

ALBERT VISSER

The fixed point calculation for modal logics with the Lewis arrow¹

Utrecht University, the Netherlands

E-mail: a.visser@uu.nl

In classical modal logic the Lewis arrow reduces to the necessity operator. This well-known fact fails over a constructive base. This last insight opens the way for the study of multiple Lewisian versions of Löb's Logic. In this talk, we zoom in on the matter of explicit fixed points.

The construction of explicit fixed points consists of two stages. In the first stage, one constructs fixed points for formulas whose main connective is modal. In the second stage, one extends the class of formulas with explicit fixed points to formulas in which the fixed point variable is modalized. In fact, one can go further as we will briefly indicate.

We study two methods of constructing explicit fixed points for formulas whose principal connective is the Lewis arrow, the de Jongh–Visser method and the de Jongh–Sambin method. We explain that these calculations are independent of one another. We discuss the matter of the minimal logic that supports each calculation (reverse mathematics of fixed point calculations). In the de Jongh–Visser case we have an elegant characterization of that logic, but in the de Jongh–Sambin case, — surprisingly, since the de Jongh–Sambin calculation is simpler — the question of an elegant characterization is open.

We show that the second stage is fairly general and can be realized over the basic logic iA .

Finally, we will discuss the case where we have the Strong Löb Principle where all constructions become much simpler.

¹ Joint work with Tadeusz Litak.