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Universal statements and Kolmogorov complexity

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The idea to classify statements of some logical form according to their "complexity" was suggested by Cristian and Elena Calude. It can be clarified as follows. Consider a universal (Π_1) arithmetical statement $A(n)$ with one integer variable n. Then, for every numeral N, we get a closed universal statement $A(N)$. The complexity $C_A(T)$ of a universal statement T with respect to A can be defined as the logarithm of a minimal N such that $A(N)$ is provably equivalent to A. The usual Solomonoff–Kolmogorov argument proves that there exist a universal statement $A(n)$ that makes the function C_A minimal up to $O(1)$ additive term.

An interesting special case of universal statements are statements of the form $C(x) > y$ where C is usual Kolmogorov complexity function and x and y are numerals. Some of them are quite strong: there exist an incompressible x such that the claim of its incompressibility implies (in PA) all true universal statements of the same or smaller complexity (up to $O(1)$ term). We will discuss the properties of these statements: for example, not every universal statement is provable equivalent to one of them, though a similar question for conditional complexity seems to be open. In this way we see some interesting interplay between recursion theory and (elementary) proof theory.