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## **A Brouwer algebra related to the Weihrauch lattice**

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We prove that the Weihrauch lattice can be transformed into a Brouwer algebra by the consecutive application of two closure operators in the appropriate order: first completion and then parallelization. The closure operator of completion is a new closure operator that we introduce. It transforms any problem into a total problem on the completion of the respective types, where we allow any value outside of the original domain of the problem. This closure operator is of interest by itself, as it generates a total version of Weihrauch reducibility, but in terms of total realizers. From a logical perspective completion can be seen as a way to make problems independent of their premises. Alongside with the completion operator and total Weihrauch reducibility we need to study pre-complete representations that are required to describe these concepts. In order to show that the parallelized completed Weihrauch lattice forms a Brouwer algebra, we introduce a new multiplicative version of an implication. While the parallelized completed Weihrauch lattice forms a Brouwer algebra with this implication, the completed Weihrauch lattice fails to be a model of intuitionistic linear logic in two different ways. In order to pinpoint the algebraic reasons for this failure, we introduce the concept of a Weihrauch algebra that allows us to formulate the failure in precise and neat terms.