"RIEMANN SURFACES AND TEICHMÜLLER THEORY"
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Titles and abstracts of talks

Vincent Alberge (Fordham Univ., NY), On a stratification of the space of projective measured laminations. We will introduce a natural stratification of the projective measured lamination space of a closed hyperbolic surface and we will prove that the group of self-homeomorphisms that preserve such a stratification is induced by the extended mapping class group.

Sorin Dumitrescu (Univ. of Nice), Holomorphic Cartan geometries on simply connected manifolds. This talk deals with holomorphic Cartan geometries on compact complex manifolds. The concept of holomorphic Cartan geometry encapsulates many interesting geometric structures including holomorphic Riemannian metrics, holomorphic affine connections or holomorphic projective connections. Conjecturally, a compact complex simply connected manifold bearing a holomorphic Cartan geometry with model the complex homogeneous space G/H, must be biholomorphic to G/H. We present here some results going toward this direction. In particular, we show that compact complex simply connected manifolds do not admit holomorphic Riemannian metrics. We also show that compact complex simply connected manifolds in Fujiki class C bearing holomorphic Cartan geometries of algebraic type are projective. Those results were obtain in collaboration with Indranil Biswas (Mumbai, TIFR).

Krishnendu Gongopadhyay (Indian Institute of Science, Mohali), Quaternionic hyperbolic Fenchel-Nielsen coordinates. We consider the Lie groups SU(n,1) and Sp(n,1) that act as isometries of the complex and the quaternionic hyperbolic spaces respectively. We classify pairs of semisimple elements in Sp(n,1) and SU(n,1) up to conjugacy. This gives local parametrization of the representations ρ from F_2 to G such that both $\rho(x)$ and $\rho(y)$ are semisimple elements in G, where $F_2 = \langle x, y\rangle$ is an abstract free group and, G=Sp(n,1) or SU(n,1).

Hrant Hakopian (Kansas State University), Limiting behavior of geodesics in the Universal Teichmller space. Abstract: We study the limiting behavior of geodesic rays in the universal Teichmller space T(D) near the Thurston boundary of T(D) that was recently defined by Bonahon and Saric. We show that geodesics corresponding to integrable quadratic differentials (Teichmller geodesics) converge at infinity in the weak * sense, and we provide an explicit description of the limiting projective measured lamination in Thurston's boundary. Moreover, distinct Teichmller geodesics converge to distinct points. In the case of rays corresponding to non-integrable quadratic differentials (generalized Teichmller geodesics), we provide a large class of geodesics which converge at infinity, but also provide examples where the limit at infinity does not exist. This is joint work with Dragomir Saric.

Sachiko Hamano (City Univ. Osaka), Rigidity of the directional moduli on pseudoconvex domains fibered by open Riemann surfaces. G.Schmieder-M.Shiba (1998) observed conformal embeddings of a fixed open Riemann surface of positive finite genus into closed Riemann surfaces and consider the extremal ones in the class. An embedding is extremal when the ideal boundary is a union of slits in the closed surface. In the case of genus one, the results are stated in terms of the moduli space of tori, and the period matrices are utilized in the higher genus case. In this talk, we consider a smooth deformation of open Riemann surfaces of the same genus with a complex parameter, and investigate elements of the period matrix for the extremal closed Riemann surface for each open surface. We show the rigidity of a directional moduli induced by elements of the period matrices on pseudoconvex domains/two dimensional Stein manifolds fibered by open Riemann surfaces of the same topological type.

Yi Huang (Tsinghua Univ., Beijing), A McShane identity for once-punctured super tori. Classical Teichmüller theory is the study of marked hyperbolic surfaces, and Penner's decorated Teichmüller theory is a particular algebraic approach which has resulted in powerful and wide-reaching generalizations via Fock and Goncharov's version of higher Teichmüller theory. There has been exciting recent progress in a different direction: super decorated Teichmüller theory, whereby the role traditionally taken up by the real numbers R is supplanted by a non-commutative Grassmann algebra. This generalised theory corresponds to super hyperbolic surfaces, and we establish McShane identities for once-punctured super tori. We also study the asymptotic behaviour of the super length spectrum for the set of simple closed geodesics for a once-punctured super torus.

Sergei Ivanov (St Petersburg), Metric space curvatures and Finsler metrics. I will discuss Alexandrov's and Busemann's definitions of curvature bounds for geodesic metric spaces, how they do or do not apply to Finsler metrics, and a result of Lythcak and myself about surprising rigidity of Busemann nonpositively curved Finsler metrics.

Rinat Kashaev (University of Geneva), Quantum dilogarithms and their applications in quantum topology. Faddeev's quantum dilogarithm is a special function of two variables that underlies the quantum theory of Teichmller spaces. It appears that one can associate quantum dilogarithms to different locally compact abelian groups together with associated representations of mapping class groups of punctured surfaces in Hilbert spaces, topological invariants of 3-manifolds and connections to hyperbolic geometry.

Inkang Kim (KIAS, Seoul), Kähler metrics on Teichmller space and beyond. We present several ways to construct Kähler metrics on Teichmller space, quasi-fuchsian space and other moduli space of geometric structures. This is a joint work with Wan and Zhang.

Georgios Kydonakis (Univ. Strasbourg) Higgs bundles in higher Teichmüller spaces for orthogonal groups. Higher Teichmüller spaces as special connected components of character varieties $\mathcal{R}(G)$ are known to exist for various cases of groups G of Hermitian type or split real forms. A unified approach introduced by O. Guichard and A. Wienhard involves a condition of positivity for fundamental group representations and it is conjectured that this condition detects such connected components of $\mathcal{R}(G)$ in general. In this talk, we shall review progress made towards verifying this conjecture using Higgs bundle methods in the case of special orthogonal groups $\mathrm{SO}(p,q)$ for $p \neq q$. Work in progress with O. Guichard.

Francois Laudenbach (Univ. Nantes), The h-principle for wrinkled maps applied to the Madsen-Weiss Theorem (after Y. Eliashberg, S. Galatius, N. Mishachev). The Madsen-Weiss Theorem which solves the so-called *generalized Mumford conjecture* states a homology equivalence between two classifying spaces. The first one classifies the bundles in surfaces F_∞, of infinite genus, with trivialized ends. The second one is the (co)-limit of some Thom spectrum! Fortunately, this latter is also a classifying space, namely it classifies *formal stable fibrations* with fibres F_∞. Here, *formal* is used in the sense of Gromov's thesis.
In their 1997 article, Eliashberg and Mishachev explained how to simplify by homotopy the singularities of a formal fibration until a *wrinkled* map, that is a map whose singularities are very simple. Actually, in the specific context, only *folds* are needed.
Then, a generalization of Harer's stability theorem for the homology of the mapping class group of surfaces allows the authors to remove all singularities not by homotopy but by bordism, until a genuine fibration. This is sufficient to get a homology equivalence.

Yukio Matsumoto (Gakushuin Univ., Tokyo), Teichmller spaces and crystallographic groups. In this talk, I will construct a "tautological orbifold atlas" on the Deligne-Mumford compactification of the moduli space of Riemann surfaces. The method is based on the mapping class groups, the curve complexes, and the Fenchel-Nielsen coordinates. As a by product, I will show that at maximally degenerate frontier points of the augmented Teichmller spaces there arise certain Euclidean crystallographic groups.

Hideki Miyachi (Univ. Kanazawa), Poisson integral formula for Teichmüller space. In this talk, we discuss the Poisson integral formula for Teichmüller space. Indeed, we will give explicit presentations of the Poisson kernel and the pluriharmonic measure for pluriharmonic functions on Teichmüller space which are continuous on the Bers closure.

Yurii Neretin (Moscow), Infinite symmetric groups and cobordisms of triangulated surfaces. Let G be the product of three copies of the infinite symmetric group $S(\infty)$. The diagonal subgroup K of G and stabilizers $K(n) \subset S(\infty)$ of the first n points. We show that double coset spaces $K(m) \setminus G/K(m)$ form a category and this category acts in a natural way in unitary representations of G. This category admits a description in terms of cobordisms of colored triangulated surfaces and the set $K(0) \setminus G/K(0)$ is in one-to-one correspondence with Belyi data.

Ken'ichi Ohshika (Gakushuin Univ., Tokyo), Thurston's bounded image theorem from today's point of view. In the process of proving his celebrated uniformisation theorem for Haken manifolds, Thurston announced the "bounded image theorem", without giving its proof. Up to now, only a proof of its weaker version, which is sufficient for the proof of uniformisation theorem, was known. In this talk, I will give a proof of the original bounded image theorem relying on modern machinery of Kleinian group theory. This is based upon joint work with Cyril Lecuire.

Gaiane Panina (St Petersburg), Diagonal complexes. (The talk is based on a joint work with J. Gordon)

Given an n-gon, the poset of all collections of pairwise non-crossing diagonals is isomorphic to the face poset of some convex polytope called *associahedron*. We replace in this setting the n-gon (viewed as a disc with n marked points on the boundary) with an arbitrary oriented surface equipped with a number of labeled marked points ("vertices"). The surface is not necessarily closed, and may contain a number of punctures. With appropriate definitions (in a sense, we mimic the construction of associahedron) we arrive at cell complexes $\mathcal{D}$ and its barycentric subdivision $\mathcal{BD}$. If the surface is closed, the complex $\mathcal{D}$ (as well as $\mathcal{BD}$) is homotopy equivalent to the space of metric ribbon graphs $RG^{met}_{g,n}$, or, equivalently, to the decorated moduli space $\widetilde{\mathcal{M}}_{g,n}$ [2], [1]. For bordered surfaces, we prove the following:
(1) Contraction of a boundary edge does not change the homotopy type of the complex.
(2) Contraction of a boundary component to a new marked point yields a forgetful map between two diagonal complexes which is homotopy equivalent to the Kontsevich's tautological circle bundle [3]. Thus, contraction of a boundary component gives a natural simplicial model for the tautological bundle. As an application, we compute the first Chern class (also its powers) in combinatorial terms. The latter result is an application of the Mnev-Sharygin local combinatorial formula [4].
(3) In the same way, contraction of several boundary components corresponds to Whitney sum of the tautological bundles.

(4) Eliminating of a puncture gives rise to a bundle which equals to a surgery on the universal curve. In particular, the bundle incorporates at a time all the M. Kontsevich's tautological S^1-bundles.

References

[1] J.L. Harer, The virtual cohomological dimension of the mapping class group of an orientable surface, Inventiones mathematicae, 84 (1986), 1, 157–176.
[2] K. Igusa, Combinatorial Miller-Morita-Mumford classes and Witten cycles, *Algebr. Geom. Topol.* 4 (1), (2004), 473–520.
[3] M. Kontsevich, Intersection theory on the moduli space of curves and the matrix Airy function. *Comm. Math. Phys.* 147 (1992), 1, 1–23.
[4] N. Mnev, G. Sharygin, On local combinatorial formulas for Chern classes of triangulated circle bundle, *Zap. Semin. PDMI*, 448 (2016), 201–235.

Joan Porti (UAB, Barcelona), Actions on products of CAT(-1)-spaces. Let X be a proper CAT(-1)-space and Γ a quasi-convex group of isometries of X. We discuss compactifications of the diagonal action of Γ on the product $X \times X$, and prove that the compactification by horofunctions of the L^{infty} metric has good properties for this actions. We also discuss to which products of proper CAT(-1) spaces $X \times Y$ these properties can be extended. This is joint work with Teresa Garca.

Makoto Sakuma (Hiroshima University), A conjectural picture of the space of Kleinian groups generated by two parabolic transformations. I will give a progress report on my project to understand the shape of Kleinian groups generated by two parabolic transformations. This is a joint project with Hirotaka Akiyoshi, Gaven Martin, Ken'ichi Ohshika, and John Parker.

Andres Sambarino (Orsay), Pressure forms on pure imaginary directions. Anosov groups are a class of discrete subgroups of semi-simple algebraic groups analogue to what is known as *convex-co-compact groups* in negative curvature. Thermodynamical constructions equip the (regular points of the) moduli space of Anosov representations from Γ to G with natural positive semi-definite bi-linear forms, known as pressure forms. Determining whether such a pressure form is Riemannian requires non-trivial work.
The purpose of the lecture is to explain some geometrical meaning of these forms, via a higher rank version of a celebrated result for quasi-Fuchsian space by Bridgeman-Taylor and McMullen on the Hessian of Hausdorff dimension on pure bending directions. This is work in collaboration with M. Bridgeman, B. Pozzetti and A. Wienhard.

Dragomir Saric (CUNY, NY), The Fenchel-Nielsen coordinates and the type problem for Riemann surfaces. A Riemann surface is said to be parabolic if it does not support Green's function. An equivalent condition is that the geodesic flow is ergodic, or that the harmonic measure of the ideal boundary is zero. The problem of deciding when an explicitly given Riemann surface is parabolic is called the type problem and it has been taken up by many authors. Most of the known explicit test for parabolicity

are given for Riemann surfaces covering a compact surface. We give sufficient conditions for parabolicity in terms of the Fenchel-Nielsen coordinates for arbitrary topological type i.e., independently of whether the Riemann surface is a cover of a compact surface.

Jean-Marc Schlenker (Univ. Luxembourg), The measured foliation at infinity of quasifuchsian manifolds. The variational properties of the renormalized volume of quasifuchsian manifolds exhibit a clear analogy with those of the volume of the convex core (or more specifically the dual volume). This analogy suggests considering a measured foliation at infinity as the analog of the measured bending lamination on the boundary of the convex core, and a number of questions on the relations between the data at infinity and on the boundary of the convex core.

George Shabat (Russian State University for the Humanities and Independent University of Moscow), On the uniformization of Fried families. A Fried family is defined as a family of curves with one-dimensional base together with a rational function on the total space, which has no more than four critical values on every fiber; such families occur as deformations of the Belyi pairs. The talk will be devoted to the uniformization of these families in terms of the Teichmüller space $\mathcal{T}_{0,5}$ the accessory parameter and the Heun equation.

Weixu Su (Fudan Univ., Shanghai), Existence of closed geodesics through a regular point on translation surfaces. We show that on any translation surface, if a regular point is contained in a simple closed geodesic, then it is contained in infinitely many simple closed geodesics, whose directions are dense in the unit circle. Moreover, the set of points that are not contained in any simple closed geodesic is finite. We also construct explicit examples showing that such points exist. For a surface in any hyperelliptic component, we show that this finite exceptional set is actually empty. The proofs of our results use Apisa's classifications of periodic points and of GL(2, R) orbit closures in hyperelliptic components, as well as a recent result of Eskin-Filip-Wright.

Leon Takhtajian (Univ. Stony Brook, and Euler Inst. St Petersburg), Symplectic forms on moduli spaces of orbifolds. I will review construction of Goldman symplectic space on character varieties of orbifold Riemann surfaces and will discuss the analog of Kawai theorem in the orbifold case, as well as its generalization for the moduli spaces of parabolic vector bundles.

Lee Peng Teo (Xiamen University, Malaysia), Liouville Action on Quasi-Fuchsian Deformation Spaces of Riemann Surfaces of Finite Type. We construct the Liouville action on the quasi-Fuchsian deformation spaces for general hyperbolic surfaces of finite type, which contain cusps and ramification points. When the group is Fuchsian, we find that the contribution of elliptic elements to the classical Liouville action can be expressed in terms of the Bloch-Wigner functions. It is shown that the classical Liouville action is a Kähler potential of the Weil-Petersson metric on the quasi-Fuchsian deformation space. There is an equality expressing the

holography principle, which relates the Liouville action and the renormalized volume for quasi-Fuchsian groups with parabolic and elliptic elements. We also construct the potential functions of the Kähler forms corresponding to the Takhtajan-Zograf metrics associated to the parabolic and elliptic elements in the quasi-Fuchsian groups.

Richard Wentworth (Univ. Maryland), Gluing determinants of Laplace operators on Riemann surfaces. I will review some gluing formulas for zeta regularized determinants of Dolbeault Laplacians on Riemann surfaces. This is used to determine exact expressions for the Mumford isomorphisms of various determinant line bundles over Teichmüller space.

Yunhui Wu (Tsinghua Univ., Beijing), Systole functions and Weil-Petersson geometry. The gradients of geodesic-length functions along systolic curves are studied. We show that the $L^p(1 \leq p \leq \infty)$-norms of them at every closed hyperbolic surface X are uniformly comparable to $(systole(X))^{1/p}$. Several applications to the Weil-Petersson geometry are discussed. For examples: (1). we reprove that the square root of the systole function is uniformly Lipschitz on the Teichmüller space endowed with the Weil-Petersson metric; (2). we also show that the minimal Weil-Petersson holomorphic sectional curvature at any closed hyperbolic surface is bounded above by a uniform negative constant independent of genus.

Sumio Yamada (Gakushuin Univ., Tokyo), Timelike geometry and generalized de Sitter spaces. Starting with the seminal work by H. Minkowski in 1908, spacetime geometry, also called chrono-geometry, has attracted attentions from different contexts over time. One example is an important but little known article by H. Busemann called "Timelike spaces" which is based on Busemann's lifelong interest in the geometry of geodesics. We discuss some aspects of the timelike geometry, and introduce a new set of timelike spaces, which can be regarded as generalizations of the de Sitter space. This is based on a collaboration with Athanase Papadopoulos.