)$, the phase portrait of the averaged system will not really depend on ε. The report presents the results of this study.

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On the Type I blow-up for the incompressible Euler equations

Dongho Chae

Chung-Ang University, Korea

donghochae@gmail.com

In this talk we discuss the Type I blow up and the related problems in the 3D Euler equations. We say a solution v to the Euler equations satisfies Type I condition at possible blow up time T_* if $\limsup_{t\nearrow T_*}(T_* - t)\|\nabla v(t)\|_{L^\infty} < +\infty$. The scenario of Type I blow up is a natural generalization of the self-similar(or discretely self-similar) blow up. We present some recent progresses of our study regarding this. We first localize previous result that "small Type I blow up" is absent. After that we show that the atomic concentration of energy is excluded under the Type I condition. This result, in particular, solves the problem of removing discretely self-similar blow up in the energy conserving scale, since one point energy concentration is necessarily accompanied with such blow up. We also localize the Beale-Kato-Majda type blow up criterion. Using similar local blow up criterion for the 2D Boussinesq equations, we can show that Type I and some of Type II blow up in a region off the axis can be excluded in the axisymmetric Euler equations. These are joint works with J. Wolf.

Exact and asymptotic solutions of 2-D linear and nonlinear wave equations with degenerated velocity and application to the water wave problems

Sergei Dobrokhotov

Ishlinsky Institute for Problem in Mechanics and Moscow Institute of Physics and Technology

s.dobrokhotov@gmail.com

In the frame of linear and nonlinear wave equations coming from the shallow water equations we discuss the problem about the propagation and interaction with the shore of the waves generated by spatially localized initial perturbations and also the the problems about standing water waves. In linear approximation this problem is 2-D wave equations with velocity degenerated on the curve organizing the boundary of the basin. We show that this boundary could be viewed as the special caustic and one can use

V.Maslov and V.Fock ideas to construct asymptotic solutions of linear problem. Near the boundary we use the Carrier-Greenspane transformation and some ideas of E.Pelinovskii and R.Mazova for construction of asymptotic solutions of nonlinear equations in the neighborhood of basin's boundary (run-up problem). We discuss the application of constructed asymptotics in tsunami wave problem and also the construction of the standing waves. This work was supported by Russian Scientific Fund, project N 16-11-10282, and was done with V.E.Nazaikinskii, A.Yu.Anikin, A.Aksenov, K.Druzhkov, D.S.Minenkov, A.A.Tolchennikov, and B.Tirozzi

On the uniqueness of the Leray-Hopf solution for a dyadic model

Nikolay Filonov

V.A. Steklov Mathematical Institute, St.-Petersburg, Russia

filonov@pdmi.ras.ru

We consider the following problem

$$\begin{cases} \dot{u}_n(t) + \lambda^{2n} u_n(t) - \lambda^{\beta n} u_{n-1}(t)^2 + \lambda^{\beta(n+1)} u_n(t) u_{n+1}(t) = f_n(t), \\ u_n(0) = a_n, \qquad n = 1, 2, \dots . \end{cases}$$

This system is similar to the system of the Navier-Stokes equations, and it can be considered as a toy model for the NSE. It is well known that the Leray-Hopf solution always exists. We study the uniqueness of such solutions. We obtain two results:

1) If RHS $f_n = 0$, and the initial data $\{a_n\}$ are "good" enough, then the Leray-Hopf solution is unique.

2) If initial data $a_n = 0$, but RHS f_n are "bad", then it is possible that the Leray-Hopf solution is not unique.

Structural instability in suspension bridges and interaction with fluids

Filippo Gazzola
Politecnico di Milano, Italy
filippo.gazzola@polimi.it

The torsional instability of suspension bridges has its origin in the nonlinear behavior of structures, see the monograph [1]. The bridge is subject to torsional oscillations whenever the amount of internal energy exceeds a critical threshold. The energy is inserted into the structure by the vortex shedding. This talk will focus on the role of the wind velocity in the resultant vortices. Depending on the strength and on the frequency of the vortices, the bridge may be either torsionally stable or unstable.
Based on a joint work with D. Bonheure (Bruxelles) and E. Moreira dos Santos (Sao Paulo).

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Fractional heat equations

Gerd Grubb
Copenhagen University, Denmark
grubb@math.ku.dk

The fractional Laplacian $P = (-\Delta)^a$, $0 < a < 1$, — and other related fractional-order operators - has been studied intensively in recent years because of its interesting applications in probability, finance, mathematical physics and differential geometry. It is a linear operator, entering also in nonlinear problems. Some of the questions that are asked are similar to those asked in connection with the Laplacian, but now with the additional difficulty that $(-\Delta)^a$ is a NONLOCAL operator. The trend among nonlinear PDE people has mostly been to use methods from potential theory and singular integral operators, whereas the fact that it is a pseudodifferential operator has not been taken much into account until recently. By ps.d.o. methods one can give exact descriptions of the solution spaces for the homogeneous Dirichlet problem (defined from a variational setup), and introduce related trace operators. Also the associated heat equation can be treated.

Interior regularity can here be described by a calculus that generalizes the calculus used in joint works with Seva Solonnikov in the '90s, but boundary regularity needs different tools.

The bidomain equations with FitzHugh-Nagumo nonlinearities in the L^q-framework

Matthias Hieber
TU Darmstadt, Germany
hieber@mathematik.tu-darmstadt.de

In this talk, we consider the bidomain equations with FitzHugh-Nagumo nonlinearities arising in the study of electrophysiology. We show that the associated bidomain operator A admits a bounded H^∞-calculus within the L^p-setting. This allows us to prove local as well as global well-posedness of this system in weak and strong settings for initial data in critical spaces. Moreover, we give stability results for spatially constant equilibria. This is joint work with Jan Prüss.

Finite-parameter feedback control for stabilizing NSV and damped nonlinear wave equations

Varga K. Kalantarov
Department of Mathematics, Koç University, Istanbul
vkalantarov@ku.edu.tr

The talk is devoted to the problem of global stabilization of solutions of the initial boundary value problems for the Navier-Stokes-Voigt (NSW) equations, the strongly damped nonlinear wave equations, the nonlinear wave equation with nonlinear damping term, the KdV-Burgers equation and related systems. A common feature of these equations is that the semigroups generated by initial boundary value problems are asymptotically compact semigroups which have finite-dimensional global attractors in corresponding phase spaces. We show that any arbitrary given solution of the initial boundary value problem for each of equations considered can be stabilized by using a feedback controller depending only on finitely many large spatial-scale parameters, such as the low Fourier modes or other finite rank spatial interpolant operators.

On three dimensional flows of pore pressure activated Bingham fluids

Josef Malek

Charles University, Prague, Czech Republic

malek@karlin.mff.cuni.cz

We are concerned with a system of partial differential equations describing internal flows of homogeneous incompressible fluids of Bingham type in which the value of activation (the so-called yield stress) depends on the internal pore pressure governed by an advection-diffusion equation. This model seems to be suitable for description of important complex processes, such as liquefaction, occuring at granular water-saturated materials. After providing the physical background of the considered model paying attention to the assumptions involved in its derivation, we focus on PDE analysis of initial and boundary value problems that are interesting from geophysical point of view. We give several equivalent descriptions for considered class of fluids of Bingham type. In particular, we exploit the possibility to write such a response as an implicit tensorial constitutive equation, involving the pore pressure, the deviatoric part of the Cauchy stress and the velocity gradient. Interestingly, this tensorial response can be characterized by two inequalities. A similar approach is used to treat stick-slip boundary conditions that includes noslip, Navier's slip and slip as special cases. Within such setting we prove long time and large data existence of weak solutions to the evolutionary problem in three dimensions. The leccture is based on joint works with Anna Abbatiello, Miroslav Bulíček, Tomáš Los and Ondřej Souček.

Global existence of solutions with non-decaying initial data 2D(3D)-Navier-Stokes IBVP in half-plane (space)

Paolo Maremonti

Università degli Studi della Campania "L.Vanvitelli", Italy

paolo.maremonti@unicampania.it

In paper [1], we investigate on the global (in time) regular solutions to the Navier-Stokes initial boundary value problem in the half-plane $\mathbb{R}^2_+$ with initial data $u_0 \in L^\infty(\mathbb{R}^2_+) \cap J^2_0(\mathbb{R}^2_+)$ or with non decaying initial data $u_0 \in L^\infty(\mathbb{R}^2_+) \cap J^p_0(\mathbb{R}^2_+)$, $p > 2$. Here and below J^q_0 :=completion of $\mathscr{C}_0$ with respect to the

$\|\nabla \cdot \|_q$. We introduce a technique that allows us to solve the two-dimesional problem, and nevertheless, it is employed to obtain weak solutions, for non decaying initial data, to the three-dimensional Navier-Stokes Cauchy problem and IBVP in the half-space. Actually, assuming that the initial data belongs to $L^\infty(\Omega) \cap J^q_0(\Omega)$, $\Omega \equiv \mathbb{R}^3$ or $\mathbb{R}^3_+$, for some $q \in (3, \infty)$, then there exists a suitable weak solution (v, π_v). The two-dimensional result is in the wake of a recent literature (see e.g.[2]-[4]). Instead, apart from the special result in [5], the three-dimensional one has only some contributions as [5, 7].

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Thermal convection in inclined fluid and porous layers

Giuseppe Mulone
Department of Mathematics and Computer Science
Città Universitaria, Catania, Italy
mulone@dmi.unict.it

An overview of linear instability and nonlinear stability results for convection problems for laminar flows in fluid-dynamics and magnetofluid-dynamics and for flows in porous and double-porous media is given. We consider both horizontal and inclined layers heated from below or above.

We analytically study stability and instability with respect to streamwise (longitudinal) perturbations. Moreover, we numerically investigate the linear instability under spanwise (transverse) and general three-dimensional perturbations to the basic state. We give sufficient nonlinear stability conditions. We also investigate numerically the Euler-Langrange system associated to the maximum problem arising from the energy equation. We prove the coincidence of linear and nonlinear stability thresholds for streamwise perturbations. In some cases these perturbations are proved to be the most destabilizing.

In the porous media case, we consider flows which satisfy Darcy or Brinkman laws and also consider double porosity media (bidispersive materials). The dependence of the stability thresholds on the boundary conditions, the inclination angle is investigated. For particular angles we find some instability islands.

Finally, in the isothermal case, we give some new nonlinear stability conditions for 2D perturbations in the case of horizontal plane Couette and Poiseuille flows.

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Homogenization of the Neumann's brush problem

François Murat

Laboratoire Jacques-Louis Lions,
Sorbonne Université and CNRS, Paris, France
murat@ann.jussieu.fr

In this lecture, I will describe joint work with Antonio Gaudiello (Naples, Italy) and Olivier Guibé (Rouen, France).

We consider a sequence of domains Ω^ε which have the form of brushes (in dimension $N = 3$) or of combs (in dimension $N = 2$): each domain Ω^ε is an open subset of $\mathbf{R}^N$ with $N \geq 2$ which is made of teeth distributed over a basis. The basis is fixed, and the teeth are vertical and cylindrical, and they have a fixed height. For each domain Ω^ε, the diameter of every tooth is less than or equal to ε, but the L^∞ weak-star limit as ε tends to zero of the characteristic functions of the sets of the teeth is assumed to be bounded from below away from zero, which means that at the limit it remains some uniformly strictly positive volume fraction of matter everywhere in the zone of the teeth. The cross sections of the teeth, which are open, can vary from one tooth to another one. They are not assumed to be smooth, and the teeth can be adjacent, i.e. they can share parts of their boundaries. Observe that no periodicity is assumed on the distribution of the teeth.

For this sequence of domains we study the asymptotic behavior, as ε tends to zero, of the solution of a linear second order elliptic equation with a zeroth order term which is bounded from below away from zero, when the homogeneous Neumann boundary condition is imposed on the whole of the boundary, and when the source term belongs to L^2. This is a classical homogenization problem since the pioneering work made by R. Brizzi and J.-P. Chalot in their Ph.D. Thesis in 1978, but our homogenization result takes place in a geometry which is much more general that the ones which have been considered since that time. Moreover we prove a corrector result which is new. I will state and prove these homogenization and corrector results.

The very goal of our work is however to study the case where in the above described geometry the source term belongs to L^1 and no more to L^2. In this case we work in the framework of renormalized solutions, and we introduce a definition of renormalized solutions for the homogenized problem. This definition is new since the homogenized problem is a degenerate elliptic equation where only the vertical derivative is involved in the zone of the teeth. In this framework we prove homogenization and corrector results which are the counterparts of the ones that we proved in the case where the source term belongs to L^2. In this lecture I will have no time to present these results, but the interested readers can find them in our paper *Homogenization of the brush problem with a source term in* L^1 published in Archive for Rational Mechanics and Analysis, volume 225 (2017), pages 1-64.

Stationary Navier-Stokes problem in $2D$ exterior domain

Konstantin Pileckas

Institute of Applied Mathematics, Vilnius University, Lithuania

konstantinas.pileckas@mif.vu.lt

Steady solutions to the Navier–Stokes problem in two dimensional exterior domain will be considered and the overview of known results will be presented.

Further, we prove that any solution with finite Dirichlet integral is bounded and uniformly converges to a constant vector at infinity. No additional condition (on symmetry or smallness, etc.) are assumed. We also prove that the problem with nonhomogeneous boundary conditions admits at least one solution with finite Dirichlet integral if the total flux over all connected components of the boundary is zero.

This talk is based on the joint work with M. Korobkov and R. Russo.

Variational problems in conformal geometry and hydroelastic waves

PAVEL PLOTNIKOV

Voronezh State university and
Lavrentyev Institute of Hydrodynamics, Novoskibirsk, Russia
piplotnikov@mail.ru

This work outlines a mathematical approach to steady periodic waves which propagate with constant velocity and without change of form on the surface of a 3-dimensional expanse of fluid which is at rest at infinite depth and moving irrotationally under gravity, bounded above by a frictionless elastic sheet. The elastic sheet is supposed to have gravitational potential energy, bending energy proportional to the square integral of its mean curvature (its Willmore functional), and stretching energy determined by the position of its particles relative to a reference configuration. The equations and boundary conditions governing the wave shape are derived by formulating the problem, in the language of geometry of surfaces, as one for critical points of a natural Lagrangian, and a proof of the existence of solutions is sketched. We focus on a 3D model where the underlying equations have Hamiltonian structure and the travelling hydroelastic waves problem can be reduced to the existence of critical points of a Lagrangian which involves the full hydroelastic energy and an inertial term. The resulting variational problem, which involves joint minimization of the Willmore functional and a Dirichlet integral, is similar to one from conformal geometry.

We assume that the flow occupies a domain $D \subset \mathbb{R}^3$ of points $x = (x^1, x^2, x^3)$ bounded by the elastic sheet S which is itself contained within a horizontal layer, and that D contains a half space. Considering only periodic waves we assume that D is periodic:

$$D + m\mathbf{t}_1 + n\mathbf{t}_2 = D \quad \text{for all} \quad (m,n) \in \mathbb{Z}^2.$$

Here the linearly independent vectors $\mathbf{t}_i = (t_i^1, t_i^2, 0),\ i = 1, 2$, form a lattice of periods in the space $\mathbb{R}^3$ and we denote by Π the fundamental cell of this lattice. As in elasticity theory, we assume the elastic shell on the surface can be parameterized as follows: $S = \{x \in \mathbb{R}^3 : x = \mathbf{r}(X),\ X \in \mathbb{R}^2\}$, where the reference variable $X = (X^1, X^2)$ is just a label for a material point. It is natural to assume that in the reference frame the shell has periodic structure with periods $\mathbf{l}_i = (l_i^1, l_i^2, 0)$, $i = 1, 2$, with fundamental cell Γ. Let $\Omega = D \cap \Pi$

and $S_\Pi = S \cap \Pi$, respectively the intersections of the flow domain and free boundary with the fundamental cell Π.

Our goal is to study hydroelastic travelling waves which propagate on the surface with constant speed. After rotation and scaling we may assume that the fluid velocity $\mathbf{v} = \nabla\varphi$, $\varphi(x) = x^1 + \Phi(x)$, where thp5e potential Φ satisfies the following equations and boundary conditions:

$$\begin{aligned}\Delta\Phi(x) = 0 \ \text{ in } \ D,\\ \Phi(x + m\,\mathbf{t}_1 + n\,\mathbf{t}_2) = \Phi(x), \quad (m,n) \in \mathbb{Z}^2, \ \text{ in } \ D,\\ \nabla\Phi(x)\cdot\mathbf{n}(x) + n^1(x) = 0 \ \text{ on } \ S,\\ \nabla\Phi(x) \to 0 \ \text{ as } \ x^3 \to -\infty,\end{aligned} \tag{0.1}$$

where $\mathbf{n}$ is the outward normal vector to S. In the absence of viscous dissipation of energy, this hydroelastic wave problem can be formulated variationally as one for critical points of the action functional $\mathcal{E}$, which is the difference between the full energy of the hydroelastic system and the work of the inertial forces. The full energy comprises the kinetic and gravitational potential energies of the fluid and the elastic energy of the sheet, and traditionally the elastic energy density of the sheet is decomposed as the sum the bending and stretching energies. Therefore, we take the action functional in the form

$$\mathcal{E} = \mathcal{E}_f + \mathcal{E}_g + \mathcal{E}_b + \mathcal{E}_s - \mathcal{E}_I, \tag{0.2}$$

where

$$\mathcal{E}_f + \mathcal{E}_g = \frac{1}{2}\int\limits_{\Omega} |\nabla\Phi|^2\,dx + \int\limits_{S_\Pi} \{\Phi n^1\,d\Sigma + \frac{1}{2}x^3n^3 + \frac{\lambda}{2}(x^3)^2n^3\,\}dS, \tag{0.3}$$

is the sum of renormalized kinetic and gravitational potential energies of the fluid per period, the bending energy $\mathcal{E}_b$ is given by

$$\mathcal{E}_b = \int_{S_\Pi} (c_a|\mathbf{A}|^2 + c_h\mathbf{H}^2)dS \equiv C_b\int\limits_{S_\Pi} |\mathbf{A}|^2\,dS \equiv 4C_b\int\limits_{S_\Pi} |\mathbf{H}|^2\,dS,$$

the difference $\mathcal{E}_s - \mathcal{E}_I$ of the stretching energy and the inertial term is defined by

$$\mathcal{E}_s - \mathcal{E}_I = c_e\int\limits_{\Gamma} C_{\alpha\beta}\,g_{\alpha\beta}\,dX. \tag{0.4}$$

Here $|\mathbf{A}|$ and $\mathbf{H}$ are the length, respectively, of the second fundamental form $\mathbf{A}$ and the mean curvature $\mathbf{H}$ of S, and the positive constants c_a, c_h, and c_e are determined by the particular hyperelastic material, the symmetric matrix $\mathbf{C} = (C_{\alpha\beta})$ satisfies $\mathbf{C} > 0$, $\det \mathbf{C} = 1$.

We assume that the lattice $\mathbf{t}_i$, the matrix $\mathbf{C}$, and the positive constant λ are given. The problem is to find the lattice $\mathbf{l}_i$, constant C_b, potential Φ, and surface S such that (Φ, S) is a critical point of $\mathcal{E}$. We prove that this problem has a nontrivial smooth solution.

This talk is based on the joint work with J.F. Toland from University of Bath.

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Exact solutions to the Navier-Stokes equations and their asymptotic character

Vladislav Pukhnachev

Lavrentyev Institute of Hydrodynamics SB RAS
and Novosibirsk State University, Novoskibirsk, Russia
pukhnachev@gmail.com

As it is well known, the Navier-Stokes equations (NSE) with potential external forces have a class of solutions, in which the velocity vector is gradient of harmonic function. Taking the fundamental solution of Laplace equation or its derivatives we obtain NSE solutions with point singularities. Let's consider the two dimensional stationary problem in a bounded domain with point source at the origin with discharge and natural boundary conditions at the flow domain boundary. Can we state that the radial flow $v_r = q/2\pi r$, $v_\varphi = 0$, gives the velocity main term near the origin? Inequality $|q| < 2\pi\nu$, where ν is viscosity, provides the sufficient conditions for this statement justice (A. Tartaglione and A. Russo; V.V. Pukhnachev, 2003). We believe that some restriction on the source intensity is essential in view of results by M.A. Goldshtik, V.N. Stern and N.I. Yavorskii (1988) and V. Shverak (2011). The similar restriction $|\Gamma| < 2\pi\nu$ arises in the similar problem, where the point vortex with circulation Γ is situated in the origin (V.V. Pukhnachev, 2014). However, we consider that above mentioned condition can be omitted. A reason for this conjecture is the absence of bifurcations in the flow in full plane induced by the point vortex. Another class of NSE exact solutions was discovered by G.B. Jeffery (1914) and G. Hamel (1916). These solutions have an essentially viscous nature. The remarkable paper by Ludmila Rivkind and V.A. Solonnikov [1] stimulated our joint research with A. Tani (2017). We studied the stationary flow in a bounded plane domain with a source or drain in the corner point of its boundary. Velocity distribution compensating a discharge in this point is given at the boundary. The boundary value problem for NSE has a solution if $|q|/\nu$ is small enough. The main term of velocity field near the corner point is described by an appropriate Jeffery-Hamel solution. We note that the Dirichlet integral of velocity vector is infinite in this solution. The obtained result allows generalization on the case of time-periodic dependence of source intensity $q(t)$ though there are no nonstationary analogs of the Jeffery-Hamel solution. Here we used an approach by V.I. Yudovich [2].

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Stable mild Navier-Stokes solutions

Reimund Rautmann

University of Paderborn, Germany

rautmann@uni-paderborn.de

The nonlinear integral equation for mild Navier-Stokes solutions in smooth bounded domains $\Omega \subset \mathbb{R}^n$, $n \geq 2$, on any compact time interval J, implies a stability equation for the difference u of two mild solutions with nearby data. Recalling Giga and Miyakawa's bounds to the convective term, in a scale $\mathbf{S}$ of Banach spaces imbedded in some spaces $\mathcal{C}^0(J; L^r(\Omega))$, $1 < r < 1$, we approximate u by iterative solution of linear singular Volterra integral equations. From the monotonicity of bounds to the resolvent kernels, by means of the singular Gronwall inequality we find the existence of a uniformly small solution u in case of sufficiently small data of u. This shows the stability of each (possibly large) mild Navier-Stokes solution inside of $\mathbf{S}$.

On some regularity criteria for axisymmetric Navier-Stokes equations

Joanna Rencławowicz
Institute of Mathematics, Polish Academy of Sciences, Warsaw, Poland
jr@impan.pl

We determine some criteria that imply regularity of axisymmetric solutions to the Navier-Stokes equations. We show that boundedness of

$$\|v_r/\sqrt{r^3}\|_{L_2(\mathbb{R}^3\times(0,T))}$$

as well as boundedness of

$$\|\omega_\varphi/\sqrt{r}\|_{L_2(\mathbb{R}^3\times(0,T))},$$

where v_r is the radial component of velocity and ω_φ is the angular component of vorticity, imply regularity of weak solutions.

Quasi-variational solutions to thick flows

Jose Francisco Rodrigues
University of Lisbon/CMAF, Portugal
rodrigue@fc.ul.pt

The thick flow problem can be regarded as a limit model for incompressible shear-thickening fluids of power-law type corresponding to the case of the absolute value of the deformation rate tensor being bounded by a threshold function. If this threshold is a priori known, it can be formulated in terms of an evolution variational inequality, but if the threshold depends on the solution itself, the model requires the study of a quasi-variational inequality of a new type. In particular, for heat conducting thick fluids that dependence is a consequence of a coupled problem. In a joint work with Lisa Santos, based on the results on the continuous dependence of the previous analysis for thick fluids, we consider the existence of weak solutions for the evolution quasi-variational inequalities, by compactness methods, and also the existence and uniqueness of strong solutions under smallness restrictions on the data.

TBA

Gregory Seregin
Oxford, UK and V.A. Steklov Mathematical Institute, St.-Petersburg, Russia
seregin@pdmi.ras.ru

Global wellposedness for some two phase problem for the Navier-Stokes equations in unbounded domains

Yoshihiro Shibata
Waseda University, Tokyo, Japan
yshibata@waseda.jp

I will talk about the global well-posedness of two phase problem for the Navier-Stokes equations in the following situation: One liquid occupies a bounded domain and another one the exterior of this bounded domain. Two liquids are separated by a sharp interface with surface tension. The tool to treat the nonlinear problem is the maximal L_p-L_q regularity and decay properties of solutions of the Stokes equations with interface conditions in the whole space. The point of proof is how to choose suitable exponents for which the L_p summability conditions hold on the time interval.

The Kato square root problem

Alexander L. Skubachevskii
RUDN University, Moscow, Russia
skub@lector.ru

In 1961, T. Kato has formulated the following problem: “Is it true that the domain of square root from regular accretive operator is equal to the domain of square root from adjoint operator?" Sufficient conditions for fulfilment of the Kato conjecture were studied by T. Kato, J. Lions and others. J. Lions has proved that strongly elliptic differential operators of $2m$ order with smooth coefficients and homogeneous Dirichlet conditions on a smooth boundary satisfy the Kato conjecture. For strongly elliptic differential operators of $2m$ order with measurable bounded coefficients corresponding result was obtained by P. Auscher, S. Hofman, A. McIntosh, and P. Tchamitchian. The Kato conjecture for strongly elliptic functional differential equations was proved in [1]. In this lecture we consider elliptic differential-difference equations with degeneration. We shall prove that these operators satisfy the Kato conjecture [2].

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Hardy-Leray inequality for curl-free vector fields

FUTOSHI TAKAHASHI

Osaka City University, Japan

futoshi@sci.osaka-cu.ac.jp

In this talk, we study the Hardy-Leray inequality for smooth vector fields $\boldsymbol{u} \in C_0^\infty(\mathbb{R}^N)^N$. The main result in this talk is the following:

Theorem 1. *Let $\varepsilon \neq 1$ be a real number and let $\boldsymbol{u} \in C_0^\infty(\mathbb{R}^N)^N$ be a curl-free vector field. Assume $\boldsymbol{u}(\boldsymbol{0}) = \boldsymbol{0}$ if $\varepsilon > 1$. Then we have*

$$\mathcal{H}_{N,\varepsilon} \int_{\mathbb{R}^N} |x|^{2-2\varepsilon-N} |\boldsymbol{u}|^2 dx \leq \int_{\mathbb{R}^N} |x|^{4-2\varepsilon-N} |\nabla \boldsymbol{u}|^2 dx$$

with the optimal constant $\mathcal{H}_{N,\varepsilon}$ given by

$$\mathcal{H}_{N,\varepsilon} = \begin{cases} (\varepsilon - 1)^2 \dfrac{\varepsilon^2 + 3(N-1)}{\varepsilon^2 + N - 1} & \text{if } -\sqrt{N+1} \leq \varepsilon - 2 \leq \sqrt{N+1}, \\ (\varepsilon - 1)^2 + N - 1, & \text{otherwise.} \end{cases}$$

This complements the former work by Costin-Maz'ya [1] on the sharp Hardy-Leray inequality for axisymmetric divergence-free vector fields.

This talk is based on a joint work with N. Hamamoto (OCU).

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Mathematical justification of paraxial approximate equations in nonlinear acoustics

Atusi Tani

Keio University, Japan

tani@math.keio.ac.jp

In this communication, we are concerned with some basic paraxial approximate equations in nonlinear and thermoviscous phenomena in acoustic fields in fluids. Such paraxial approximate equations are derived from nonlinear media using acoustical properties of beam's propagation:

1. the beams are concentrated near the x_1-axis (say);
2. The beams propagate along the x_1-axis;

3. The beams are generated either by an initial conditions or by a forcing term on the boundary $x_1 = 0$.

Such approximations lead to the equations similar to the well-known Khokhlov–Zabolotskaya–Kuznetsov (KZK) like equation and nonlinear progressive wave like equation (NPE), which are considered as the first order approximation of modified Kuznetsov-Blackstock equation or equivalently the second order approximation of a compressible Navier-Stokes system. The contents of my talk are as follows:

- To derive the equations from modified Kuznetsov-Blackstock equation through paraxial approximations from the real approximation standpoint;
- Global-in-time existence of a small solution to the paraxial approximate equations mentioned above;
- Mathematical justification of the approximations.

PDEs in domains with non-smooth boundaries

Maria Vivaldi
Sapienza Universit'a di Roma, Italy
maria.vivaldi@sbai.uniroma1.it

This talk concerns regularity properties in weighted Sobolev spaces for solutions of PDEs in domains with non-smooth boundaries. I will also briefly mention some works in progress.

Inflow-outflow problem to the Navier-Stokes equations

Wojciech Zajaczkowski
Institute of Mathematics, Polish Academy of Sciences, Warsaw, Poland
wz@impan.pl

We consider Navier-Stokes motions in a cylindrical domain with inflow and outflow on the top and the bottom of the cylinder. On the lateral part of the cylinder we assume the slip boundary conditions. Our aim is to prove the existence of global regular solutions without restrictions on the magnitude of the flux. For this purpose we need smallness of L_2 norms of derivatives along the axis of the cylinder of the initial velocity and the external force. We need also the slip boundary conditions. The proof is divided into the following steps:

1. First we prove existence of weak solutions for the inflow-outflow case using appropriate weighted Sobolev spaces
2. Second we prove existence of long time regular solutions using the above smallness conditions
3. Extension in time by applying the step by step in time procedure. This is possible having sufficiently long time existence of solutions from 2.

Thermoviscoelastic model of polymer solutions motion

Andrey V. Zvyagin
Voronezh State University, Russia
zvyagin.a@mail.ru

Let $\Omega \subset \mathbb{R}^n$, $n = 2, 3$, be a bounded domain with a sufficiently smooth boundary $\partial\Omega$. In $Q_T = [0, T] \times \Omega$ we consider the following initial-boundary value problem

$$\frac{\partial v}{\partial t} + \sum_{i=1}^{n} u_i \frac{\partial v}{\partial x_i} - \nu\Delta v - \varkappa\frac{\partial \Delta v}{\partial t} - 2\varkappa \mathrm{Div}\big(v_i \frac{\partial \mathcal{E}(v)}{\partial x_i}\big) + \nabla p = f, \qquad (0.5)$$

$$u = (I - \alpha^2\Delta)^{-1} v, \qquad (0.6)$$

$$\mathrm{div}\, v(t, x) = 0, \quad t \in [0, T], \quad x \in \partial\Omega, \qquad (0.7)$$

$$v|_{\partial\Omega} = 0, \quad v|_{t=0} = a. \qquad (0.8)$$

Here, $v = (v_1(t, x), \dots, v_n(t, x))$ is the velocity vector function, u is the modified velocity vector function, $p = p(t, x)$ is the function of the pressure, $f = f(t, x)$ is the density of applied forces, $\mathcal{E} = (\mathcal{E}_{ij}(v))$, $\mathcal{E}_{ij}(v) = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$, $i, j = 1, \dots, n$, is the strain rate tensor, $\varkappa > 0$ is the time of retardation, ν is the fluid viscosity, $\alpha > 0$ is the scalar parameter.

Let consider the space $E_1 = \{v : v \in L_\infty(0, T; V^1),\ v' \in L_2(0, T; V^{-1})\}$.

Theorem. *Let $f \in L_2(0, T; V^{-1})$, $a \in V^1$. Then the initial–boundary value problem (1)–(4) has a weak solution $v \in E_1$.*

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Attractors of hydrodynamic equations

Victor G. Zvyagin
Voronezh State University, Russia
zvg_vsu@mail.ru

In the talk we present the theory of trajectory attractors for autonomous and non-autonomous systems in the case of trajectory space which is not invariant under the translation semigroup. Exactly, this situation arises under the investigation of attractors for most models describing the motion of non–Newtonian media.

This theory generalizes the trajectory attractors theory, which was proposed independently by Russian scientists M.I. Vishik and V.V. Chepyzhov [1] and by American scientist G.R. Sell [2] for the study of nondeterministic problems (the solutions to these problems are not uniquely determined by the initial values). However, in the Vishik–Chepyzhov–Sell's theory the requirement of invariance of trajectory space under the translation semigroup is necessary. This requirement arises due to the fact that the proof of the abstract existence theorem for trajectory attractors uses the construction of the omega–limit set which is taken from the attractors theory of dynamical systems. This construction requires the invariance of the trajectory space.

Using the above theory of invariant trajectory space the existence of a trajectory and global attractors for the three–dimensional Navier–Stokes system was proved. Unfortunately the condition of invariance of the trajectory space is an obstruction to the study of other fluid dynamics equations.

In the paper [3] another construction of the attractors theory was proposed. This construction does not require the invariance of trajectory space and is based on the topological Shura–Bura's lemma. Also, this construction removes a lot of restrictions in the case of uniform attractors for non-autonomous systems and as well it allows to construct a theory of pullback–attractors for different models of fluid dynamics [4].

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List of all speakers

1. Ken Abe (Japan) kabe@math.osaka-cu.ac.jp
2. Herbert Amann (Switzerland) herbert.amann@math.uzh.ch
3. Stanislav Antontsev (Portugal) antontsevsn@mail.ru
4. Hugo Beirao da Veiga (Italy) bveiga@dma.unipi.it
5. Vladimir Belonosov (Russia) belonosov@academ.org
6. Dongho Chae (Korea) donghochae@gmail.com
7. Sergei Dobrokhotov (Russia) s.dobrokhotov@gmail.com
8. Nikolay Filonov (Russia) filonov@pdmi.ras.ru
9. Filippo Gazzola (Italy) filippo.gazzola@polimi.it
10. Gerd Grubb (Denmark) grubb@math.ku.dk
11. Matthias Hieber (Germany) hieber@mathematik.tu-darmstadt.de
12. Varga Kalantarov (Turkey) vkalantarov@ku.edu.tr
13. Josef Malek (Czech Republic) malek@karlin.mff.cuni.cz
14. Paolo Maremonti (Italy) paolo.maremonti@unina2.it
15. Giuseppe Mulone (Italy) mulone@dmi.unict.it
16. Francois Murat (France) murat@ann.jussieu.fr
17. Konstantin Pileckas (Lithuania) konstantinas.pileckas@mif.vu.lt
18. Pavel Plotnikov (Russia) piplotnikov@mail.ru
19. Vladislav Pukhnachev (Russia) pukhnachev@gmail.com
20. Reimund Rautmann (Germany) rautmann@uni-paderborn.de
21. Joanna Renclawowicz (Poland) jr@impan.pl
22. Jose Francisco Rodrigues (Portugal) rodrigue@fc.ul.pt
23. Gregory Seregin (Russia) seregin@pdmi.ras.ru
24. Yoshihiro Shibata (Japan) yshibata@waseda.jp
25. Alexander Skubachevskii (Russia) skub@lector.ru
26. Futoshi Takahashi (Japan) futoshi@sci.osaka-cu.ac.jp
27. Atusi Tani (Japan) tani@math.keio.ac.jp
28. Maria Agostina Vivaldi (Italy) maria.vivaldi@sbai.uniroma1.it
29. Wojciech Zajaczkowski (Poland) wz@impan.pl
30. Andrey Zvyagin (Russia) zvyagin.a@mail.ru
31. Victor Zvyagin (Russia) zvg_vsu@mail.ru