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The finitary content of sunny nonexpansive retractions[1]
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The goal of proof mining is to extract quantitative information out of
proofs in mainstream mathematics which are not necessarily fully constructive. Often, such proofs make use of strong mathematical principles, but a
preliminary analysis may show that they are not actually needed, so the proof
may be carried out in a system of strength corresponding roughly to first-order arithmetic. Following a number of significant advances in this vein by
Kohlenbach in 2011 and by Kohlenbach and Leuştean in 2012, we now tackle
a long-standing open question: the quantitative analysis of the strong convergence of resolvents in classes of Banach spaces more general than Hilbert
spaces.

This result was proven for the class of uniformly smooth Banach spaces
by Reich in 1980. What we do is to analyze a proof given in 1990 by Morales,
showing that adding the hypothesis of the space being uniformly convex, and
thus still covering the case of L^p spaces, can serve to eliminate the strongest
principles used in the proof by way of a modulus of convexity for the squared
norm of such spaces. A further procedure of arithmetization brings the proof
down to System T so that the proper analysis may proceed. After obtaining a
non-effective realizer of the metastability statement, this is majorized in order
to obtain the desired rate. Subsequent considerations calibrate this bound to
T_1. In particular, this result completes some analyses that had previously
been obtained only partially, yielding rates of metastability within the above-considered class of Banach spaces for the Halpern and Bruck iterations.

[1] Joint work with Ulrich Kohlenbach.